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VERTICAL INTEGRATION AND THE LICENSING OF INNOVATION WITH A FIXED FEE OR A ROYALTY

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Vertical integration and the licensing of innovation with a fixed fee or a royalty *

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Abstract

In this paper, we analyse a situation where a patent holder is considered as an upstream firm who can license its innovation to some downstream companies that compete on a final market with differentiated products. Licensing contract may be based either on a royalty or a fixed fee. The patent holder can either be independant or vertically integrated with one of the downstream companies. We show that a license based on a royalty works better with vertical integration, and that consequently, the patent holder have some interest to vertically integrate if it enables him to apply a royalty based license. The effect of vertical integration on the social surplus can be either positive or negative.

Keywords: Licensing, Innovation, Vertical Integration

JEL: D45, L22, L42, O31, O32

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Introduction

It is generally recognised that licensing can represent a large share of the firms' profit, especially in the technology intensive sectors. This fact is associated with the development of markets for technologies [Arora et al., 2002], where the upstream actors develop some new technologies, and the downstream actors incorporate them into new products or processes in order to improve their competitive position on the final market. In such a setting, a patent holder may choose either to be only an upstream technology provider (or an outsider innovator), or to vertically integrate some downstream activities (becoming then, an insider or an incumbent innovator). With no vertical integration, the patent holder earns profit only from licensing its innovation. With vertical integration, the patent holder can either license its innovation or foreclose its downstream competitor(s). The aim of this paper is to synthesise some previous finding about the effect of vertical integration on the optimal licensing practices, and to analyse the interest of vertical integration both from the patent holder and the social perspective.

The comparison of different payment structure (royalty, fixed fee, auction, two-part tariff) is a major topic in the literature on the licensing of innovation. The first contributions were more particularly concerned with the interest of a royalty *vs* a fixed fee or an auction mechanism. On the one hand, the theoretical analysis highlights the interest of a fixed fee (or auction) compared to a royalty, in a context where the innovator is not vertically integrated [Kamien and Tauman, 1986, Kamien et al., 1992]. On the other hand, the empirical literature shows that royalty based licenses are rather frequent [Rostocker, 1983, Caves et al., 1983]. Several more recent theoretical contributions enable to understand this contradiction. Muto [1992] and Poddar and Sinha [2004] show that a royalty can be preferred to a fixed fee when considering a market with product differentiation. It is shown also that there the innovator has more interest for a royalty when it is vertically integrated with one downstream licensee [Wang, 1998, Wang and Yang, 1999, Wang, 2002, Kamien and Tauman, 2002]. At last, the innovator has some interest to introduce a royalty in a context with uncertainty or information asymmetry on the innovation quality [Bousquet et al., 1998, Gallini and Wright, 1990].

Some recent theoretical analysis have also introduced more complex payment structure where a royalty can be combined with a fixed fee or an auction [Erutku and Richelle, 2000, Faulí-Oller and Sandonís, 2003, Sandonís and Faulí-Oller, 2003, Sen and Tauman, 2003]. Such contract is generally preferred by the innovator because he can then extract a larger share and sometime all the surplus generated by the innovation.[1] This result is not fully coherent with the empirical literature that

[1]This result can be derived from the fact that corner solutions with only a royalty of a fixed fee

shows that the use of a simple payment structure is rather frequent:

- Rostocker [1983] analyses the licensing practices of 37 US firms, and show that licensing contracts based only on a royalty or a fixed fee occur in 52% of the cases, while two-part tariff occur in 46% of the cases.

- Macho-Stadler et al. [1996] analyse a sample of 241 licensing contracts between Spanish firms (licensees) and foreign companies (licensor), for a large range of sector at the beginning of the 1990's. In this sample, more than 80% of the contract are based only on a royalty or a fixed fee, while only 10% of the contract are based on two-part tariff.

- Jensen and Thursby [2001] study the licensing practices of 62 universities in the US between 1991 and 1995. Their survey shows that the universities frequently combine two types of payment, and in particular a fixed fee and a royalty.

It is difficult to exclude, *a priori* some type of licensing contract. Hence, the analysis of the licensing strategy should be done with a large range of contract.

All the theoretical contributions mentioned before, except one [Sandonís and Faulí-Oller, 2003], consider that vertical integration is exogenous. The aim of this paper is to analyse the optimal licensing strategy when the patent holder can choose whether or not to vertically integrate in a preliminary stage. By doing so it is possible to better understand to what extend vertical merger can be explained by motivations related to the licensing of intellectual property rights, and to analyse the welfare impact of such mergers.

The analysis is based here on a simple model with no uncertainty and no information asymmetry. Competition on the final market occurs between two firms selling one differentiated product each. Both Cournot and Bertrand competition are considered. One of these two firms is the subsidiary of the upstream patent holder in the case with vertical integration. Licensing contract may be based either on a royalty or a fixed fee. This model is identical to the one studied by Muto [1992] with no vertical integration and Wang and Yang [1999] and Wang [2002] with vertical integration. Hence the results from these three articles will be synthesised in this paper, and completed with an analysis of the interest for vertical integration This model is also identical to the one used by Sandonís and Faulí-Oller [2003], except that these authors consider only a payment structure based on two-part tariff. The combined two papers provide interesting conclusions about vertical integration and licensing with a large range of payment structures.

generally do not appear at the equilibrium.

The first important result of this paper is that *the patent holder always integrates vertically when only a royalty based license can be used*. Two mechanisms explain this result. First, a royalty is a poor instrument for extracting the net profit let to the independent downstream licensee. Vertical integration enables a better extraction of the licensee profit, because he then faces a more competitive environment. Second, vertical integration enables to better preserve the industry profit when the royalty is high enough, simply because the double margin is less important (it concerns one non integrated firm instead of two).

The second important result is that, when vertical integration is endogenous and either a royalty or a fixed fee based license can be used, *integrated company choose a royalty while non-integrated company choose a fixed fee*. Vertical integration has no effect on the patent holder profit with a fixed fee (except in some particular cases which are not relevant here). Hence, the patent holder chooses to vertically integrate only when a royalty is preferred to a fixed fee.

The paper is organised as follow. The model is presented in the section 1. We first consider the case where the patent holder can only use a royalty based license and show that he has then always an interest for vertical integration (section 2). However we know from the literature that a fixed fee can be preferred to a royalty. Hence, the analysis of the interest for vertical integration is extended to the case where the patent holder can apply a fixed fee license only (section 3), or the best type of license between a royalty and a fixed fee (section 4). Extensions with alternative specification of the demand and alternative licensing contracts are addressed in the section 5.

1 The model

The model is first presented in a configuration with no vertical integration. The (minor) modifications made with vertical integration are presented after.

We consider a final market with two differentiated and competing products. Each product is produced and sold by a specific firm ($i = 1$ or $i = 2$). Two versions of each product can be produced depending on whether or not it incorporates an innovation. In practice this innovation can be considered as an improvement of one characteristic that increases the propension to pay of the consumers for the product. It is supposed that each firm can produce and sell only one version of its product. The dummy variable θ_i is used to indicate whether or not the product sold by the firm i incorporates ($\theta_i = 1$) or not ($\theta_i = 0$) the innovation. The property right of the innovation is owned by a third actor, called either the patent holder or the upstream firm. In contrast, the two firms 1 and 2 are called the downstream firms.

The inverse demand function on the final market is defined as follow: [2]

$$p_i = a + \theta_i \delta - b q_i - \lambda b q_j \tag{1}$$

a is the highest propension to pay for the product when it does not incorporate the innovation ($a > 0$), and δ is the additional propension to pay for the product when it incorporates the innovation ($\delta > 0$). b reflects the own price elasticity of the demand ($b > 0$). λ reflects the degree of substituability between the product ($\lambda \in [0, 1]$). The case where $\lambda = 0$ corresponds to a maximum differentiation of the product: the two products are sold on two different and independent final markets. Conversely, when $\lambda = 1$ the two products are perfect substitutes.

The innovation is patent-protected and can be incorporated in one product if a license contract is signed with the patent holder. Two types of licensing contract are considered in this paper: (i) a royalty based license where the licensee pays w to the patent holder for each unit he sells, (ii) a fixed fee based license, where the licensee pays F to the patent holder whatever the quantity he sells. With both types of agreement, the contracts are supposed to be public. The marginal production cost of the firm i is $c + \theta_i w$. Note also that since the upstream firm only sells a right to access to some intellectual property, its marginal cost is equal to zero[3].

The interactions between the actors are structured in three stages. First the patent holder decides the type of licensing contract and the corresponding variable (w with a royalty, F with a fixed fee). The same contract is proposed to the two downstream firms. Second, each of the downstream firm decides whether he accepts or refuses the license contract (decision variable θ_i). Third, competition occurs on the final market with either Cournot or Bertrand competition.

An alternative configuration will be considered, where the patent holder and one of the two downstream firms are vertically integrated. The subscript v is then used to refer to the vertically integrated firm and the subscript s is used to refer to the independent downstream firm. The vertically integrated company gives a free access to the innovation to its subsidiary. The three stages presented before are still valid with vertical integration. The only difference is that, at the stage 2, we are concerned

[2] This demand function can be derived from the utility function of a representative consumer defined as follow:

$$U(q_1, q_2) = (a + \theta_1 \delta) q_1 + (a + \theta_2 \delta) q_2 - \frac{b}{2}(q_1^2 + 2\lambda q_1 q_2 + q_2^2)$$

See Singh and Vives [1984] for a detailed analysis of the duopoly equilibrium with such a demand function.

[3] The cost of the research that leads to the patent is supposed to be already spend because we are not considering the research stage in this model.

with the decision of only one downstream firm (the firm s), instead of two with no vertical integration.

