Laboratoire d'Economie Appliquée de Grenoble

CHOICE OF AN ORDERING STRATEGY TAKING INTO ACCOUNT OF RISKS ABOUT CUSTOMER SERVICE LEVELS AND ON-HAND INVENTORIES

BABOLI Arman ; LLERENA Daniel ; MOYAUX Thierry

March 2009

Working Paper GAEL ; 2009-03

Institut National de la Recherche Agronomique - Université Pierre Mendès France
Unité Mixte de Recherche 1215
Domaine Universitaire - BP 47 - 38040 GRENOBLE Cedex 9
Tél. : 33 (0) 4 76 82 54 39 - Fax : 33 (0) 4 76 82 54 55
E-mail : vertier@grenoble.inra.fr - http://www.grenoble.inra.fr

Choice of an ordering strategy taking account of risks about customer service levels and on-hand inventories*

Arman BABOLI [a], Daniel LLERENA [b],
Thierry MOYAUX [a]

[a] *LIESP, INSA de Lyon, Bâtiment Blaise Pascal, 69621 Villeurbanne Cedex, France*
[b] *GAEL, INRA/UPMF de Grenoble, BP 47, 38040 Grenoble Cedex 09, France*

March 2009

Abstract

This paper proposes a methodology to study what ordering strategy will be chosen by companies in a supply chain when risk is taken into account. Here, risks are measured as the standard deviation of the customer service and on-hand inventory levels induced by the three considered strategies. We apply this methodology to investigate the conditions under which optimisation-based and stream management-based strategies are preferred. We find that the considered traditional optimisation-based strategy appears more often in Nash equilibria than any of our two stream management-based strategies. To our knowledge, the methodology itself is one of the first to demonstrate how to take several constraints (market demand, and preferences of companies over customer service and inventory levels) into account when choosing an ordering strategy.

Key words: Game theory, Supply chain management, Decision analysis, Risk analysis, Simulation

1 Introduction

Ordering policies describe decisions at operational level about when and how much to order, while the choice of what ordering policy to use is a tactical or, even, strategical, decision. This operational decision on ordering usually assumes the company alone, while every company is embedded in (at least) one supply chain. Consequently, every placed order not only depends on the state (i.e., inventory level, products currently shipped from suppliers, etc.) and ordering policy of the considered company, but also on the ordering policy and state of the other companies in the supply chain. Therefore, the tactical/strategical decision on what ordering strategy to use must take account of both the internal constraints of the considered company and the constraints imposed by the rest of the supply chain in which this company is embedded. Game theory allows taking such various constraints into account while making such tactical/strategical decisions.

This paper proposes a methodology based on game theory in order to address this tactical/strategical decision of what ordering strategy to choose. Application of game theory to supply chains is far from new – see, for instance, [2] – but, to our knowledge, the study of what ordering strategy is better suited to some context has never been addressed in the literature. Actually, we do not even know of any work comparing the efficiency of several ordering strategies, in particular if risks about decisions are taken into account. For this, we use a simple linear Markowitz's formulation [7] in order to describe the utility function of companies.

* Corresponding author: Thierry Moyaux, LIESP, INSA de Lyon.
Email addresses: thierry.moyaux@insa-lyon.fr.

Next, the contribution of this paper is a methodology taking the demand of end customers, and the preferences and ordering strategies of all companies in the supply chain into account. We call preferences both the behaviour toward risk λ, and the service level μ (measured in this paper as backorders) delivered by companies. The goal of the proposed methodology is to find the conditions under which the strategies used by all companies in a supply chain are a Nash equilibrium. This methodology is developed in this paper on a supply chain with a retailer r and its wholesaler w. Technically, we determine the conditions on σ_m (standard deviation of market demand), λ_r and λ_w (attitude toward risk), and μ_r and μ_w (attitude toward service level) under which every configuration of ordering strategies is a Nash equilibrium.

The rest of this paper is organised as follows. Section 2 presents our model of supply chains and the ordering strategies available to the companies. Section 3 summarises the data obtained by numerical experimentations, then Section 4 introduces how these data are used in the utility function of the companies. Next, Section 5 is the core of the paper since it applies game theory to analyse the behaviour of the companies. Finally, Section 6 discusses our methodology, and Section 7 concludes.

2 Model

We shall present our methodology on an example of supply chains modelled as Sterman's Beer Game [8]. We use this model because it is one of the simplest while it has very complex dynamics and our research question has links with the impact of supply chain dynamics (e.g., bullwhip effect) on the choice of an ordering strategy. Beer Game is played by turns, where each turn represents a day, and is played in six steps. These six steps are played in parallel by the two company-players w and r (see Figure 1): (1) advance the shipping delay, (2) fulfil the incoming order, (3) advance the order delay, (4) place an order, (5) add the incoming shipping to inventory, and (6) record inventory.

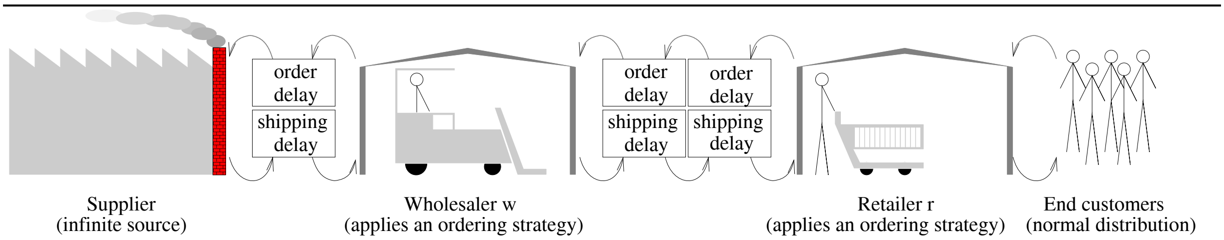


Fig. 1. The short version of the Beer Game considered in this paper

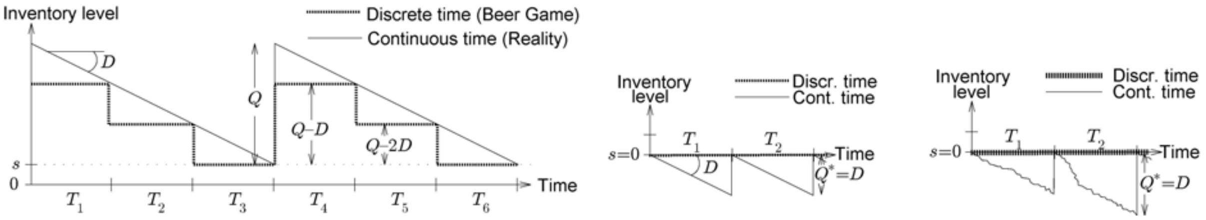
Our model differs from the original one as follows. First, instead of four companies, we only consider a retailer r and its wholesaler w, as shown in Figure 1. We aim to study different topologies of supply chains, but start with the simplest one in order to set up our methodology and the associated tools. The second modification deals with the sharing of demand information which is forbidden to the players in the original Beer Game. Specifically, the players in Sterman's game can see the number of items (represented as tokens) in the other players' inventory, but can see neither the demand addressed by the market to the retailer, nor the demand (orders) placed by every player to their supplier. On the contrary, we will see that two of the three strategies considered in this paper (namely, β and γ) allow the sharing of information on market demand.

