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Commodity R&D and Promotion

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Considerable evidence exists of high returns to public and private investment in commodity research and development programs. This study investigates the potential returns to product research, development, and marketing in a dynamic commodity-market model. Theoretical hypotheses derived from the solution to this model are tested in an empirical example of Washington apples. Estimation results show that, despite significant spillovers to research and promotion expenditure in this industry, there is nonetheless considerable latitude to increase annual sales.

Key Words: advertising, commodity, innovation, optimal control, Poisson model, research and development

JEL Classifications: L15, M37, Q13, Q16

Economists commonly cite publicly funded agricultural research and development (R&D) as the primary reason for the sustained relatively high rate of productivity growth in U.S. agriculture (Fuglie et al.). As evidence of this success, many empirical studies estimate social rates of return to agricultural R&D to be far higher than the cost of capital invested. However, there remain several issues regarding the economic impacts of R&D that attract considerable public and academic interest, including the returns to R&D under alternative market structures (Hamilton and Sunding; Huang and Sexton; Moschini and Lapan), the returns to R&D in a multimarket setting (Lemieux and Wohlgenant), and the relative merits of investments in R&D and commodity promotion (Fang and Goddard; Wohlgenant). However, most of these studies consider only process R&D, or R&D designed to lower production costs.

Alternatively, product R&D is becoming increasingly important as seed companies, growers, and retailers alike seek to differentiate, add value, and even brand many products that have previously been regarded as mere commodities. In fact, product R&D, defined in general terms as efforts directed at developing food products with new attributes that consumers demand such as sweetness, improved texture, or storability, may soon become the largest component of agricultural R&D spending. As recently as 1992, fully 40% of the $3.4 billion spent by private industry on R&D went toward designing and testing new products (Fuglie et al.). Even with this amount of spending, relatively little finds its way to fruit varieties and even less to successful endeavors. This relatively low level of activity may be due to the fact that most commodity R&D is conducted by government agencies and relatively little by the commodity organizations and their members who perhaps stand to reap the greatest benefit from jointly developing and promoting new products.[1] Consequently,

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[1] Notable exceptions to this observation for com-

understanding the role of product R&D, or the design and development of new and better agricultural products, is increasingly important as the search for value-added products intensifies amid declining returns for traditional or bulk commodities.[2]

The limited role played by U.S. commodity organizations is particularly surprising given that similar grower associations in countries such as New Zealand, Australia, or South Africa actively sponsor product R&D and, in fact, often hold trademarks on the results. U.S. commodity organizations, on the other hand, tend to focus on either purely generic promotion efforts or local programs designed to create a regional "brand" of a commodity. Collective promotion programs, as opposed to privately funded ones, are necessary because of the free-rider problem—if a product is difficult to brand, then the promotion efforts of one producer will benefit all others whether or not they help fund the promotion. Consequently, no individual producer has an incentive to promote his or her product. Numerous empirical studies demonstrate the ability of commodity organizations sanctioned under the 1937 Agricultural Marketing Agreement Act or similar state organizations to overcome this free-rider problem and, in fact, provide positive returns to their members (Alston et al.; Vande Kamp and Kaiser; and others). Given that the primary obstacle to private funding of R&D consists of a very similar free-rider problem (Alston and Pardey; Huffman and Evenson; Katz)—the ownership of intellectual property rights over the output of applied research—it seems that commodity organizations can potentially play a similar role in helping growers develop and market new products.

Recognizing the potential benefits to this new role becomes even more important when the possible complementarities between commodity promotion and development within a complete marketing program are considered (Chou and Shy). For other consumer goods, firms such as Procter and Gamble or Gillette would not consider developing a new product without heavily promoting it. Similarly, products that fail to provide attributes that consumers value cannot be made financially viable simply through heavy promotion and advertising. Clearly, these companies recognize the value of product development and promotion as inseparable parts of an effective marketing strategy. As commodity organizations become more sophisticated in their approach to marketing, their exploitation of the benefits potentially available to both developing and promoting new products seems inevitable. In fact, many commodity groups are beginning to adopt elements of an efficient consumer response (ECR) program, two components of which are efficient promotion and efficient development of new products. The incentives to take advantage of the possible synergies available become even more apparent as plant breeders' rights are strengthened. By developing a new variety and obtaining a patent on it, a commodity organization may be able to erect an effective barrier to entry, thus vastly improving the return to their commodity promotion efforts. Despite these potential benefits, there has been little concerted effort among growers to develop, trademark, and promote their own products.

There are many potentially valid economic reasons why this is so. First, R&D is an inherently risky endeavor, involving long lags between investment and return, low probabilities of successful innovation, and high probability of imitation. Second, grower organizations are well aware of the federal government's commitment to agricultural re-

modities other than produce include the National Livestock and Meat Board, the Cotton Council, and the National Dairy Board.

[2] This distinction is finer than this introduction suggests. While many new products, such as hybrid rice and corn varieties, are technically the result of product R&D, their primary benefit has been in increasing yields and hence reducing per bushel production costs. However, product R&D is narrowly defined here as activities that lead to products with improved characteristics demanded by consumers, such as convenience, taste, or nutritional characteristics. As demonstrated in the economic model of this article, the distinction is made clear by differentiating between supply-side and demand-side R&D, where the former is assumed to include all process and cost-reducing product R&D while the latter includes all demand-enhancing product R&D.

search dating back to the 1862 Morrill Act, so they are reluctant to displace existing federal research activities. Third, and perhaps most important, growers' own realizations that marketing involves more than just generic promotion is a relatively recent phenomenon. Much of this realization owes itself to a long history of extension and education efforts by universities, agribusiness supply and marketing firms, and commodity groups themselves. Consequently, the critical research question is whether a coordinated program in product research, development, and promotion may indeed be economically beneficial to a private organization.

Thus, the objective of this study is to determine whether there exists empirical evidence of complementarities between commodity research and promotion expenditures. To achieve these objectives, the study develops a simple economic model of a representative firm engaged in both research and promotion. We use this model to demonstrate how these two activities interact and how the incentives to engage in one depend on the effectiveness of the other. From this theoretical model, we develop a simultaneous, dynamic empirical model of promotion, R&D, and patent productivity. Using historical data describing U.S. apple sales, promotion activity, and new variety development, we test the implications of our theoretical model. We use the results from this empirical exercise to draw implications for activities of both apple grower organizations and commodity groups more generally.