The choice of whether or not to integrate vertically is done before the three basic stages described before. We suppose here that the patent holder has an interest to integrate vertically one licensee if it leads to an increase of their joint profit. Note that, with such an hypothesis, there is no cost associated with vertical integration.

The resolution is made by backward induction. The three basic stages will be solved first and the choice concerning vertical integration will be analysed after.

Note also that the stage 1 resolution indirectly leads the patent holder to choose a certain level of access restriction to the innovation. Access restriction is defined as follow: there is no access restriction when the innovation is incorporated in the two products, a partial access restriction when the innovation is incorporated in only one product, and a complete access restriction when no product incorporate the innovation. By construction, complete access restriction never appears at the equilibrium with vertical integration because the innovation is sold at least by v.

2 Analysis with a royalty based license

The resolution is presented first in details with Cournot competition (section 2.1 to 2.3). The differences that appears with Bertrand competiton are briefly presented after (section 2.4)

2.1 Stages 3 and 2 subgames

The stages 3 and 2 subgames determine the behavior of the firms on the final market with given licensing contract (i.e. given value of w).

2.1.1 Stage 3

The stage 3 equilibrium depends on the vertical structure, the incorporation of the innovation in the product (value of θ_i or θ_s) and the production cost.

Two types of equilibrium are possible at the stage 3 (table 1): a duopoly equilibrium where the two competitors have positive sales and profit, and a monopoly equilibrium where only one competitor have positive sales and profit. The monopoly equilibrium can appear if only one firm incorporates the innovation and if the innovation is important enough. δ_M is the minimum value of δ over which the innovation is drastic[4] ($\delta_M = (2-\lambda)(a-c)/\lambda$). A drastic innovation is a necessary (but not

[4]Remind that, following Arrow [1962], an innovation is drastic if the competitor of the innovator cannot make positive profit when the innovator applies the monopoly price.

Table 1: Condition for having the different types of stage 3 equilibrium

	Duopoly	Monopoly
With no vertical integration	$\theta_i = \theta_j$ or $\theta_i \neq \theta_j$ and $\delta < \delta_M + w$	$\theta_i \neq \theta_j$ and $\delta \geq \delta_M + w$
With vertical integration	$\theta_s = 1$ or $\theta_s = 0$ and $\delta < \delta_M$	$\theta_s = 0$ and $\delta \geq \delta_M$

sufficient) condition for having a monopoly equilibrium.

The detailed characteristics of the stage 3 equilibrium are given in the appendix A. With no vertical integration, the gross profit of the firm i at the stage 3 is $\pi_i(w, \theta_i, \theta_j)$ when its competitor j chooses θ_j at the stage 2. With vertical integration, the gross profit of the firm s (resp. v) is $\pi_s(w, \theta_s)$ (resp. $\pi_v(w, \theta_s)$). The notations p and q (instead of π) are used to refer to the prices and quantities at the stage 3 equilibrium.

At last, note that the stage 3 equilibrium would be identical with a model where the innovation enables to decrease the production cost from c to $c - \delta$, but does not affect the product characteristics. This alternative modelling is the one used by most of the contribution on licensing [c.f. for example Wang, 2002]. This equivalence for the stage 3 implies that all the other results concerning the previous stages are also identical.

2.1.2 Stage 2

With no vertical integration, and a given choice of the firm j, the firm i accepts the license if its profit increases. Formally, we have:

$$\pi_i(w, 1, \theta_j) \geq \pi_i(w, 0, \theta_j) \qquad \Leftrightarrow \qquad w \leq \hat{w}_{NI} \quad \text{with} \quad \hat{w}_{NI} = \delta$$

Two equilibrium are possible: if $w \leq \hat{w}_{NI}$ the two firms accept the license because the best response of each firm is to accept the license whatever the choice of its competitor; and conversely, if $w > \hat{w}_{NI}$ the two firms reject the license because the best response of each firm is to reject the license whatever the choice of its competitor. Partial access restriction never appears at the equilibrium with a royalty and no vertical integration.

With vertical integration and a non drastic innovation, the firm s accepts the license if:

$$\pi_s(w, 1) \geq \pi_s(w, 0) \qquad \Leftrightarrow \qquad w \leq \hat{w}_I \quad \text{with} \quad \hat{w}_I = \delta$$

With a drastic innovation the firm s accepts the license because he earns no profit otherwise.

2.2 Stage 1 subgame

At the stage 1, the patent holder chooses the royalty level that maximises its profit. From now on, the patent holder profit is defined as the difference between the industry profit and the downstream and non-integrated firm. Such a definition enables to better understand the decisions of the patent holder by analysing the effect on these two components.

The industry profit respectively with no vertical integration and with vertical integration is defined as follow:

$$\Pi_{NI}(w,\theta_1,\theta_2) = w(\theta_1 \cdot q_1(w,\theta_1,\theta_2) + \theta_2 \cdot q_2(w,\theta_2,\theta_1)) + \pi_1(w,\theta_1,\theta_2) + \pi_2(w,\theta_2,\theta_1)$$
$$\Pi_I(w,\theta_s) = \pi_v(w,\theta_s) + \pi_s(w,\theta_s)$$

The patent holder profit is $\Pi_{NI}(w,\theta_1,\theta_2) - \pi_i(w,\theta_1,\theta_2) - \pi_i(w,\theta_2,\theta_1)$ with no vertical integration, and $\Pi_I(w,\theta_s) - \pi_s(w,\theta_s)$ with vertical integration.

The stage 1 resolution is made as follow: first, we search for the optimal royalty level with no access restriction and, second, we analyse the interest for access restriction. In-between, we analyse the effect of the royalty level on the industry profit.

2.2.1 Stage 1 with no access restriction

Solving the stage 1 with no access restriction corresponds to the search for a local optimal royalty level, such that $w < \hat{w}_{NI}$ with no vertical integration and such that $w < \hat{w}_I$ with vertical integration.

With no vertical integration, we have:

$$\text{argmax}_w \left[\Pi_{NI}(w,1,1) - 2\pi_i(w,1,1)\right] = \tilde{w}_{NI} \quad \text{with} \quad \tilde{w}_{NI} = \frac{a-c+\delta}{2}$$

This royalty level is lower than $\hat{w}_{NI}$ if δ is high enough. Consequently, with no access restriction, the optimal royalty level is:

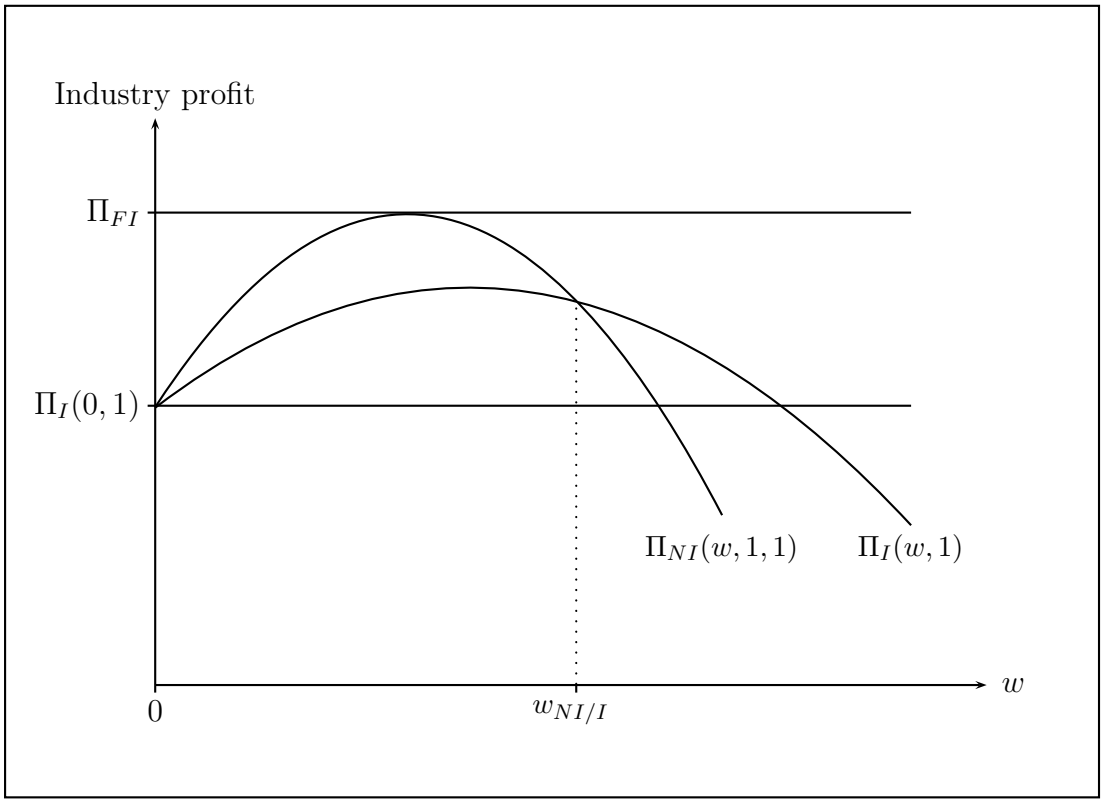
$$w^*_{NI} = \begin{cases} \hat{w}_{NI} & \text{if } \delta < \delta_{NI} \text{ with } \delta_{NI} = a-c \\ \tilde{w}_{NI} & \text{otherwise} \end{cases} \tag{2}$$

Thereafter, the optimal royalty level is qualified as *constrained* if $w^*_{NI} = \hat{w}_{NI}$ and *unconstrained* if $w^*_{NI} = \tilde{w}_{NI}$.

