Measuring and Testing Advertising-Induced Rotation in the Demand Curve

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Abstract: Advertising can rotate the demand curve if it changes the dispersion of consumers' valuations. We provide an elasticity form measure of the advertising-induced demand curve rotation in five demand models and test for its presence in the U.S. non-alcoholic beverage market. The AIDS model reveals that doubling advertising spending rotates the demand curves clockwise for milk, and coffee and tea with associated slope changes of 7.3% and 11.6%. Soft-drink advertising rotates its demand curve counterclockwise. Our policy suggestion is that milk and soft-drink firms might enhance profits by timing advertising to coincide with high- and low-price periods, respectively.

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Introduction

The goal of this research is to model and measure the effects of advertising allowing for both outward (parallel) shifts and advertising-induced rotation in demand curves, with an application to the U.S. non-alcoholic beverage market. Largely viewed as being persuasive or informative (Bagwell 2005), advertising has received a large number of studies on its shift effects on demand (Nelson and Moran 1995; Dong, Chung, and Kaiser 2004). Simply enough, advertising, however, can rotate the demand curve if it changes the dispersion of consumers' valuations. Surprisingly, as Johnson and Myatt (2006, p. 756) pointed out, "While demand rotation is an elementary concept, it has received remarkably little formal study." Johnson and Myatt (2006) further proposed a new taxonomy of advertising in which hype shifts demand by emphasizing the product's existence and real information rotates demand by matching the product's characteristics with the consumer's subjective preferences.[1] Quilkey (1986, p. 51) provided another theoretical explanation for the demand curve rotation by arguing that advertising can rotate demand by stressing either a product's "substitutability for other products in its end uses" (counterclockwise) or uniqueness (clockwise). If advertising rotates the demand curve, two empirical questions follow and should be answered. First, to which direction and by how much would advertising rotate the demand curve? Second, what are the marketing implications for producers who advertise their products? We address the first question by providing an elasticity form measure of the advertising-induced demand curve rotation in five demand models and testing for its presence in non-alcoholic beverages, thereafter we address the second question using Frisch's (1959) duality relation.

Despite a long-standing hypothesis that the advertising of farm products alters own-price demand elasticities (Waugh 1959; Quilkey 1986), and the importance of the hypothesis for allocation decisions (Chung and Kaiser 1999) and producer returns (Zhang and Sexton 2002), there is little research to date that has tested this hypothesis. The only known tests in the agricultural economics literature are the studies of domestic cotton promotion by Ding and Kinnucan (1996) and of fluid milk and cheese advertising by Schmit and Kaiser (2004) in which the hypothesis of curve rotation was both rejected, and a study by Chung and Kaiser (2000) in which advertising was found to make demand less elastic for New York City fluid milk market.[2] Furthermore, in the marketing literature where the hypothesis has received greater attention there is evidence that advertising can indeed influence consumers' sensitivity to price. In particular, Wittink (1977) found that of 20 studies that addressed the issue 15 showed evidence of curve rotation, with seven indicating a more elastic demand due to advertising and eight a less elastic demand.

The purpose of this research is to address the direction, size, and marketing implications of the advertising-induced rotation in the demand curve for non-alcoholic beverages. Compared with alcoholic beverage or tobacco advertising (Saffer and Dave 2002; Keeler *et al.* 2004), non-alcoholic beverage advertising received much less attention in the literature. Recent research by Kinnucan *et al*. (2001) firmly rejected the hypothesis that non-alcoholic beverage advertising has no effect on the *level* of demand for the individual beverages. Specifically, they found advertising redistributed demand within the non-alcoholic beverage group (juices benefited the most from advertising), but had no effect on the overall group demand. What is not known is whether the advertising affects the slopes of the demand curves. Given the firm rejection of no shift effect, this would appear to be an especially promising group in which to test whether there is a rotation effect.

Prior to model specification we explain how advertising affects slopes of the demand curve based on Johnson and Myatt's work (2006) and distinguish between curve rotation and elasticity change based on the theoretical paper by Kinnucan and Zheng (2004); thereafter we introduce price-advertising interaction terms into five different demand models and develop methods to measure curve rotation caused by the interaction, and derive some propositions about price-advertising interaction using Frisch's duality relationship.[3] The results of the hypothesis tests are then presented employing time-series data. The article concludes with a brief summary of the key findings. Overall, this research is a full empirical extension of the work of Kinnucan and Zheng (2004).

How Advertising Rotates the Demand Curve

This section illustrates advertising's shift and pivotal effects on demand using a numerical example. According to Johnson and Myatt (2006), advertising can shift a demand curve by shifting the location of consumers' willingness-to-pay (WTP); it can rotate a demand curve as well by changing the spread of the WTP. The former case is illustrated in figures (1a-1c), and the latter case is illustrated in figures (1d-1f), wherein the dotted and boldfaced curves denote scenarios before and after a hypothetical successful advertising campaign for milk, respectively. Suppose the WTP for milk before the advertising follows a normal distribution with a mean of three and a variance of one. The dotted curve in figure (1a) represents the probability distribution function (pdf) of the milk WTP. A successful milk advertising campaign was usually assumed to shift the pdf of the milk WTP outward without changing its spread, implying that the advertising increased the WTP of all milk consumers unanimously. If the milk advertising increases every consumer's WTP by two (a large number to make the curves before and after advertising look distinct), then the pdf of the milk WTP after the advertising follows a normal distribution with a mean of five and a variance of one, represented by the boldfaced curve in figure (1a). An outward shift of the pdf results in an outward shift of the cumulative distribution function (cdf), which is shown in figure (1b). Note that any point cdf(WTP_0) on the cdf curve indicates the proportion of consumers that will not purchase milk since their WTP's are less than WTP_0. Therefore, q (=1- cdf(WTP_0)) is the proportion of consumers that will purchase milk for a given price of WTP_0. Mapping the WTP to the vertical axis and corresponding q to the horizontal axis, we have the familiar inverse demand curves in figure (1c), which shows that an outward shift of the pdf caused by the advertising finally leads to an outward shift of the demand curve.

At issue here is that advertising may change the spread of the pdf by influencing consumers' WTP by a varying degree. As Johnson and Myatt (2006) argues, if advertising is unambiguously persuasive, then it will shift the demand curve outward; however, it may discourage some customers from purchasing while encouraging others, which leads to a rotation in the demand curve.

Figure (1d) presents two pdf's of the milk WTP before and after another hypothetical milk advertising campaign. The two pdf's are normally distributed with a common mean of three but different variances at one and 1.5, respectively. The flatter and boldfaced pdf indicates that advertising increases the proportion of customers that have high WTP for milk, as well as the proportion of customers that have low WTP for milk. As an example, a milk advertising campaign emphasizing the contribution of drinking milk to weight loss may increase milk lovers' valuations of milk; however, it may reduce the valuations of milk among those who seek nutritional elements from milk as a cheap source. Figure (1e) shows that if the pdf gets flatter, then the corresponding cdf rotates clockwise and intersects the original cdf at the mean of WTP, WTP* (they intersect at the mean of WTP because the advertising does not shift the pdf in this case). As a result, the demand curve rotates clockwise around WTP* in figure (1f). Overall, if advertising is able to shift and change the spread of the WTP simultaneously, the effects of advertising on demand curves reduce to a rotation effect. Note that if the milk advertising campaign induces those who like milk to become milk lovers without changing the proportion of those who dislike or hate milk, it leads to a kinked demand curve instead.