Eventually, the market m has a demand represented by a normal distribution of integers with average μ_m=50 and standard deviation σ_m. σ_m is an integer between 1 and 20. Each simulation is run over 365 days. The initial inventory level of both companies is set to zero, which corresponds to the optimal level of Strategy α, as now presented.

We now detail each of the three strategies considered to illustrate our methodology.

2.1 *Strategy α: No information sharing*

Ordering policy α is locally optimal for the company which uses it when (i) the inventory system of this company incurs the classic cyclical behaviour shown by the thin continue line in Figure 2a. Hax and Candea [6] add other assumptions about the typical behaviour of such a cyclical system: (ii) demand is continuous at a constant rate; (iii) the process continues infinitely; (iv) no constraints are imposed (on quantities ordered, storage capacity, available capital, etc.); (v) replenishment is instantaneous (the entire order quantity is received all at once as soon as the order is released); (vi) all costs are time-invariant; (vii) no shortages are allowed; (viii) quantity discounts are not available, and (ix) no ordering costs are considered. The notations in this figure are *s* for the inventory level at which a new order is placed, *Q* for the quantity ordered by the considered company, and *D* for the demand rate, i.e., the number of products demanded by the client per time unit T_d. (We call *d* the day such as T_d is Day *d*.)



(a) General behaviour of the inventory system.

(b) Figure 2a for optimal Q^*=D and s^*=0.

(c) Figure 2c when demand D is stochastic.

Fig. 2. Cycles for an inventory system when the supplier has no backorder and thus ships ordered quantity *Q*

Besides the thin continuous line, Figure 2a also shows a thick dotted line representing the behaviour of the same inventory system when time is discrete. The difference between these two lines reflects the fact that the inventory level in discrete time corresponds to the inventory level at the end of a period in continuous time. For instance, the company considered in Figure 2a receives $s+Q$ products at the beginning of T_4 but pays for holding only $s+Q-D$ products in inventory (cf. the level indicated by the thick line during T_4) because *D* products are shipped to the client in this period. Precisely, $s+Q$ products are in inventory during a part of T_4, but this is ignored because time is discrete, and we only consider the inventory level at the end of T_4. Next, cycle $\{T_1, T_2, T_3\}$ is identical to cycle $\{T_4, T_5, T_6\}$ – cf. assumption (i). The duration of each cycle is Q/D and a 365-day simulation thus has $365*D/Q$ cycles. The inventory level charged on the d^{th} day of a cycle is $(s+Q-dD)$: see $s+Q-D$ in T_4, $s+Q-2D$ in T_5, and $s=s+Q-3D$ in T_6 in Figure 2a. Then, the inventory on the d^{th} day of a cycle costs $h(s+Q-dD)$, where *h* is the inventory carrying cost per item per day. Finally, the annual total cost *TC* of the inventory system is:

$$TC = 365\frac{D}{Q}\sum_{t=1}^{Q/D} h(s+Q-tD) = 365\frac{D}{Q}\left[h(s+Q)\frac{Q}{D} - hD\frac{\frac{Q}{D}\left(\frac{Q}{D}+1\right)}{2}\right] = 365h(s+\frac{Q-D}{2}) \quad (1)$$

There are two decision variables in Equation 1, viz., *s* and *Q*. Clearly, *TC*=0 when $Q^*=D$ (the company orders what is ordered by its client) and $s^*=0$ (no safety stocks). An order is placed whenever the inventory level becomes negative. Such behaviour of the inventory level is shown in Figure 2(b). In this figure, the inventory in continuous time has backorders all the time (thin continuous line) except when a shipping is received, but this is ignored by discrete time (thick dotted line). To be precise, only the inventory level at the end of a period is taken into account, and this level is equal to zero because the inventory receives the quantity $Q=D$ corresponding to backorders just before recording the inventory level.

Since we cannot obtain a lower annual cost than *TC*=0, we do not try to relax assumption (vii) by checking if companies should sometimes incur backorders in order to save money. Of course, we may also try to relax other assumptions. For instance, relaxing assumption (ii) would allow taking the stochastic demand in our simulation into account, as presented in Figure 2(c). In this figure, assumption (v) still applies, i.e., replenishment is instantaneous. Relaxing assumption (v) as well would slightly increase the safety stock by setting $s>0$ in

Figure 2c. However, we think that taking the (ordering and shipping) lead times into account in order to calculate precisely the value of s depending on μ_m is not as effective as relaxing assumption (i). In fact, the main reason for which the theoretical α presented in this subsection does not follow the simulated α is due to the fact that assumption (i) does not hold. We mean that the inefficiency of retailer's α is due to the backorders of the wholesaler, but these backorders are not modelled in Figures 2. For instance, the wholesaler may not ship Q items as expected in these three figures, but, for instance, only $Q/2$ now, then $Q/2$ later on. As a consequence, the inventory system of the retailer does not always behaves cyclically as assumes by hypothesis (i).

Finally, if we call O^{i-1}_{d-2} the order O placed by client i-1 two days earlier and arriving by the considered company i on day d, $InvPos^{i}_{d}$ the inventory position (defined as the sum of on-hand inventory plus on-order products minus backordered orders), then we can formally describe the order O^{i}_{d} placed by the considered company i using Strategy α as:

$$O_d^i = \begin{cases} 0 & \text{if } InvPos_d^i > 0, \\ O_{d-2}^{i-1} - InvPos_d^i & \text{if } InvPos_d^i \le 0. \end{cases} \tag{2}$$

As we can see, Strategy α is based on the optimisation of the operation of the inventory system. We now present Strategies β and γ which are based on another paradigm, i.e., stream management. [3, 5]

