Estimation and Inference for Threshold effects in panel data stochastic frontier models

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Abstract

One of the most enduring problems in cross-section or panel data models is heterogeneity among individual observations. Different approaches have been proposed to deal with this issue, but threshold regression models offer intuitively appealing econometric methods to account for heterogeneity. We propose three different estimators that can accommodate multiple thresholds. The first two, allowing respectively for fixed and random effects, assume that the firms' specific inefficiency scores are time-invariant while the third one allows for time-varying inefficiency scores. We rely on a likelihood ratio test with $m-1$ regimes under the null against m regimes. Testing for threshold effects is problematic because of the presence of a nuisance parameter which is not identified under the null hypothesis. This is known as Davies' problem. We apply procedures pioneered by Hansen (1999) to test for the presence of threshold effects and to obtain a confidence set for the threshold parameter. These procedures specifically account for Davies problem and are based on non-standard asymptotic theory. Finally, we perform an empirical application of the fixed effects model on a panel of Quebec dairy farms. The specifications involving a trend and the Cobb-Douglas and Translog functional forms support three thresholds or four regimes based on farm size. The efficiency scores vary between 0.95 and 1 in models with and without thresholds. Therefore, productivity differences across farm sizes are most likely due to technological heterogeneity.

Key words: Stochastic frontier models; threshold regression; technical efficiency; bootstrap; dairy production.

Journal of Economic Literature classification: C12, C13, C23, C52.

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1 Introduction

Structural change and threshold effects are two related issues that have motivated considerable empirical and theoretical research in time series econometrics (e.g. Tsay (1989, 1998), Enders and Granger (1998), Hansen (2000*b*, 2000*a*)). This paper considers statistical inference methods for threshold effects in panel data stochastic frontier models. One of the most enduring problems in cross-section or panel data models is heterogeneity among individual observations. One approach to address the heterogeneity issue is to compare a regression function that is identical across all observations in a sample to a set of regression functions that allow for observations to fall into discrete classes as in Hansen (1999).

Threshold regression models offer intuitively appealing econometric methods to account for heterogeneity. In the context of stochastic production frontier models, the question may be whether large firms use a production technology that differs from that of small firms. This would allow researchers to determine whether the higher productivity of large firms stems from the use of a different technology or simply a more efficient use of inputs given the constraints imposed by the common technology as measured by technical efficiency scores (see Tran and Tsionas (2006)). Related methods that allow for heterogeneity in stochastic frontier models include latent class models (Greene (2002, 2005); Orea and Kumbhakar (2004)), random coefficients models (Tsionas (2002); Greene (2002, 2005)) and Markov switching frontier models (Tsionas and Kumbhakar (2004)). The distinguishing feature of threshold models is that they assume that heterogeneity is induced by an observable exogenous variable, e.g. firm size, while in the other methods cited above heterogeneity is introduced in the models through exogenous variables or unobservable random terms.

Recently, Tsionas and Tran (2006) have proposed various models to allow for heterogeneity in technology and in the distribution of technical inefficiency. Bayesian inference methods are proposed for the estimation of these models and for model comparisons. Bayesian tools such as the posterior odds ratio and the Bayes factor are proposed for model selection, including the comparison of a threshold model against a model without threshold effects. These statistics are used as evidence pertaining to the presence of threshold effects in the data. However, from a classical inference approach, such evidence needs to be based on a test of the null hypothesis of no threshold effect. Testing for threshold effects is problematic and requires non standard tools because of the presence of a nuisance parameter which is not identified under the null hypothesis. This is known as Davies' problem and appropriate techniques have been proposed in Davies (1987), Andrews (1993) and Hansen (1996, 1999, 2000*a*). For our specific threshold effects problem, the nuisance parameter is the value of the threshold. In this paper, we consider one of the threshold models analyzed in Tsionas and Tran (2006), the simple threshold stochastic frontier model and provide a testing strategy for the presence of threshold effects in a parametric stochastic frontier model with panel data.

Our methodology is anchored on three formulations of the panel data stochastic frontier model, which differ by the time dependence of the inefficiency term as follows: (i) a fixed effect time invariant inefficiency term, (ii) a random effect time invariant inefficiency term, and (iii) a random time varying inefficiency term. For specifications (i)-(ii), we assume that the technical inefficiency term is a firm-specific constant, so we obtain a fixed effects or random effects panel data model as in Schmidt and Sickles (1984), Horrace and Schmidt (1996) and Greene (1997). These specifications of the panel data stochastic frontier model have the advantage of not requiring any distributional assumption for technical inefficiency. Therefore, for the fixed effects case we apply procedures pioneered by Hansen (1999) to test for the presence of threshold effects and to obtain a confidence set for the threshold parameter. These procedures are based on non-standard asymptotic theory and specifically account for Davies' problem. We then examine the extension of these procedures to random effects the case. However, these time invariant specifications for the inefficiency term may not be adequate for panel data with a number of time periods large enough to jeopardize the validity of the assumption of constant technical inefficiency. For long panels, our alternative specification (iii) is more appropriate. With this specification, we assume a half-normal distribution for the inefficiency term and a normal distribution for the two-sided error term of the model. We consider sup-type tests initially proposed by Davies (1987) and extended by Andrews (1993) and Hansen (1996). Given a known specific value for the threshold parameter, the model is estimated by the maximum likelihood method without threshold effects (the model under the null hypothesis) and with threshold effects (the model under the alternative hypothesis). For both models, we measure technical inefficiency using the Jondrow, Lovell, Materov and Schmidt (1982) estimator. As in Hansen (1999, 2000*a*), our test statistic is a LR-type statistic defined from the residuals sums of squares under the null and the alternative hypotheses respectively. Since the value of the threshold is unknown, we consider a supremum of the test statistic over a relevant subset of values of the threshold parameter. The problem under consideration is more complex than the one considered in Hansen (1999, 2000*a*) because we address Davies' problem for a highly nonlinear model. As a result, the asymptotic theory for inference on the threshold parameter is non-standard and we propose a bootstrap strategy to obtain an asymptotic p-value and to construct a confidence set. Our bootstrap method involves a combination of bootstrap techniques used for the stochastic frontier model (Hall, Härdle and Simar (1995), Simar and Wilson (2000), Kim, Kim and Schmidt (2006)) and the bootstrap procedure proposed in Hansen (2000*a*). The test procedures discussed in this paper have wide-ranging empirical applications. To illustrate the applicability of the proposed tests, we report results from one empirical application involving a panel of 302 dairy farms located in the province of Quebec and observed during 11 years, over the period 1993-2003. For this application, the threshold variable is the number of dairy cows, a proxy for farm size.

