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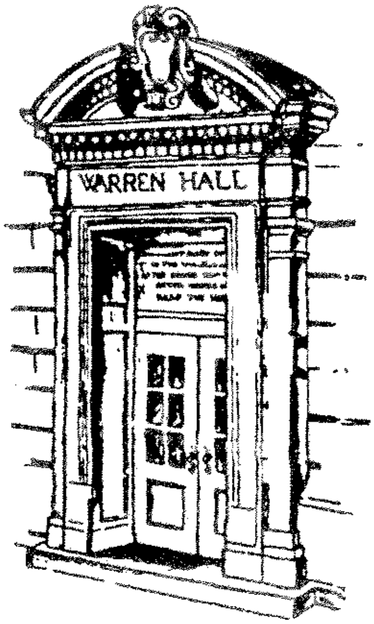

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Valuing Electricity Assets in Deregulated Markets: A Real Options Model with Mean Reversion and Jumps

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ABSTRACT

Valuing Electricity Assets in Deregulated Markets: A Real Options Model with Mean Reversion and Jumps

Valuation of electricity generating assets is of central importance as utilities are forced to spin-off generators with the introduction of competitive markets. A continuous-time mean reverting price path with stochastic upward jumps is proposed as an appropriate model for long-run competitive electricity prices faced by a generator. A real options model is derived via dynamic programming using infinite series solutions. The derived model produces asset values which are uniformly higher than those produced by existing models, and which accurately predict observed generator sale prices. The model has favorable implications for stranded cost recovery and generator entry in competitive markets.

Keywords: real options, electricity deregulation, mean reversion, jump processes, asset valuation.

I. Introduction and Overview

The recent sale of the Homer City electricity generating plant by New York State Electric and Gas (NYSEG) and GPU, Inc. to Edison Mission Energy surprised many industry observers with its high sale price of $1.8 billion. For example, the $955/KW sale price of Homer City (a 1884 MW coal plant on the New York-Pennsylvania border) is nearly triple that of the (older and less well-located) Dunkirk and Huntley plants recently sold by Niagara Mohawk Power Corporation (NMPC) in upstate New York. At first glance, the Homer City sale might seem to be an example of the Winner's Curse (e.g. Kagel and Levin 1986), with Edison Mission the unlucky winner. This paper argues that if the proposed electricity-specific real options model is appropriate, the price paid by Edison Mission is reasonable. The proposed model takes into account the unusual character of electricity supply and transport in selecting a price path. The electricity-specific model produces higher asset values than do traditional real options models. An implication is that many plants sold to date which seem appropriately valued by traditional models could well have been undervalued because traditional models fail to capture the true, favorable nature of competitive electricity prices. Under-valuation, while a boon to buyers, would negatively impact existing utilities, their shareholders and ratepayers.

The sale of the Homer City plant was prompted by the deregulation of the electricity industry in both New York and Pennsylvania.[1] As competition is introduced, existing regulated utilities are being forced to spin off their generating assets in competitive auctions to prevent combined generating and transmission corporations from exercising the potential market power which would come with the ability to shut out competitors via the transmission network. The spread of competition means that well over $300 billion in utility generating assets could ultimately be sold at auction.[2]

What is the value of a generating unit which is able to compete in deregulated electricity markets? An appropriate tool for asset valuation in a world of uncertain prices is a real options model (Dixit and Pyndick 1994, Trigeorgis 1996). Such models generally produce higher asset values than traditional discounted cash flow techniques. Given that the detailed asset valuation models currently used by the industry can easily cost hundreds of thousands of dollars to implement, a sparsely parameterized real options model is a useful tool. A prerequisite for valuing a generating asset with a real options model is a stochastic price path appropriate for electricity prices in a wholesale market.[3]

A continuous-time mean reverting process with stochastic upward jumps is presented as an appropriate model of prices faced by generators in competitive wholesale electricity markets. Such a process has not be employed in the real options literature, and comes at a time when alternatives to the standard assumption of geometric Brownian motion (GBM) have drawn increased interest. Lund (1993), for example, argues that GBM is an inappropriate price path characterization for exhaustible resources. Mean reverting price paths have been suggested as more appropriate for commodities such as oil and copper (Pindyck and Rubenfeld 1991, Schwartz 1997, and Baker, Mayfield and Parsons 1998). The price path used here encompasses both GBM and mean reversion as special cases, while allowing random, and temporary, upward jumps in prices. The special characteristics of electricity production, transmission and demand suggest a price path which has these characteristics, and that is generally distinct from that of other commodities. As shown by Schwartz (1997), price path specification significantly influences resulting asset values.