2.2 *Strategy β: Point-to-point information sharing*

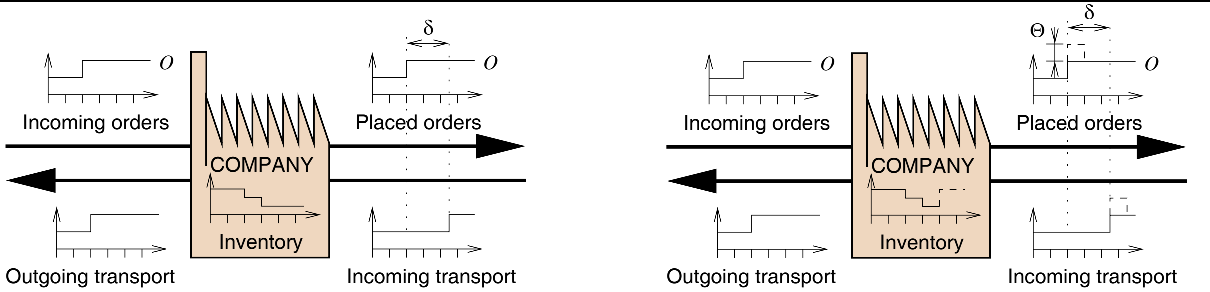
Figure 3 describes how (O, θ) orders were designed in (blinded for review) [9] as a way to manage the flows of products and orders, conversely to α which only uses O (i.e., θ=0 in α). For that purpose, Strategy β replaces the single number of traditional orders by a vector (O, θ) in order to share demand information. Technically, orders are vectors of the two components O and θ in order to carry out information sharing. This sharing of information is obtained by having companies apply the lot-for-lot ordering policy to choose the part O of an order, so that retailers transmit the market demand to their wholesalers in O, then, these wholesalers also transmit the demand information to their suppliers in O, etc. Shortly, O (in β and γ, not in α) contains the demand information at the point of sale.

Applying the lot-for-lot policy for O not only performs information sharing in β (and γ), but also prevents the bullwhip effect, as illustrated by Figure 3a: incoming orders are as variable as placed orders. Unfortunately, this does not manage inventories well, since the inventory level in Figure 3a decreases as demand increases because outgoing transport is larger than incoming transport during a period of time corresponding to (ordering and shipping) lead times. This is the reason for the part θ of orders in Figure 4b. θ is used by companies to order more or less products than the quantity ordered in O. In other words, both O and θ are orders of equal importance, and a company receiving an order (O, θ) has to ship the quantity $O+\theta$ to its client. Next, the method to choose θ was designed in order to avoid the bullwhip effect to appear in θ by following the principle that non-zero θs are possible only when market demand changes, i.e., $\frac{dO}{dt} = 0 \Rightarrow \theta \neq 0$ and $\frac{dO}{dt} \neq 0 \Rightarrow \theta = 0$.

If we use the notations in Equation 2, call λ=4 a constant defined as the sum of the durations of ordering and shipping delays, and add θ^{i-1}_{d-2} as the order θ arriving on the current day d and placed two days earlier by client i-1, then β can be described as:

$$\left(O_d^i, \theta_d^i\right) = \left(O_{d-2}^{i-1}, \theta_{d-2}^{i-1} + \lambda\left(O_{d-2}^{i-1} - O_{d-3}^{i-1}\right)\right) \tag{3}$$

In real life, all companies are not always connected by the same information system, and it may therefore be necessary for companies to carry out some action in order to transmit information from their client to their supplier. This is the slow information sharing just presented in β. On the contrary, information may travel instantaneously, which is implemented by γ, as now described.



(a) The transmission of end customer demand by the *O* part of (*O*, *θ*) orders.

(b) The management of inventory level by the *θ* part of (*O*, *θ*) orders.

Fig. 3. Information shared with (*O*, *θ*) orders in Strategy β (blinded for review)

2.3 *Strategy γ: Information centralisation*

Shortly, we may say that "γ = β + information centralisation." More precisely, γ is very similar to β, except that information centralisation (defined as the sharing in real-time of market consumption by the retailer to the rest of the supply chain) is added. The difference between sharing demand information with β and γ is the celerity of this transmission, which is as slow as orders with β, or instantaneous with γ. Besides, γ uses (*O*, *θ*) orders as β, but introduces a decoupling between the quantity to ship to the client and the quantity to consider to place orders to the supplier. Specifically, the quantity to ship corresponds to the quantity *O*+*θ* previously ordered in an (*O*, *θ*) order by the client, while the quantity to order corresponds to the market demand on the current day *d*.

It is worth noticing that information centralisation is only carried out when both companies use γ, that is, when the retailer agrees to transmit its sales in real-time (rather than delayed in time by two days, as done with β) and the wholesaler uses this information by also using γ. Otherwise γ operates with the same delay as β.

With the notations in Equations 2 and 3, except λ=2 because information centralisation removes the ordering lead time, and calling O^{m}_{d-2} the demand arriving at the retailer from market m on day *d*, γ may be written as:

$$\left(O^i_d, \theta^i_d\right) = \left(O^{\mathrm{m}}_{d-2}, \theta^{i-1}_{d-2} + \lambda\left(O^{m}_{d-2} - O^{\mathrm{m}}_{d-3}\right)\right) \qquad (4)$$

for retailer i=r, as well as for the wholesaler i=w when the retailer also uses γ, otherwise w uses Equation 2.

3 Simulation outcomes

After that, we run the 3^2=9 combinations of these 3 strategies among our 2 companies. We may also call "joint strategy" or "configurations" each of these 9 combinations, e.g., (α, γ). Every joint strategy is run 15,000 times in order to measure the average μ and the standard deviation σ of the inventory i and of the backorder b. For instance, $\mu_{i,r}(\alpha, \gamma)$ is the average over 15,000 simulations of the inventory level of the retailer when this company uses Strategy α while the wholesaler uses γ. Then, each of the 9*15,000 simulations is repeated under 20 different values of the standard deviation σ_m of market demand, so that $\mu_{i,r}(\alpha, \gamma)$ is also a function of σ_m. Two weeks (precisely, 21,133 minutes) were necessary to compute all the 20*9*15,000 simulations analysed in this paper on a 3 GHz Pentium 4. Finally, each of the four measures (i.e., μ_i, σ_i, μ_b, σ_b) per company and per joint strategy is approximated by a polynomial function dependent on σ_m. The rest of this paper only considers these approximations. Please notice that R^2 denotes the determination coefficient, which should be as close to 1 as possible in order to have a good approximation of the simulation outputs in the following functions:

$$\mu_{i,r}(\alpha, \alpha, \sigma_m) = -0.0016\sigma_m^3 + 0.0629\sigma_m^2 - 0.1395\sigma_m + 32.903 \qquad (R^2 = 0.9939) \qquad (5)$$
$$\sigma_{i,r}(\alpha, \alpha, \sigma_m) = -0.0023\sigma_m^3 + 0.1136\sigma_m^2 - 0.6157\sigma_m + 23.565 \qquad (R^2 = 0.9999) \qquad (6)$$
$$\mu_{i,w}(\alpha, \alpha, \sigma_m) = -8*10^{-5}\sigma_m^3 + 0.0586\sigma_m^2 - 0.0885\sigma_m + 61.915 \qquad (R^2 = 0.9987) \qquad (7)$$
$$\sigma_{i,w}(\alpha, \alpha, \sigma_m) = -0.0017\sigma_m^3 + 0.1192\sigma_m^2 - 0.5127\sigma_m + 43.833 \qquad (R^2 = 0.9996) \qquad (8)$$