An Economic Model of Product Development and Promotion

Typically, studies that compare the incentives to invest in cost-reduction or process R&D and commodity promotion treat the former as a supply shifter and the latter as a demand shifter (Fang and Goddard; Levin and Reiss; Wohlgenant). Lemieux and Wohlgenant and Voon and Edwards, however, consider examples of product R&D where agricultural research is assumed to shift domestic and export demand, respectively, without the aid of promotion. Similarly, Wohlgenant assumes that the economic effects of product R&D and promotion are observationally equivalent. However, differentiating between these two activities is critical because, as we argue above, researchers direct a significant part of total R&D spending toward product, and not process, innovation. Further, it is an oversimplification to assume that because they both serve to change demand that their effects are indistinguishable. Rather, we assume both activities affect commodity demand but that their influences differ in strength, persistence, and appropriability—three key factors in determining the returns to investment in each. By considering each as a separate activity, we model not only their unique impact on demand but also the possibility of synergistic effects among them. The objective of this section is, therefore, to develop a theoretical model of firms' investment in advertising and product R&D that accounts for these differences.

This study follows Nerlove and Arrow; Jacquemin; and Vidale and Wolfe in constructing an optimal dynamic model of investments in promotion and R&D but extends this work by allowing for varietal patenting, spillover in both research and promotion, and interaction between them. Determining optimal investments in advertising and promotion is an inherently dynamic problem due to the importance of trial, experience, and learning (Kotowitz and Mathewson) or the formation of "goodwill" (Nerlove and Arrow). However, the creation of goodwill requires more than words and images, but also requires a reputation for quality, value, and effectiveness that comes only from investments in product design. Research and promotion, therefore, both contribute to the creation and maintenance of goodwill but in different ways and at different rates. Whereas advertising often produces only short-term gains in market share, improved product attributes often take months or years to become established, particularly in the case of new plant varieties, but can result in sustained increases in demand. The theoretical model described below incorporates these differences in a simple, dynamic demand framework.

This model also extends the existing literature by explicitly considering spillovers to both activities. Producers of agricultural commodities often face both rivalry and duplicity with respect to their advertising activities and imitation in their efforts to create new varieties. Similar to Levin and Reiss, Carey and Bolton, and Jacquemin, we include these spillovers in the model as simple "conjectural variations" or, to avoid the confusion surrounding this term (Geroski), simply aggregate response parameters, φ.[3] In the case of advertising, this assumption allows the model to include all types of advertising as special cases, from pure brand advertising ($\varphi = \partial A_t/\partial a_{it} > 1$) to generic ($\varphi = \partial A_t/\partial a_{it} < 1$). In the extreme, the generic case can even lead to a reduction in the stock of industry advertising if free-riding is severe. By including such aggregate response parameters for both advertising (φ) and R&D (θ), the model captures the effect of varying degrees of appropriability of both advertising and research on the optimal demand for each. Similarly, the process governing goodwill accumulation incorporates other ways in which advertising and R&D differ—differences that are only apparent in a dynamic marketing model.

Although there are many alternative specifications for goodwill dynamics that are common in the literature (see Sethi for a review), Nerlove and Arrow assume that the stock of goodwill grows with new investment but decays by a constant proportion each period. Reflecting likely differences in the persistence of each activity's impact on demand, this study differentiates between the decay paths of the stock of R&D and advertising by defining two state variables, or types of goodwill, each with its own rate of depreciation:

$$(1)\qquad \dot{B}_t = b_{it} - \delta_{iB}B_t$$
$$\dot{A}_t = a_{it} - \delta_{iA}A_t,$$

[3] Although this assumption is necessary to focus on the objectives of the model, d'Aspermont and Jacquemin demonstrate that various assumptions about R&D and output rivalry can lead to markedly different results. In the current context, "spillovers" refer to competitive responses from outside the domestic industry, e.g., responses from New Zealand apple marketers to generic promotion programs or new varieties developed in the United States.

where $\dot{B}_t$, $\dot{A}_t$ are the rates of growth of each stock and δ_k is the constant rate of depreciation of stock k. Chintagunta and Jain use a similar specification for the evolution of goodwill at two levels of a marketing channel but capture their interaction through a single-period demand function. In the present case, however, the interaction is not between promotion and spending on R&D but rather between promotion and the output of an R&D process—new product innovations. To capture this effect, we assume that R&D generates innovations according to a patent productivity function. Because patents are discrete random variables, we assume they are generated according to a Poisson process (Cincera; Griliches; Hall, Griliches, and Hausman; Hausman, Hall, and Griliches; Lanjouw, Pakes, and Putnam; Wang, Cockburn, and Puterman), so the probability of observing N_{it} innovations for commodity i in period t is

$$(2)\qquad P[N_{it}] = \frac{\lambda_{it}^{N_{it}}e^{-\lambda_{it}}}{N_{it}!},$$

which implies that the expected number of patents each time period is

$$(3)\qquad E[N_{it}] = \lambda_{it} = \exp\left[\beta_0 + \sum_j \beta_j X_{jt}\right].$$

Studies that focus on estimating a patent production function similar to equation (3) typically include such factors as R&D spending, interindustry R&D spillovers (Cincera), indicators of market structure (Gopinath and Vasavada), or a simple time trend in the vector of determinants, X_t. Because the objective of this article is more limited than this and because each of these other factors is not likely to be important for an agricultural commodity, X_t includes only annual public spending on R&D for the commodity in question, B. Consequently, demand depends not on R&D spending directly but on the number of new products.

We model commodity demand as a function of both promotion and new product introductions in a manner similar to Chintagunta and Jain. As in their study, firms compete using nonprice methods, so prices are assumed to be parametric to their decision. To simplify the notation, define N as its expected value and suppress the time and individual firm subscripts, so the direct demand curve is

$$(4)\quad q(A, N) = \alpha_0 A + \alpha_1 A^2 + \alpha_2 N + \alpha_3 N^2 + \alpha_4 AN,$$

which, as opposed to a linear demand specification, allows demand to be concave in A and N and is a second-order approximation to any arbitrary demand function. Notice that this function models the possible complementarity between A and B in a very simple way as the marginal productivity of one is a linear function of the other. Further, assuming advertising and R&D costs are convex and separable, cost functions for each are

$$(5)\quad C_A = \gamma_{1A} a + (1/2)\gamma_{2A} a^2,$$
$$C_B = \gamma_{1B} b + (1/2)\gamma_{2B} b^2,$$

where γ_{1k} is the unit price of activity k. Combining each of these elements of the model, the objective function (Hamiltonian) becomes

$$(6)\quad H(A_t, B_t, q_t | \Psi) = p_t q_t(A_t, N_t) - C_{At} - C_{Bt} + \lambda_t \dot{A}_t + \eta_t \dot{B}_t,$$