The rest of the paper is organized as follows. Section 2 describes the basic framework under which our estimators and testing procedures are developed. The three different estimators are presented in Section 3 while Sections 4 describes the test statistic about a single regime/technology. Section 5 focuses on inference issues pertaining to the threshold parameter and methods to address them. Section 6 presents results from an application involving Quebec dairy farms. This section showcases our fixed effects estimator and our testing procedure to identify the presence of one or more thresholds. The concluding section summarizes our contribution to the literature and discusses future research avenues.

2 Framework

We consider the following threshold effects panel data stochastic frontier model

$$y_{it} = \alpha + \beta_1' x_{it} I\left(q_{it} \leq \gamma\right) + \beta_2' x_{it} I\left(q_{it} > \gamma\right) - u_{it} + v_{it}, \quad u_{it} \geq 0, \tag{2.1}$$

where for firm i at time period t, $i = 1, ..., N$, $t = 1, ..., T$, y_{it} is the logarithm of output, $x_{it} \in \mathbb{R}^k$ is a vector of logarithm of inputs, $I(.)$ is the indicator function, β_1 and β_2 are two vectors of parameters associated with two different technologies Γ_1 and Γ_2. q_{it} is an exogenous and observable threshold variable that governs the technology regime of firms. γ is the threshold value such that if $q_{it} \leq \gamma$ then firm i adopts the technology Γ_1 at time period t, otherwise firm i adopts technology Γ_2. v_{it} is statistical error term, and $u_{it} \geq 0$ represents technical inefficiency. We assume throughout that the error term v_{it} is independent and identically distributed with mean zero and finite variance σ_v^2. For $\beta_1 = \beta_2$, we get the basic panel data stochastic frontier model (see Pitt and Lee (1981), Schmidt and Sickles (1984), Cornwell and Schmidt (1995), Greene (1997)). As in Hansen (1999), this model can be written in a more compact form as follows. Let

$$x_{it}(\gamma) = \begin{pmatrix} x_{it} I\left(q_{it} \leq \gamma\right) \\ x_{it} I\left(q_{it} > \gamma\right) \end{pmatrix},$$

and $\beta = \left(\beta_1', \beta_2'\right)'$. With this notation, equation (2.1) can be written as

$$y_{it} = \alpha + \beta' x_{it}(\gamma) - u_{it} + v_{it}, \quad u_{it} \geq 0. \tag{2.2}$$

Statistical procedures to test for threshold effects in this model will strongly depend on distributional and time dependence assumptions made on the inefficiency term u_{it}. Our analysis considers in turn the following cases:

Case 1 u_{it} *is a fixed time invariant effect,* $u_{it} \equiv \mu_i$, *for all* $t = 1, ..., T$.

Case 2 u_{it} *is a time-invariant random variable* u_i.

Case 3 u_{it} *is a time-varying random variable.*

Under Case 1, model (2.2) can be written as a fixed effects panel data model. Let $\alpha_i = \alpha - \mu_i$; then $\alpha_i \leq \alpha$ for all i and α_i may take positive or negative values. Therefore, we can re-write model (2.2) as the following non-dynamic panel model with firm-specific fixed effects:

$$y_{it} = \alpha_i + \beta' x_{it}(\gamma) + v_{it};\ i = 1, ..., N, t = 1, ..., T. \tag{2.3}$$

Model (2.3) assumes absence of any unmeasured time invariant heterogeneity across firms (for further details see Greene (2005, p. 277))[1]. The time invariance assumption for technical inefficiency may be an unreasonable one in long panels. Kumbhakar (1990) argued that this assumption is inadequate because firms aware of their relative inefficiency would take steps to catch-up over time. However, this fixed effects formulation is standard in the panel data stochastic frontier literature and has the obvious advandage that no distributional or independence assumption on inefficiency terms is needed (Schmidt and Sickles (1984), Greene (1997), Horrace and Schmidt (1996), Kim et al. (2006)). For least squares estimation and asymptotic inference on threshold effects in this model, we rely on Hansen (1999).

Under Case 2, we get the random effects stochastic frontier model (see Pitt and Lee (1981), Schmidt and Sickles (1984))

$$y_{it} = \alpha + \beta' x_{it}(\gamma) - u_i + v_{it}, \quad u_i \geq 0;\ i = 1, ..., N, t = 1, ..., T. \tag{2.4}$$

One further assumes that inefficiencies u_i are uncorrelated with the regressors, which implies that any unmeasured heterogeneity across firms must be independent of the inputs variables.

Finally, Case 3 represents a more flexible and realistic model by having inefficiencies vary over time for each firm. This is an obvious advantage when dealing with long panels. For simplicity, we assume in addition that u_{it} and v_{it} are independent over time and across individuals, so no specific panel data treatment is needed (Greene (1997)). For various formulations and specifications for the time dependence of technical inefficiency, see Cornwell, Schmidt and Sickles (1990), Kumbhakar (1990), Lee and Schmidt (1993) and Battese and Coelli (1992, 1995) among others; we defer the extension of our test methods to accomodate these models to future research.

[1] This model is different from the true fixed effects stochastic frontier model, which is subject (i) to practical estimation problems as the number of firms in the sample is very large, and (ii) to the incidental parameters' problem Greene (2005, p. 277).