$$\mu_{b,r}(\alpha, \alpha, \sigma_m) = 0.0003\sigma_m^3 + 0.0017\sigma_m^2 + 0.1488\sigma_m - 0.0052 \qquad (R^2 = 0.9999) \qquad (9)$$
$$\sigma_{b,r}(\alpha, \alpha, \sigma_m) = 0.0003\sigma_m^3 + 0.0058\sigma_m^2 + 0.597\sigma_m - 0.0081 \qquad (R^2 = 1) \qquad (10)$$
$$\mu_{b,w}(\alpha, \alpha, \sigma_m) = 0.0004\sigma_m^3 - 0.0064\sigma_m^2 + 0.218\sigma_m + 1.2723 \qquad (R^2 = 0.9986) \qquad (11)$$
$$\sigma_{b,w}(\alpha, \alpha, \sigma_m) = -0.0002\sigma_m^3 + 0.0244\sigma_m^2 + 0.2082\sigma_m + 7.1487 \qquad (R^2 = 0.9999) \qquad (12)$$

$$\mu_{i,r}(\alpha, \beta, \sigma_m) = -0.0007\sigma_m^3 + 0.0377\sigma_m^2 + 0.1975\sigma_m + 32.97 \qquad (R^2 = 0.9982) \qquad (13)$$
$$\sigma_{i,r}(\alpha, \beta, \sigma_m) = -0.0021\sigma_m^3 + 0.1081\sigma_m^2 - 0.5088\sigma_m + 23.503 \qquad (R^2 = 0.9999) \qquad (14)$$
$$\mu_{i,w}(\alpha, \beta, \sigma_m) = -0.0052\sigma_m^3 + 0.5133\sigma_m^2 - 4.5834\sigma_m + 152.81 \qquad (R^2 = 0.9969) \qquad (15)$$
$$\sigma_{i,w}(\alpha, \beta, \sigma_m) = -0.0021\sigma_m^3 + 0.1674\sigma_m^2 + 0.0289\sigma_m + 81.699 \qquad (R^2 = 0.9993) \qquad (16)$$
$$\mu_{b,r}(\alpha, \beta, \sigma_m) = 0.0002\sigma_m^3 + 0.0004\sigma_m^2 + 0.1192\sigma_m + 1.0895 \qquad (R^2 = 0.9997) \qquad (17)$$
$$\sigma_{b,r}(\alpha, \beta, \sigma_m) = 0.0002\sigma_m^3 + 0.0107\sigma_m^2 - 0.0175\sigma_m + 11.446 \qquad (R^2 = 0.9999) \qquad (18)$$
$$\mu_{b,w}(\alpha, \beta, \sigma_m) = -5*10^{-6}\sigma_m^5 + 0.0003\sigma_m^4 - 0.0071\sigma_m^3 + 0.0819\sigma_m^2 - 0.4096\sigma_m + 2.9774 \qquad (R^2 = 0.9751) \qquad (19)$$
$$\sigma_{b,w}(\alpha, \beta, \sigma_m) = -0.0006\sigma_m^3 + 0.0203\sigma_m^2 - 0.1692\sigma_m + 18.441 \qquad (R^2 = 0.9785) \qquad (20)$$

$$\mu_{i,r}(\beta, \alpha, \sigma_m) = 0.0008\sigma_m^3 + 0.0327\sigma_m^2 + 1.0089\sigma_m + 0.891 \qquad (R^2 = 0.9999) \qquad (21)$$
$$\sigma_{i,r}(\beta, \alpha, \sigma_m) = 0.0004\sigma_m^3 + 0.0042\sigma_m^2 + 2.0883\sigma_m + 0.6849 \qquad (R^2 = 1) \qquad (22)$$
$$\mu_{i,w}(\beta, \alpha, \sigma_m) = -0.0017\sigma_m^3 + 0.0406\sigma_m^2 + 4.9204\sigma_m + 33.093 \qquad (R^2 = 0.9999) \qquad (23)$$
$$\sigma_{i,w}(\beta, \alpha, \sigma_m) = -0.0033\sigma_m^3 + 0.0807\sigma_m^2 + 4.6594\sigma_m + 21.93 \qquad (R^2 = 0.9998) \qquad (24)$$
$$\mu_{b,r}(\beta, \alpha, \sigma_m) = 0.0033\sigma_m^3 - 0.168\sigma_m^2 + 2.7501\sigma_m + 1.3075 \qquad (R^2 = 0.9993) \qquad (25)$$
$$\sigma_{b,r}(\beta, \alpha, \sigma_m) = -0.0009\sigma_m^3 + 0.0076\sigma_m^2 + 1.0138\sigma_m + 14.041 \qquad (R^2 = 0.999) \qquad (26)$$
$$\mu_{b,w}(\beta, \alpha, \sigma_m) = -0.0001\sigma_m^3 - 0.0007\sigma_m^2 + 0.7753\sigma_m + 0.3243 \qquad (R^2 = 0.9998) \qquad (27)$$
$$\sigma_{b,w}(\beta, \alpha, \sigma_m) = -0.0024\sigma_m^3 + 0.0914\sigma_m^2 + 0.589\sigma_m + 9.4998 \qquad (R^2 = 0.9997) \qquad (28)$$

$$\mu_{i,r}(\beta, \beta, \sigma_m) = 0.0002\sigma_m^3 + 0.0726\sigma_m^2 + 0.2618\sigma_m + 0.4934 \qquad (R^2 = 1) \qquad (29)$$
$$\sigma_{i,r}(\beta, \beta, \sigma_m) = -0.001\sigma_m^3 + 0.0699\sigma_m^2 + 1.2319\sigma_m + 0.4362 \qquad (R^2 = 1) \qquad (30)$$
$$\mu_{i,w}(\beta, \beta, \sigma_m) = -0.0005\sigma_m^3 + 0.0678\sigma_m^2 + 3.9635\sigma_m + 0.5934 \qquad (R^2 = 1) \qquad (31)$$
$$\sigma_{i,w}(\beta, \beta, \sigma_m) = -0.001\sigma_m^3 + 0.0117\sigma_m^2 + 5.2792\sigma_m + 0.4969 \qquad (R^2 = 1) \qquad (32)$$
$$\mu_{b,r}(\beta, \beta, \sigma_m) = 0.0089\sigma_m^3 - 0.3878\sigma_m^2 + 5.0044\sigma_m + 1.3853 \qquad (R^2 = 0.9894) \qquad (33)$$
$$\sigma_{b,r}(\beta, \beta, \sigma_m) = 0.0056\sigma_m^3 - 0.2649\sigma_m^2 + 4.5621\sigma_m + 2.1428 \qquad (R^2 = 0.9982) \qquad (34)$$
$$\mu_{b,w}(\beta, \beta, \sigma_m) = 0.006\sigma_m^3 - 0.2585\sigma_m^2 + 3.7752\sigma_m + 1.9329 \qquad (R^2 = 0.9923) \qquad (35)$$
$$\sigma_{b,w}(\beta, \beta, \sigma_m) = 0.0051\sigma_m^3 - 0.2399\sigma_m^2 + 5.082\sigma_m + 1.9993 \qquad (R^2 = 0.9994) \qquad (36)$$