A quasi-analytic real options asset valuation model is derived via dynamic programming using the proposed mean reverting price process with jumps. The mean reverting with jumps price path has not previously been examined in a real options framework. The solution requires solving a non-homogeneous functional differential equation with the use of linearly independent infinite series, with the model easily implemented in a spreadsheet. The model developed here adds to the limited portfolio of analytic real options models and demonstrates a flexible solution technique using infinite series which might be used to develop future models. The calculated asset values are much higher under the mean reverting with jumps specification when compared with the nested models of GBM and simple mean reversion. One interesting result is that while at low prices increased volatility in electricity prices increases asset values, at high prices increased volatility lowers asset values. This is in contrast to GBM based models, where increased volatility generally increases values. Using plausible price parameter values and using the Homer City plant physical characteristics, the derived model produces asset values close to the observed sale price for Homer City. The derived model implies higher than anticipated levels of electricity industry investment, increased stranded cost recovery and delayed nuclear plant retirement when compared with conventional models.

While much work has been done on real options generally (e.g. Dixit and Pindyck 1994, Trigeorgis 1995, 1996), little has been done in the electricity sector. Pindyck (1993) explored investment in nuclear power plants with cost uncertainty. However, until recently revenue uncertainty has not been an important issue in utility investment planning. Generally the electricity-specific option models developed for competitive markets have been focused on financial options or short-run Monte Carlo models (Deng, Johnson and Sogomonian 1998, Deng 1998, Tseng and Barz 1998), though each has recognized the unique character of electricity price paths.

The next section provides the stochastic price path and detailed justification, as well as model set up and assumptions. Section III presents the model derivation, numerical evaluation of the model, and a comparison of asset values under alternative stochastic processes with calculations specific to the Homer City plant. Section IV presents conclusions.

II. The Model

It is assumed that the price for a unit of electricity sold by a generator evolves according to a mean-reverting process with jumps. The stochastic process differential is given by:

$$dP = \eta(\overline{P} - P)Pdt + \sigma Pdz + Pdq \tag{1}$$

where dt is a small increment of time, dz is an increment of a standard Wiener process and:

$$dq = \begin{cases} 0 \ w/\ prob.\ (1-\lambda)dt \\ u \ w/\ prob.\ \lambda dt \quad u > 0 \end{cases}$$

P is the average yearly on-peak price of electricity in \$/KWh, $\overline{P}$ is the reverted-to electricity price, η is the rate of reversion, u is the jump size (scaled by P), λ is the jump frequency, and σ is the standard deviation (scaled by P). If λ=0, the price process becomes simple mean reversion. If η=0 as well, then the model becomes GBM without drift. Thus the model is a general price process which will allow flexible parameterization. Note that in this formulation the actual rate of mean reversion is high when P is high and low when P is low. This might imply, for example, relatively rapid industry entry when high prices are observed, but relatively slow exit.

Choosing the Price Path

This price path combines elements of a simple form of jump diffusion with geometric mean reversion, each of which has been explored separately (Dixit and Pindyck 1994, Schwartz 1997, Saphores and Carr 1998). Why is this an appropriate price path specification for electricity? The mean reverting component is consistent with other commodity price paths. Pindyck and Rubenfeld (1991), Schwartz (1997), and Baker, Mayfield and Parsons (1998), among others, note that the real prices of commodities such as oil and copper are mean reverting in the long run.[4] Strong mean reversion is also widely assumed for short and medium term (daily and monthly) prices in competitive electricity markets (see Pilipovic 1997, Barz and Johnson 1998, or Deng, Johnson and Sogomonian 1998). This is because the entry of new generating capacity, through either greenfield installations, increased capacity from existing generators, or increased sales from adjacent grid areas, suggests the persistence of competitive markets. The history of electricity production under regulation suggests stable electricity prices, not the unbounded growth provided by GBM. The mix of available fuels (coal, oil, gas, solar, wind, etc.) and an existing generator stock which utilizes this range of options allows primary fuel substitution, thus insulating electricity prices from the long term vagaries of commodity markets and oligopolistic behavior in those markets (e.g. OPEC in oil markets in the 1970's).