where $e^{-rt}\lambda$, $e^{-rt}\eta$ are the marginal present values of an increment to the stock of product knowledge and product quality, respectively. Assuming firms choose current levels of investment in advertising (a_t) and R&D (b_t) and regard prices as given, we derive the solution to equation (6) using standard methods of optimal control (see Appendix A). More important, this solution provides structural demand equations for both advertising and R&D expenditures,

$$(7)\quad a^* = (pq_A\varphi - w_A) + (r + \delta_A)^{-1} \times [pq_A - pq_{AA}\varphi\dot{A} - pq_{AB}\varphi\dot{B}], \text{ and}$$

$$(8)\quad b^* = (pq_N N_B\theta - w_B) + (r + \delta_B)^{-1} \times [pq_N N_B - p(q_{NN}N_B + q_N N_{BB})\theta\dot{B} - pq_{NA}\theta\dot{A}],$$

where $q_A = \partial q/\partial A = \alpha_0 + 2\alpha_1 A + \alpha_4 N$, $q_N = \partial q/\partial N = \alpha_2 + 2\alpha_3 N + \alpha_4 A$, $N_B = \partial N/\partial B = \beta_{1N}$, and the second derivatives follow straightforwardly. Although not in reduced form, these structural equations define the basic rules for current expenditure on advertising and R&D, conditional on changes in the stock of each over time. Specifically, equation (7) implies that optimal annual advertising investment depends on the current marginal value product of advertising, net of its price, plus the present value of all future advertising investments net of any spillovers (negative or positive) that may exist between advertising and R&D. Equation (8) has a similar interpretation for investments in R&D, but here the marginal value product of both current and future R&D spending depends critically on the productivity of R&D. Differentiating these equations with respect to structural parameters and the stocks of advertising and R&D permits us to derive hypotheses with regard to the likely effect of persistence, appropriability, and complementarity between advertising and new product development.[4]

In a dynamic model, there are many plausible definitions of complementarity. For the purposes of this article, we define net dynamic complementarity as the case where the demand for a particular investment rises in the stock of the other. Consequently, the degree of complementarity between advertising and R&D is found by, first, differentiating the demand for advertising with respect to the stock of R&D,

$$(9)\quad \frac{da}{dB} = pq_{NA}\varphi N_B + (r + \delta_A)^{-1} \times [pq_{NA}(N_B + \varphi(N_B\dot{B} - N_{BB}\delta_B))].$$

Assuming the marginal impact on firm sales of introducing new products rises in the amount of advertising that accompanies their

[4] Given the complexity of the full reduced-form solutions for a and b, numerical simulations of each comparative static are required. These are interpreted in Appendix A. The conclusions there, however, differ little from the qualitative conclusions found using the conditional demand functions, equations (7) and (8).

introduction ($\alpha_4 > 0$) and the patent productivity function is concave ($0 < \beta_1 < 1$), the demand for advertising rises in the stock of R&D only if

$$(10) \quad N_B\dot{B} - N_{BB}\delta_B > 0$$

or if the net rate of productivity growth of R&D investments exceeds the rate at which previous increases in R&D efficiency become obsolete. These assumptions also ensure that the complementarity between a and B rises in the value of φ or the extent of rivalry among advertisers. In other words, the more a given message is seen as generic, thereby promoting all products in the category, the less incentive an individual firm has to develop and market new products within that category.

A similar exercise determines whether a similar complementarity applies in the opposite direction—from advertising to R&D. Differentiating the demand for current R&D spending by the stock of advertising goodwill gives

$$(11) \quad \frac{db}{dA} = pq_{NA}N_B\theta + (r + \delta_B)^{-1} \times [pq_{NA}(N_B - N_{BB}\dot{B}\theta + \theta\delta_A)],$$

which implies that the rate of investing in R&D rises in the stock of advertising goodwill only if

$$(12) \quad (N_B - N_{BB}\dot{B}\theta) + \theta\delta_A > 0.$$

A sufficient condition for equation (12) to hold requires either the productivity of R&D in creating new patents ($\beta_1 = N_B$) or the appropriability of R&D investment to be low enough, so that $(\beta_1\theta)^{-1} > \dot{B}$. These conditions imply that a firm will only conduct more R&D as its stock of advertising goodwill rises if they are unable to create new products or retain their benefits at a rate sufficient to take advantage of their greater ability to advertise. Although this is an indirect effect of appropriability, it also has a direct effect that reflects one of the fundamentally different ways in which advertising and R&D impact demand.

First, the extent to which advertising and R&D by an individual firm spillover to the rest of the industry are likely to differ from each other. While the appropriability of benefits from advertising depend on whether it is perceived as generic or brand-specific, R&D activities that involve basic science rather than patentable products are more likely to spillover. To show this, we first differentiate the demand for R&D in the parameter θ and find

$$(13) \quad \frac{db}{d\theta} = pq_NN_B - (r + \delta_B)^{-1} \times [p(q_{NN}N_B + q_NN_{BB})\dot{B} + pq_{NA}\dot{A}].$$

Recall that higher values of θ mean that a given firm's R&D investment has a relatively large impact on the total stock of R&D or product development knowledge. Therefore, equation (13) implies that, if the current marginal value product of adding to the stock of R&D is greater than the discounted value of an increment to the stock of R&D in the future, then a higher value of θ will induce higher levels of individual firm investment. As a corollary, the more investment by one firm displaces investment by others ($\theta < 1$), the less incentive a firm has to invest in R&D on its own. Clearly, a firm need invest less today to achieve a given level of sales the more other members of the industry invest in knowledge that it can use as well. However, we are unable to determine *a priori* whether this is indeed the case, so the sign of this effect remains an empirical question. A similar exercise defines equivalent conditions for the impact of advertising spillover on the demand for advertising. Differentiating the demand for advertising in the aggregate response parameter gives

$$(14) \quad \frac{da}{d\varphi} = pq_A - (r + \delta_A)^{-1}[pq_{AA}\dot{A} + pq_{AN}N_B\dot{B}],$$

which is directly analogous to equation (13). Specifically, if the present value of increasing current advertising and R&D stocks is greater than the discounted marginal value of higher future sales, then "brand" advertising, or advertising with a higher value of φ, will induce more individual investment in advertising. Because higher values of φ imply a strong aggregate response to investments made by a single firm, then *ceteris paribus,* a rising φ will lead to more advertising by an individual firm. On the other hand, lower values of φ, which are consistent with aggregate free-riding behavior, will lead to lower levels of individual investment. Such is the case with "generic" commodity promotion.