$$\mu_{i,r}(\gamma, \gamma, \sigma_m) = 0.0011\sigma_m^3 + 0.039\sigma_m^2 + 0.2332\sigma_m + 0.463 \qquad (R^2 = 0{,}9999) \qquad (37)$$
$$\sigma_{i,r}(\gamma, \gamma, \sigma_m) = -3*10^{-5}\sigma_m^3 + 0.049\sigma_m^2 + 0.9621\sigma_m + 0.5393 \qquad (R^2 = 0.9999) \qquad (38)$$
$$\mu_{i,w}(\gamma, \gamma, \sigma_m) = 0.0065\sigma_m^3 - 0.308\sigma_m^2 + 4.5468\sigma_m - 0.3971 \qquad (R^2 = 0.9984) \qquad (39)$$
$$\sigma_{i,w}(\gamma, \gamma, \sigma_m) = 0.003\sigma_m^3 - 0.1664\sigma_m^2 + 3.6395\sigma_m + 0.6878 \qquad (R^2 = 0.9995) \qquad (40)$$
$$\mu_{b,r}(\gamma, \gamma, \sigma_m) = 0.0017\sigma_m^3 - 0.0668\sigma_m^2 + 3.0595\sigma_m + 0.3759 \qquad (R^2 = 0.9999) \qquad (41)$$
$$\sigma_{b,r}(\gamma, \gamma, \sigma_m) = 0.0015\sigma_m^3 - 0.0982\sigma_m^2 + 4.2008\sigma_m + 0.0615 \qquad (R^2 = 1) \qquad (42)$$
$$\mu_{b,w}(\gamma, \gamma, \sigma_m) = 0.0027\sigma_m^3 - 0.1436\sigma_m^2 + 3.3396\sigma_m + 0.1745 \qquad (R^2 = 0.9997) \qquad (43)$$
$$\sigma_{b,w}(\gamma, \gamma, \sigma_m) = 0.0019\sigma_m^3 - 0.1127\sigma_m^2 + 4.1305\sigma_m + 0.3828 \qquad (R^2 = 0.9999) \qquad (44)$$

Notice that the 5 joint strategies covered by these 40 equations are enough to describe all 9 joint strategies, because some configurations induce the same behaviour of the supply chain due to the similarity of Strategies β and γ. In fact, we saw above that information centralisation is achieved only when both r and w use γ at the same time, otherwise γ operates as β. As a consequence, configurations (β, α) and (γ, α) induce the same behaviour of the supply chain because r's γ operates as β: the (O, θ) orders sent by r in both (β, α) and (γ, α) are processed in the same way by w (i.e., the demand information broadcast by r in (β, α) is ignored by w's α). Similarly, (α, β) and (α, γ) also induce the same behaviour of the supply chain because w's γ operates as β: this is the opposite to the previous situation, that is, w's β or γ listens to the information broadcast by information sharing, but r's α does not transmit this information. Finally, (β, β), (β, γ) and (γ, β) make up a third set of similar joint strategies in which γ operates as β, because γ implements information centralisation only when both companies use this same strategy. This explains why, for example, no results about joint strategies (β, γ) and (γ, β) are shown in Equations 5-44; they are the same as for (β, γ), e.g., $\mu_{i,r}(\beta, \beta, \sigma_m)=\mu_{i,r}(\beta, \gamma, \sigma_m)=\mu_{i,r}(\gamma, \beta, \sigma_m)$.

4 Utility function and game

We next apply Markowitz's mean-variance model [7] to build utility functions u_r and u_w from the simulation data summarised in Equations 5-44. Shortly, Equation 45 shows our definition of the utility function u_c of Company c ($c \in \{r, w\}$) when retailer r plays s_r, wholesaler w plays s_w and the standard deviation of market demand is σ_m:

$$u_c(s_r, s_w, \sigma_m) = -[\mu_{i,c}(s_r, s_w, \sigma_m) + \varepsilon_c.\mu_{b,c}(s_r, s_w, \sigma_m)] + \lambda_c.[\sigma_{i,c}(s_r, s_w, \sigma_m) + \varepsilon_k.\sigma_{b,c}(s_r, s_w, \sigma_m)] \quad (45)$$
with $\varepsilon_c \geq 0$ and $\lambda_c \in [-2; 2]$

In this definition, $\mu_c(s_r, s_w, \sigma_m)$, $\mu_{b,c}(s_r, s_w, \sigma_m)$, $\sigma_{i,c}(s_r, s_w, \sigma_m)$ and $\sigma_{b,c}(s_r, s_w, \sigma_m)$ represent the simulation outputs summarised in Equations 5-44. Equation 45 contains two additional parameters, viz., ε_c and λ_c:

- ε_c models the importance given by Company c to backorders: $\varepsilon_c=0$ means that c ignores its backorders while a more or less large positive value of ε_c models a company c which takes more or less customer service into account. Specifically, some markets or industries do not pay much attention in serving clients as soon as possible (small ε_c), while others force company c to prefer high inventory levels rather than backorders, thus the high price given to backorders when ε_c is high. Indeed, Equation 45 implies $u_c \leq 0$ when $\lambda_c=0$ because $\mu_{i,c} \geq 0$ and $\mu_{b,c} \geq 0$, which means that u_c only contains costs without profits (i.e., $\mu_{i,c}$ is the inventory holding cost and $\mu_{i,c}$ the backorder cost).

- λ_c represents the importance given to risk by Company c according to a simple linear Markowitz's formulation [7]. In fact, $\lambda_c=0$ means that agent c is risk neutral, that is, c assumes its payoff to be the average level of inventory ($\mu_{i,c}$) and backorder levels ($\mu_{b,c}$). Nevertheless, the simulation does not always obtain these averages, as shown by $\sigma_{i,c} \neq 0$ and $\sigma_{b,c} \neq 0$ in Equations 5-44. A risk loving company c may think it will get a payoff higher than the average value obtained by simulations (thus, $\lambda_c>0$), while a risk averse company c' may fear to get a payoff lower than this average value (thus, $\lambda_{c'}<0$). Finally, $|\lambda_c| \leq 2$ because the probability to get an actual payoff greater than $\mu+2\sigma$ or lower than $\mu-2\sigma$ is quite low.