The similar resolution can be made with vertical integration. The patent holder chooses an optimal unconstrained royalty level ($w^*_I = \tilde{w}_I$) if the innovation is important enough ($\delta \geq \delta_I$), and a constrained level ($w^*_I = \hat{w}_I$) otherwise:

$$\tilde{w}_I = \frac{8-4\lambda^2+\lambda^3}{8-3\lambda^2} \cdot \frac{a-c+\delta}{2} \quad \text{and} \quad \delta_I = \frac{8-4\lambda^2+\lambda^3}{8-2\lambda^2-\lambda^3} \cdot (a-c) \tag{3}$$

Figure 1: Effect of the royalty level on the industry ($\lambda = 0.8$)



2.2.2 Industry profit with no access restriction

We first discuss the effect of the royalty level on the industry profit, and then analyse the industry profit when the patent holder chooses the optimal unconstrained royalty level.

Lemma 1 *With no access restriction the industry profit is concave in w. Moreover, the industry profit is strictly greater with vertical integration if and only if the royalty level is high enough:*

$$\Pi_I(w,1) > \Pi_{NI}(w,1,1) \quad \Leftrightarrow \quad w > w_{NI/I}$$

The figure 1 illustrates this result. Π_{FI} is the profit of the fully integrated firm and corresponds to the benchmark case where the patent holder vertically integrates the two downstream companies.

A qualitative explanation of this result is provided here (see appendix B for a detailed proof). The industry profit is affected by the royalty level through two opposite effects:

- *A competition relaxing effect.* An increase of w leads to an increase of the production cost of the licensees, and to an increase of their prices on the final market.

- *A double margin effect [Spengler, 1950].* The royalty causes a negative vertical externality between the licensee and the patent holder. More precisely, the decision taken by the licensee is based on a higher production cost compared to the one of a vertically integrated company ($c+w$ instead of c). As a consequence, the licensee's strategy leads to too low quantities on the final market.

These two effects explain why the industry profit is concave in w. When w is low enough, the double margin effect is very small, and the industry profit is increasing in w because of the competition relaxing effect. Conversely, when the royalty level is very high the double margin effect becomes dominant, and the industry profit is decreasing in w. Note that the maximum industry level (Π_{FI}) can be reached with no vertical integration and an intermediary royalty level.

These two effects are more important with no vertical integration because the royalty affects the production cost of two licensees instead of one. When $w = 0$, the industry profit is equal with or without vertical integration because there is no competition relaxing effect and no double margin effect (the formal proof will be given latter with the lemma 6). If w increases from 0, the industry profit increases more rapidly with no vertical integration, because the competition relaxing effect is more important in this case. In other words, there is always a small enough value of w such that the industry profit is greater with no vertical integration[5]. Conversely, when w is very high, the industry profit is higher with vertical integration because the double margin effect is less important in this case. By the continuity of the profit function, we can finally conclude that there is an intermediary level of w such that the industry profit is equal with or without vertical integration.

Lemma 2 *The optimal unconstrained royalty level ($\tilde{w}_I$ or $\tilde{w}_{NI}$) is greater than the royalty level that maximises the industry profit.*

The main interest of this lemma is to give a simple explanation of the determination of the optimal unconstrained royalty level. With vertical integration, $\tilde{w}_I$ is such

[5]rigorously, one should observe that this result holds only when $\lambda > 0$. Conversely, when $\lambda = 0$, the industry faces two independent downstream markets, each downstream firm being a monopoly on each downstream market. There is no competition relaxing effect, and consequently, the industry profit is always greater with vertical integration for any value of w.

that:

$$\frac{\partial(\Pi_I(w,1)-\pi_s(w,1))}{\partial w}=0 \quad \Leftrightarrow \quad \frac{\partial \Pi_I(w,1)}{\partial w}=\frac{\partial \pi_s(w,1)}{\partial w}$$

Since the profit of the licensee is decreasing with the royalty level, the industry profit is necessarily decreasing when $w=\tilde{w}_I$. In other words, the optimal unconstrained royalty level is such that the marginal gain from reducing the profit of the licensee is equal to the marginal lost of industry profit. Because of the concave form of the profit function, when the industry profit is decreasing, the royalty level is greater than the royalty level that maximises the industry profit. The equivalent argument can be made with no vertical integration.

2.2.3 Interest for access restriction

The objective here is to search for the optimal global royalty level at the stage 1. With vertical integration, we compare the profit with w_I^* (no access restriction) and the profit with $w>\hat{w}_I$ (partial access restriction). With no vertical integration, we compare the profit with w_{NI}^* (no access restriction) and the profit with $w>\hat{w}_{NI}$ (complete access restriction).

Lemma 3 *With a royalty based license, the patent holder prefers not to restrict the access at the stage 1 equilibrium, with either vertical or no vertical integration.*

With vertical integration, this result has already been established in the literature [Wang, 2002, Wang and Yang, 1999] with process innovation. This result is also valid here because the characteristics of the stage 3 equilibrium are similar. In addition to these contributions, we discuss here the effect of access restriction on the industry and licensee profits.

The patent holder prefers not to restrict the access if:

$$\begin{aligned} & \Pi_I(w_I^*,1)-\pi_s(w_I^*,1)>\Pi_I(0,0)-\pi_s(0,0) \\ \Leftrightarrow \quad & \Pi_I(w_I^*,1)-\Pi_I(0,0)>\pi_s(w_I^*,1)-\pi_s(0,0) \end{aligned}$$

The term on the right hand side of the second inequality is positive (otherwise the firm s would refuse the licence agreement at the stage 2): access restriction enables to decrease the profit of the independent firm s. However, access restriction leads also to a decrease of the industry profit for two reasons:

- First, the innovation is distributed less widely, and consequently less profit is earned from it. This effect is more important when the products are weak substitute (λ close to 0). In such a case the market share of the product sold by s, that does not incorporate the innovation, can still be large with access restriction.

- Second, foreclosure prevents the patent holder from using the competition relaxing effect of the royalty described before (cf. lemma 1). Access restriction leads to a more intensive competition on the final market and to a lower industry profit. This effect is more important when the products are close substitutes (λ close to 1).

Finally, foreclosure is never interesting with a royalty because it leads to a more important decrease of the industry profit, compared to the decrease of the profit let to the firm s.

With no vertical integration, the interest for no access restriction is straightforward: partial access restriction cannot appear at the stage 2 equilibrium, and the patent holder earns no profit with complete access restriction. One could wonder however if partial access restriction could be interesting with a modified version of the model such that partial access restriction can appear at the equilibrium. We can suppose for example that the patent holder can credibly commit to sign only one license agreements at the beginning of the stage 1, before choosing the level of the royalty. However, it can be shown that the lemma 3 still holds, with this modified version of the model.

2.3 The effects of vertical integration

The objective of this section is to show that a royalty based license works better with vertical integration, so that the patent holder always prefers to integrate vertically. Note first that the comparison needs to be made with different royalty level, since vertical integration affects the optimal royalty level. With Cournot competition on the final market, the optimal royalty level with vertical integration is always lower or equal to the level with no vertical integration but the difference between the two values is small ($w^*_{NI}/w^*_I \in [0.97, 1]$).

Lemma 4 *With a royalty based license (and no access restriction), vertical integration leads to a decrease of the licensee profit:*

$$\pi_s(w^*_I, 1) < \pi_i(w^*_{NI}, 1, 1)$$

The intuition behind this result is simple (see appendix C for a detailed proof). Since the two products sold on the final market incorporate the innovation, the difference of the licensee profit is caused by the difference of production cost. With vertical integration the licensee (firm s) faces a competitor with a lower production cost than with no vertical integration, because the competitor has a free access to the innovation in the former case, while he pays a royalty in the last case. Rigorously,

one have to observe also that the production cost of the licensee is lower with vertical integration, because of the lower royalty level. However this last effect is negligible compared to the first one.

Proposition 1 *With royalty based licensing contract only, the patent holder has always an interest for vertical integration.*

Vertical integration leads to an increase of the social surplus (Cournot specific).

The patent holder has an interest for vertical integration if:

$$\begin{aligned} & \Pi_I(w_I^*,1) - \pi_s(w_I^*,1) > [\Pi_{NI}(w_{NI}^*,1,1) - 2\pi_i(w_{NI}^*,1,1)] + \pi_i(w_{NI}^*,1,1) \\ \Leftrightarrow \quad & \Pi_I(w_I^*,1) - \Pi_{NI}(w_{NI}^*,1,1) > \pi_s(w_I^*,1) - \pi_i(w_{NI}^*,1,1) \end{aligned}$$

The term on the right hand side of the first inequality is the joint profit between the patent holder (in square brackets) and one licensee. We have just shown (lemma 4), that the term on the right hand side of the second inequality is negative. Two cases have to be considered in order to check the second inequality:

- With important enough innovations ($\delta \geq \delta_{NI}$), the royalty level is unconstrained. We have seen (lemma 2) that the unconstrained optimal royalty level is then high in order to moderate the profit let to the licensee(s). We have seen also (lemma 1) that the industry profit is greater with vertical integration if the royalty levels is greater than $w_{NI/I}$. With Cournot competition, it can be shown that the unconstrained optimal royalty level is greater than $w_{NI/I}$ (cf. appendix D). Hence, the patent holder prefers to integrate vertically because it enables both to increase the industry profit and to decrease the profit let to the licensee.

- With small innovations ($\delta < \delta_{NI}$), the royalty level is constrained at least with no vertical integration. With very small innovation, the industry profit can be lower with vertical innovation but, because of the constraint on the royalty level, the difference of profit let to the licensee is then more important, so that the patent holder still prefers to integrate vertically (cf. appendix D).