Curve Rotation and Elasticity Change

Kinnucan and Zheng (2004) showed that the effect of advertising on the own-price elasticity in absolute value (η) depends not only on the extent to which advertising expenditure (A) rotates the demand curve (a rotation effect), but also on the shift in the curve (a shift effect). Specifically, when prices are assumed exogenous, this relation can be written as

$$(1) \qquad \frac{\partial \ln \eta}{\partial \ln A} = \frac{\partial \ln \Delta}{\partial \ln A} - \alpha$$

where q and p stand for quantity and prices, respectively, $\Delta = -\partial q / \partial p$ is the demand curve's slope in absolute value, $\partial \ln$ stands for logarithmic partial differential, and $\alpha = \partial \ln q / \partial \ln A$ is the *horizontal* relative shift in the demand curve due to a small change in advertising, i.e., the shift in the quantity direction holding prices constant.[4] A clockwise (counterclockwise) rotation, for example, implies that $\partial \ln \Delta / \partial \ln A$ is less than (greater than) zero.

Because this shift effect (the commonly known "advertising elasticity") is generally positive, it will either reinforce or offset the rotation effect depending on the latter's sign. For example, if $\partial \ln \Delta / \partial \ln A > 0$, the effect of this type of advertising on the own-price elasticity is ambiguous, dependent on the relative magnitude of α . Conversely, if $\partial \ln \Delta / \partial \ln A < 0$, then $\partial \ln \eta / \partial \ln A$ is unambiguously negative in the presence of a positive shift effect. The upshot is that the shift effect complicates the interpretation of advertising's effect on the own-price elasticity, especially in situations where the advertising is designed to make demand more price elastic. Stated differently, the shift effect biases the results in favor of making demand *less* price elastic, regardless of the advertising's original intent.

Empirically, a rotation in the demand curve can be determined by estimating a demand equation (or system of equations) with price and advertising entered as interaction terms and testing whether the interaction terms are significant. Following Cramer (1973) five widely-used demand models − a linear model, a semi-log model, a compensated double-log model motivated by Alston, Chalfant, and Piggott (2002), a Rotterdam model, and a linear approximate almost ideal demand system (AIDS) − are specified to test whether advertising rotates demand curves as follows:

(2) $$q_i = a_i + b_i Y + \sum_j^n c_{ij} p_j + \sum_j^n d_{ij} A_j + \gamma_i p_i A_i + e_i Age5 + f_i Fafh$$

(3) $$q_i = a_i + b_i \ln(Y/P^*) + \sum_j^n c_{ij} \ln p_j + \sum_j^n d_{ij} \ln A_j + \gamma_i \ln p_i \ln A_i + e_i \ln Age5 + f_i \ln Fafh$$

(4) $$\ln q_i = a_i + b_i \ln(Y/P^*) + \sum_j^n c_{ij} \ln p_j + \sum_j^n d_{ij} \ln A_j + \gamma_i \ln p_i \ln A_i + e_i \ln Age5 + f_i \ln Fafh$$

(5) $$w_i d \ln q_i = a_i + b_i d \ln Q + \sum_j^n c_{ij} d \ln p_j + \sum_j^n d_{ij} d \ln A_j + \gamma_i d \ln p_i d \ln A_i + e_i d \ln Age5 + f_i d \ln Fafh$$

(6) $$w_i = a_i + b_i \ln(Y/P^*) + \sum_j^n c_{ij} \ln p_j + \sum_j^n d_{ij} \ln A_j + \gamma_i \ln p_i \ln A_i + e_i \ln Age5 + f_i \ln Fafh$$

where *i* indexes the four beverages ($n = 4$) in the non-alcoholic group (fluid milk, juices, soft drinks (including carbonated soft drinks and bottled water), and coffee and tea; p_i, q_i, w_i, and A_i, are the price, demand, budget share, and advertising for group *i*; *d ln* denotes the logarithmic first-difference operator; $Y = \sum_{i=1}^4 p_i q_i$ is group expenditure; P^* denotes Stone's geometric price index ($\ln P^* = \sum_{i=1}^4 w_i \ln p_i$); the term $d\ln Q = \sum_{i=1}^{4} w_i d\ln q_i$ is the Divisia volume index; *Age5* is the proportion of the U.S. population less than age five; *Fafh* is the ratio of food-away-from home expenditures to food-at-home expenditures; time subscript for each variable is suppressed here for ease of derivation of the rotation effect. For ease of discussion, equations (2) - (6) are denoted as models A - E if the price-advertising interaction terms are not included and models F - J otherwise. Following Kinnucan *et al*. (2001, p. 5), these (conditional) models treat non-alcoholic beverages as a weakly separable group since Moschini *et al*. (1994) found empirical evidence supporting the commonly used separability assumption in modelling food demand.

Table 1 summarizes the own-price elasticities in absolute values, advertising's effects on the own-price elasticities which are derived in the appendices A and B, and their decomposition into the rotation and shift effects. The first column lists the own-price elasticities (η_i*'s*).[5] Taking the logarithmic partial differential of η_i with respect to advertising expenditure A_i yields $\partial \ln \eta_i / \partial \ln A_i$, which is reported in the second column. Finally, by adding the shift effect (α_{ii}) to the $\partial \ln \eta_i / \partial \ln A_i$, we have the rotation effect ($\partial \ln \Delta_i / \partial \ln A_i$) according to equation (1).

Implications from table 1 are threefold. First, an econometric test of whether advertising affects the own-price elasticity is a joint test of a curve rotation and shift (Kinnucan and Zheng 2004), echoing our conclusion made in the beginning of the second section that curve rotation is neither necessary (as shown in model C) nor sufficient (as shown in model A) for advertising to alter the own-price elasticity.

Second, advertising can rotate the demand curve even when the price-advertising interaction terms are *not* included. For a double-log model featuring constant elasticity (model C), the demand curve must rotate to offset the advertising-induced shift effect to keep the own-price elasticity unchanged in most cases. As an illustration let two demand curves be $Q_1 = P^{-\eta}$ and $Q_2 = P^{-\eta} A^{\alpha}$ with the own-price elasticity (absolute value), advertising expenditure, and advertising elasticity taking the hypothetical values of 2, 500, and 0.05, respectively. A horizontal comparison (Q is the horizontal axis) of slopes between Q_1 and Q_2 clearly shows that a positive and advertising-induced shift in demand makes the demand curve flatter. The shift effect and shift-related rotation effect, in this case, are both 0.05. Relaxing the assumption of fixed prices will alter the magnitude of the shift and shift-related rotation effects, but will not change the fact that advertising rotates the demand curve unless supply elasticity is unitary. Advertising can also rotate the demand curve through its influence on budget shares (in models D and E). We, therefore, consider the rotation effects as shift-related if they are induced by a shift in demand caused by advertising. All the rotation effects in models A - E are shift related. Note the shift-related rotation effects are function specific, as they arise in models C - E due to constraints on functional forms (e.g., constant elasticity in model C) and could disappear otherwise (as in models A and B).

The last implication builds upon the second one. Combining the rotation effects with and without the price-advertising interaction effects yields the interaction-related rotation effects. In the presence of a price-advertising interaction term, the rotation effects in models F - J combine an interaction-related rotation effect, as well as a shift-related rotation effect. Once segregated from the shift-related rotation effects, the interaction-related rotation effects for models F - J are $\gamma_i/(c_{ii}+\gamma_i A_i)$, $\gamma_i/(c_{ii}+\gamma_i \ln A_i)$, $\gamma_i/(c_{ii}+\gamma_i \ln A_i)$, $\gamma_i/(c_{ii}+\gamma_i d \ln A_i)$, and $(\gamma_i+\gamma_i \ln A_i \sum_j^n w_j\alpha_{ij})/(c_{ii}+\gamma_i \ln A_i - w_i)$, respectively. Taking the Rotterdam model (model I) as an example, the effect of advertising on the own-price elasticity can be decomposed into three parts: an interaction-related rotation effect of $\gamma_i/(c_{ii}+\gamma_i \ln A_i)$ or $-\gamma_i/(\eta_i w_i)$, a shift-related rotation effect of $\sum_j^n w_j\alpha_{ij}$, and the negative of a shift effect of α_{ii}.[6] Using the above interaction-related rotation effects to measure the advertising-induced demand curve rotation is advantageous because it reflects the "true" rotation effects indicated by the price-advertising interaction terms *γ_i's*, and because of the ease of interpretation and comparison across demand models since they are in the form of elasticities.