Figure 4 shows how these two functions u_r and u_w may be used to build a game in the normal form. In this figure, we can read that joint strategy (α, β) corresponds to a situation in which retailer r plays α and wholesaler w chooses β, and r incurs payoff $u_r(\alpha, \beta, \sigma_m)$ while w gets $u_w(\alpha, \beta, \sigma_m)$ in this configuration. We may notice that these two payoffs also depend on the standard deviation of market demand σ_m. It is also important to note that the relationship between β and γ gives the particular structure of the game in Figure 4. In fact, we have just seen that, for instance, (β, β), (β, γ) and (γ, β) are simulated in the same way, which explains why entries (β, β), (β, γ) and (γ, β) in Figure 4 all contain payoffs $u_r(\beta, \beta, \sigma_m)$ and $u_w(\beta, \beta, \sigma_m)$, that is, $u_r(\beta, \beta, \sigma_m) = u_r(\beta, \gamma, \sigma_m) = u_r(\gamma, \beta, \sigma_m)$ and $u_r(\beta, \beta, \sigma_m) = u_w(\beta, \gamma, \sigma_m) = u_w(\gamma, \beta, \sigma_m)$.

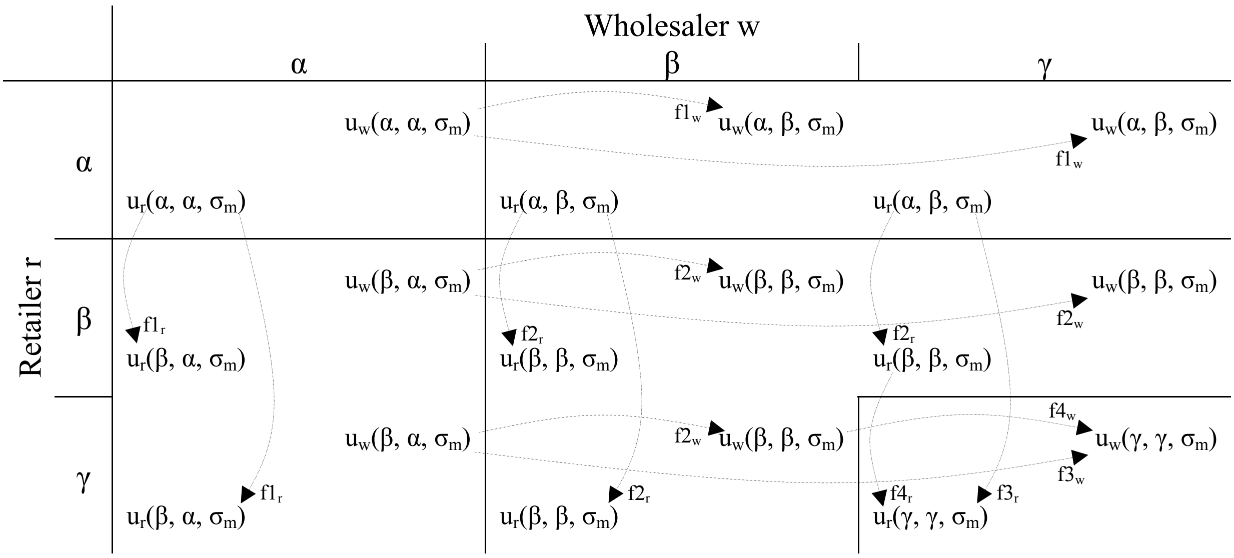


Fig. 4. The game analysed in this paper with the eight functions characterising all its Nash equilibriums

5 Analysis of the simulation outcomes

Figure 4 also contains arrows named by a function as $f1_r$, $f1_w$ or $f2_r$. These functions describe the comparisons to carry out in order to find the Nash equilibria in Figure 4. To be precise, every entry in Figure 4 may be an equilibrium depending on the value of ε_r, ε_w, λ_r, λ_w and σ_m. The idea to find the Nash equilibria is to assume that Company c plays its best response strategy s_c, then to calculate the conditions under which $s_{\hat{c}} \in \{\alpha, \beta, \gamma\}$ is a best response of the other company ĉ to s_c. Let us illustrate this method by assuming that the wholesaler (thus, c=w in the previous sentence) plays its best response $s_c=\alpha$. Then, we may define function $f1_r=u_r(\alpha, \alpha, \sigma_m)-u_r(\beta, \alpha, \sigma_m)$, so that α is the best response of the retailer when $f1_r \geq 0$, and β and γ are the best responses of the retailer when $f1_r \leq 0$. In fact, according to Figure 4, if the wholesaler uses α, then the retailer gets a higher payoff by also using α when $f1_r \geq 0$, and the retailer gets the same payoff by playing β or γ. We now solve $f1_r \geq 0$ by putting ε_r on the left hand side of the equation. For that purpose, we first rewrite $f1_r$:

$$f1_r = -[\mu_{i,r}(\alpha, \alpha, \sigma_m) + \varepsilon_r.\mu_{b,r}(\alpha, \alpha, \sigma_m)] + \lambda_r.[\sigma_{i,r}(\alpha, \alpha, \sigma_m) + \varepsilon_r.\sigma_{b,r}(\alpha, \alpha, \sigma_m)]$$
$$+ [\mu_{i,r}(\beta, \alpha, \sigma_m) + \varepsilon_r.\mu_{b,r}(\beta, \alpha, \sigma_m)] - \lambda_r.[\sigma_{i,r}(\beta, \alpha, \sigma_m) + \varepsilon_r.\sigma_{b,r}(\beta, \alpha, \sigma_m)]$$

Summarising Equations 5-44 by $\mu_{i,r}(s_r, s_w, \sigma_m)=\sum_j \mu_{j,i,r}(s_r, s_w, \sigma_m).\sigma_m^j$ and $\sigma_{i,r}(s_r, s_w, \sigma_m)=\sum_j \sigma_{j,i,r}(s_r, s_w, \sigma_m).\sigma_m^j$, we obtain:

$$f1_r = \sum_j [-\mu_{j,i,r}(\alpha, \alpha, \sigma_m) - \varepsilon_r.\mu_{b,r}(\alpha, \alpha, \sigma_m) + \lambda_r.\sigma_{i,r}(\alpha, \alpha, \sigma_m) + \lambda_r.\varepsilon_r.\sigma_{b,r}(\alpha, \alpha, \sigma_m)$$
$$+ \mu_{j,i,r}(\beta, \alpha, \sigma_m) + \varepsilon_r.\mu_{j,b,r}(\beta, \alpha, \sigma_m) - \lambda_r.\sigma_{j,i,r}(\beta, \alpha, \sigma_m) - \lambda_r.\varepsilon_r.\sigma_{j,b,r}(\beta, \alpha, \sigma_m)].\sigma_m^j$$