3 Estimation methods

3.1 Time-invariant fixed effects model

Under Case 1, the stochastic frontier model, written in the form (2.3), is the standard threshold regression for non-dynamic panel with individual-specific fixed effects discussed by Hansen (1999). Estimates for threshold and slopes parameters can be obtained using a least squares estimation. Specifically, the estimation proceeds as follows. Assume that γ is known and let

$$\overline{y}_i = T^{-1} \sum\nolimits_{t=1}^{T} y_{it},\ \ \overline{x}_i(\gamma) = T^{-1} \sum\nolimits_{t=1}^{T} x_{it}(\gamma),\ \ \overline{v}_i = T^{-1} \sum\nolimits_{t=1}^{T} v_{it};\ \ i = 1, ..., N.$$

If we apply a fixed-effect transformation to (2.3) in order to remove firm-specific means, we get

$$y_{it}^{*} = \beta' x_{it}^{*}(\gamma) + v_{it}^{*}, \tag{3.5}$$

where

$$y_{it}^{*} = y_{it} - \overline{y}_i,\ \ x_{it}^{*}(\gamma) = x_{it}(\gamma) - \overline{x}_i(\gamma),\ \ v_{it}^{*} = v_{it} - \overline{v}_i;\ \ i = 1, ..., N, t = 1, ..., T.$$

Model (3.5) can be written in matrix form as

$$Y^{*} = X^{*}(\gamma)\,\beta + v^{*}, \tag{3.6}$$

where Y^{*}, $X^{*}(\gamma)$ and v^{*} are the data stacked over all N firms and over T time periods as follows: for Y^{*}, form $Y^{*} = (y_1^{*}, ..., y_N^{*})'$ where $y_i^{*} = (y_{i1}^{*}, y_{i2}^{*}, ..., y_{iT}^{*})'$; proceed similarly to obtain $X^{*}(\gamma)$ and v^{*}. From (3.6), the ordinary least squares estimator of β as a function of γ is given by

$$\hat{\beta}_F(\gamma) = \left[X^{*}(\gamma)' X^{*}(\gamma)\right]^{-1} X^{*}(\gamma)' Y^{*},$$

and the residual sum of squares is

$$\begin{aligned} S_F(\gamma) &= \left[Y^{*} - X^{*}(\gamma)\,\hat{\beta}(\gamma)\right]' \left[Y^{*} - X^{*}(\gamma)\,\hat{\beta}(\gamma)\right] \\ &= Y^{*\prime} \left(I - X^{*}(\gamma)' \left[X^{*}(\gamma)' X^{*}(\gamma)\right]^{-1} X^{*}(\gamma)'\right) Y^{*}. \end{aligned} \tag{3.7}$$

Since γ is unknown, it must be estimated from the data set. Least squares estimation of γ can be done by minimization of the residual sum of squares as

$$\hat{\gamma}_F = \arg \min_{\gamma \in \bar{\Gamma} \subset \Gamma} S_F(\gamma). \tag{3.8}$$

The minimization in (3.8) can be restricted to a specific subset $\bar{\Gamma} \subset \Gamma$, where Γ is the set of all possible values of γ, if we want a minimal percentage of the observations to lie in each of the two technology regimes defined by the threshold. A grid search over values in $\bar{\Gamma}$ is used in practice to solve this problem; see Hansen (1999, pp. 349-350) for details. The final estimate of the regression coefficients β is $\hat{\beta}_F = \hat{\beta}_F(\hat{\gamma}_F)$; the vector of residuals is $\hat{v}_F^{*} = Y^{*} - X^{*}(\hat{\gamma}_F)\,\hat{\beta}_F(\hat{\gamma}_F)$ and the error variance is estimated by $\hat{\sigma}_{vF}^2 = (1/NT)\, S_F(\hat{\gamma}_F)$.

3.2 Time-invariant random effects model

We now consider the stochastic frontier model defined by (2.4). For any given γ, the inefficiency terms u_i are assumed to be uncorrelated with the inputs variables $x_{it}(\gamma)$. In addition, we assume that the u_i are *i.i.d.* with $E(u_i) = \mu$ and $Var(u_i) = \sigma_u^2$ and that u_i are independent of the v_{it}. It is convenient to rewrite the model as follows. Let $\alpha^* = \alpha - \mu$, and $u_i^* = u_i - \mu$. Then, (2.4) is equivalent to

$$y_{it} = \alpha^* + \beta' x_{it}(\gamma) - u_i^* + v_{it};\ i = 1, ..., N, t = 1, ..., T.$$

where the error terms u_i^* and v_{it} have zero mean.

Assuming that N is large, we can obtain a consistent estimate $\hat{\sigma}_u^2(\gamma)$ of σ_u^2, and we also assume that a consitent estimator $\hat{\sigma}_v^2(\gamma)$ of σ_v^2 is available. Then, the regression coefficients β can be estimated by $\hat{\beta}_R(\gamma)$ using feasible generalized least squares. Provided $T \longrightarrow \infty$, for firm i, $\alpha_i = \alpha^* - u_i^*$ can be consistently estimated by

$$\widehat{\alpha}_i(\gamma) = \frac{1}{T} \sum\nolimits_{t=1}^{T} \left(y_{it} - \hat{\beta}_R(\gamma)' x_{it}(\gamma) \right);\ i = 1, ..., N.$$

Then, we form the residual and the residual sum of squares of the random effects model as

$$\hat{v}_{itR}(\gamma) = y_{it} - \hat{\beta}_R(\gamma)' x_{it}(\gamma) - \widehat{\alpha}_i(\gamma),\ S_R(\gamma) = \sum\nolimits_{t=1}^{T} \sum\nolimits_{i=1}^{N} \hat{v}_{itR}(\gamma).$$

As is the case of the fixed effects model, γ needs to be estimated from the data set, and we also rely on least squares estimation method. Thus, $\hat{\gamma}_R$ is defined by

$$\hat{\gamma}_R = \arg \min_{\gamma \in \bar{\Gamma} \subset \Gamma} S_R(\gamma). \tag{3.9}$$

The final estimator of the regression coefficients β is $\hat{\beta}_R = \hat{\beta}_R(\hat{\gamma}_R)$; the error variance σ_v^2 is estimated by $\hat{\sigma}_{vR}^2 = (1/NT)\, S_R(\hat{\gamma}_R)$.