Additional insight can be obtained by noting that the second-order cross partial derivatives of any particular function are unaffected by the order in which the derivative is taken. Thus, in the simple case where quantity demanded q_D is defined to be a function of price and advertising:

(7) $q_D = D(p, A)$

the following "duality relation" (Frisch 1959, p. 180) holds:

$$\frac{\partial^2 D}{\partial p \partial A} = \frac{\partial^2 D}{\partial A \partial p},$$

or, in elasticity notion,

$$(8) \qquad \frac{\partial \eta}{\partial \ln A} = -\frac{\partial \alpha}{\partial \ln p},$$

where η is (as before) expressed in absolute value. Thus, if advertising has no effect on the own-price elasticity, then by (8) it must also be true that price has no effect on the advertising elasticity. The latter inference contradicts an argument underlying Chung and Kaiser's (1999) analysis, namely that advertising would be more effective at shifting the demand curve when prices are low than when prices are high. As noted by Frisch (p. 180) equations such as (8) are invariant under a general (non-linear) transformation of the utility function. Hence, the hypothesis based on (8) that the advertising-price interaction effect should be non-zero is quite general.

Data and Estimation Procedures

The models F - J were estimated using U.S. annual time series data for the period 1970-2004. Variable definitions and some description statistics of the data are reported in table 2. The price and quantity data were obtained from the U.S. Department of Labor's *CPI Detailed Report* (price of bottled water was obtained from Beverage Marketing Corporation) and the U.S. Department of Agriculture's *Food Disappearance Data*, respectively; the advertising data were obtained from private sources, chiefly *Ad $ Summary* published by Leading National Advertisers, Inc. The price data were divided by the CPI for all items (1982−1984 = 100) to remove the effects of inflation. A complete description of the data covering the period 1970−1994, including sources, is available in Kinnucan *et al*. (2001, pp. 24-28). Their data were updated in three aspects for use in this article: *(i)* ten more years of data were collected to extend the data period to 2004, *(ii)* advertising expenditures were deflated by a media cost index (2004 = 100) computed from annual changes in promotion and advertising costs by media type provided by Dairy Management Inc., and *(iii)* bottled water was added to the soft drinks group to reflect its current place of the second-largest non-alcoholic beverage category by volume.[7] Some years of bottled-water data − consumption prior to 1976, price prior to 1984, and advertising prior to 1985 − were not available (note its consumption per capita was very low at 1.6 gallons in 1976 and its advertising expenditures were only $12 million in 1985, compared with its counterparts of 23.2 gallons and $116 million in 2004) and therefore interpolated linearly using data from their most adjacent years.

Equations were estimated using the PROC MODEL procedure in SAS version 9.13, and as a system using Seemingly Unrelated Regressions to account for contemporaneous correlation among individual equation errors (Griffiths, Hill, and Judge 1993, p. 551).[8] In the case of the Rotterdam and AIDS models one equation (juices) was dropped to avoid singularity in the variance-covariance matrix. As indicated, the Rotterdam and AIDS models were estimated with homogeneity and symmetry imposed on both prices and advertising expenditures (Selvanathan 1989), and adding-up was used to recover the coefficients from the omitted equation.

Results and Marketing Implications

Table 3 reports the parameters estimates for models F - J. Estimation results are satisfactory in the sense that the adjusted R^2s range from 0.83 to 0.99 in the AIDS model to between 0.38 and 0.53 in the Rotterdam model. The majority of the Durbin-Watson statistics center around two with some falling into the inconclusive region regarding serial correlation (especially in the AIDS model). Overall, the models appear to do a better job of explaining milk, soft-drink and coffee and tea demand than juice demand. All own-price parameters (except the one for coffee and tea in model J) in models I and J are statistically significant (at the 5% level unless noted otherwise) with correct signs, while only few of them are statistically significant in models F - H, indicating the advantage of using demand system over single equation. The own-advertising parameter is statistically significant only for soft drinks in models F, G, and J. The price-advertising interaction term is found weakly significant (at the 10% level) for soft drinks and coffee and tea in model F, significant for soft drinks in model G, and significant for milk, soft drinks, and coffee and tea model J. Furthermore, most of the models show higher proportion of population under age five leads to higher demand for milk, and more dining out (higher *Fafh*) leads to higher demand for soft drinks and lower demand for milk, and coffee and tea, which are all consistent with expectation.

Base on the estimates in table 3, we calculate the own price and advertising elasticities, compute interaction-related rotation effects according to the formulae in table 1, and report them in table 4. Wald statistics for the null hypothesis that the estimated interaction effects are jointly zero are also reported.[9] We report own advertising elasticities when γ_i or d_{ii} is found significant and report interaction-related rotation effects when γ_i is found significant. The linear, semi-log, and AIDS models reject the null hypothesis that the estimated interaction effects are jointly zero at the 5% level based on the Wald statistics. To put the results of rotation effect into perspective, we focus on interpreting the results of the AIDS model given the theoretical advantages inherent in demand systems, the overall satisfactory significance in its estimates, and a more reasonable size of the rotation effect. In addition, Duffy (2001) found that the AIDS model provided the "most suitable framework for investigating advertising effects" in U.K. alcoholic drinks markets, a finding that helps to justify our selection of the AIDS model.

For milk, the computed interaction-related rotation effect is -0.073, indicating a 10% increase in the milk advertising (note that most of the milk advertising is generic advertising) would reduce the slope (in absolute value) of milk demand by 0.73%, a number not seen in the literature. Similarly, a 10% increase in the advertising of coffee and tea would decrease the slope of its demand by 1.16%. Conversely, advertising is found to increase the slope of soft-drink demand. A 10% increase in the soft-drink advertising would increase the slope of its demand by 0.49%. As a robustness check, the AIDS model was estimated with the data prior to 1976 deleted. The price-advertising interaction terms hold significant at the 5% level, and D.W. statistics come closer to two.

To put the results into perspective, figure 2 plots the two representative cases of demand curve rotation due to advertising, clockwise rotation for milk and counterclockwise rotation for soft drinks. In figure (2a), a 10% increase in the milk advertising rotates the demand curve D_0 clockwise to D_R by reducing the size of its slope by 0.73% (measured at the mean advertising level). When measured at the mean price level, the 10% increase in the milk advertisings increases milk demand by 0.19%. What (2a) implies is that since advertising makes milk demand less elastic, it must also be true that an increase in price increases advertising's ability to shift the demand curve, which is an illustration of equation (8). In other words, milk advertising is more effective in shifting milk demand when milk prices are *higher* (the dispersion between D_R and D_0 gets wider). In this instance, a satiation phenomenon may be at work whereby the advertising elasticity increases as the quantity consumed decreases. From a policy perspective, the positive term γ in the milk equation would imply that milk producers might enhance profits by timing advertising to coincide with high-price periods. Conversely, figure (2b) indicates that the soft-drink advertising shifts its demand outward (from D_0 to D_S) but it is more effective in doing so when soft-drink prices are *lower* (from D_S to D_R). The point is that the duality relation permits a richer interpretation of the interaction parameter than otherwise possible.