Vertical integration leads to an increase of the social surplus because the prices on the final market are lower with vertical integration. Two results explain this property. First, with a given royalty level, the prices are lower with vertical integration ($p_v(w,1) < p_s(w,1) < p_i(w,1,1)$) because the competition is then more intense. Second, the optimal royalty level is lower with vertical integration ($w_I^* \leq w_{NI}^*$). Note however that this last result is specific to the Cournot competition.

2.4 Analysis with Bertrand competition on the final market

With Bertrand competition, three types of equilibrium appear at the stage 3: a duopoly and a monopoly equilibrium like with Cournot competition, and, in-between, a constrained monopoly. In the constrained monopoly, only one firm has positive sales, but he is constrained to apply a limit price lower than the monopoly price in order to keep the competitor out of the market [6].

All the lemma and proposition established with Cournot competition are still valid with Bertrand competition, except one: vertical integration can lead to a surplus decrease if the products are close substitutes and the innovation is minor. This result comes from the fact that the maximum royalty level defined from the stage 2 resolution can be much higher with vertical integration ($\hat{w}_I > \hat{w}_{NI}$) especially when the product are close substitutes. As a consequence, when the optimal royalty level with no vertical integration is constrained (i.e. when δ is small), the optimal royalty can be much higher with vertical integration. Such a high royalty level can leads to higher prices on the final market, and to a surplus decrease.

3 Analysis with a fixed fee based license

The stage 3 resolution made with a royalty is valid with a fixed fee: the gross profit with a fixed fee is equal to the gross profit with a royalty when $w = 0$. All the analysis made in this section is valid both with Cournot and Bertrand competition on the final market.

3.1 Stages 2 and 1 resolution

As in the case with a royalty, we first search for the local optimal fixed fee level with a given level of access restriction. We then analyse the interest for access restriction, which corresponds to a search for the global optimal fixed fee.

3.1.1 Fixed fee with a given access restriction

At the stage 2, the decision of one potential licensee is based on its net profit, defined as the difference between the gross profit and the fixed fee F.

- With no vertical integration, the downstream firm i accepts the license in the following cases:

[6] More details on this equilibrium with limit pricing can be found in Muto [1992]

- If its competitor j also accepts the license, then we need to have:

$$\pi_i(0,1,1) - F \geq \pi_i(0,0,1) \quad \Leftrightarrow \quad F \leq F_2 \quad \text{with:} \quad F_2 = \pi_i(0,1,1) - \pi_i(0,0,1)$$

- If j reject the license, then i accepts the license if $F \leq F_1$ with $F_1 = \pi_i(0,1,0) - \pi_i(0,0,0)$

After observing that F_2 is always lower than F_1 (cf. appendix E), we can conclude that three equilibrium are possible at the stage 2:

- If $F < F_2 < F_1$, then the best response of i is to accept the license whatever the choice of its competitor. At the equilibrium the two downstream firms accept the license (no access restriction). The net profit of the licensee is then $\pi_i(0,0,1)$.

- If $F_2 < F < F_1$, then the best response of i is to accept the license if its competitor rejects it, and to reject the license if its competitor accepts it. At the equilibrium, only one of the downstream firm accepts the license (partial access restriction). Two equilibrium are possible ($(\theta_1, \theta_2) \in \{(1,0),(0,1)\}$) but they both lead to the same results because the two downstream firms are symmetric. The net profit of the licensee is $\pi_i(0,0,0)$ and the profit of the excluded downstream firm is $\pi_i(0,0,1)$.

- If $F_2 < F_1 < F$, then the best response of the firm i is to reject the license whatever the choice of its competitor. At the equilibrium both downstream firms reject the license (complete access restriction).

- With vertical integration, the firm s accepts the license if:

$$\pi_s(0,1) - F \geq \pi_s(0,0) \quad \Leftrightarrow \quad F \leq F_I \quad \text{with:} \quad F_I = \pi_s(0,1) - \pi_s(0,0)$$

The net profit of the licensee is $\pi_s(0,0)$ whatever its choice at the period 2. Note that we always have $F_I = F_2$. This result will be established in the proof of the lemma 6.

- Synthesis. At the stage 1, the patent holder choose $F = F_2 = F_I$ if he decides not to restrict the access (with both vertical structure). If he decides to have a partial access restriction, the patent holder choose $F = F_1$ with no vertical integration and $F > F_I$ with vertical integration. The table 2 synthesises the net profit of the firms with the different restriction level and vertical structure. As we made before with royalty license, the patent holder profit is expressed as the difference between the industry profit and the net profit of the downstream independent firms.

Table 2: Net profit of the firms with a fixed fee

	With no vertical integration	With vertical integration
No access restriction ($F = F_2$ or $F = F_I$)		
Patent holder	$\Pi_{NI}(0,1,1) - 2\pi_i(0,0,1)$	$\Pi_I(0,1) - \pi_s(0,0)$
Each licensee	$\pi_i(0,0,1)$	$\pi_s(0,0)$
Partial access restriction ($F = F_1$ or $F > F_I$)		
Patent holder	$\Pi_{NI}(0,0,1) - \pi_i(0,0,0) - \pi_i(0,0,1)$	$\Pi_I(0,0) - \pi_s(0,0)$
Licensee	$\pi_i(0,0,0)$	
Excluded firm	$\pi_i(0,0,1)$	$\pi_s(0,0)$

Compared to the royalty based license, a new equilibrium with partial access restriction can appear with no vertical integration. This equilibrium exists because there is a range of value for the fee ($F_2 < F \leq F_1$) where the best response of each downstream firm is to make the converse choice compared to the competitor.

3.1.2 Interest for access restriction

Lemma 5 *With a fixed fee license, the patent holder prefers to restrict (partially) the access if the innovation is important enough and if the products are close enough substitutes.*

The figure 2 illustrates this result. This result has already been established in the literature in the case with vertical integration [Wang, 2002], and can be extended to the case with no vertical integration.

Note first that complete access restriction is excluded from the analysis either because it leads to no profit for the patent holder (with no vertical integration) or because it is not possible by construction (with vertical integration).

With vertical integration, the patent holder prefers to restrict the access if it leads to an increase of the industry profit:

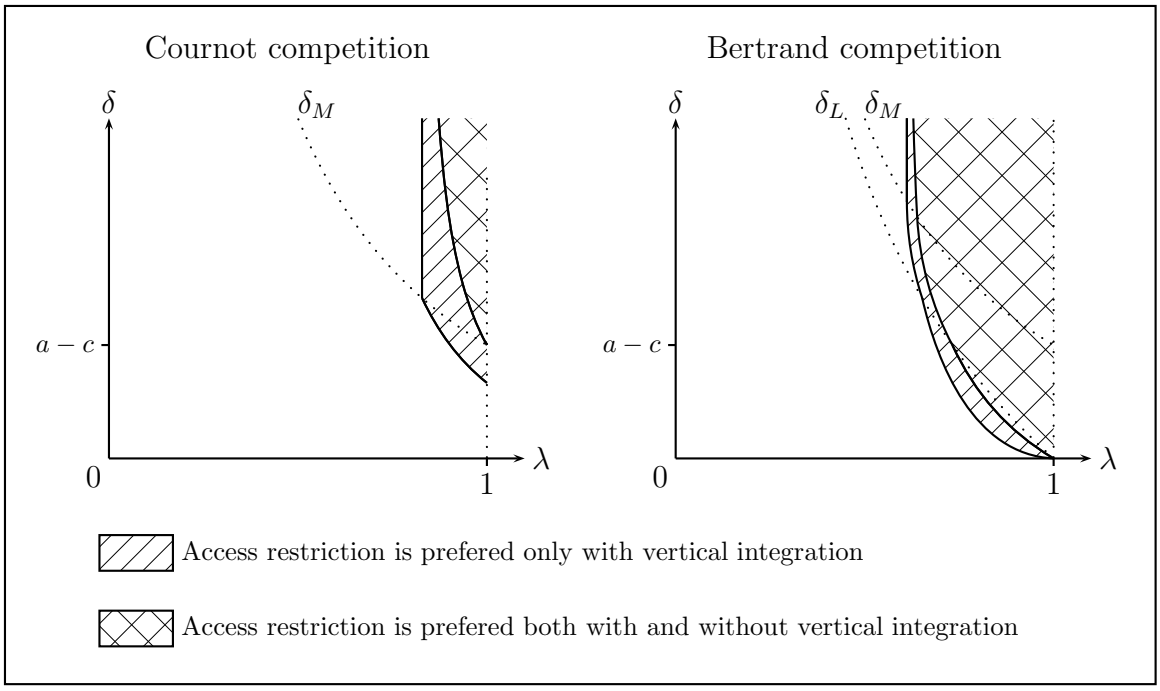
$$\Pi_I(0,0) - \pi_s(0,0) > \Pi_I(0,1) - \pi_s(0,0) \quad \Leftrightarrow \quad \Pi_I(0,0) > \Pi_I(0,1)$$

With no vertical integration, the condition is different but the increase of the industry profit is still a necessary condition.

Access restriction affects the industry profit through two opposite effects:

- On the one hand, access restriction enables to increase the market power of the firm that distributes exclusively the innovation. Formally, we have $p_v(0,0) >$

Figure 2: Interest of the patent holder for partial access restriction with a fixed fee



$p_v(0,1)$ with vertical integration and $\pi_i(0,1,0) > \pi_i(0,1,1)$ with no vertical integration. The variation of market power caused by access restriction is more important when the products are close substitute because the industry profit with no access restriction is then lower. Moreover, the market power provided by the access restriction increases when the innovation is important enough.

- On the other hand, some demand are harder to be satisfied with access restriction. In other words, the quantity of innovation sold is more important when the innovation is incorporated in two products (non-restricted access) instead of one (partial access restriction). This negative effect is less important when the product are close substitute and when the innovation is important enough.

Finally, with close enough substitute and important enough innovation, the positive effect of access restriction on the industry profit increases while the negative effect decreases.