Unlike advertising spillover, which is not subject to direct control, the amount of R&D spillover can be controlled by establishing a patent.[5] If a firm is able to patent a new variety that is clearly superior to existing strains, then others must conduct their own research, often resulting in a process of "patenting around" the initial innovation (Choi). In this framework, patents cause θ to take a binary rather than continuous value—a firm either owns a patent or does not. Levin and Reiss consider a logical outcome of such an R&D game with appropriability as one that maintains "constant market shares," where $\theta = n$ as each firm invests just enough in R&D to maintain their current position. The result in equation (13) suggests that the lack of a spillover effect causes a firm to invest more in R&D, thus increasing the incentive to advertise. However, this is not necessarily the case in a dynamic framework. Specifically, comparing "patent" and "no patent" versions of equation (8), it is evident that the net effect once again depends on the growth rate of each stock relative to its annualized opportunity cost and the strength of the marginal product (α_2 and α_3) and interaction parameters (α_4):

$$(15)\quad b_1 - b_0 = pnq_N N_B - (r + \delta_B)^{-1} \times [pn(q_{NN}N_B + q_N N_{BB})\dot{B} + pnq_{NA}\dot{A}],$$

where b_1 is the investment in R&D with patenting and b_0 is investment without patent laws. Investment will rise, therefore, if the current aggregate value of new products created is greater than the capitalized value of the investment in R&D and advertising required to bring them about. Resolution of the sign of these comparative static derivatives, however, requires knowledge of the parameters of both the commodity demand function and the equations of motion governing the growth of both advertising and R&D stocks.

Econometric Model of Promotion and R&D

Our econometric model consists of both the structural equations for new product development and aggregate demand (equations (3) and (4)) as well as the first-order conditions for optimal expenditure on advertising and R&D (equations (7) and (8)). Specification of the full model is necessary both to recover the aggregate response parameters and to ensure that the estimated parameters are consistent with the theory outlined above. To make the econometric model tractable, we convert the continuous structural equations above into discrete form and follow a two-stage estimation procedure. Assuming patents occur according to a Poisson process, in the first stage, we estimate a single-equation Poisson model using maximum likelihood methods

$$(16)\quad \ln N_t = \beta_0 + \beta_1 B_t,$$

where N_t is the number of patents granted in year t and B_t is the amount of public R&D expenditure on deciduous tree fruits in the U.S. in year t. Empirical Poisson models, however, often find evidence of overdispersion (variance greater than the mean), which leads to inconsistent estimates of β_1 above. Generalizations of equation (16) take this into account, wherein the Poisson parameter varies according to a random disturbance term,

$$(17)\quad \ln N_t = \beta_0 + \beta_1 B_t + \mu_t,$$

where the distribution of μ_t determines the

[5] Of course, where existing patent laws are insufficient to protect intellectual property rights in plant development, the policy scenario becomes one of establishing patent laws by a government rather than a firm establishing a patent.

specific form of the alternative model. Specifically, if $g(\mu_t)$ is gamma distributed, then N_t follows a negative binomial distribution with density

$$(18)\quad f(N_t) = \left(\frac{1}{\Gamma(v_t)}\right)\left(\frac{v_t N_t}{\psi_t}\right)^{v_t} \exp\left(\frac{-v_t N_t}{\psi_t}\right)\frac{1}{\psi_t},$$

where ψ_t is the mean of the process, v_t is the precision parameter, and Γ is the gamma density function (Cameron and Trivedi). Cameron and Trivedi develop a simple regression-based test for overdispersion that is useful in selecting between a Poisson and the more general negative binomial models. Under the null hypothesis of no overdispersion, the variance of N_t is equal to its mean, but under the alternative, the variance is some function of the mean,

$$(19)\quad H_0:\ V[N_t] = \psi_t$$
$$H_1:\ V[N_t] = \psi_t + \gamma h(\psi_t),$$

where they assume simple linear or quadratic functional forms for $h(\psi_t)$. With either of these assumptions, testing for overdispersion then involves running linear regressions of the variance of N_t on each $h(\psi_t)$ and conducting t-tests for the significance of γ. If this parameter is significantly different from zero, we reject the Poisson specification in favor of the negative binomial. The second-stage model consists of equations (3), (7), and (8), which we estimate simultaneously to account for the likely endogeneity of advertising stocks and patent counts. Writing equation (7) in estimable form gives an expression for investments in advertising,

$$(20)\quad a_t = \alpha_0(k_1 + \varphi) + 2\alpha_1(k_1 + \varphi)A_t + \alpha_4(k_1 + \varphi)N_t + 2\alpha_1 k_1\varphi(A_t - A_{t-1}) - \alpha_4\beta_1 k_1\varphi N_t(B_t - B_{t-1}) + \varepsilon_2,$$

while equation (8) gives the estimated form of the R&D or new product development equation,

$$(21)\quad b_t = \alpha_2\beta_1\theta N_t + 2\alpha_3\beta_1(k_2 + \theta)N_t^2 + \alpha_4\beta_1(k_2 + \theta)A_t N_t - k_2\theta(2\alpha_3\beta_1 + \alpha_2\beta_1^2 + \alpha_4\beta_1^2) \times N_t(B_t - B_{t-1}) - 2k_2\theta\alpha_3\beta_1^2 N_t^2(B_t - B_{t-1}) - k_2\theta\alpha_4(A_t - A_{t-1}) + \varepsilon_3,$$

where $k_1 = (r + \delta_A)^{-1}$ and $k_2 = (r + \delta_B)^{-1}$ and we allow for a full variance/covariance matrix among equation residuals. Nonlinear three-stage least squares provides consistent estimates of all structural parameters in the system, given prior estimates of β_1 and estimates of δ_A and δ_B found using the grid-search procedure defined below. The remaining parameters, of which there are only seven, are easily identified given the parsimony of this system. This is an important characteristic of this model because the data on R&D investments is notoriously scarce.

Data and Methods

In order to both test the hypotheses described above and provide parameter estimates for the indeterminate comparative static results, we apply our modeling framework to data from the Washington apple industry and publicly funded R&D programs. Specifically, the data consist of annual advertising funded by Washington apple growers through the Washington Apple Commission (WAC), R&D investments by the U.S. Department of Agriculture (USDA), and patent awards to both private breeders and USDA scientists (U.S. Patent and Trademark Office) for the period 1973–1998. Although the WAC is responsible for a dominant share of media advertising of apples in the United States, the development of new varieties may be attributable to a large number of sources. Private seed companies and nurseries, private–public consortia, universities, and individual growers have all been responsible for the development of new varieties over the sample period. However, none of these entities provides data on R&D spending specifically on apples, so expenditures by the USDA are used as an indicator of the industry's efforts to develop new varieties. Although these sources provide data on annual gross investments in R&D and advertising, the Nerlove–Arrow goodwill model requires the definition of a "stock" of R&D and advertising.