Figure 3 plots the retail prices of the four beverages in real terms. Milk has the second highest prices in the group. Although milk has stable prices in the most recent 15 years, its relative price to other non-alcoholic beverages rose gradually from 1 in 1995 to 1.28 in 2005, a large increase of 28% (Kaiser 2006). The current high prices of milk, coupled with our finding that milk advertising is more effective when milk prices are high, warrant the continuous existence of the milk check-off program, which funds the generic advertising for milk. On the other hand, since soft-drink prices have been low and declining in the past 30 years, our finding that soft-drink advertising makes its consumers more sensitive to the price decline indicates that soft-drink producers enhanced their profits by the advertising-induced rotation in the demand curve. Conversely, since the prices of coffee and tea have been the lowest in the group and have shown downward trend in the past 10 years, the ideal advertising should make demand more elastic for coffee and tea. In this sense, the coffee-and-tea advertising is not considered successful, which might already have been reflected in the historical expenditures on coffee. The real advertising expenditure on coffee and tea in 2004 was only about a quarter of those in 1984 and a third of those in 1974. Overall, advertising seems to have done the right job for milk and soft-drink producers.

Conclusions

Showing that advertising can influence own-price elasticity through combinations of its shift effect, shift-related and interaction-related rotation effects, this article provides an elasticity form measure of the interaction-related rotation effect in five demand models and tests for its existence in the non-alcoholic beverages. Results are mixed in that compensated double-log model and the Rotterdam models fail to reject the null hypothesis of no price-advertising interaction while linear model, semi-log model, and the AIDS model indicate rejection. Interaction-related rotation effects were found to be not robust to a change in model specification. This confirms Hauser and Wernerfelt's (1989) result that functional forms used to model advertising and price interactions influence conclusions about its direction.

Since both model I and Kinnucan *et al*. (2001) use the Rotterdam model with similar data − although the former allows a price-advertising interaction effect and the latter does not, their results are comparable.[10] For example, model I's estimated own-price parameters of -0.029, -0.067, -0.059, and -0.040 for the four beverages in their respective order compare favorably to their counterparts of -0.047, -0.057,

-0.060, and -0.032 obtained by Kinnucan *et al*. (2001). All of the own-price parameters are statistically significant, confirming the strong influence of prices on the allocation of consumer spending. The intercepts terms in both models reveal a positive consumption trend for soft drinks and a negative consumption trend for milk, and coffee and tea. The main difference lies in that Kinnucan *et al*. (2001) found that advertising enhanced demand for juices, while model I does not report any statistically significant own advertising effect or price-advertising interaction effect.

Results of the best-performing AIDS model indicate that advertising might have the ability to make the demand curve steeper for milk, and coffee and tea, as well as the ability to make the demand curve flatter for soft drinks. For milk and coffee-and-tea advertising, this is the case depicted in figures (2d) – (2f), where advertising flattens the probability distribution of WTP. For soft-drink advertising, it's the reverse. The implications are, although this might not be the true intention of producers who advertise their products, advertising of milk, and coffee and tea appeals better to consumers who have high WTP for them, while soft-drink advertising appeals better to consumers who have low WTP for it. Our policy suggestion based on the AIDS model, therefore, is that milk and soft-drink firms might enhance profits by timing advertising to coincide with high- and low-price periods, respectively.

Notes:

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[1] Real information actually plays a role similar to the match-products-to-buyers effect discussed by Bagwell (2005, p.19).

[2] The two models used by Schmit and Kaiser and Chung and Kaiser were the same in functional form. The latter study used per capita fluid milk sales as the dependent variable while the former one used per capita retail fluid milk/cheese demand instead.

[3] Since Farr *et al.* (2001) and Tremblay and Okuyama (2001) argued that advertising could affect equilibrium consumption through its influence on supply (price competition), we don't rule out the possibility of a price-advertising interaction relationship on the supply side. The analysis done in this article is strictly on the demand side.

[4] Exogenous price is a common finding in the empirical literature (e.g., Brester and Schroeder, 1995; Kinnucan *et al*., 1997).

[5] Green and Alston (1990) show that all of the previously reported formulae for AIDS elasticities are incorrect when LA-AIDS is estimated instead of the true AIDS with a few exceptions including constant group price, i.e., d ln P* is independent of individual goods' prices. This condition is satisfied since this article assumes exogenous prices.

[6] Note $-(c_{ii} + \gamma_i d \ln A_i)/w_i$ is the Hicksian own-price elasticity (η_i^h) for the Rotterdam model. The Marshallian own-price elasticity (η_i^m) is equal to $-(c_{ii} + \gamma_i d \ln A_i)/w_i - b_i$. The effect of advertising on the Marshallian own-price elasticity is derived as $\partial \ln \eta_i^m / \partial \ln A_i = (\eta_i^h / \eta_i^m) \partial \ln \eta_i^h / \partial \ln A_i$.

[7] In 2005, the volume shares of the three largest non-alcoholic beverage categories by volume were carbonated soft drinks (43.8%), bottled water (21.6%), and fluid milk (17.8%), according to Beverage Marketing Corporation.

[8] Results of t-statistics were much improved from using OLS to SUR, but remained alike from SUR to iterative SUR.

[9] For completeness purpose, all c_{ii}*'s*, d_{ii}*'s*, and γ_i*'s* were used to calculate the own demand elasticities but only significant c_{ii}*'s*, d_{ii}*'s*, and γ_i*'s* were used to calculate the own advertising elasticities and the interaction-related rotation effects except the d_{ii} in model J to avoid a negative own advertising elasticity for coffee and tea; price, demand, and advertising took their mean levels when they were needed.

[10] The difference of the two datasets is reported in detail in the section IV.

Table 1. Decomposition of Advertising's Effects on the Own-Price Elasticities with Fixed Prices

Model Name (Model version in parenthesis)	$\lvert\text{Own - Price Elasticity}\rvert$ η_i	$\partial \ln \eta_i / \partial \ln A_i$ =Rotation effect − Shift effect	Rotation effect $\partial \ln \Delta_i / \partial \ln A_i$		Shift effect α_{ii}
Without Interaction					
Linear (A)	$-c_{ii}(p_i / q_i)$	$-\alpha_{ii}$	0		α_{ii}
Semi-log (B)	$-c_{ii} / q_i$	$-\alpha_{ii}$	0		α_{ii}
Double-log (C)	$-c_{ii}$	0	α_{ii}		α_{ii}
Rotterdam (D)	$-c_{ii} / w_i$	$\sum_{j}^{n} w_j \alpha_{ij} - \alpha_{ii}$	$\sum_{j}^{n} w_j \alpha_{ij}$		α_{ii}
AIDS (E)	$-(c_{ii} / w_i - 1)$	$\dfrac{c_{ii}(\sum_{j}^{n} w_j \alpha_{ij} - \alpha_{ii})}{c_{ii} - w_i}$	$\dfrac{c_{ii} \sum_{j}^{n} w_j \alpha_{ij} - w_i \alpha_{ii}}{c_{ii} - w_i}$		v
With interaction			Interaction-related	Shift-related	
Linear (F)	$-(c_{ii} + \gamma_i A_i)(p_i / q_i)$	$\dfrac{\gamma_i A_i}{c_{ii} + \gamma_i A} - \alpha_{ii}$	$\dfrac{\gamma_i A_i}{(c_{ii} + \gamma_i A_i)}$	0	α_{ii}
Semi-log (G)	$-(c_{ii} + \gamma_i \ln A_i) / q_i$	$\dfrac{\gamma_i}{c_{ii} + \gamma_i \ln A_i} - \alpha_{ii}$	$\dfrac{\gamma_i}{c_{ii} + \gamma_i \ln A_i}$	0	α_{ii}
Double-log (H)	$-(c_{ii} + \gamma_i \ln A_i)$	$\dfrac{\gamma_i}{(c_{ii} + \gamma_i \ln A_i)}$	$\dfrac{\gamma_i}{(c_{ii} + \gamma_i \ln A_i)}$	α_{ii}	α_{ii}
Rotterdam (I)	$-(c_{ii} + \gamma_i d \ln A_i) / w_i$	$\dfrac{\gamma_i}{(c_{ii} + \gamma_i d \ln A_i)} + \sum_{j}^{n} w_j \alpha_{ij} - \alpha_{ii}$	$\dfrac{\gamma_i}{(c_{ii} + \gamma_i d \ln A_i)}$	$\sum_{j}^{n} w_j \alpha_{ij}$	α_{ii}
AIDS (J)	$-((c_{ii} + \gamma_i \ln A_i) / w_i - 1)$	$\dfrac{\gamma_i + (c_{ii} + \gamma_i \ln A_i)(\sum_{j}^{n} w_j \alpha_{ij} - \alpha_{ii})}{(c_{ii} + \gamma_i \ln A_i - w_i)}$	$\dfrac{\gamma_i + \gamma_i \ln A_i \sum_{j}^{n} w_j \alpha_{ij}}{(c_{ii} + \gamma_i \ln A_i - w_i)}$	$\dfrac{c_{ii} \sum_{j}^{n} w_j \alpha_{ij} - w_i \alpha_{ii}}{(c_{ii} + \gamma_i \ln A_i - w_i)}$	α_{ii}