Therefore:

$$f1_r \geq 0 \iff \sum_j [-\varepsilon_r.\mu_{j,b,r}(\alpha, \alpha, \sigma_m) + \lambda_r.\varepsilon_r.\sigma_{j,b,r}(\alpha, \alpha, \sigma_m) + \varepsilon_r.\mu_{j,b,r}(\beta, \alpha, \sigma_m) - \lambda_r.\varepsilon_r.\sigma_{j,b,r}(\beta, \alpha, \sigma_m)].\sigma_m^j \geq \sum_j [\mu_{j,i,r}(\alpha, \alpha, \sigma_m) - \lambda_r.\sigma_{j,i,r}(\alpha, \alpha, \sigma_m) - \mu_{j,i,r}(\beta, \alpha, \sigma_m) + \lambda_r.\sigma_{j,i,r}(\beta, \alpha, \sigma_m)].\sigma_m^j$$

$$\iff \varepsilon_r \geq \frac{\sum_j \left[\lambda_r.(\sigma_{j,i,r}(\beta,\alpha,\sigma_m) - \sigma_{j,i,r}(\alpha,\alpha,\sigma_m)) + \mu_{j,i,r}(\alpha,\alpha,\sigma_m) - \mu_{j,i,r}(\beta,\alpha,\sigma_m)\right]\sigma_m^j}{\sum_j \left[\lambda_r.(\sigma_{j,b,r}(\alpha,\alpha,\sigma_m) - \sigma_{j,b,r}(\beta,\alpha,\sigma_m)) + \mu_{j,b,r}(\beta,\alpha,\sigma_m) - \mu_{j,i,r}(\alpha,\alpha,\sigma_m)\right]\sigma_m^j}$$

Next, replacing polynomial coefficients μ_j and σ_j by their value in equations 5-44, e.g., $\mu_{2,i,r}(\beta, \alpha, \sigma_m)=0.0327$ in Equation 21, we find:

$$\iff \varepsilon_r \geq \frac{(0.0027\lambda_r - 0.0024)\sigma_m^3 + (-0.1094\lambda_r + 0.0302)\sigma_m^2 + (-2.704\lambda_r - 1.1484)\sigma_m + (-22.8801\lambda_r + 32.012)}{(-0.0006\lambda_r + 0.003)\sigma_m^3 + (0.0134\lambda_r - 0.1697)\sigma_m^2 + (1.6108\lambda_r + 2.6013)\sigma_m + (14.0329\lambda_r + 1.3127)} \quad (46)$$

Inequality 46 states the conditions on ε_r, λ_r, and σ_m for which α is a best response of the retailer when α is the best response of the wholesaler. In other words, if Inequality 46 holds while playing α is also a best response of the wholesaler (i.e., $f1_w \geq 0$), then (α, α) is a Nash equilibrium, otherwise (β, α) and (γ, α) are equilibria.

Similarly, we may repeat this reasoning for every entry in Figure 4 in order to find when each corresponding configuration is a Nash equilibrium. As previously said, the arrows in Figure 4 show all the comparisons to carry out. In particular, (γ, γ) is an equilibrium when $f3_r=u_r(\alpha, \beta, \sigma_m)-u_r(\gamma, \gamma, \sigma_m) \leq 0$, $f4_r=u_r(\beta, \beta, \sigma_m)-u_w(\gamma, \gamma, \sigma_m) \leq 0$, $f3_w=u_w(\beta, \alpha, \sigma_m)-u_w(\gamma, \gamma, \sigma_m) \leq 0$ and $f4_w=u_w(\beta, \beta, \sigma_m)-u_w(\gamma, \gamma, \sigma_m) \leq 0$. This process provides us with sets of inequalities similar to Inequality 46.

These sets of inequalities are relations between three variables, viz., either ε_r, λ_r and σ_m (e.g., in Inequality 46) or ε_w, λ_w and σ_m. Figure 5 shows the areas described by these sets of inequalities for $\sigma_m=5$. Since each set of inequalities describes the conditions under which a joint strategy is a best response, the areas in Figure 5 enumerates the Nash equilibria when every company is in its coloured area. Let us take an example to explain how to read Figure 5. In this example, retailer r has *preferences* $\lambda_r=-1$ (moderately risk averse) and $\varepsilon_r=4$ (moderately dislike backorders), while the preferences of wholesaler w are represented by $\lambda_w=1$ (moderately risk lover) and $\varepsilon_w=1$ (slightly dislike backorders). These two instances of companies are represented by letters "r" and "w" in each of the 9 graphs in Figure 5.

This representation makes Nash equilibria obvious, that is, a configuration is an equilibrium if and only if both players are in their area of best reply at the same time:

- both (α, β) and (α, γ) are equilibria because r and w are in their respective area of best reply;
- (α, α) is not an equilibrium because r is in its area of best reply, but not w;
- (β, β), (β, γ) and (γ, β) are not equilibria because r is not in its area of best reply (conversely to w);
- (β, α), (γ, α) and (γ, γ) are not equilibria because neither r nor w is in its area of best reply.