3.2 Effects of vertical integration

We can observe from the figure 2 that the conditions where the patent holder prefers access restriction are different depending on whether or not there is vertical integration. This result will now be explained, and we will see that it determines the condition for having an interest for vertical integration.

3.2.1 Effect of vertical integration on the industry profit

Lemma 6 *With a fixed fee and a given restriction of the access to the innovation, the profit of the industry is identical with or without vertical integration:*

$$\Pi_I(0,\theta) = \Pi_{NI}(0,1,\theta) = \Pi_{NI}(0,\theta,1)$$

$\theta = 0$ *with partial access restriction, and* $\theta = 1$ *with no access restriction.*

With a given access restriction, the level of the fixed fee does not affect the production cost of the two downstream companies, and consequently does not affect the equilibrium on the final market. Fixed fee affects only the transfer among firms but not the industry profit.

The profit of v is equal to the gross profit of one licensee with no vertical integration. Similarly, the gross profit of s is equal to the gross profit of one downstream firm with no vertical integration, knowing that its competitor is a licensee. Formally we have:

$$\pi_v(0,\theta) = \pi_i(0,1,\theta) \qquad \text{and} \qquad \pi_s(0,\theta) = \pi_i(0,\theta,1) \tag{4}$$

In passing, note that one consequence of this result is that $F_2 = F_I$.

The industry profit are also identical because:

$$\Pi_{NI}(0,\theta,1) = \pi_i(0,\theta,1) + \pi_i(0,1,\theta) = \pi_s(0,\theta) + \pi_v(0,\theta) = \Pi_I(0,\theta)$$

3.2.2 Effect of vertical integration on the interest for access restriction

Proposition 2 *With fixed fee, if the patent holder prefers to restrict (partially) the access with no vertical integration, then he prefers also to restrict the access with vertical integration. Equivalently, if the patent holder prefers not to restrict the access with vertical integration, then he prefers also not to restrict the access with no vertical integration.*

Suppose that the patent holder prefers to restrict the access with no vertical integration:

$$\begin{aligned} & \Pi_{NI}(0,1,0) - \pi_i(0,0,0) - \pi_i(0,0,1) > \Pi_{NI}(0,1,1) - 2\pi_i(0,0,1) \\ \Leftrightarrow \quad & \Pi_{NI}(0,1,0) - \Pi_{NI}(0,1,1) > \pi_i(0,0,0) - \pi_i(0,0,1) \end{aligned}$$

The term on the right hand side of the second inequality is positive: access restriction leads to an increase of the licensee's net profit with no vertical integration. The term on the left hand side is equivalent with vertical integration: $\Pi_I(0,0) - \Pi_I(0,1) = \Pi_{NI}(0,1,0) - \Pi_{NI}(0,1,1)$ (cf. lemma 6). The previous inequality implies that:

$$\Pi_I(0,0) - \Pi_I(0,1) > \pi_i(0,0,0) - \pi_i(0,0,1) > 0$$

The industry profit with vertical integration increases with access restriction, and consequently the patent holder prefers to restrict the access with vertical integration.

In summary, access restriction is more often preferred with vertical integration because it does not lead to an increase of the licensee net profit as it does with no vertical integration.

Corollary 1 *With a fixed fee, there is a set of parameters value for λ and δ such that the patent holder has an interest for a partial access restriction with vertical integration but no interest for access restriction with no vertical integration.*

The figure 2 illustrates this corollary.

3.2.3 Interest of the patent holder for vertical integration

Proposition 3 *With a fixed fee, there is an interest for vertical integration only if it enables the patent holder to move from a strategy with no access restriction (with no vertical integration) to a strategy with access restriction (with vertical integration).*

Vertical integration leads to a decrease of the social surplus.

If vertical integration does not affect the equilibrium on the final market, then there is no interest for vertical integration because it does not affect the joint profit of the patent holder and one licensee[7].

Suppose now that the parameters are such that there is an interest for access restriction only with vertical integration (cf. corollary 1). We have:

$$\Pi_I(0,0) - \pi_s(0,0) > \Pi_I(0,1) - \pi_s(0,0)$$

Using the lemma 6, this expression is equivalent to:

$$\Pi_I(0,0) - \pi_s(0,0) > [\Pi_{NI}(0,1,1) - 2\pi_i(0,0,1)] + \pi_i(0,0,1)$$

The term on the right hand side is the joint profit of the patent holder and one licensee with no vertical integration and no access restriction. Hence, the patent holder prefers to integrate vertically in this case.

With a fixed fee, the surplus is maximum with no access restriction because the innovation is then integrated in the two products and the the marginal cost of the licensee is minimal. Vertical integration leads to a decrease of the social surplus because it is chosen only if it enables to move from no access restriction (i.e. maximum surplus) to access restriction (i.e. non-maximum surplus).

[7] More precisely, if no access restriction if preferred in both cases, it leads to the same joint profit:

$$[\Pi_{NI}(0,1,1) - 2\pi_i(0,0,1)] + \pi_i(0,0,1) = \Pi_I(0,1) - \pi_s(0,0)$$

On the left hand side, the first term in square brackets is the patent holder profit and the second term is the profit of the downstream firm that can be integrated. The term on the right hand side is the profit of the vertically integrated company.

If partial access restriction is preferred in both cases, the joint profit is not affected by vertical integration:

$$[\Pi_{NI}(0,1,0) - \pi_i(0,0,1) - \pi_i(0,0,0)] + \pi_i(0,0,0) = \Pi_I(0,0) - \pi_s(0,0)$$

4 Analysis with a royalty or a fixed fee based license

We consider here that the patent holder chooses first whether of not to integrate vertically and then the best type of license between a royalty and a fixed fee. Thereafter, the three basic stages of the model take place. Consequently, the best type of license is analyzed first and the analysis of the interest for vertical integration after. All the analysis presented here is valid both with Cournot and Bertrand competition on the final market.

4.1 Interest for fixed fee *vs* royalty

Lemma 7 *With no access restriction, the net profit of a licensee is lower with a fixed fee compared to a royalty, either with or without vertical integration:*

$$\left| \begin{array}{l} \pi_s(0,0) \leq \pi_s(w_I^*, 1) \\ \pi_i(0,0,1) \leq \pi_i(w_{NI}^*, 1, 1) \end{array} \right.$$

With a royalty, the net profit of a licensee is equal to its gross profit. With a fixed fee, the net profits are defined in the table 2.

With vertical integration and a royalty, the licensee accepts the contract if the royalty is lower than a maximum level ($\hat{w}_I$ such that $\pi_s(\hat{w}_I, 1) = \pi_s(0,0)$). Moreover, knowing that $w_I^* \leq \hat{w}_I$ and that the profit of the licensee is decreasing in w, we have:

$$\pi_s(w_I^*, 1) \geq \pi_s(\hat{w}_I, 1) = \pi_s(0,0)$$

With no vertical integration, the net profit of the licensee with a fixed fee is lower compared to the case where nobody accepts the license ($\pi_i(0,0,1) \leq \pi_i(0,0,0)$). We have then:

$$\pi_i(w_{NI}^*, 1, 1) \geq \pi_i(\hat{w}_{NI}, 1, 1) = \pi_i(0,0,0) \geq \pi_i(0,0,1)$$

Proposition 4 *If the patent holder prefers to apply a fixed fee rather than a royalty with vertical integration, then he prefers to apply a fixed fee rather than a royalty with no vertical integration. Equivalently, if the patent holder prefers to apply a royalty rather than a fixed fee with no vertical integration, then he prefers to apply a royalty rather than fixed fee with vertical integration.*

This property is the indirect consequence of the fact that a royalty license works better with vertical integration, while the profit with a fixed fee (and no access restriction) are equivalent with or without vertical integration. Hence the patent holder prefers more often to use a royalty with vertical integration. Interestingly, the proof is based only on the previous lemma and propositions.

We suppose that the patent holder prefers to use a fixed fee rather than a royalty with no vertical integration.

Note first that, with vertical integration, a fixed fee with partial access restriction cannot be preferred to a royalty. With vertical integration and access restriction, the patent holder earns the same profit ($\Pi_I(0,0) - \pi_s(0,0)$) with any type of contract because there is no contract. We have seen that there is no interest for access restriction with a royalty (lemma 3). Hence, the patent holder profit with vertical integration is always greater with a royalty (and no access restriction) compared to a fixed fee and partial access restriction.