Table 2. Variable Definitions and Summary Statistics, 1970-2004

Variable	Definition	Mean	Minimum	Maximum	*s.d.*
q_1	Per capita fluid milk consumption, gallons/person	26.13	21.20	31.30	2.92
q_2	Per capita juice consumption, gallons/person	7.66	5.60	9.10	1.02
q_3	Per capita soft-drink consumption, gallons/person	49.63	24.50	75.50	16.04
q_4	Per capita coffee-and-tea consumption, gallons/person	34.21	28.20	40.90	3.42
p_1	Retail price for fluid milk, $/gallons, CPI deflated	1.86	1.55	2.51	0.29
p_2	Retail price for juices, $/gallons, CPI deflated	3.03	2.73	3.43	0.19
p_3	Retail price for soft drinks, $/gallons, CPI deflated	1.37	0.97	1.88	0.26
p_4	Retail price for coffee and tea, $/gallons, CPI deflated	0.71	0.53	1.37	0.18
A_1	Advertising expenditures for fluid milk, million $, MCI deflated	98.19	17.45	243.31	55.92
A_2	Advertising expenditures for juices, million $, MCI deflated	428.00	85.40	702.04	128.36
A_3	Advertising expenditures for soft drinks, million $, MCI deflated	845.63	258.96	1216.92	198.06
A_4	Advertising expenditures for coffee and tea, million $, MCI deflated	498.75	150.39	823.16	189.08
w_1	Budget share for fluid milk, conditional	0.30	0.23	0.47	0.07
w_2	Budget share for juices, conditional	0.14	0.11	0.17	0.02
w_3	Budget share for soft drinks, conditional	0.40	0.25	0.50	0.08
w_4	Budget share for coffee and tea, conditional	0.15	0.11	0.23	0.03
Fafh (%)	U.S. food-away-from home expenditures / total food expenditures	42.23	33.41	48.47	4.90
Age5 (%)	Proportion of the U.S. population less than age five	7.37	6.78	8.37	0.41

Table 3. SUR Parameters Estimates for Models F - J

	Price coefficients				Advertising coefficients				Interaction	Intercept	Expend.	*Age5*	*Fafh*		
Equations	c_{i1}	c_{i2}	c_{i3}	c_{i4}	d_{i1}	d_{i2}	d_{i3}	d_{i4}	γ_i	a_i	b_i	e_i	f_i	Adj. R^2	*D.W.*
Linear (Model F)															
Milk	-0.479	1.378*	0.907	-2.044*	0.002	0.001	-0.002*	0.001	0.001	1.710	0.099**	2.437**	-0.288**	0.99	2.06
	(1.200)	(0.673)	(1.456)	(1.168)	(0.010)	(0.001)	(0.001)	(0.001)	(0.006)	(8.481)	(0.029)	(0.406)	(0.108)		
Juices	-4.143**	-0.417	-1.638	-2.746*	0.001	0.010	-0.001	0.000	-0.003	19.485*	0.079**	-0.715	-0.133	0.86	1.97
	(1.267)	(1.530)	(1.626)	(1.343)	(0.002)	(0.009)	(0.001)	(0.001)	(0.003)	(10.955)	(0.033)	(0.457)	(0.122)		
Soft drinks	-1.125	-0.600	-14.04**	-3.916	0.000	0.002	0.017**	-0.007*	-0.009*	64.428**	0.013	-7.144**	1.429**	0.99	1.81
	(4.007)	(2.372)	(5.786)	(3.955)	(0.007)	(0.005)	(0.007)	(0.004)	(0.005)	(29.401)	(0.099)	(1.415)	(0.377)		
Coffee & tea	-3.718	0.438	-8.352**	-26.009**	-0.003	-0.003	0.002	-0.008	0.009*	44.371*	0.421**	0.129	-1.005**	0.93	2.33
	(3.068)	(1.840)	(3.996)	(3.987)	(0.005)	(0.004)	(0.003)	(0.005)	(0.005)	(22.923)	(0.076)	(1.106)	(0.292)		
Semi-log (Model G)															
Milk	4.658	7.020**	6.748**	-0.584	0.610	0.975*	-2.528**	0.693	-0.426	-28.423	14.947**	15.994**	-14.629**	0.99	2.10
	(4.329)	(1.611)	(1.283)	(0.674)	(0.597)	(0.475)	(0.821)	(0.463)	(0.915)	(20.908)	(4.115)	(3.199)	(4.049)		
Juices	-4.481*	-6.799	1.093	0.046	0.262	-0.495	-0.227	-0.341	0.573	27.235	-0.069	-4.553	0.371	0.82	2.23
	(2.540)	(27.463)	(1.997)	(0.983)	(0.253)	(4.893)	(1.093)	(0.608)	(4.449)	(45.164)	(5.743)	(4.669)	(5.714)		
Soft drinks	-2.912	-1.405	22.708	-3.170	0.130	-0.628	6.196**	-2.177	-7.553**	-185.715**	23.807*	-32.981**	46.944**	0.99	1.99
	(5.803)	(4.865)	(19.095)	(2.070)	(0.562)	(1.484)	(2.644)	(1.385)	(2.727)	(65.622)	(13.055)	(10.038)	(12.815)		
Coffee & tea	10.848*	12.115**	17.766**	-9.759	-0.094	-0.864	0.231	0.077	-0.322	-176.385**	72.147**	9.516	-45.787**	0.93	2.10
	(5.464)	(4.560)	(3.454)	(9.406)	(0.532)	(1.394)	(2.297)	(1.476)	(1.466)	(61.440)	(12.197)	(9.372)	(12.009)		
Double-log (Model H)															
Milk	-0.056	0.223**	0.258**	-0.008	-0.011	0.026	-0.110**	0.045**	0.037	0.908	0.593**	0.685**	-0.486**	0.99	2.10
	(0.179)	(0.060)	(0.049)	(0.025)	(0.025)	(0.018)	(0.031)	(0.017)	(0.039)	(0.773)	(0.152)	(0.118)	(0.149)		
Juices	-0.646*	-0.256	0.223	-0.001	0.032	0.066	-0.021	-0.047	-0.034	3.865	0.014	-0.689	0.054	0.84	2.27
	(0.324)	(3.701)	(0.260)	(0.126)	(0.032)	(0.659)	(0.140)	(0.078)	(0.600)	(5.954)	(0.732)	(0.599)	(0.729)		
Soft drinks	-0.133	-0.110	-0.637	-0.009	-0.006	-0.025	0.089	-0.012	0.040	-3.761**	0.668**	-0.331	1.370**	0.99	2.13
	(0.133)	(0.111)	(0.595)	(0.047)	(0.013)	(0.034)	(0.064)	(0.032)	(0.086)	(1.495)	(0.298)	(0.229)	(0.292)		
Coffee & tea	0.330*	0.387**	0.546**	-0.074	-0.009	-0.026	0.013	-0.016	-0.043	-2.847	2.128**	0.237	-1.267**	0.92	1.97
	(0.166)	(0.138)	(0.104)	(0.383)	(0.016)	(0.042)	(0.069)	(0.049)	(0.060)	(1.855)	(0.368)	(0.283)	(0.363)		
Rotterdam (Model I)															
Milk	-0.029**	0.024**	0.000	0.004	-0.002	0.008**	-0.003	-0.002	-0.046	-0.004**	0.116**	0.043	-0.048	0.52	1.73