3.3 Independent time-varying technical inefficiency model

Under Case 3 and under the assumption that the inefficiency terms u_{it} are serially and contemporaneously uncorrelated we get, for any given γ, the panel data version of the standard stochastic frontier model. These assumptions correspond to that maintained in the various threshold stochastic frontier models discussed in Tsionas and Tran (2006) and imply that despite its variation over time, there is non persistance effect in technical inefficiency. Estimation proceeds as set in Aigner, K. and Schmidt (1977) and Jondrow et al. (1982) for the case of cross-sectional data.

Assuming that γ is known, let $\varepsilon_{it} \equiv= v_{it} - u_{it}$, where $v_{it} \overset{i.i.d.}{\sim} \mathrm{N}(0, \sigma_v^2)$, and $u_{it} = |U_{it}|\,,\ U_{it} \sim \mathrm{N}(\mu, \sigma_u^2),\ i = 1, ..., N, t = 1, ..., T$. Under these distributional assumptions, the parameters of the models can be estimated using the maximum likelihood (ML) method.

Let $\left(\hat{\alpha}_I(\gamma), \hat{\beta}_I(\gamma), \hat{\sigma}_{uI}^2(\gamma), \hat{\sigma}_{vI}^2(\gamma)\right)$ denote the ML estimates of $\left(\alpha, \beta, \sigma_u^2, \sigma_v^2\right)$, given a specified value γ. The technical inefficiency term can then be estimated by the ML estimate of the conditional expectation $\mathrm{E}(u_{it}|\varepsilon_{it} = e_{it})$, where $\mathrm{E}(.|\varepsilon_{it} = e_{it})$ is the conditional expectation operator conditioned on $\varepsilon_{it} = e_{it}$. The result is as follows:

$$\hat{u}_{it}(\gamma) = \mathrm{E}(u_{it}|\varepsilon_{it} = e_{it}(\gamma)) = \left(\frac{\phi\left(e_{it}\hat{\lambda}(\gamma)/\hat{\sigma}(\gamma)\right)}{1 - \Phi\left(e_{it}\hat{\lambda}(\gamma)/\hat{\sigma}(\gamma)\right)} - \frac{e_{it}\hat{\lambda}(\gamma)}{\hat{\sigma}(\gamma)}\right)\hat{\sigma}^*(\gamma),$$

where ϕ and Φ denotes the standard normal density and cumulative distribution function and

$$\begin{aligned} \hat{\lambda}(\gamma) &= \hat{\sigma}_{uI}(\gamma)/\hat{\sigma}_{vI}(\gamma), \quad \hat{\sigma}^2(\gamma) = \hat{\sigma}_{uI}^2(\gamma) + \hat{\sigma}_{vI}^2(\gamma), \\ \hat{\sigma}^*(\gamma) &= \frac{\hat{\sigma}_{vI}^2(\gamma)\hat{\sigma}_{uI}^2(\gamma)}{\hat{\sigma}_{vI}^2(\gamma) + \hat{\sigma}_{uI}^2(\gamma)}, \quad e_{it}(\gamma) = y_{it} - \hat{\alpha}_I(\gamma) + \hat{\beta}_I'(\gamma)x_{it}(\gamma). \end{aligned}$$

We define the residual and the residual sum of squares as

$$\hat{v}_{itI}(\gamma) = y_{it} - \hat{\alpha}_I(\gamma) + \hat{\beta}_I'(\gamma)x_{it}(\gamma) - \hat{u}_{it}(\gamma),\ S_I(\gamma) = \sum\nolimits_{t=1}^{T}\sum\nolimits_{i=1}^{N}\hat{v}_{itI}(\gamma).$$

The least squares estimator $\hat{\gamma}_I$ of γ is defined by

$$\hat{\gamma}_I = \arg\min_{\gamma \in \bar{\Gamma} \subset \Gamma} S_I(\gamma). \tag{3.10}$$

The final estimator of the model parameters are obtained as

$$\left(\hat{\alpha}_I, \hat{\beta}_I, \hat{\sigma}_{uI}^2, \hat{\sigma}_{vI}^2\right) = \left(\hat{\alpha}_I(\hat{\gamma}_I), \hat{\beta}_I(\hat{\gamma}_I), \hat{\sigma}_{uI}^2(\hat{\gamma}_I), \hat{\sigma}_{vI}^2(\hat{\gamma}_I)\right).$$

4 Testing for a threshold

The model formulation (2.1) and the estimation methods discussed in the previous section assumed that there exists some threshold effect in the data. However, since this formulation introduces an extra (threshold) parameter in the model, estimation problems may arise due to specification error when there is actually no threshold effects in the data. Therefore, it is important to assess the presence of a threshold using a formal statistical test. We rely on the likelihood ratio test proposed in Hansen (1999).

The null hypothesis of no threshold effect in the model (2.1) can be written as

$$\mathrm{H}_0 : \beta_1 = \beta_2. \tag{4.11}$$

Clearly, under H_0 the model (2.1) takes the form

$$y_{it} = \alpha + \beta_1' x_{it} - u_{it} + v_{it}, \quad u_{it} \geq 0, \tag{4.12}$$
$$i = 1, ..., N, t = 1, ..., T,$$

which does not involve the threshold parameter γ. So for the problem at hand, the parameter γ is not identified under the null hypothesis and usual test statistics have non-standard distributions. This is the so-called Davies' Problem (Davies (1977, 1987)). For this problem, Hansen (1999) suggested to simulate the non-standard asymptotic distribution of the likelihood ratio (LR) test using a bootstrap method. The test procedure proposed in Hansen (1999) works as follows.