Because patents in this area expire after 20 years, it seems natural to assume a discrete depreciation schedule of 100% after 20 years. However, the state variable is not the number of patents but the stock of R&D investments that are used to generate new patents. By assuming the knowledge generated by these investments deteriorates at a constant linear rate, we presume a continuous rate of obsolescence. With respect to the stock of advertising goodwill, other studies (Cox; Rickertsen) develop lag structures that capture the fact that advertising does not have its greatest impact until several weeks into the campaign, but then the effects decline geometrically over time as the message is forgotten by consumers. However, in annual data, these intraperiod effects are likely to be lost. Therefore, we assume goodwill accumulates according to equation (1) above. Slade and Ehrlich and Fisher discuss the problems associated with estimating depreciation rates for both state variables in the Nerlove–Arrow framework. In this study, we follow Ehrlich and Fisher by estimating the entire model over a range of assumed depreciation rates and choosing the pair (δ_A, δ_B) that maximizes the log-likelihood function of the entire model. In this way, the rate at which advertising depreciates is not independent of the decay in product development knowledge and vice versa.

Although it would be preferable to estimate the patent productivity and investment demand models together, there is no known estimator for a nested count data and nonlinear three-stage least squares problem, so we use the two-stage procedure described above. With this approach, we estimate the patent-generation process in the first stage. Because patents arrive in a discrete manner and as a function of R&D investments, we estimate patent production as a count-data regression as in Hausman, Hall, and Griliches; Cincera; Crepon and Duguet; or Cameron and Trivedi. These studies provide thorough discussions of the many issues involved in applying this approach to estimating patent-productivity models so will not be repeated here. Assuming the arrival parameter is gamma distributed implies a negative binomial model, although we test against a Poisson alternative. In general, empirical studies of patent generation tend to find a positive, if not always uniformly strong, relationship between R&D and the number of patents granted (Wang, Cockburn, and Puterman). Therefore, we adopt a similar approach and include R&D as an explanatory variable in the arrival rate function.

In the second stage, we estimate the demand for output (equation (4)), along with the investment demand for advertising (equation (7)) and R&D (equation (8)) in a simultaneous equations procedure. Specifically, we use nonlinear 3SLS in order to account for simultaneity in the change in advertising and R&D stocks as well as to impose the cross-equations restrictions implied by the theoretical model and the prior estimates of β_1, the patent productivity parameter. The instruments for this procedure consist of all exogenous and predetermined variables in the model. Of course, we first convert all time derivatives to discrete first differences to facilitate estimation. Once we obtain these parameter estimates, we then conduct numerical simulations to determine the signs of the comparative static effects that are *a priori* indeterminate.

Results and Discussion

Table 1 provides the results from estimating both stages of the R&D and advertising model. In the first stage, we first conduct specification tests of the maintained Poisson model relative to a negative binomial alternative (Cameron and Trivedi; Cincera). The estimation results provide t-ratios of 2.488 and 2.144 for the linear and quadratic $h(\psi)$ terms, respectively. Therefore, we reject the Poisson model in favor of a negative binomial specification. This misspecification test says nothing, however, about the goodness of fit of the negative binomial alternative. One common measure of goodness of fit involves comparing

Table 1. Patent Productivity and Demand for Advertising and R&D: Washington Apples

Coefficient	Variable	Estimate	t-ratio	Elasticity	t-ratio
Stage 1: patent productivity negative binomial estimates[1]					
β_0	Constant	.425	0.712	—	—
β_1	B	.768	2.435	.209	2.635
α	$g(N_t)$	.114	0.7	—	—
Stage 2: demand for advertising and R&D—3SLS estimates[2]					
α_0	A	.046	2.971	.248	1.367
α_1	A^2	−.002	−2.581	—	—
α_2	N	.016	1.965	.035	1.808
α_3	N^2	−.006	−2.73	—	—
α_4	AN	.004	1.941	.024	1.941
θ	dB/db	.341	0.887	—	—
ϕ	dA/da	.595	3.317	—	—

[1] The likelihood ratio test statistic comparing the estimated against a null model is 15.166 with 1 df. The critical chi-square value at a 5% level of significance is 3.84.
[2] The pseudo-R^2 values for each equation are as follows: advertising demand, .649; R&D demand, .766; apple sales, .683.

the estimated model to a null model using a likelihood ratio test. With a chi-square value of 15.166, compared with a critical value of 3.84, we are confident that this model does an acceptable job of explaining patent variation over time. Importantly, however, the interpretation of β_1 remains consistent with the simpler Poisson model. From the results reported in Table 1, the estimate of β_1 implies an elasticity of .21, so that we expect a 10% increase in funding to generate a 2% increase in patent formation. It remains, however, to determine whether patent activity causes an increase in sales volume.[6]

Table 1 also provides estimates of the simultaneous demand model. Given that the majority of the parameters are significantly different from zero and are of the expected sign, we are confident that the model provides a reasonable fit to the data. However, because the coefficient of determination is not well defined in a nonlinear 3SLS model, we calculate a pseudo-R^2, or the coefficient of determination between observed and predicted dependent variable values, as further evidence. As shown in Table 1, the pseudo-R^2 is .649 for the advertising demand equation, .766 for the R&D equation, and .683 for the output equation. With the paucity of data available for this analysis, these values suggest a relatively good fit to the data, so we interpret each with some confidence.

According to our simple model of demand, higher sales result from either creating a perception of higher quality through advertising or from generating new and improved products. Implicitly, therefore, we assume new apple varieties embody traits that consumers prefer over their existing choices. The second panel of Table 1 supports this assumption. Generic apple advertising causes sales to increase with a partial elasticity of .248 at the mean of sample advertising and sales but at a declining rate as advertising increases. This result is consistent with previous studies' assessment of the effect of generic advertising on apple demand (Richards and Patterson; Ward). The development of new varieties, on the other hand, also causes sales to increase at a declining rate. Again at the mean of the data, the partial elasticity of sales with respect to new variety generation is .035. Further, we expect

[6] Note that this model is similar to Chintagunta and Jain in assuming short-run industry equilibrium, so that grower welfare is determined by sales quantity. In the long run, both price and quantity effects would have to result in order for there to be a steady-state rise in producer surplus (Kinnucan). A model of long-run industry equilibrium is, however, beyond the scope of this research but is easily derived from the steady-state solution to the dynamic model presented herein.

that commodity promotion and new variety creation exhibit synergistic, or complementary, effects. Although positive complementarities are statistically significant only in a one-sided hypothesis test, the implied partial elasticity of .024 suggests that the creation and promotion of new varieties can indeed increase total apple sales more effectively than either activity conducted in isolation. Each of these elasticities, however, measures the response of sales to stock variables but not to annual investments in advertising and R&D.