But there are complications caused by the transmission grid, especially when node specific on-peak, not just average regional, prices are considered.[5] Electricity markets facilitate exercise of market power in general (see, for example, Rudkevich, Duckworth, and Rosen 1998), and create isolated regions where market power might exist in an otherwise competitive system. It is important to note that each generator is likely to face a node-specific locational price. Nodal prices are much more volatile than system average prices and allow a generator to have market power which significantly influences its own price but not that of competitors.[6] The PJM region, for example, uses locational spot prices which have large variability both spatially and through time (Hogan 1998). California uses zonal average prices which allow regional variation.

Price jumps are enabled by two other characteristics of electricity markets. First is that electricity storage is generally infeasible, thus there is little or no ability to arbitrage across time. This reduces the ability of markets to dampen price shocks. Second, inelastic demand means that there is little consumption reduction in response to a rise in prices. The yearly price elasticity of electricity demand in New York State, for example, has been estimated at –0.042 for the residential sector to –0.261 for the industrial sector (Ethier and Mount 1998).

Upward, localized (either nodal or zonal) jumps in electricity prices could happen for a number of reasons. Plant shutdowns, like the Millstone nuclear plant shutdowns in New England in the summer of 1997, would temporarily raise electricity prices for an entire region. [7] Locational prices faced by individual generators would fluctuate more dramatically. Line constraints or outages also create load pockets, allowing generators inside the pocket to exploit market power. Bernard et al (1998) show that dramatic price differentials can occur between a load pocket and the remainder of a region. There is strong evidence of market power within a load pocket being exploited in the England and Wales market by an individual generator when line constraints are present (Newbery 1995, p.58). In this case, offer prices, and subsequent payments, increased by a factor of nearly five and persisted before being addressed by regulators. Changing demand patterns or new load can also create load pockets. Oligopolistic behavior may also develop, evidenced through capacity withholding or inflated offer prices. Evidence of oligopolistic behavior in the England and Wales electricity market suggests that this might happen in a systematic fashion before it is recognized and reigned in by regulators (see Wolak and Patrick 1997). This too would lead to a significant price rise.

The stylized facts above suggest that while long-run electricity prices might be mean reverting, generators are likely to also experience localized (or even plant specific) upward jumps which allow temporarily increased profits. Capturing this in the stochastic price path will prove to have a large effect on asset values.

Modeling Generating Assets

A generating plant has operating costs of C ($/Kwh) per unit of output. The yearly profit function for the plant can then be written:

$$\Pi(P) = (P-C)\,M \tag{2}$$

where M is total on-peak electricity production per year. P and C must be the average yearly on-peak price and cost of electricity to the generator, including amortized capital costs for repair and refurbishment in $/Kwh. Thus M is on-peak Kwh per year. The implicit assumption is that off-peak hours are 'break even' hours for the plant, which given the low and stable off-peak prices observed in off-peak electricity markets, is not particularly restrictive. Clearly this ignores complexities involved in electricity production, such as generator ramping constraints (the speed at which a generator is physically able to increase or decrease production) and start-up costs.

Note that increased detail might be incorporated in the model by breaking up a year into smaller time blocks (e.g. quarters), with operation each quarter viewed as functionally independent of other quarters. Thus there would be a separate price parameterization for each quarter, with *M* (and *C*) adjusted accordingly. This would allow a richer variety of price processes, some without jumps (for fall and spring, perhaps), and others with jumps (summer and winter). It would also allow patterns in seasonal generation, input prices, or outages. The value of the plant would then be the sum of the value in each quarter.[8] For simplicity the model presented uses a yearly value.