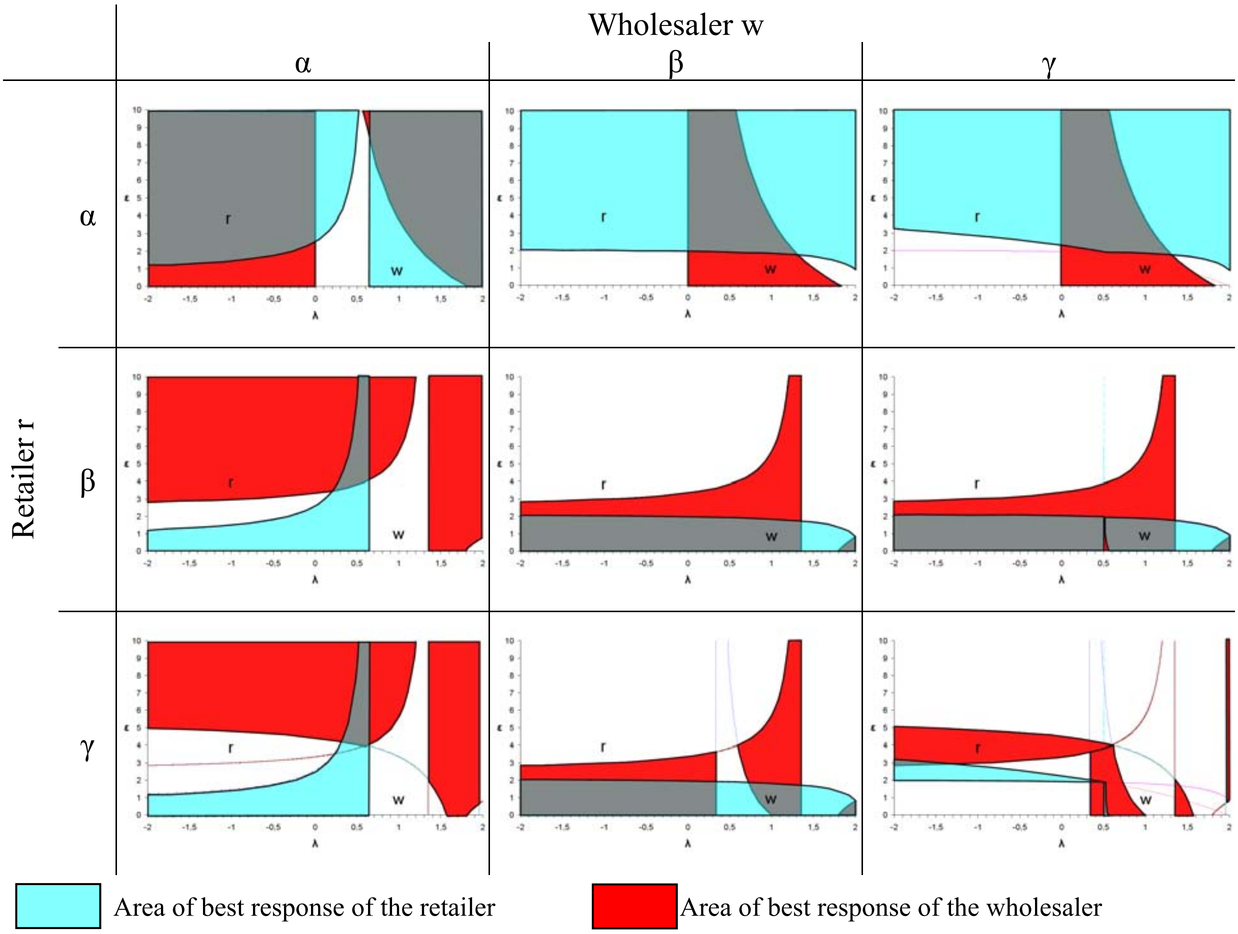


Fig. 5. Areas of best response of both companies in every joint strategy for σ_m=5. In each of the 9 configurations,
"r" denotes an example of retailer with preferences λ_r=-1 and ε_r=4, and "w" a wholesaler with λ_w=1 and ε_w=1

More generally, the observation of Figure 5 seems to indicate that the use of (α, α) – thus, no sharing of demand information – may occur the more often because the areas of best reply of r and w both have an area larger in this configuration than in any of the other 8 configurations in Figure 5. This result about α, β and γ does not imply that optimisation-based strategies such as α are always better than stream management-based strategies such as β and γ. The first reason for such an assertion is that Nash equilibria are not supposed to be efficient in any way, but only to be stable. This is what the famous Prisoner's Dilemma demonstrates – the best configuration for both players is not equilibrium. The other reason for the previous assertion is that we only compared the three instances of ordering strategies α, β and γ. Specifically, our methodology allows covering a large space of preferences (λ and ε) of the companies very easily, but enlarging the space of ordering strategies quickly increases the complexity of computing Nash equilibria with our method (graphically, the number of comparisons – represented by arrows in Figure 4 – increases as the square of the number of strategies).

6 Discussion

The goal of this paper is to propose a methodology to compare several replenishment strategies, such as α, β and γ. Our methodology should improve the understanding about ordering strategies in supply chains, e.g., regarding customer service levels, behaviour against market demand, risk about on-hand inventory levels, etc. In fact, most studies focus on the efficiency of an ordering strategy for the entire supply chain – see textbooks as [1] and [6]. On the contrary, our methodology focuses on the benefits for individual companies by relying on game theory.

Our methodology aims to be as generic as possible. For instance, Figure 5 may easily be obtained for other values of the standard deviation of market demand σ_m, since this figure is obtained by evaluating inequalities similar to Inequality 46 at a specific value of σ_m. Next, the average value of market demand μ_m=50 is less important than σ_m, since the results in this paper would linearly scale as μ_m, while Inequality 46 shows that qualitative changes would occur for other values of σ_m. Consequently, the results presented in the previous section may easily be extended to any other kind of normal distribution of market demand.

The main difficulty we see with our methodology is the use of Nash equilibrium. In fact, increasing the number of companies and/or strategies quickly makes our methodology intractable. Actually, computing Nash equilibria, and even the complexity of this task, is still an issue under investigation. [4]

Beside, we expected to obtain continuous areas of best response in Figure 5 while some are not at all, e.g., for the wholesaler in configuration (γ, γ) – its area is made up of three separated areas. This implies that the sensitivity of some equilibria may be difficult to assess, since a little change in λ or ε may suddenly change the equilibria. In fact, a company in the middle of the area representing its best response is less likely to “hesitate” about the strategy to use than a company on the edge of this area.

7 Conclusion

This paper proposes a methodology based on game theory to explore the conditions under which an ordering strategy can be used in a supply chain. These conditions include market demand, and attitudes toward service level (here, backorders) and risk about the level of on-hand inventory. This methodology is applied to a supply chain with two members who may use one of three strategies, one strategy based on traditional concepts from optimisation [1, 6], and two strategies implementing recent concepts from stream management [3, 5].

Some similarities between the two considered stream management-based strategies lead to a game with a particular structure, i.e., several joint strategies with the same payoffs. Nevertheless, this particular structure is not reflected by the Nash equilibria of this game. In addition, our experimental results imply that our optimisation-based strategy is used more often than our stream management-based ones. These latter strategies are not only based on stream management, but also imply the sharing of demand information. Our experimental results also show that quick information sharing (with information centralisation) is preferred less often than slow information sharing (with point-to-point transmission).

As future work, we plan to apply our methodology to models of supply chains including transportation and ordering costs to our Beer Game, then to adapt decision making accordingly. We also would like to investigate how the results obtained with Markowitz’s analysis would evolve with another criterion.

8 References

[1] Sven Axsäter. Inventory Control. 2nd edition. Springer, New York, NY, USA, 2006.
[2] Gérard P. Cachon, Martin A. Lariviere. Contracting to assure supply: How to share demand forecasts in a supply chain. Management Science. 47(5). 629-46, 2001.
[3] Carlos F. Daganzo. A theory of supply chains. Lecture Notes in economics and mathematical systems 526. Springer Verlag. Berlin, Heidelberg, New York. 2003.
[4] Constantinos Daskalakis, Paul W. Goldberg, Christos H. Papadimitriou. The complexity of computing a Nash equilibrium. Proc. 38th annual ACM symp. on Theory of computing table of contents. P. 71-78. Seattle, WA, USA, 2006.
[5] Jeroen Dejonckheere, S. M. Disney, M. R. Lambrecht, D. R. Towill (2003). Measuring and avoiding the bullwhip effect: A control theoretic approach. European Journal of Operational Research, 147:567–590.
[6] Arnoldo C. Hax, Dan Candea. Production and Inventory Management. Prentice-Hall, New York, NY, USA, 1984.
[7] Harry M. Markowitz. Portfolio Selection: Efficient Diversification of Investments. John Wiley & Sons, NJ, USA, 1959. (Reprinted in 1970 by Yale University Press.)
[8] John D. Sterman. Modeling managerial behavior: Misperceptions of feedback in a dynamic decision making experiment, Management Science, 35(3):321–339, 1989.
[9] Thierry Moyaux, Brahim Chaib-draa, Sophie D'Amours (2007). The impact of information sharing on the efficiency of an ordering approach in reducing the bullwhip effect, *IEEE Trans. on Systems, Man, and Cybernetics, Part C (SMC-C)*, 37(3), 396-409..