Response elasticities for annual investments differ from their cumulative counterparts primarily due to the more volatile nature of annual investments and the spillover effects of each. In the case of R&D investments, the point estimate of θ in Table 1 suggests that the stock of aggregate "new product knowledge" increases by 34% of every new dollar. This implies that, while R&D is not a perfect public good, there is a significant amount of leakage. As a result, the short-run elasticity of individual investments in R&D is only .001 (t-ratio = 2.458). With respect to advertising, the spillover parameter is also less than one (φ = .595) but is significantly higher than the equivalent R&D parameter, perhaps because it is easier to develop brand awareness for a particular type of fruit (Washington apples) than it is to establish an entirely new variety. Given that apple advertising is funded by levies deducted from individual growers' sales receipts, this level of free-riding may indeed explain much of the recent grower dissatisfaction with these programs.[7] Calculating the short-run elasticity of sales with respect to advertising clearly illustrates the disincentive effect on others' advertising created by a generic program, as the elasticity is fully 58% lower than for the stock of advertising goodwill, or .103 (t-ratio = 2.494). This result suggests that if growers were able to effectively brand their product, thereby appropriating more of the benefits from both developing and promoting new varieties, then the overall effectiveness of commodity organizations' programs would likely improve both directly and indirectly through a reduction in the incentives to free-ride on others' participation. In addition to the differences in appropriability and impact between advertising and R&D, it is also likely that they differ in terms of persistence as well.

Defining persistence in terms of the estimated rate of goodwill depreciation, we estimate the value of δ_i as .51 for advertising and .11 for R&D. Interpreting these parameters in terms of a "partial adjustment" process, advertising takes approximately 2 years to arrive at its desired value in response to a shock, whereas the impact of a one-time investment in R&D lasts for almost 10 years. This is consistent with our expectations as there exists a fundamental tradeoff between a high short-run advertising impact that disappears relatively quickly and a small short-run impact of new product investments that rises only after the product gains acceptance. This result in itself may also offer part of the explanation for why commodity organizations do not conduct more basic variety research. Pressed to achieve short-run return-on-investment results by both growers and government regulators, these organizations are likely to choose a myopic strategy and avoid investments, the payoff for which does not arrive until relatively far in the future. With the entire set of estimated parameters, it is possible to determine, numerically, the comparative static derivatives that could not be determined analytically in the theoretical model above.

First, we estimate the effect of changing the stock of advertising goodwill on investment demand for advertising and R&D. Numerical simulation shows that, as expected, higher levels of advertising goodwill cause a greater demand for annual investments in each.[8] Because of the positive complementar-

[7] For a pointed description of both the most recent case involving mushrooms in Tennessee and perhaps the highest profile case of Wileman Bros. versus Glickman, see the NICPRE quarterly newsletter (April 2000). In the latter, the Supreme Court of the United States held that mandatory assessments were indeed constitutional and do not violate growers' First Amendment rights.

[8] Detailed results for all numerical simulations are available from the authors, as space limitations prevent including the relevant figures here.

ity between each activity, higher levels of one lead to higher marginal productivity and hence demand for the other. However, higher stocks of advertising goodwill have more of an impact on the demand for R&D than on advertising itself. Although many factors within the model potentially contribute to this result, the dominant influence is the fact that the diminishing marginal returns to advertising limit the own-effect on advertising demand, while the complementarity with R&D stocks accentuates the effect on research productivity. A second simulation shows that the effect of R&D on advertising is less pronounced than the opposite case due to the many sources of "leakage" of R&D from the system—the patent productivity coefficient is less than one, the spillover rate is higher, and a more concave new product–demand relationship. Of these factors, it is perhaps of greatest interest to consider the impact of appropriability on demand for either activity.

To this end, a third numerical simulation shows the effect of a rising θ parameter on the demand for both advertising and R&D investments. The simulation results show that a higher degree of appropriability of research benefits leads to greater demand for current R&D services but slightly lower demand for advertising. Although higher levels of θ cause the marginal product of additional investments in both A and B to rise, the increase in productivity is greater for B than for A, even after allowing for the effect of diminishing marginal returns. Additional simulations show that this effect is symmetric between advertising and R&D as the demand for advertising rises with φ, but the demand for R&D is left virtually unchanged. This is a "partial" effect, however, because rising values of A as φ increases will cause the demand for b to rise, just not through the φ mechanism. Each of these results suggests that it is perhaps the degree of complementarity that determines the strength of the cross-activity effects.

To examine this possibility, we simulate the impact of stronger complementarity (α_4) on the demand for advertising and R&D. As expected, greater complementarity causes a higher demand for each, but the effect is approximately three times as strong in R&D as in advertising. Although the elasticity of demand, with respect to new products, the productivity of R&D, and the appropriability of R&D output are all relatively low, the persistence of R&D investment is relatively high. Thus, although advertising promises a far greater short-run impact on sales, product R&D can potentially have a significant role in increasing commodity demand, particularly if advertising and R&D form parts of a coordinated marketing strategy.

Conclusions and Implications

There are many reasons why fruit and vegetable commodity organizations conduct or fund relatively little research and development activities or strive to patent their own varieties. Economic infeasibility, however, is likely not one of them. Rather, these reasons may instead lie more in legal, institutional, and budgetary issues. This study shows that commodity organizations or grower cooperatives may be particularly effective in developing successful new varieties primarily because they are able to promote the products that they help create.

Specifically, this study finds significant complementarities between commodity research and promotion. Therefore, this suggests that managers of commodity organizations may be well advised to direct a significant part of their checkoff budget to developing marketable new varieties. This, in turn, describes a model for commodity commissions that admits a far broader set of responsibilities than is now the case. Rather than simply help to promote the commodities that are grown by its members, the commission would take an active role in defining, developing, and sustaining the market for a particular variety. With control over the distribution of seed stock from its R&D efforts, commissions would have a measure of supply control that they don't currently possess.

However, given that the enforcement of quality standards has raised allegations of cartel-power abuse in the past, these new responsibilities would require regulators to define a new role for commissions as activist agents of producer welfare. Such a role may be more palatable to society as a whole if couched in terms of the new global agricultural economy, where other nations' growers have been using similar practices for many years. If commodity organizations take a more active role in funding and licensing their own varieties, then growers themselves would clearly benefit from having a closer relationship to those doing the research and would have a greater degree of certainty that the promotion they fund goes to promote the specific commodity that they grow. In this respect, future legal challenges to generic promotion programs will be mute as organizations will have a direct, proprietary link to the "brand" that they promote.