We now suppose that the patent holder prefers to use a fixed fee and no access restriction rather than a royalty with vertical integration:

$$\Pi_I(0,1) - \pi_s(0,0) > \Pi_I(w_I^*,1) - \pi_s(w_I^*,1)$$

Using the proposition 1, we know that the profit with a royalty and vertical integration (right hand side) is greater to the joint profit of the patent holder and one licensee with a royalty and no vertical integration. Consequently, we have:

$$\Pi_I(0,1) - \pi_s(0,0) > \Pi_{NI}(w_{NI}^*,1,1) - \pi_i(w_{NI}^*,1,1)$$

Using the lemma 6, we can replace the term on the left hand side by there equivalence with no vertical integration:

$$\Pi_{NI}(0,1,1) - \pi_i(0,0,1) > \Pi_{NI}(w_{NI}^*,1,1) - \pi_i(w_{NI}^*,1,1)$$

At last, we subtract $\pi_i(0,0,1)$ on both sides, and we use the lemma 7, to get:

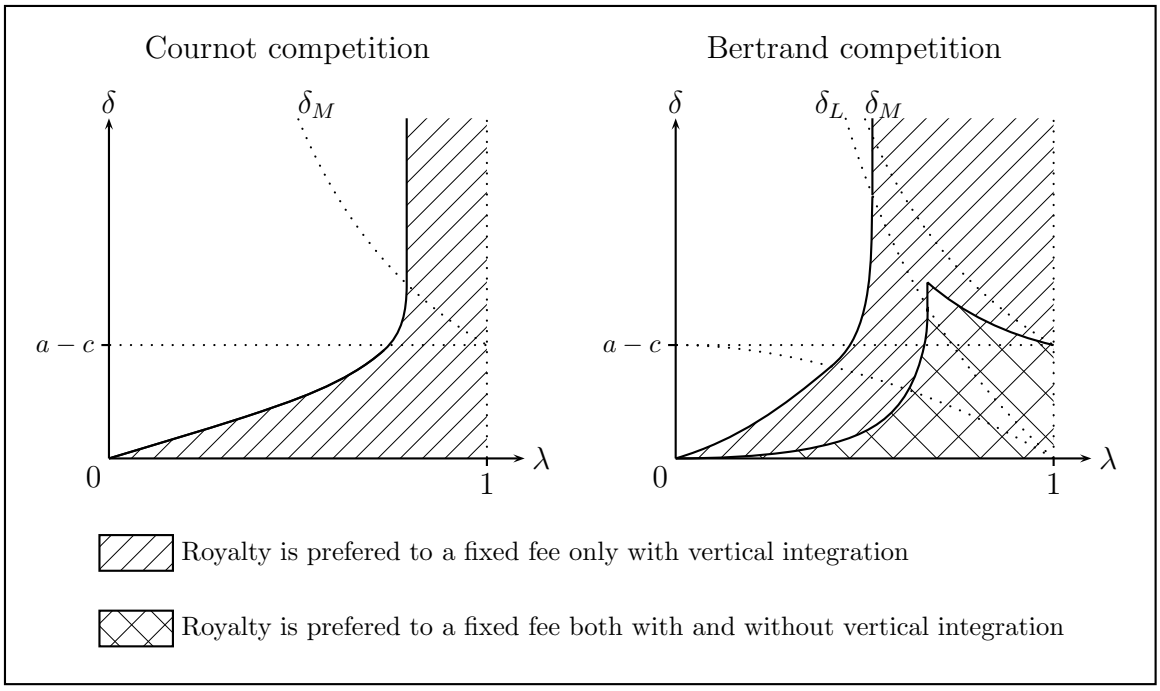
$$\begin{aligned} \Pi_{NI}(0,1,1) - 2\pi_i(0,0,1) &> \Pi_{NI}(w_{NI}^*,1,1) - \pi_i(w_{NI}^*,1,1) - \pi_i(0,0,1) \\ &> \Pi_{NI}(w_{NI}^*,1,1) - 2\pi_i(w_{NI}^*,1,1) \end{aligned}$$

This last inequality indicates that the patent holder prefers to use a royalty rather than a the fixed fee with no vertical integration.

Corollary 2 *There is a set of parameters value for λ and δ such that the patent holder uses a royalty based license rather than a fixed fee based license with vertical integration, and the converse (a fixed fee rather than a royalty) with no vertical integration.*

The figure 3 illustrates this corollary.

Figure 3: Interest of the patent holder for a royalty rather than a fixed fee



4.2 Interest for vertical integration

Proposition 5 *If the patent holder can use either a fixed fee or a royalty based license, there is a net interest for vertical integration if and only if it leads the patent holder to choose a royalty based licensing contract.*

Suppose first that the patent holder prefers a fixed fee rather than a royalty with vertical integration. He has then no interest for access restriction with both types of license. By the proposition 4, the patent holder prefers also a fixed fee rather than a royalty with no vertical integration (with no access restriction in the two case). Using the proposition 3, we conclude that the patent holder has no interest for vertical integration.

Suppose now that the patent holder prefers to use a royalty rather than a fixed fee with vertical integration. Two sub-cases have to be considered:

- If he prefers also to use a royalty rather than a fixed fee with no vertical integration, then by proposition 1 we conclude he prefers to integrate vertically.
- If he prefers also to use a fixed fee rather than a royalty with no vertical integration, then the preference of a royalty over a fixed fee with vertical integration is equivalent to the preference of a royalty with vertical integration to a fixed fee with no vertical integration because the profit with a fixed fee and a given access restriction is not affected by vertical integration. Hence the patent holder prefers to integrate vertically (more details can be found in the appendix F).

The effect of vertical integration on the social surplus can be either positive or negative. With Cournot competition, the patent holder always prefers to apply fixed fee rather than royalty with no vertical integration. Vertical integration leads to an increase (resp. a decrease) of the social surplus if the patent holder would choose to restrict (resp. not to restrict) the access with no vertical integration (cf. appendix F).

5 Discussion

5.1 Alternative demand function

The demand function used in this paper is common in the economic literature and in most of the paper on the licencing of innovation with differentiated products. However over specification of the demand with product differentiation are possible.

It can be observed that the results with a fixed fee or a royalty (section 4) are all derived from the previous results with only one type of license (either a fixed fee or a royalty). We expect the result with a fixed fee only (section 3) to be robust

with a large range of demand function[8]. The results with only a royalty based license (section 2) are specific to this demand function. However, only two particular results are important for the rest of paper: the patent holder never prefers to restrict the access (lemma 3) and always prefers to integrate vertically (proposition 1). Finally, the main results from this paper are valid with alternative demand function if these two particular results are checked.

A counter-example can be find in Poddar and Sinha [2004] who consider a demand function based on the hotelling model where the two downstream firms are positioned at the two extremes of the segment.[9] With such a model, the optimal royalty level corresponds to the value of the innovation (δ) either with or without vertical integration. The patent holder does not increase its profit by integrating vertically a licensee. This results comes from the fact that the whole demand is served with any value of the royalty. Hence there is no double margin effect with such a model, and consequently no interest for vertical integration.

5.2 Alternative licensing contract

Auctions and two-part tariff are the two alternative types of licensing contract commonly considered in the literature on the licensing of innovation.

Consider first the case where the patent holder can sell a certain number of licenses through a first price auction mechanisms. Applying such a mechanism here makes sense only when there is at least two potential bidders (i.e. with no vertical integration). We know from the literature [Katz and Shapiro, 1986] that auction and fixed fee lead to the same profit with no access restriction if the patent holder can impose a minimum bid. With access restriction, auction is more efficient than fixed fee because it enables to reduce the net profit let to the licensee (from $\pi_i(0,0,0)$ to $\pi_i(0,0,1)$). Hence, the net profit of the downstream firms with an auction is identical with or without access restriction. Access restriction is preferred if and only if it leads

[8]The lemma 6 is robust with any demand function. The proposition 2 is valid as long as $\pi_i(0,0,1) < \pi_i(0,0,0)$, a basic and rather general property. The proposition 3 follows from the proposition 2.

[9]Caballero-Sanz et al. [2002] consider a similar model based on a circular rather than a linear layout of the consumers, and a quadratic rather than a linear transportation cost. Despite these differences, the results are qualitatively identical to those obtained by Poddar and Sinha [2004]

to an increase of the industry profit (either with or without vertical integration).[10] Finally, vertical integration cannot lead to a change of the access restriction as it is a case with a fixed fee (cf. proposition 3). The patent holder has no interest for vertical integration with auction.

Sandonís and Faulí-Oller [2003] consider the same model than the one used in this paper, but with two-part tariff. They show first that there is no interest for access restriction either with or without vertical integration. The optimal royalty level is a compromise between two opposite effects: on the one hand, the patent holder has an interest to decrease the royalty level in order to decrease the net profit let to the licensee; but on the other hand, the patent holder has an interest to keep the royalty level at a rather high level in order to moderate the competition on the final market and to preserve the industry profit. The more important is the innovation, the lower is the net profit of the licensee, and the easier it is for the patent holder to get a good compromise. The interest of vertical integration is to decrease the profit let to the licensee, but the drawback is that it leads to lower industry profit.[11] Finally, the patent holder prefers to integrate vertically with a small innovation and not to integrate vertically with large enough innovation. This choice leads to a decrease of the social surplus. Indeed, with moderate innovation, the surplus is greater with no vertical integration because the patent holder is then forced to apply a lower royalty level which leads to a lower prices on the final market. Conversely, with important innovation, the surplus is greater with vertical integration because it decreases the loss associated with the double margin.

Conclusion

This paper analyses the interest of an upstream patent holder to integrate vertically a downstream potential licensee in a context where either a fixed fee or a royalty based license can be used. Two main cases have to be distinguished in order to summarise the results. In the first case, only a royalty based license is possible. The patent

[10] Formally we have:

$$\begin{aligned}\Pi_{NI}(0,1,1) - 2\pi_i(0,0,1) &> \Pi_{NI}(0,1,0) - 2\pi_i(0,0,1)\\ &\Leftrightarrow \Pi_{NI}(0,1,1) > \Pi_{NI}(0,1,0) \Leftrightarrow \Pi_I(0,1) > \Pi_I(0,0)\\ &\Leftrightarrow \Pi_I(0,1) - \pi_s(0,0) > \Pi_I(0,0) - \pi_s(0,0)\end{aligned}$$

[11] The profit let to the licensee is lower with vertical integration because he faces a the subsidiary of the vertically integrated company who has access to the innovation for free. Industry profit could be higher with vertical integration when the royalty level is high, but this situation never appears at the equilibrium with two-part tariff.

holder has then always an interest for vertical integration and this choice generally leads to an increase of the social surplus. For the patent holder, vertical integration has two positive effects (part of the double margin is eliminated and the profit let to the licensee decreases) that always overcomes one negative effect (the more intense competition that leads to a decrease of the industry profit). The second case is when a fixed fee can be used. Vertical integration is preferred when it enables the patent holder to change qualitatively its licensing strategy: to move from no access restriction to access restriction when only a fixed fee is possible, or to move from a fixed fee to a royalty when both types of license are possible. Vertical integration leads either to an increase or a decrease of the social surplus.