	(0.009)	(0.010)	(0.008)	(0.004)	(0.002)	(0.003)	(0.002)	(0.002)	(0.050)	(0.001)	(0.030)	(0.039)	(0.036)		
Juices	0.024**	-0.067**	0.033*	0.010	0.008**	-0.006	-0.001	-0.010	0.103	-0.003	0.354**	0.074	0.027	0.38	2.53
	(0.010)	(0.031)	(0.017)	(0.014)	(0.003)	(0.014)	(0.007)	(0.008)	(0.173)	(0.002)	(0.111)	(0.148)	(0.132)		
Soft drinks	0.000	0.033*	-0.059**	0.026**	-0.003	-0.001	-0.006	0.010**	-0.069	0.010**	0.198**	0.002	0.114*	0.53	1.84
	(0.008)	(0.017)	(0.015)	(0.007)	(0.002)	(0.007)	(0.008)	(0.006)	(0.209)	(0.001)	(0.055)	(0.073)	(0.065)		
Coffee & tea	0.004	0.010	0.026**	-0.040**	-0.002	-0.001	-0.010	0.002	0.012	-0.003**	0.332**	-0.119	-0.093	0.51	2.95
	(0.004)	(0.014)	(0.007)	(0.012)	(0.002)	(0.007)	(0.008)	(0.009)	(0.048)	(0.002)	(0.080)	(0.103)	(0.091)		
AIDS (Model J)															
Milk	0.125**	-0.008	-0.080**	-0.037**	0.001	0.007	-0.006	-0.003	0.010**	2.032**	-0.128	0.160**	-0.409**	0.99	1.19
	(0.027)	(0.029)	(0.020)	(0.011)	(0.004)	(0.004)	(0.004)	(0.002)	(0.005)	(0.320)	(0.079)	(0.062)	(0.065)		
Juices	-0.008	0.064**	-0.075*	0.019	0.007	0.010	-0.019	0.002	-0.007	0.035	0.035	-0.092	0.026	0.83	1.90
	(0.029)	(0.029)	(0.039)	(0.022)	(0.004)	(0.010)	(0.011)	(0.006)	(0.004)	(0.337)	(0.087)	(0.064)	(0.067)		
Soft drinks	-0.080**	-0.075*	0.177**	-0.022	-0.006	-0.019	0.031**	-0.007	-0.013**	-1.749**	-0.037	-0.027	0.654**	0.98	1.31
	(0.020)	(0.039)	(0.046)	(0.014)	(0.004)	(0.011)	(0.014)	(0.007)	(0.006)	(0.376)	(0.112)	(0.066)	(0.082)		
Coffee & tea	-0.037**	0.019	-0.022	0.040	-0.003	0.002	-0.007	0.007	0.010**	0.682**	0.130**	-0.041	-0.271**	0.97	1.31
	(0.011)	(0.022)	(0.014)	(0.026)	(0.002)	(0.006)	(0.007)	(0.005)	(0.004)	(0.212)	(0.061)	(0.039)	(0.046)		

Note: ** and * denote estimates are significant at the 5% level or less and at the 10%, respectively.

Table 4. Own Price and Advertising Elasticities, and Interaction-Related Rotation Effects

Model/Commodity	Own-price elasticity	Own-adv. elasticity	Interaction-related rotation effect	Wald stat.	No rotation: $\gamma_i = 0$ Reject at 5%?
Linear (Model F)				11.11	Yes
Milk	-0.026	--	--		
Juices	-0.720	--	--		
Soft drinks	-0.604	0.075	0.357		
Coffee & tea	-0.444	0.097	-0.218		
Semi-log (Model G)				19.08	Yes
Milk	0.106	--	--		
Juices	-0.439	--	--		
Soft drinks	-0.563	0.079	0.149		
Coffee & tea	-0.343	--	--		
Double-log (Model H)				2.29	No
Milk	0.108	--	--		
Juices	-0.462	--	--		
Soft drinks	-0.366	--	--		
Coffee & tea	-0.337	--	--		
Rotterdam (Model I)				0.98	No
Milk	-0.102	--	--		
Juices	-0.427	--	--		
Soft drinks	-0.428	--	--		
Coffee & tea	-0.269	--	--		
AIDS (Model J)				11.76	Yes
Milk	-0.447	0.019	-0.073		
Juices	-0.843	--	--		
Soft drinks	-0.772	0.069	0.049		
Coffee & tea	-0.324	0.023	-0.116		