These results, however, clear in the case of Washington apples, do apply directly only to this particular commodity. The strength and generality of our conclusions would benefit from future research that is able to apply our methods to a wider set of products involving deeper, more detailed data.

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References

Alston, J.M., J.A. Chalfant, J.E. Christian, E. Meng, and N.E. Piggott. *The California Table Grape Commission's Promotion Program: An Evaluation.* Oakland, CA: Giannini Foundation, Monograph Number 43, 1997.

Alston, J.M., and P.G. Pardey. *Making Science Pay: The Economics of Agricultural R&D Policy.* Washington, DC: AEI Press, 1996.

Cameron, A.C., and P.K. Trivedi. "Econometric Models Based on Count Data: Comparisons and Applications of Some Estimators and Tests." *Journal of Applied Econometrics* 1(1986):29–53.

Carey, C., and D.L. Bolton. "Brand Versus Generic Advertising and the Decision to Advertise Collectively." *Review of Industrial Organizations* 11(1996):93–105.

Chintagunta, P.K., and D. Jain. "A Dynamic Model of Channel Member Strategies for Marketing Expenditures." *Marketing Science* 11(Spring 1992):168–88.

Choi, J. "Cooperative R&D with Product Market Competition." *International Journal of Industrial Organization* 11(1993):553–71.

Chou, C., and O. Shy. "New Product Development and the Optimal Duration of Patents." *Southern Economic Journal* 57(1991):811–21.

Cincera, M. "Patents, R&D, and Technological Spillovers at the Firm Level: Some Evidence from Econometric Count Models for Panel Data." *Journal of Applied Econometrics* 12(1997):265–80.

Cox, T. "A Rotterdam Model Incorporating Advertising Effects: The Case of Canadian Fats and Oils." *Commodity Advertising and Promotion.* H.W. Kinnucan, S.R. Thompson, and H.S. Chang, eds. Ames, IA: Iowa State Press, 1992.

Crepon, B., and E. Duguet. "Estimating the Innovation Function from Patent Numbers: GMM on Count Panel Data." *Journal of Applied Econometrics* 12(1997):243–63.

d'Aspermont, C., and A. Jacquemin. "Cooperative and Noncooperative R&D in Duopoly with Spillovers." *American Economic Review* 78(1988):1133–37.

Ehrlich, I., and L. Fisher. "The Derived Demand for Advertising: A Theoretical and Empirical Investigation." *American Economic Review* 72(June 1982):366–88.

Fang, C., and E. Goddard. "Optimal Investment Allocation in Generic Advertising and Research: The Case of the Canadian Beef, Egg, Pork and Milk Industries." Paper presented at the American Agricultural Association Annual Meetings, Indianapolis, IN, August 1995.

Fuglie, K., N. Ballenger, K. Day, C. Klotz, M. Olinger, J. Reilly, U. Vasavada, and J. Yee. *Agricultural Research and Development: Public and Private Investments Under Alternative Markets and Institutions.* Washington, DC: U.S. Department of Agriculture, Economic Research Service, Agricultural Economic Report No. 735, 1996.

Geroski, P.A. "In Pursuit of Monopoly Power: Recent Quantitative Work in Industrial Economics." *Journal of Applied Econometrics* 3(April 1988):107–23.

Gopinath, M., and U. Vasavada. "Patents, R&D, and Market Structure in the U.S. Food Processing Industry." *Journal of Agricultural and Resource Economics* 24(July 1999):127–39.

Griliches, Z. "Patent Statistics as Economic Indicators: A Survey." *Journal of Economic Literature* 28(1990):1661–707.

Hall, B.H., Z. Griliches, and J.A. Hausman. "Patents and R&D: Is There a Lag?" *International Economic Review* 27(1986):265–83.

Hamilton, S.F., and D.L. Sunding. "Returns to Public Investments in Agriculture with Imperfect

Downstream Competition." *American Journal of Agricultural Economics* 80(November 1998): 830–38.

Hausman, J.A., B.H. Hall, and Z. Griliches. "Econometric Models for Count Data with an Application to the Patents-R&D Relationship." *Econometrica* 52(1984):909–38.

Huang, S., and R. Sexton. "Measuring Returns to an Innovation in an Imperfectly Competitive Market: Application to Mechanical Harvesting of Processing Tomatoes in Taiwan." *American Journal of Agricultural Economics* 78(August 1996):558–71.

Huffman, W.E., and R.E. Evenson. *Science for Agriculture—A Long-Term Perspective.* Ames, IA: Iowa State Press, 1993.

Jacquemin, A.P. "Optimal Control and Advertising Policy." *Metroeconomica* 25(1973):200–9.

Kamien, M.I., and N.L. Schwartz. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management.* New York, NY: North Holland, 1981.

Katz, M. "An Analysis of Cooperative Research and Development." *Rand Journal of Economics* 17(1986):527–43.

Kinnucan, H. "Advertising Traded Goods." *Journal of Agricultural and Resource Economics* 24(July 1999):38–56.

Kotowitz, Y., and F. Mathewson. "Advertising, Consumer Information, and Product Quality." *The Bell Journal of Economics* 10(Autumn 1979):566–88.

Lanjouw, J.O., A. Pakes, and J. Putnam. "How to Count Patents and Value Intellectual Property: The Uses of Patent Renewal and Application Data." *The Journal of Industrial Economics* 46(December 1998):405–32.

Lemieux, C.M., and M.K. Wohlgenant. "*Ex-Ante* Evaluation of the Economic Impact of Agricultural Biotechnology: The Case of Porcine Somatotropin." *American Journal of Agricultural Economics* 71(November 1989):903–14.

Levin, R.C., and P.C. Reiss. "Tests of a Schumpeterian Model of R&D and Market Structure." *R&D, Patents, and Productivity.* Z. Griliches, ed. Chicago: University of Chicago Press, 1984.

Moschini, G., and H. Lapan. "Intellectual Property Rights and the Welfare Effects of Agricultural R&D." *American Journal of Agricultural Economics* 79(November 1997):1229–42.

Nerlove, M., and K. Arrow. "Optimal Advertising Policy Under Dynamic Conditions." *Economica* (May 1962):129–42.

Richards, T.J., and P.M. Patterson. "New Varieties and the Returns to Commodity Promotion: Washington Fuji Apples." *Agricultural and Resource Economics Review* 29(2000):10–23.