These results complete those from Sandonís and Faulí-Oller [2003] who show first that vertical integration can be preferred when the innovation is small and second that such a choice leads to a decrease of the social surplus. This analysis by Sandonís and Faulí-Oller [2003] leads to the clear-cut policy recommendation that vertical mergers motivated by patent licensing should be forbidden *per se*. In this paper, we show the result with two-part tariff is not robust with respect to the type of licensing contract. More precisely, if for some reason the patent holder cannot combine a royalty and a fixed fee, but is constrained to use only one of the two instruments, then some vertical mergers should not be forbidden. Hence, a rule of reason approach should be used to analyse such vertical merger cases, and a particular attention should be paid to the type of licensing contract used by the patent holder.

Policy recommendation can be strengthened by several development of the current model. First, one have to analyse the robustness of the result with respect to the number of downstream firms. The optimal licensing contract with an arbitrary number of downstream firms has been analysed in the literature either with or without vertical integration and with different types of payment structure[12]. However, no contribution analyses the interest of the patent holder for vertical integration in such a context. Second, when considering product innovation as we do in this paper, one should consider the case with vertical differentiation where the two types of products (the one that incorporate the innovation, and the one that does not) have positive sales. Avenel and Caprice [2003] consider a very similar case in a context with a manufacturer and two retailers and two part tariff contacts. Some results are qualitatively different compared to those obtained by by Sandonís and Faulí-Oller [2003][13], but these differences could be explained by the use of different demand func-

[12]With a fixed fee or a royalty, the optimal contract has been analysed by Kamien and Tauman [1986] in a context with no vertical integration and by Kamien and Tauman [2002] in a context with vertical integration. With two-part tariff, Sen and Tauman [2003] analyse the optimal contract.

[13]In Avenel and Caprice [2003] the production cost of the high quality product by the upstream firm can be higher compared to the low quality product. The case considered here with innovation licensing is a particular case of there paper, where the production costs are identical. In such a case,

tions. Third, the dynamic effect on the incentive to invest in research should also be addressed. More precisely, what would be the effect of the vertical integration, after a first generation of innovation, on the incentive to invest in research for a second generation of innovation?

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Avenel and Caprice [2003] conclude that there is always an interest for vertical integration and that it always leads to a decrease of the social surplus.

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Appendix (Cournot competition)

A Prices and quantities at the stage 3 equilibrium

- With no vertical integration. *At the duopoly equilibrium,* the quantity, price and profit are:

$$q_i(w,\theta_i,\theta_j) = \frac{(2-\lambda)(a-c)+(2\theta_i-\lambda\theta_j)(\delta-w)}{b(4-\lambda^2)}$$
$$p_i(w,\theta_i,\theta_j) = c+\theta_j w + \frac{(2-\lambda)(a-c)+(2\theta_i-\lambda\theta_j)(\delta-w)}{4-\lambda^2}$$
$$\pi_i(w,\theta_i,\theta_j) = \frac{((2-\lambda)(a-c)+(2\theta_i-\lambda\theta_j)(\delta-w))^2}{b(4-\lambda^2)^2}$$

We can check that if i does not incorporate the innovation but its competitor j does it ($\theta_i = 0$ et $\theta_j = 1$), then, at the stage 3 equilibrium, i supply a positive quantity with a positive margin if $\delta < \delta_M + w$ with $\delta_M = (2-\lambda)(a-c)/\lambda$. If $\theta_i = \theta_j = 1$, the quantities are positive if $w < a - c + \delta$. This condition is always fulfilled at the equilibrium because, otherwise, the patent holder who defines w, would make no profit.

At the monopoly equilibrium, the quantities, prices and profits are:

$$q_i(w,1,0) = \frac{a-c+\delta-w}{2b} \qquad q_i(w,0,1) = 0$$
$$p_i(w,1,0) = c+w+\frac{a-c+\delta-w}{2} \qquad p_i(w,0,1) = c$$
$$\pi_i(w,1,0) = \frac{(a-c+\delta-w)^2}{4b} \qquad \pi_i(w,0,1) = 0$$

This case appears only if $\delta \geq \delta_M + w$. The quantity sold by i is positive if $w < a - c + \delta$. This condition is always fulfilled at the equilibrium because, otherwise, the patent holder who defines w, would make no profit.

- With vertical integration. *With a duopoly equilibrium,* the quantity, price and profit of the firm s are:

$$q_s(w,\theta_s) = \frac{(2-\lambda)(a-c)-\lambda\delta+2\theta_s(\delta-w)}{b(4-\lambda^2)}$$
$$p_s(w,\theta_s) = c+\theta_s w + \frac{(2-\lambda)(a-c)-\lambda\delta+2\theta_s(\delta-w)}{4-\lambda^2}$$
$$\pi_s(w,\theta_s) = \frac{((2-\lambda)(a-c)-\lambda\delta+2\theta_s(\delta-w))^2}{b(4-\lambda^2)^2}$$

If $\theta_s = 0$, s sells positive quantities with a positive markup if $\delta < \delta_M$ (otherwise there is a monopoly equilibrium). If $\theta_s = 1$, s sells positive quantities with a positive markup if $w < (a-c+\delta)(2-\lambda)/2$. This condition is always fulfilled at the equilibrium.

The quantity, price and profit of the firm v are:

$$q_v(w,\theta_s) = \frac{(2-\lambda)(a-c)+2\delta-\lambda\theta_s(\delta-w)}{b(4-\lambda^2)}$$
$$p_v(w,\theta_s) = c + \frac{(2-\lambda)(a-c)+2\delta-\lambda\theta_s(\delta-w)}{4-\lambda^2}$$
$$\pi_v(w,\theta_s) = q_v(w,\theta_s)(p_v(w,\theta_s)-c) + q_s(w,\theta_s)\cdot w$$

There is no simple expression of the profit of the firm v.

With a monopoly equilibrium, quantities, prices and profits are:

$$q_v(0,0) = \frac{a-c+\delta}{2b} \qquad q_s(0,0) = 0$$
$$p_v(0,0) = c + \frac{a-c+\delta}{2} \qquad p_s(0,0) = c$$
$$\pi_v(0,0) = \frac{(a-c+\delta)^2}{4b} \qquad \pi_s(0,0) = 0$$

This equilibrium appears if $\theta_s = 0$ and $\delta \geq \delta_M$.

B Analysis of industry profit with a royalty and no access restriction

We have only to consider a duopoly equilibrium because there is no access restriction. We present here the simplest proof based on analytical resolution. A more instructive (but longer) proof can be provided by the author.

Since prices and quantities are linear function of w, the profit of the patent holder and the two licensees are quadratic in w, and consequently the industry profit is also quadratic in w.

With no vertical integration, the industry profit is:

$$\Pi_{NI}(w,1,1) = \frac{2}{b(2+\lambda)^2}\left((a-c+\delta)^2 + (a-c+\delta)\lambda w - (1+\lambda)w^2\right)$$

With vertical integration, the industry profit is:

$$\Pi_I(w,1) = \frac{2}{b(2+\lambda)^2}\left((a-c+\delta)^2 + \frac{1}{2}(a-c+\delta)\lambda w - \frac{4-3\lambda^2}{2(2-\lambda)^2}w^2\right)$$

We can check that the industry profit is concave in w. Moreover, the industry profit is greater with vertical integration if:

$$\begin{aligned} & \Pi_I(w,1) > \Pi_{NI}(w,1,1) \\ \Leftrightarrow \quad & \frac{1}{2}(a-c+\delta)\lambda w - \frac{4-3\lambda^2}{2(2-\lambda)^2}w^2 > (a-c+\delta)\lambda w - (1+\lambda)w^2 \\ \Leftrightarrow \quad & w > w_{NI/I} \qquad \text{with:} \quad w_{NI/I} = (a-c+\delta)\frac{\lambda(2-\lambda)^2}{4-3\lambda^2+2\lambda^3} \end{aligned}$$

C Effect of vertical integration on the licensee's profit with a royalty

We show here that the profit of the licensee with a royalty (and no access restriction) is lower with vertical integration. Three cases need to be distinguished:

1) If $\delta > \delta_{NI}$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \tilde{w}_{NI}$. After compilation we have:

$$\pi_i(\tilde{w}_{NI}, 1, 1) - \pi_s(\tilde{w}_I, 1) = \frac{\lambda(64 - 32\lambda - 7\lambda^2)}{4b(2+\lambda)^2(8-3\lambda^2)^2} \cdot (a - c + \delta)^2 > 0$$

2) If $\delta_I < \delta < \delta_{NI}$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. We have $\pi_i(\hat{w}_{NI}, 1, 1) > \pi_i(\tilde{w}_{NI}, 1, 1)$ because $\hat{w}_{NI} < \tilde{w}_{NI}$ and because profit of the licensee is decreasing in w. Moreover we showed that $\pi_i(\tilde{w}_{NI}, 1, 1) > \pi_s(\tilde{w}_I, 1)$ in the previous paragraph. Consequently, we have $\pi_i(\hat{w}_{NI}, 1, 1) > \pi_s(\tilde{w}_I, 1)$.

3) If $\delta < \delta_I$, then $w_I^* = \hat{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. After compilation we have:

$$\pi_i(\hat{w}_{NI}, 1, 1) - \pi_s(\hat{w}_I, 1) = \frac{\delta\lambda(2(2-\lambda)(a-c) - \lambda\delta)}{b(4-\lambda^2)}$$

This difference is always positive when $\delta < \delta_I$.

D Interest of the patent holder for vertical integration with a royalty

With a royalty license, we have seen that the patent holder have no interest for partial or complete access restriction. Three cases need to be considered as in the previous section.