For Case 1 (it is similar for Cases 2 and 3), we estimate the fixed-effects panel data stochastic frontier model associated to model 4.12 under Case 1 using the fixed-effect transformation as described in section 3.1. Let us write the model after the within transformation as

$$y_{it}^* = \beta_1' x_{it}^* + v_{it}^*, \tag{4.13}$$

where y_{it}^*, x_{it}^*, and v_{it}^* are the within transformation version of y_{it}, x_{it}, and v_{it} respectively (see section 3.1). For further reference, let $\tilde{\beta}_{1F}$ denote the within estimator of β_1. Let $\tilde{v}_F^*$ denote the vector of residuals and $S_{0F} = (\tilde{v}_F^*)' (\tilde{v}_F^*)$ be the residual sum of squares under H_0 .The LR test statistic may be defined as

$$\mathrm{LR}_F = (S_{0F} - S_F(\hat{\gamma}_F)) / \hat{\sigma}_{vF}^2. \tag{4.14}$$

The statistic LR_F has a non-standard asymptotic distribution whose characteristics may be affected by the asymmetric distribution of the technical efficiency terms. This is likely to be problematic in the case of random-effects and time varying technical inefficiency models. We rely on the bootstrap procedure proposed by Hansen (1999) for the standard fixed-effects panel model, even though its validity has not been established yet for the latter two cases. The resampling is based on the sample of firms, and once a firm is selected all its observations over the T periods are included in the bootstrap sample. We resample residuals as follows. Let $\hat{v}_{F,i}^* = \left(\hat{v}_{F,i1}^*, \hat{v}_{F,i2}^*, ..., \hat{v}_{F,iT}^*\right)'$, $i = 1, ..., N$, denote the $T \times 1$ vector of residuals computed for firm i from the model assuming threshold effects. Then form the sample $\left(\hat{v}_{F,1}^*, \hat{v}_{F,2}^*, ..., \hat{v}_{F,N}^*\right)$. The empirical distribution of $\left(\hat{v}_{F,1}^*, \hat{v}_{F,2}^*, ..., \hat{v}_{F,N}^*\right)$ is used for bootstrap resampling, *i.e.* we draw randomly with replacement a sample of size N from $\left(\hat{v}_{F,1}^*, \hat{v}_{F,2}^*, ..., \hat{v}_{F,N}^*\right)$. These draws are treated as errors to be used to create a bootstrap sample under H_0. For each bootstrap replication $b = 1, ..., B$, let $\left(v_1^{(b)}, ..., v_i^{(b)}, ..., v_N^{(b)}\right)$ represents the bootstrap draw. We should generate the output variable using

$$y_{it}^{(b)} = \hat{y}_{it} + v_{it}^{(b)},$$

where $\hat{y}_{it}$ is the predicted value of y_{it} under H_0. In the case of the fixed-effects model, we consider $\hat{y}_{it} \equiv \tilde{y}^*_{it} = \tilde{\beta}'_{1F} x_{it}$, while for the random-effects and time varying technical inefficiency models, prediction of y_{it} under H_0 should explicitly account for the estimated value of the inefficiency term u_i or u_{it}. Using the bootstrap sample data $\left(y^{(b)}_{it}, x_{it}\right)$, we estimate in turn the model under H_0 and without imposing H_0. For the fixed-effects model, these correspond to models (4.13) and (3.5) respectively. We compute the bootstrap value $\mathrm{LR}^{(b)}_F$ of the LR test statistic using 4.14. If we let LR^0_F denote the value of the test statistic calculated from the observed data, we can define the approximate bootstrap p-value $\hat{p}_B\left(\mathrm{LR}^0_F\right)$ as

$$\hat{p}_B\left(\mathrm{LR}^0_F\right) = \frac{B\hat{G}_B\left(\mathrm{LR}^0_F\right) + 1}{B+1}, \tag{4.15}$$

where $B\hat{G}_B\left(\mathrm{LR}^0_F\right)$ is the number of bootstrap statistics $\mathrm{LR}^{(b)}_F$ greater than or equal to LR^0_F. A test of level α, $0 < \alpha < 1$, is defined by the critical region $\hat{p}_B\left(\mathrm{LR}^0_F\right) \leq \alpha$; that is, we reject the null hypothesis at level α if $\hat{p}_B\left(\mathrm{LR}^0_F\right) \leq \alpha$, $0 < \alpha < 1$.

5 Inference about the threshold parameter

In the presence of threshold effects, it would be useful to make a statistical inference about the threshold parameter in addition to have its point estimate discussed in section 3. Indeed, in the related time series structural change literature a confidence set for the break date can be constructed using the asymptotic distribution of the estimator of the break point parameter (see Bai, Lumsdaine and Stock (1998)). In Hansen (2000a), it is shown that the asymptotic distribution of the threshold estimator $\hat{\gamma} = \hat{\gamma}_F, \hat{\gamma}_R, \hat{\gamma}_I$ is highly non-standard and this distribution depends on unknown parameters. In such contexts, a confidence set based on the inversion of Wald or t statistics may behave very poorly in finite sample.

5.1 Inverting a likelihood ratio test

The asymptotic distribution of $\hat{\gamma} = \hat{\gamma}_F, \hat{\gamma}_R, \hat{\gamma}_I$ is highly non-standard and Wald or t statistics-based confidence sets may not be reliable, particularly in finite sample; Hansen (2000a) recommended confidence set estimation based on inverting likelihood ratio tests on γ. Inverting a test with respect to a parameter means that we collect all the values of this parameter for which the test is not significant. So we consider the test of the hypothesis $\mathrm{H}_0\left(\gamma_0\right) : \gamma = \gamma_0$, where γ_0 is any specified value for γ. The LR statistic to test $\mathrm{H}_0\left(\gamma_0\right)$ is

$$\mathrm{LR}_m\left(\gamma_0\right) = \left(S_m\left(\gamma_0\right) - S_m\left(\hat{\gamma}_m\right)\right)/\hat{\sigma}^2_{vm}, \; m = F, R, I, \tag{5.16}$$

where we index on m to emphasize that the test statistic is defined for any of the three model formulations and corresponding estimation methods. Hansen (1999, 2000a) shows that the asymptotic distribution of $\mathrm{LR}_m(\gamma_0)$ under $\mathrm{H}_0(\gamma_0)$ is non-standard and free of nuisance parameters.