Rickertsen, K. "The Effects of Advertising in an Inverse Demand System." Discussion paper No. D-04/1996, As, Norway: Department of Economics and Social Sciences, Agricultural University of Norway, 1996.

Sethi, S.P. "Dynamic Optimal Control Models in Advertising: A Survey." *SIAM Review* 19(October 1977):685–25.

Slade, M.E. "Product Rivalry with Multiple Strategic Weapons: An Analysis of Price and Advertising Competition." *Journal of Economics and Management Strategy* 4(Fall 1995):445–76.

United States Department of Agriculture—CSREES. *Current Research Information System.* Washington, DC, various years.

United States Patent and Trademark Office. *Patent and Trademark Data.* Internet site: http://www.uspto.gov/web/offices/ac/ido/oeip/taf/reports.htm (Accessed May 15, 1999b).

Vande Kamp, P., and H. Kaiser. "Optimal Temporal Policies in Fluid Milk Advertising." *American Journal of Agricultural Economics* 82(May 2000):274–86.

Vidale, M.L., and H.B. Wolfe. "An Operations Research Study of Sales Response to Advertising." *Operations Research* 5(1957):370–81.

Voon, T.J., and G.W. Edwards. "Research Payoff from Quality Improvement: The Case of Protein in Australian Wheat." *American Journal of Agricultural Economics* 74(1992):564–72.

Wang, P., I.M. Cockburn, and M.L. Puterman. "Analysis of Patent Data—A Mixed-Poisson-Regression-Model Approach." *Journal of Business and Economic Statistics* 16(January 1998): 27–41.

Ward, R. *Evaluation of Washington Apple Advertising Programs.* Wenatchee, WA: Washington Apple Commission, 1993.

Washington Apple Commission. *Summary of Program Expenditures.* Wenatchee, WA, 1998.

Wohlgenant, M. "Distribution of Gains from Research and Promotion in Multi-Stage Production Systems: The Case of U.S. Beef and Pork Industries." *American Journal of Agricultural Economics* 75(August 1993):642–51.

Appendix A. Derivation of Reduced-Form Investment Demand Equations

Pontryagin's Maximum Principle (Kamien and Schwartz) applied to the Hamiltonian yields first-order conditions,

$$(22)\quad p_t\left(\frac{\partial q_t}{\partial N_t}\frac{\partial N_t}{\partial B_t}\frac{dB_t}{db_t}\right) - w_{Bt} - b_t + \eta_t = 0,$$

$$(23)\quad p_t\left(\frac{\partial q_t}{\partial A_t}\frac{dA_t}{da_t}\right) - w_{At} - a_t + \lambda_t = 0,$$

$$(24)\quad \dot{\eta}_t = \frac{-\partial H_t}{\partial B_t} + r\eta_t$$

$$= -p_t\left(\frac{\partial q_t}{\partial N_t}\frac{\partial N_t}{\partial B_t}\right) + \delta_B\eta_t + r\eta_t,$$

$$(25)\quad \dot{\lambda}_t = \frac{-\partial H_t}{\partial A_t} + r\lambda_t = -p_t\left(\frac{\partial q_t}{\partial A_t}\right) + \delta_A\lambda_t + r\lambda_t,$$

and differentiating equations (22) and (23) with respect to time and substituting the resulting expressions for $\dot{\eta}_t$ and $\dot{\lambda}_t$ along with equation (3) into equations (24) and (25), respectively, eliminate both λ and η. Then, after suppressing time subscripts, we solve for the continuous dynamic demand functions for advertising and R&D shown in

$$(26)\quad a^* = (pq_A\varphi - w_A) + (r + \delta_A)^{-1}$$

$$\times [pq_A - pq_{AA}\varphi\dot{A} - pq_{AB}\varphi\dot{B}], \quad \text{and}$$

$$(27)\quad b^* = (pq_N N_B\theta - w_B) + (r + \delta_B)^{-1}$$

$$\times [pq_N N_B - p(q_{NN}N_B + q_N N_{BB})\theta\dot{B}$$

$$- pq_{NA}\theta\dot{A}],$$

where $q_A = \partial q/\partial A = \alpha_0 + 2\alpha_1 A + \alpha_4 N$, $q_N = \partial q/\partial N = \alpha_2 + 2\alpha_3 N + \alpha_4 A$, $N_B = \partial N/\partial B = \beta_1 N$, and the second derivatives follow straightforwardly. Although not in reduced form, these structural equations define the basic rules for current expenditure on advertising and R&D, conditional on changes in the stock of each over time. To arrive at reduced-form solutions for a and b, it is necessary to eliminate $\dot{A}$ and $\dot{B}$ between the structural equations above and solve simultaneously for the control variable values. In addition to the necessary conditions presented above, the maximum principle requires that the costate equations be satisfied such that

$$(28)\quad \frac{\partial H}{\partial \lambda_t} = \dot{B}_t = b_{it} - \delta_B B_t$$

$$\frac{\partial H}{\partial \mu_t} = \dot{A}_t = a_{it} - \delta_A A_t.$$

Substitute these equations into equations (26) and (27) and simplify the resulting expressions by defining

$$(29)\quad R = p\theta q_N N_B - w_B$$

$$S = p\theta(q_{NN}N_B + q_N N_{BB})$$

$$T = p\theta q_{NA} \qquad U = pq_N N_B$$

$$V = p\varphi q_A - w_A \qquad W = pq_A$$

$$X = pq_{AB}\varphi \qquad Y = pq_{AA}\varphi.$$

$$(30)\quad a^* = \{[(r + \delta_B) + S]$$

$$\times [V(r + \delta_A)$$

$$+ (W + X\delta_B B + X\delta_A A)]$$

$$- X[R(r + \delta_B)$$

$$+ (U + S\delta_B B + T\delta_A A)]\}$$

$$\div \{[(r + \delta_A) + Y][(r + \delta_B) + S]$$

$$\times (1 - TX)\}.$$

Solving these simultaneously for the control variable values gives

$$(31)\quad b^* = \{[(r + \delta_A) + Y]$$

$$\times [R(r + \delta_B) + (U + S\delta_B B + X\delta_A A)]$$

$$- T[U(r + \delta_A)$$

$$+ (W + X\delta_B B + Y\delta_A A)]\}$$

$$\div \{[(r + \delta_B) + S]$$

$$\times [(r + \delta_A) + Y](1 - TX)\}.$$

Simulating these solutions over various values of the state and costate variables produces comparative static results similar to those discussed above. However, these solutions provide long-term results wherein the state or costate variable is not assumed to be fixed, as is the case in the analysis above. These solutions also offer an investigation of the effect of greater complementarity (α_4) on the demand for both advertising and R&D investments. These are interpreted in the text.