1) If $\delta < \delta_I < \delta_{NI}$, then $w_I^* = \hat{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. Vertical integration increases the profit of the patent holder:

$$(\Pi_I(\hat{w}_I, 1) - \pi_s(\hat{w}_I, 1)) - (\Pi_{NI}(\hat{w}_{NI}, 1, 1) - \pi_i(\hat{w}_{NI}, 1, 1)) = \frac{\delta(\delta(4 - 4\lambda + \lambda^3) + \lambda^2(2-\lambda)(a-c))}{b(4-\lambda^2)^2} > 0$$

2) If $\delta_I < \delta < \delta_{NI}$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. The patent holder prefers to integrate vertically because of the the following inequalities:

$$\Pi_I(\tilde{w}_I, 1) - \pi_s(\tilde{w}_I, 1) > \Pi_I(\hat{w}_I, 1) - \pi_s(\hat{w}_I, 1) > \Pi_{NI}(\hat{w}_{NI}, 1, 1) - \pi_i(\hat{w}_{NI}, 1, 1)$$

The first part of the inequality comes from the fact that the profit of v is greater when the optimal royalty level is unconstrained rather than constrained. The second part of the inequality was established in the previous paragraph for any value of δ.

3) If $\delta_I < \delta_{NI} < \delta$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \tilde{w}_{NI}$. The patent holder prefers to vertically integrate because it leads to an increase of the industry profit and a decrease of the profit let to the licensee. The decrease of the licensee profit comes from the lemma 4. The increase of the industry profit can be shown has follow:

$$\Pi_{NI}(\tilde{w}_{NI}, 1, 1) < \Pi_{NI}(\tilde{w}_I, 1, 1) < \Pi_I(\tilde{w}_I, 1)$$

The first inequality comes from the fact $\tilde{w}_I < \tilde{w}_{NI}$ and that the industry profit is decreasing when the optimal royalty level is unconstrained (lemma 2). The second inequality comes from the fact that the unconstrained royalty with vertical is greater than $w_{NI/I}$:

$$\tilde{w}_I - w_{NI/I} = \frac{(2-\lambda)(16 - 24\lambda + 14\lambda^3 + \lambda^4 - 2\lambda^5)}{2(8-3\lambda^2)(4-3\lambda^2+2\lambda^3)} \cdot (a - c + \delta) > 0$$

Remind that if $w > w_{NI/I}$ we have $\Pi_{NI}(w, 1, 1) < \Pi_I(w, 1)$ (lemma 1).

E Definition and properties of the fixed fee with no vertical integration

Since $w = 0$, the monopoly equilibrium appears if only one firm incorporates the innovation ($\theta_i \neq \theta_j$) and if the innovation is drastic ($\delta \geq \delta_M$).

- Definition of F_2. Remind that $F_2 = \pi_i(0, 1, 1) - \pi_i(0, 0, 1)$ and that the net profit of the licensee is $\pi_i(0, 0, 1)$. With a non-drastic innovation, the patent holder cannot extract all the profit of the licensees ($\pi_i(0, 0, 1) > 0$). Conversely, with a drastic innovation, the patent holder extracts all the profit of the licensees ($\pi_i(0, 0, 1) = 0$). The detailed expression of F_2 is:

$$F_2 = \begin{cases} \dfrac{4\delta((2-\lambda)(a-c) + \delta(1-\lambda))}{b(4-\lambda^2)^2} & \text{if } \delta < \delta_M \\ \dfrac{(a-c+\delta)^2}{b(2+\lambda)^2} & \text{otherwise} \end{cases} \tag{5}$$

- Definition of F_1. Remind that $F_1 = \pi_i(0, 1, 0) - \pi_i(0, 0, 0)$ and that the net profit of the licensee is $\pi_i(0, 0, 0)$. Whatever the type of innovation, the patent holder cannot extract all the profit of the licensees. ($\pi_i(0, 0, 0) > 0$). $\pi_i(0, 1, 0)$ corresponds to the duopoly profit if δ is small enough ($\delta < \delta_M$), and to the monopoly profit otherwise. The detailed expression of F_1 is:

$$F_1 = \begin{cases} \dfrac{4\delta((2-\lambda)(a-c) + \delta)}{b(4-\lambda^2)^2} & \text{if } \delta < \delta_M \\ \dfrac{(\lambda(a-c) + (2+\lambda)\delta)((4+\lambda)(a-c) + (2+\lambda)\delta)}{4b(2+\lambda)^2} & \text{otherwise} \end{cases} \tag{6}$$

- Comparison of F_2 and F_1. With a non-drastic ($\delta < \delta_M$) innovation we have:

$$F_1 - F_2 = \frac{4\lambda\delta^2}{b(4-\lambda^2)^2} > 0$$

With a drastic innovation, we have:

$$F_1 - F_2 = \frac{\lambda(4+\lambda)}{4(2+\lambda)^2}\delta^2 + \frac{\lambda(4+\lambda)}{2(2+\lambda)^2}\cdot(a-c)\delta - \frac{4-4\lambda-\lambda^2}{4(2+\lambda)^2}\cdot(a-c)^2$$

This expression is positive because it is convex in δ and the two roots are lower than δ_M.

F Interest of vertical integration with a fixed fee or a royalty

- Interest for the patent holder

This section provides the proof that the patent holder always prefers to integrate vertically in the case where he prefers to apply a royalty (rather than a fixed fee) with vertical integration and the converse with no vertical integration. Two sub-cases have to be considered depending on whether the patent holder prefers to restrict the access restriction with no vertical integration (and a fixed fee).

Consider first the case where the patent holder does not restrict the access with no vertical integration. With vertical integration the patent holder prefers to apply a royalty, so we have:

$$\begin{aligned} & \Pi_I(w_I^*,1) - \pi_s(w_I^*,1) \geq \Pi_I(0,1) - \pi_s(0,0) \\ \Leftrightarrow \quad & \Pi_I(w_I^*,1) - \pi_s(w_I^*,1) \geq [\Pi_{NI}(0,1,1) - 2\pi_i(0,0,1)] + \pi_i(0,0,1) \end{aligned}$$

The equivalence is based on the fact that the profit with a fixed fee are equivalent with or without vertical integration (cf. lemma 6). The term on the right hand side of the second inequality is the joint profit of the patent holder (in squared brackets) and the licensee with fixed fee, no vertical integration, and no access restriction. Hence the patent holder prefers to integrate vertically.

The same type of equivalence can be established when the patent holder prefers to restrict the access with no vertical integration. There is also an interest for vertical integration because we have:

$$\begin{aligned} & \Pi_I(w_I^*,1) - \pi_s(w_I^*,1) \geq \Pi_I(0,0) - \pi_s(0,0) \\ \Leftrightarrow \quad & \Pi_I(w_I^*,1) - \pi_s(w_I^*,1) \geq [\Pi_{NI}(0,1,0) - \pi_i(0,0,0) - \pi_i(0,0,1)] + \pi_i(0,0,0) \end{aligned}$$

- Welfare impact of vertical integration

With no vertical integration, the patent holder always prefers a fixed fee to a royalty, but he can either or not prefers to restrict the access. With vertical integration, the patent holder can either prefers a fixed fee to a royalty or the converse, but he has no interest for access restriction.

Four potential potential cases have to be considered. Using the proposition 2, we can observe that it is not possible to have a case where the patent holders prefers a fixed fee and access restriction with no vertical integration, and a fixed fee plus access restriction with vertical integration. Hence, only three cases have to be considered.

1) If the patent holders prefers a fixed fee and no access restriction both with or without vertical integration, then the stage 3 equilibrium are identical both with or without vertical integration. The patent holder has no interest for vertical integration, and the social surplus is identical with or without vertical integration.

2) If the patent holders prefers a fixed fee and no access restriction with no vertical integration and a royalty with vertical integration, then he prefers to integrate vertically. The two products incorporate the innovation, but the prices are higher with vertical integration because of the royalty. Hence vertical integration leads to a decrease of the social surplus.

3) If the patent holders prefers a fixed fee and access restriction with no vertical integration, then he prefers a royalty to a fixed fee with vertical integration. With no vertical integration, the production cost is at the minimum level but the innovation is incorporated in only one product. With vertical integration, the innovation is incorporated in the two products but the production cost of the licensee is higher than the marginal cost because of the royalty. The compilation of the social surplus is necessary to analyse the welfare effect of vertical integration. Social surplus is defined as follow:

$$W(q_i, q_j, \theta_i, \theta_j) = q_i(a + \theta_i\delta - c) + q_j(a + \theta_j\delta - c) - \frac{b}{2} \cdot (q_i^2 + 2\lambda q_i q_j + q_j^2)$$

Vertical integration leads to an increase of the social surplus because:

$$W(q_v(w_I^*, 1), q_s(w_I^*, 1), 1, 1) - W(q_i(0, 1, 0), q_i(0, 0, 1), 1, 0) = \frac{(2-\lambda)(1-\lambda)(14 - 3\lambda - 6\lambda^2)}{2(8 - 3\lambda^2)^2} \cdot \frac{(a + \delta - c)^2}{b} > 0$$

Note that the patent holder prefers to restrict the access with a fixed fee only if the innovation is radical ($\delta > \delta_M$), so that $q_i(0, 0, 1) = 0$.

In conclusion, if the patent holder prefers to integrate vertically, it leads either to an increase or a decrease of the social surplus depending on the access restriction he would decide with no vertical integration. Vertical integration leads to an increase (resp. decrease) of the social surplus if the patent holder would decide to restrict the access (resp. not to restrict the access) with no vertical integration. Moreover, there is no loss of surplus when the patent holder prefers not to integrate vertically.