Under regularity conditions, $\mathrm{LR}_m(\gamma_0) \overset{asy}{\sim} \omega$, where ω is a random variable with distribution function $P(\omega \leq x) = (1 - \exp(-x/2))^2$. The critical value of the latter distribution at level α, $0 < \alpha < 1$, is $c(\alpha) = -2\ln\left(1 - \sqrt{1-\alpha}\right)$. An asymptotic test of $\mathrm{H}_0(\gamma_0)$ rejects at level α if $\mathrm{LR}_m(\gamma_0) > c(\alpha)$. A $(1-\alpha)$-level confidence set for γ can be defined by the 'no-rejection region' of the LR test as

$$\mathrm{CS}(\gamma;\alpha) = \{\gamma_0 : \mathrm{LR}_m(\gamma_0) \leq c(\alpha)\}. \tag{5.17}$$

The asymptotic validity of this confidence set requires, among other conditions (Hansen (2000a, p. 579)), that the difference in the slope parameters between the two regimes be small and tend to zero as the sample size increases. This confidence set is rather asymptotically conservative if the error terms v_{it} are $i.i.d.$ $\mathrm{N}\left(0, \sigma_v^2\right)$ and strictly independent of the regressors and of the threshold variable (see Hansen (2000a, Theorem 3)). Even if the gaussian errors assumption is not unusual in the literature on parametric stochastic frontier models, we also consider an alternative bootstrap approach to confidence set estimation of the threshold parameter.

5.2 Bootstrap confidence set

In spite of the presence of unknown parameters in the asymptotic distribution of $\hat{\gamma}$, we suggest the use of a bootstrap method to obtain an approximation to the sampling distribution of $\hat{\gamma}$. The validity of the bootstrap in this context can be justified using the same arguments as for the case of bootstrapping the asymptotic distribution of the statistic LR_F defined in (4.14) (see Hansen (1999, 2000a)). We suggest using an $i.i.d.$ resampling scheme as opposed to resampling regression residuals. The $i.i.d.$ resampling has been recently used by Seo and Linton (2007) for bootstrap inference on any scalar function of the parameters of a threshold regression model estimated through a smoothed least squares estimator; note, however, that in Seo and Linton (2007), the estimator is shown to be asymptotically normal and thus its asymptotic distribution is free of nuisance parameters.

Let $\{Z_{it} : i = 1, ..., N;\ t = 1, ..., T\}$ denote the data set, with $Z_{it} = (y_{it}, x_{it}')'$. Then, let $Z_i = (Z_{i1}, Z_{i2}, ..., Z_{iT})$. In order to account for the panel structure, the empirical distribution to be used for bootstrapping is $(Z_1, Z_2, ..., Z_N)$; that is, resampling is based on firms and once a firm is resampled, all its observations over the T time periods enter the bootstrap sample. For $b = 1, 2, ..., B$, where B is the number of bootstrap replications used, let $\left(Z_1^{(b)}, Z_2^{(b)}, ..., Z_N^{(b)}\right)$ be a random sample drawn with replacement from $(Z_1, Z_2, ..., Z_N)$; let $Z_i^{(b)} = \left(Z_{i1}^{(b)}, Z_{i2}^{(b)}, ..., Z_{iT}^{(b)}\right)$ for all $i = 1, 2, ..., N$.

Then, using the bootstrap data set $\left\{Z_{it}^{(b)} : i = 1, ..., N;\ t = 1, ..., T\right\}$, estimate the stochastic frontier model 2.1 using any of the three formulations and corresponding estimation techniques; let $\hat{\gamma}^{(b)}$ denote the bootstrap estimate of γ. The key result of the bootstrap is that, conditionally on the observed data $\{Z_{it} : i = 1, ..., N;\ t = 1, ..., T\}$, the asymptotic distribution of $N^{1/2}\left(\hat{\gamma}^{(b)} - \hat{\gamma}\right)$ approximates the asymptotic sampling distribution of $N^{1/2}(\hat{\gamma} - \gamma)$ for any $b = 1, ..., B$. The conditional distribution of the bootstrap estimator $N^{1/2}\left(\hat{\gamma}^{(b)} - \hat{\gamma}\right)$ can be approximated by Monte Carlo replication of the resampling procedure. So, the collection $\left\{\hat{\gamma}^{(b)} - \hat{\gamma} : b = 1, 2, ..., B\right\}$ can be treated as a random sample from the asymptotic distribution of $\hat{\gamma} - \gamma$. So, this sample can be used to construct a confidence interval for γ.

To obtain a confidence interval based on the percentile method, we need to compute the quantiles $q_\gamma(\alpha)$ of the empirical distribution $\left\{\hat{\gamma}^{(b)} : b = 1, 2, ..., B\right\}$ as $q_\gamma(\alpha) = G_{\gamma,B}^{-1}(\alpha)$, $0 \le \alpha \le 1$, where $G_{\gamma,B}$ denotes the empirical cumulative distribution function of $\left\{\hat{\gamma}^{(b)} : b = 1, 2, ..., B\right\}$. For $0 \le \alpha \le 1$, a confidence set of asymptotic level $(1 - \alpha)$ for γ is given by

$$\left[\hat{\gamma} + q_\gamma(\alpha/2),\ \hat{\gamma} + q_\gamma(1 - \alpha/2)\right]. \tag{5.18}$$

Moreover, due to bias in the sample estimate $\hat{\gamma}$, there is some bias in the position of the bootstrap estimates $\hat{\gamma}^{(b)}$ relative to $\hat{\gamma}$. Therefore, generally it does not hold that $G_{\gamma,B}(\hat{\gamma}) = 1/2$, which means that the bootstrap sample $\left\{\hat{\gamma}^{(b)} : b = 1, 2, ..., B\right\}$ is not centered around the sample estimate $\hat{\gamma}$. We can construct a bias-corrected confidence interval for γ as follows. Let Φ be the standard normal cumulative distribution function and z_α denote the standard normal cut-off point of level α, $0 \le \alpha \le 1$; then, $q_\gamma(\alpha) = G_{\gamma,B}^{-1}(\Phi(z_\alpha))$. Define

$$q_\gamma^{\mathrm{bc}}(\tau) = G_{\gamma,B}^{-1}\left[\Phi\left(m_{\hat{\gamma}} + (m_{\hat{\gamma}} + z_\tau)\right)\right] = G_{\gamma,B}^{-1}\left[\Phi\left(2m_{\hat{\gamma}} + z_\tau\right)\right], \quad 0 < \tau < 1, \tag{5.19}$$

where $m_{\hat{\gamma}} = \Phi^{-1}\left(G_{\gamma,B}(\hat{\gamma})\right)$ is a bias-correction term. Then, the lower and upper confidence limits of a bias-corrected confidence interval for γ with asymptotic confidence level $(1 - \alpha)$, $0 \le \alpha \le 1$ are respectively given by

$$\gamma_{\mathrm{L},\alpha}^{\mathrm{bc}} = q_\gamma^{\mathrm{bc}}(1 - \alpha/2), \quad \gamma_{\mathrm{U},\alpha}^{\mathrm{bc}} = q_\gamma^{\mathrm{bc}}(\alpha/2). \tag{5.20}$$

The accuracy of these confidence intervals in term of coverage rate strongly relies on the quality of the bootstrap approximation.