Figure 1. Advertising’s Shift and Rotation Effects on Demand Curves

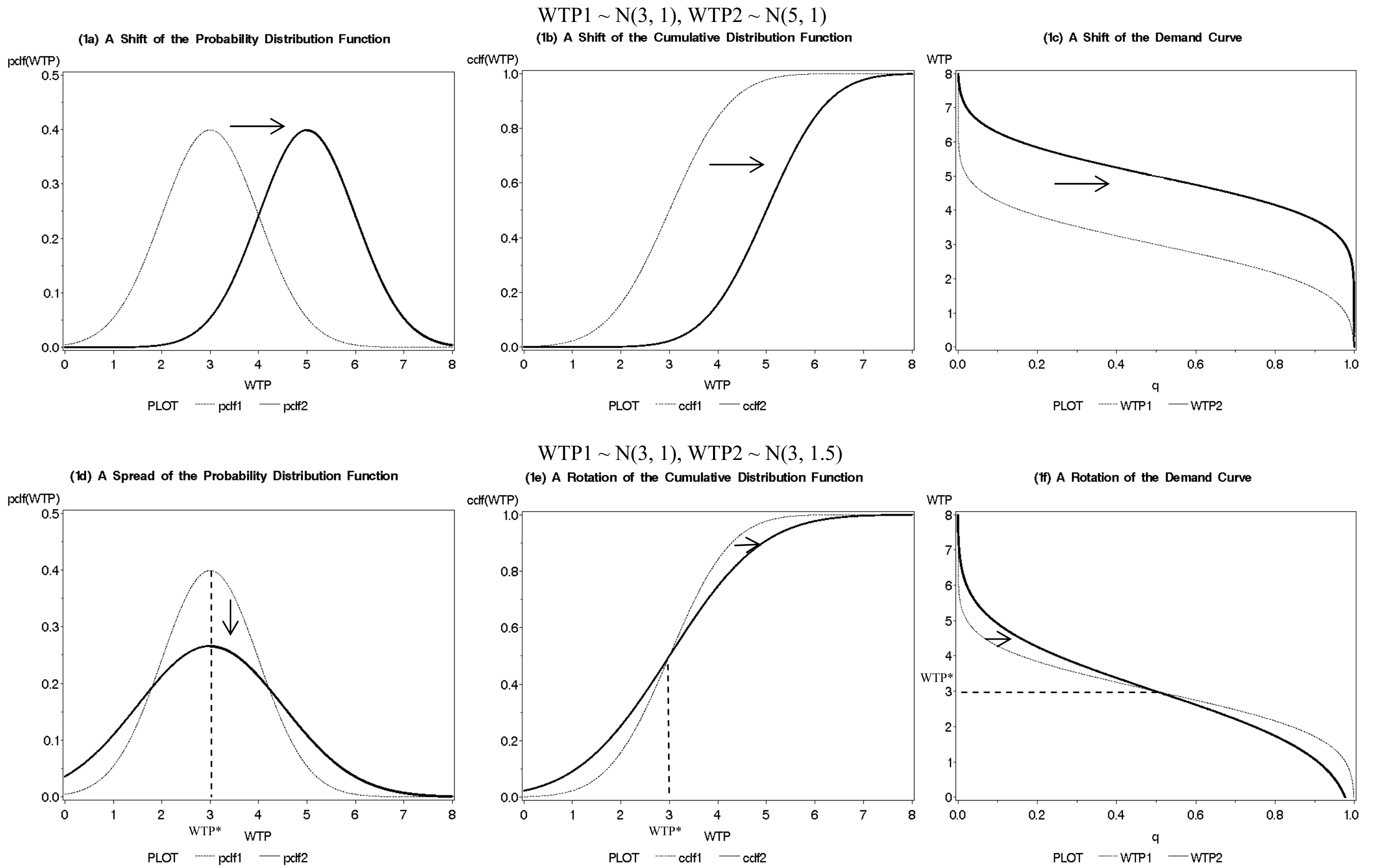
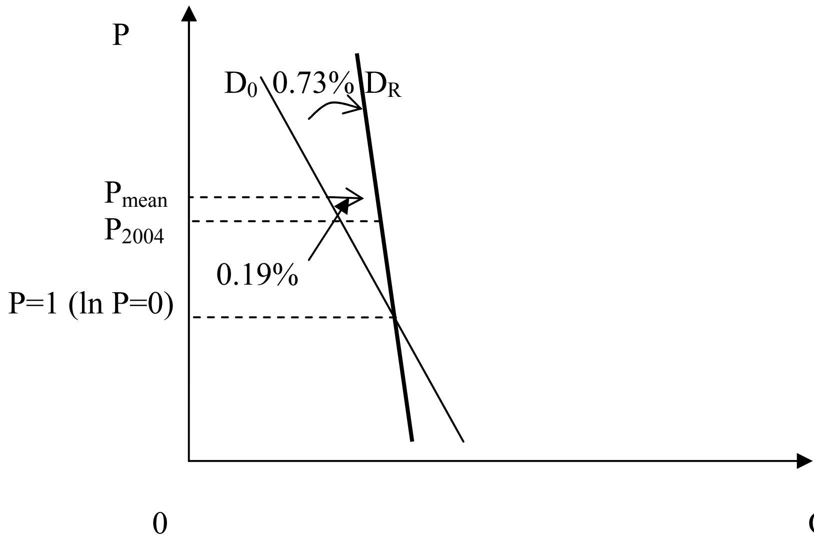
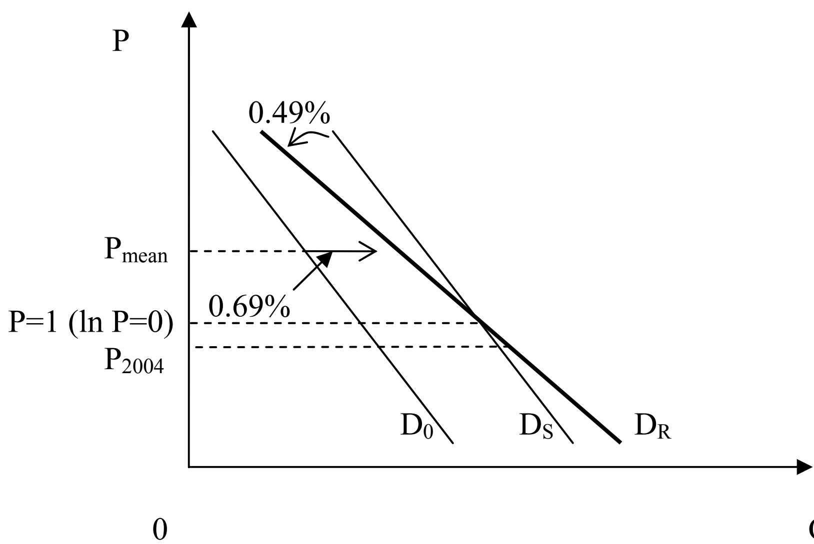


Figure 2. The Effects of a 10% Increase in Milk or Soft-Drink Advertising Expenditures on Its Respective Demand

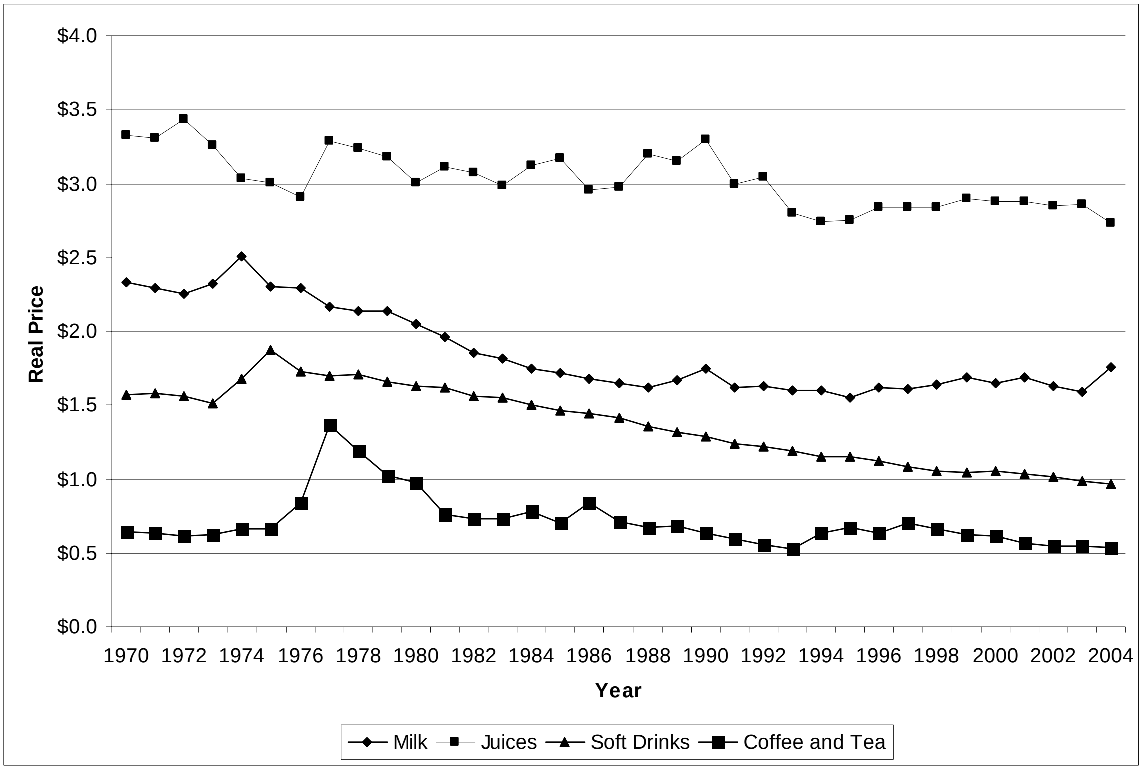


(2a) Milk



(2b) Soft Drinks

Figure 3. Real Retail Prices of Non-Alcoholic Beverages



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Appendix A

When price-advertising interaction terms are not present in equations (2) – (6), the own demand elasticities (in absolute values) are η_i^{LM} $(=-c_{ii}(p_i/q_i))$, η_i^{SL} $(=-c_{ii}/q_i)$, η_i^{DL} $(=-c_{ii})$, η_i^{RM} $(=-c_{ii}/w_i)$, and η_i^{AIDS} $(=-(c_{ii}/w_i-1))$, respectively. The $\partial\ln\eta/\partial\ln A$ term in table 1 is derived as follows:

(A1) $$\frac{\partial\ln\eta_i^{LM}}{\partial\ln A_i}=\frac{\partial\ln(-c_{ii})}{\partial\ln A_i}-\frac{\partial\ln p_i}{\partial\ln A_i}+\frac{\partial\ln q_i}{\partial\ln A_i}=-\alpha_{ii},$$

(A2) $$\frac{\partial\ln\eta_i^{SL}}{\partial\ln A_i}=\frac{\partial\ln(-c_{ii})}{\partial\ln A_i}+\frac{\partial\ln q_i}{\partial\ln A_i}=-\alpha_{ii}\text{ and}$$

(A3) $$\frac{\partial\ln\eta_i^{DL}}{\partial\ln A_i}=\frac{\partial\ln(-c_{ii})}{\partial\ln A_i}=0.$$

Since the demand elasticities for the Rotterdam and AIDS models include a budget share, $\partial w_i/\partial\ln A_i$ is derived beforehand. Note that:

(A4) $$\frac{\partial w_i}{\partial\ln A_i}=\frac{w_i\partial\ln w_i}{\partial\ln A_i}=\frac{w_i(\partial\ln p_i+\partial\ln q_i-\partial\ln Y)}{\partial\ln A_i}.$$

Under the assumption of fixed price, we have:

(A5) $$\frac{\partial w_i}{\partial\ln A_i}=\frac{w_i(\partial\ln q_i-\partial\ln\sum_j^n p_jq_j)}{\partial\ln A_i}=w_i(\alpha_{ii}-\frac{\partial\sum_j^n p_jq_j}{Y\partial\ln A_i})=w_i(\alpha_{ii}-\frac{\sum_j^n p_j\partial q_j}{Y\partial\ln A_i}).$$

Since $\sum_j^n p_j\partial q_j$ is identical to $\sum_j^n p_jq_j\partial\ln q_j$, (A5) leads to:

(A6) $$\frac{\partial w_i}{\partial\ln A_i}=w_i(\alpha_{ii}-\sum_j^n w_j\alpha_{ij})\text{, or }\frac{\partial\ln w_i}{\partial\ln A_i}=(\alpha_{ii}-\sum_j^n w_j\alpha_{ij}).$$

It follows that:

(A7) $$\frac{\partial \ln \eta_i^{RM}}{\partial \ln A_i} = \frac{\partial \ln(-c_{ii})}{\partial \ln A_i} - \frac{\partial \ln w_i}{\partial \ln A_i} = \sum_j^n w_j \alpha_{ij} - \alpha_{ii}$$ and

(A8) $$\frac{\partial \eta_i^{AIDS}}{\partial \ln A_i} = \frac{-c_{ii}\partial w_i}{w_i^2 \partial \ln A_i} = \frac{-c_{ii}}{w_i}(\sum_j^n w_j \alpha_{ij} - \alpha_{ii}).$$

Dividing (A8) by η_i^{AIDS} yields:

(A9) $$\frac{\partial \ln \eta_{AIDS}^i}{\partial \ln A_i} = \frac{(-c_{ii}/w_i)(\sum_j^n w_j \alpha_{ij} - \alpha_{ii})}{-(c_{ii}/w_i - 1)} = \frac{(c_{ii}/w_i)(\sum_j^n w_j \alpha_{ij} - \alpha_{ii})}{(c_{ii}/w_i - 1)}$$

Appendix B

With price-advertising interaction terms included in equations (2) – (6), the corresponding own demand elasticities (in absolute values) are η_i^{LM} $(= -(c_{ii} + \gamma_i A_i)(p_i / q_i))$, η_i^{SL} $(= -(c_{ii} + \gamma_i \ln A_i)/q_i)$, η_i^{DL} $(= -(c_{ii} + \gamma_i \ln A_i))$, η_i^{RM} $(= -(c_{ii} + \gamma_i d \ln A_i)/w_i)$, and η_i^{AIDS} $(= -((c_{ii} + \gamma_i \ln A_i)/w_i - 1))$, respectively. The $\partial \ln \eta / \partial \ln A$ term in table 1 is derived as follows:

(B1) $$\frac{\partial \ln \eta_i^{LM}}{\partial \ln A_i} = \frac{\partial \eta_i^{LM}}{\eta_i^{LM} \partial \ln A_i} = \frac{-p_i}{\eta_i^{LM}} \left[\frac{-c_{ii} \partial q_i}{q_i^2 \partial \ln A_i} + \frac{\gamma_i (q_i \partial A_i / \partial \ln A_i - A_i \partial q_i / \partial \ln A_i)}{q_i^2} \right]$$

$$= \frac{-p_i q_i}{-(c_{ii} + \gamma_i A_i) p_i} \left[\frac{-c_{ii} \partial \ln q_i}{q_i \partial \ln A_i} + \frac{\gamma_i A_i q_i (\partial \ln A_i / \partial \ln A_i - \partial \ln q_i / \partial \ln A_i)}{q_i^2} \right]$$

$$= \frac{\gamma_i A_i - (c_{ii} + \gamma_i A_i) \alpha_{ii}}{c_{ii} + \gamma_i A},$$

(B2) $$\frac{\partial \ln \eta_i^{SL}}{\partial \ln A_i} = \frac{\partial \ln[-(c_{ii} + \gamma_i \ln A_i)]}{\partial \ln A_i} - \frac{\partial \ln q_i}{\partial \ln A_i} = \frac{\partial[-(c_{ii} + \gamma_i \ln A_i)]}{-(c_{ii} + \gamma_i \ln A) \partial \ln A_i} - \alpha_{ii}$$

$$= \frac{\gamma_i}{c_{ii} + \gamma_i \ln A_i} - \alpha_{ii},$$

(B3) $$\frac{\partial \ln \eta_i^{DL}}{\partial \ln A_i} = \frac{\partial \eta_i^{DL}}{\eta_i^{DL} \partial \ln A_i} = \left(\frac{-1}{c_{ii} + \gamma_i \ln A_i}\right) \left[\frac{\partial(-c_{ii})}{\partial \ln A_i} - \frac{\gamma_i \partial \ln A_i}{\partial \ln A_i} \right] = \frac{\gamma_i}{c_{ii} + \gamma_i \ln A_i}$$ and

(B4) $$\frac{\partial \ln \eta_i^{RM}}{\partial \ln A_i} = \frac{\partial \eta_i^{RM}}{\eta_i^{RM} \partial \ln A_i} = \frac{-1}{\eta_i^{RM}} \left[\frac{-c_{ii} \partial w_i}{w_i^2 \partial \ln A_i} + \frac{\gamma_i (w_i - d \ln A_i \partial w_i / \partial \ln A_i)}{w_i^2} \right]$$

$$= \frac{-1}{\eta_i^{RM}} \left[\frac{-c_{ii} \partial \ln w_i}{w_i \partial \ln A_i} + \frac{\gamma_i (1 - d \ln A_i \partial \ln w_i / \partial \ln A_i)}{w_i} \right].$$

Plugging (A6) into (B4) yields:

(B5) $$\frac{\partial \ln \eta_i^{RM}}{\partial \ln A_i} = \frac{\gamma_i + (c_{ii} + \gamma_i d \ln A_i)(\sum_j^n w_j \alpha_{ij} - \alpha_{ii})}{c_{ii} + \gamma_i d \ln A_i}.$$

Finally,

(B6) $$\frac{\partial \ln \eta_i^{AIDS}}{\partial \ln A_i} = \frac{\partial \eta_i^{AIDS}}{\eta_i^{AIDS} \partial \ln A_i} = \frac{\gamma_i / w_i - (c_{ii} + \gamma_i \ln A_i)(\alpha_{ii} - \sum_j^n w_j \alpha_{ij}) / w_i}{(c_{ii} + \gamma_i \ln A_i) / w_i - 1}$$

$$= \frac{\gamma_i + (c_{ii} + \gamma_i \ln A_i)(\sum_j^n w_j \alpha_{ij} - \alpha_{ii})}{c_{ii} + \gamma_i \ln A_i - w_i}.$$