We next report results from an empirical application of one of the methods discussed previously to an empirical data set featuring a panel of dairy farms located in the province of Quebec.

Table 1. Summary statistics for dairy production variables

Variables	mean	Std. dev.	Min.	Max.
Production function:				
Total volume of milk/cow (litre)	8304.03	1281.12	4557.87	12253.09
Concentrates (kg)	2879.73	741.77	632.30	6417.81
Forages (kg)	5273.25	949.78	390.44	9270.93
Capital ($)	4801.67	2545.28	372.84	34917.92
Labor (hour)	57.28	13.92	23.49	120.93
Threshold:				
Number of cows	51.64	25.58	18.70	451.90

6 Empirical application

6.1 Data sources and descriptive statistics

We consider a balanced panel covering 11 annual observations for 302 dairy farms that were in business between 1993 and 2003. Thus, our data set has a total of 3322 observations. This so-called Agritel database was collected by the Federation of Management Clubs in the province of Quebec. Summary statistics on the different variables used in our stochastic frontier production models and the threshold variable are presented in Table 1.

Canada's dairy production is governed by a supply management policy featuring tight import controls and domestic production quotas to insure a "fair" return for dairy producers. Basically, supply is constrained to achieve a domestic price target (Larue, Gervais and Pouliot (2007)). Individual production licences or quotas are traded between producers within the province of Quebec through a double-auction. The value of these individual quotas has steadily increased over time and represents a significant financial barrier deterring entry and expansion. This explains why the average number of cows is low compared to U.S. standards and why there are so few large dairy farms in Quebec[2]. The inputs selected as arguments of the production function are the most important ones in terms of cost shares. The standard deviations are much smaller than the means because there is a significant proportion of farms that are quite similar size-wise. We begin our investigation with a fixed effects stochastic frontier model without threshold(s).

[2] According to http://www.dairyfarmingtoday.org/DairyFarmingToday/Learn-More/Facts-And-Figures/ consulted on May 30, 2007, the average herd size in the U.S. is 135 cows. See also Romain and Sumner (2001) on comparisons between the Canadian and U.S. dairy industries.

Table 2. Summary statistics for estimated technical efficiency scores derived from a fixed-effects production frontier without threshold(s)

Specification	Cobb-Douglas		Translog	
Statistics	No trend	Trend	No trend	Trend
Mean	96.03	96.64	95.69	96.58
Stand. dev.	.69	.65	.72	.64
Median	96	96.65	95.62	96.60
Minimum	94.27	95.09	94.04	95

Note. This table reports descriptive statistics for technical efficiency scores (in %) estimated in the framework of a panel data stochastic production frontier model with fixed-effects inefficieny terms. The estimation method assumes that there is at least one fully efficient firm in the sample, so the maximum value is 100 for all model specifications.

6.2 A stochastic production frontier with a homogenous technology

The fixed effects stochastic frontier model without threshold can be considered as our benchmark. We estimated four different versions to assess the robustness of the results. We consider two different functional forms for the production technology which could be specified with or without a trend. The most popular functional forms used in the applied literature are the Cobb-Douglas and the Translog. The latter is more flexible than the former, but it involves the estimation of more parameters which increases the risk of convergence problems. The presence of a trend allows for dynamic effects or structural change. The summary statistics for estimated technical efficiency scores derived from the four competing specifications are presented in Table 2. Our results suggest that the choice of the functional form does not have much influence on the central tendency and dispersion statistics of the (time-invariant) efficiency scores. The mean and median are very close to 96% in all cases. The standard deviations are very small, which is not surprising given that the minima vary between 94% and 95%. Such high efficiency scores for Quebec dairy farms are to be expected because the supply management policy has been in place for a long time and, despite all of its flaws, it cannot be denied that it has contributed to create a stable environment for dairy farmers. Technical efficiency is a relative concept since the frontier is defined by the firms included in the sample. The Quebec dairy industry is subject to far less volatility than the U.S. dairy industry and this should make management easier.

Table 3. Tests of $m-1$ thresholds against m in a fixed-effects production frontier: bootstrap p-values

Specification	Cobb-Douglas		Translog	
m	No trend	Trend	No trend	Trend
1	.627	.007	.076	.004
2	.406	.001	.650	.004
3	.771	.006	.720	.018

Note. The numbers in this table are bootstrap p-values for the test of the null hypothesis that there exists $m-1$ threshold values for the production function against the alternative of m, $m = 1, 2, 3$. For a test of level α, the null hypothesis is rejected if the reported p-value is less than or equal to α.

6.3 A stochastic production frontier with threshold(s)

Even though Quebec has a high proportion of small dairy farms, not all of the farms use the same milking system. Some farms are large enough to mix their feed on the farm. Some have little land or are located in areas where it is difficult to produce corn. Hence, it is not inappropriate to entertain the possibility that farms need not have the exact same technology. In this section, we posit that technological jumps occur at various farm sizes. The methodology presented previously focused on a single threshold parameter allowing for two regimes or production technologies. However, it is easy to accommodate multiple thresholds and to use the LR statistic to find the appropriate thresholds consistent with the data (see Hansen (1999, Section 5)). We find numerically the least squares estimates of the threshold parameters through a grid search over 500 quantiles of the empirical distribution of the threshold variable; we trimmed out top and bottom 1% or 5%. We used 500 replications for the bootstrap tests, which implies that 250000 regressions were needed to run a test.

In our application, we allowed for up to three thresholds supporting four different regimes. Table 3 reports test results pertaining to the number of thresholds. Under the null hypothesis, the model has $m-1$ thresholds while the alternative has m thresholds. The presence of a trend in the specification makes a huge difference and in the Cobb-Douglas and Translog cases, there is empirical evidence for three thresholds. For the Translog without trend, there is apparently only one threshold (interpreting a p-value of 0.08 as rejection at 10% level). For the Cobb-Douglas case without trend, the tests results suggest that there is no evidence for the presence of any threshold value in the model.

The point estimates for the threshold parameters are presented in Table 4 along with lower

Table 4. Point estimates and 95% level confidence set for threshold parameters in a m thresholds fixed-effects production frontier

Specification		Cobb-Douglas	Translog	
Parameter		Trend	No trend	Trend
	$\hat{\gamma}_1$	34.4	42.6	34.1
γ_1	γ_{1L}	34.0	42.5	34.9
	γ_{1U}	67.3	48.4	34.6
	$\hat{\gamma}_2$	45.1	-	44.7
γ_2	γ_{2L}	44.7	-	26.2
	γ_{2U}	50.0	-	45.5
	$\hat{\gamma}_3$	66.3	-	66.7
γ_3	γ_{3L}	65.6	-	44.7
	γ_{3U}	68.1	-	67.7

Note. This table reports the point estimates and the lower and upper bounds of 95% level confidence sets for the threshold parameters constructed by inverting an LR test statistic in a model with fixed-effects inefficiency terms. The threshold parameters are $\gamma_1, \gamma_2, \gamma_3$; $\hat{\gamma}_i$, $i = 1, ..., 3$ denote the point estimate of γ_i; γ_{iL} and γ_{iU} respectively denote the lower and upper bounds of the confidence set.

Table 5. Regression estimates: triple threshold model for Cobb-Douglas technology with a trend under fixed-effects inefficiency

	regime 1		regime 2		regime 3		regime 4	
Variables	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
Concent.	.1185	7.57	.1495	14.22	0.1612	15.30	0.0950	5.14
Forages	.0487	3.75	.0317	3.87	0.0362	3.79	0.0588	4.72
Capital	-.0034	-.53	.0042	0.89	0.0029	0.67	0.0152	2.45
Labor	.0911	5.22	.0424	3.26	0.0150	1.11	0.0582	3.29
Trend	.0257	22.32	.0256	33.24	0.0205	25.89	0.0252	21.67

Note. Results for the estimation of stochastic production frontier with three thresholds values with fixed effects inefficiency; the production function relies on a Cobb-Douglas technology with a trend; t-ratios based on White-corrected standard errors are in parentheses.

Table 6. Summary statistics for estimated technical efficiency scores derived from a threshold effects stochastic production frontier with fixed-effects inefficiency

Specification	Cobb-Douglas	Translog	
Statistics	Trend	No trend	Trend
Mean	96.68	95.93	96.64
Stand. dev.	.62	.73	.66
Median	96.72	95.84	96.67
Minimum	95.11	94.31	95.03

Note. This table reports descriptive statistics for technical efficiency scores (in %) estimated in the framework of a threshold panel data stochastic production frontier model with fixed-effects inefficieny terms. The estimation method assumes that there is at least one fully efficient firm in the sample, so the maximum value is 100 for all model specifications.

and upper bounds of the corresponding 95% confidence sets for the Cobb-Douglas and Translog forms with and without a trend. The presence of thresholds in the Cobb-Douglas model without a trend did not significantly improve the model without threshold and this is why there are no thresholds reported. In contrast, the Cobb-Douglas frontier with trend has three thresholds whose point estimates are 34, 45 and 66. The second and third thresholds have narrow confidence sets, but the first threshold has a high upper bound. The point estimates obtained from the Translog with a trend are nearly identical, but the confidence sets differ. In this instance, the confidence set for the first threshold is very narrow while the second and third thresholds have low lower bounds. The Translog frontier without a trend supports a single threshold. The latter's point estimate is 48 with a lower bound of 46 and an upper bound of 49. Some of our confidence sets are skewed, as either the lower bound or the upper bound of the bootstrap confidence set are very close to the reported point estimate. This is also apparent in Hansen (1999) but to a lesser degree. The implication is that the probability that the true threshold be far away from the point estimate is quite low. This is why for instance the null of two thresholds is soundly rejected (p-value equals .006) even though the confidence set of the first threshold spans the confidence set of the second threshold.

Table 5 reports estimates of the coefficients characterizing the production technologies of the four regimes associated with the Cobb-Douglas with trend frontier. The concentrate coefficients vary between 0.095 and 0.161 across regimes while the range for the forage coefficients is 0.031-0.059. The coefficients on capital are small and not significantly different from zeros for the three smallest categories of farms. In contrast, labour is most important for the smallest farm group.

The labour coefficient for the smallest farms is roughly 50% larger than that for the largest farms. The trend coefficients are very similar across regimes.

Results about the efficiency scores associated with the threshold models are presented in Table 6. The mean efficiency level is close to 96% in all cases. This is what we got with the estimation of a stochastic frontier without thresholds. This suggests that productivity advantage of larger dairy farms over smaller farms are due to technological advantages and not to technical efficiency.

7 Conclusion

Heterogeneity among individual observations in cross-section or panel data models is an issue that has motivated a rapidly-increasing literature. Applied econometricians estimating panel data stochastic frontier models are routinely confronted to this problem. In this paper, we propose three different estimators allowing for multiple thresholds to address the heterogeneity issue. Inference is problematic in threshold models because of nuisance parameters not identified under the null hypothesis. We built on procedures developed by Hansen (1999) in developing a likelihood ratio test enabling us to test for $m-1$ regimes under the null against m regimes. We also develop a bootstrap procedure to conduct statistical inference about the threshold parameters.

Our empirical application features the estimation of a fixed effects stochastic frontier model on a panel of Quebec dairy farms. We found evidence of threshold effects, but the latter depend on the presence or absence of a trend and the choice of functional form. The efficiency scores are highly concentrated at the top for models with and without thresholds. We conclude that productivity differences across farm sizes are most likely due to technological heterogeneity.

Future version of this paper will showcase applications of the other proposed estimators and analyse the distributions of efficiency scores within and between regimes.

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