To calculate plant value by dynamic programming, the value function for an operating generator *V(P)* must satisfy the Bellman equation:

$$\delta V(P) = \Pi(P) + \frac{1}{dt} E_t\{dV\} \tag{3}$$

with discount rate δ, so the yearly return to the plant's value equals the yearly profit plus the expected capital gain. Note that this assumes an infinitely lived plant. While that is not strictly true, if the plant can be expected to last many years, modeling a finitely-lived plant as infinitely lived is a reasonable approach. Dixit and Pindyck (1994) finds the difference to be negligible for a life span greater than ten years (p.401). The finitely lived plant problem involves a functional partial differential equation which requires numerical solution. The focus here is on obtaining an analytic formula.

To solve the Bellman equation (3), dV can be expanded as:

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial P} dP + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(dP)^2 + \ldots$$

Substituting for *dP* from above, and applying Ito's Lemma, results in:

$$dV = \left[\frac{\partial V}{\partial t} + \eta(\overline{P} - P)P\frac{\partial V}{\partial P} + \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2}\right]dt + \sigma P \frac{\partial V}{\partial P} dz + P\frac{\partial V}{\partial P} dq$$

Taking the expectation operator, where $\frac{\partial V}{\partial t}$ equals zero and $E\{dz\}=0$ leads to:

$$E\{dV\}=\left[\eta(\overline{P}-P)P\frac{\partial V}{\partial P}+\frac{1}{2}\sigma^2P^2\frac{\partial^2 V}{\partial P^2}\right]dt+E_{dq}\{\lambda[V(P+uP,t)-V(P,t)]\}dt$$

Substituting back into the Bellman equation and using V' and V'' to denote the first and second derivatives of *V(P)* yields:

$$\delta V(P)=\Pi(P)+\eta(\overline{P}-P)PV'(P)+\frac{1}{2}\sigma^2P^2V''(P)+\lambda[V(P+uP)-V(P)]$$

Rearranging and combining terms gives:

$$\frac{1}{2}\sigma^2P^2V''(P)+\eta(\overline{P}-P)PV'(P)-(\delta+\lambda)V(P)+\lambda V((1+u)P)=-\Pi(P) \qquad (4)$$

which is a non-homogeneous functional differential equation to be solved for *V(P)*.[9]

The General Solution.

Standard practice is to solve the homogeneous equation first to find the 'general' solution to the equation.[10] The homogeneous equation corresponding to (4) is:

$$\frac{1}{2}\sigma^2P^2V''(P)+\eta(\overline{P}-P)PV'(P)-(\delta+\lambda)V(P)+\lambda V((1+u)P)=0$$

A solution to the homogeneous equation is given by the infinite series representation:

$$V(P) = \sum_{n=0}^{\infty} a_n P^{n+r}$$

$$V'(P) = \sum_{n=0}^{\infty} a_n (n+r) P^{n+r-1}$$

$$V''(P) = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) P^{n+r-2}$$

Substituting *V(P)* into the homogeneous equation, combining powers of *P*, reindexing and rearranging results in:

$$\left[\frac{1}{2}\sigma^2 a_0 r(r-1) + r\eta\overline{P}a_0 - (\lambda+\delta)a_0 + \lambda a_0(1+u)^r\right]P^r$$
$$+\sum_{n=1}^{\infty}\left[\begin{array}{l}\left(\frac{1}{2}\sigma^2 a_n(n+r)(n+r-1) + \eta\overline{P}a_n(n+r) - (\lambda+\delta)a_n + \lambda a_n(1+u)^{n+r}\right) \\ -\eta a_{n-1}(n+r-1)\end{array}\right]P^{n+r} = 0$$

For the above equation to hold for all $P>0$ requires:

$$\frac{1}{2}\sigma^2 r(r-1) + r\eta\overline{P} - (\lambda+\delta) + \lambda(1+u)^r = 0$$

for a_0 not equal to zero. This equation can be solved numerically for *r*. It can be shown that *r* is strictly increasing for *r* greater than (less than) the positive (negative) solution to the implicit equation for *r* and that two roots result. The following equation must hold for every $n>0$:

$$\left(\frac{1}{2}\sigma^2 a_n(n+r)(n+r-1) + \eta\overline{P}a_n(n+r) - (\lambda+\delta)a_n + \lambda a_n(1+u)^{n+r}\right) - \eta a_{n-1}(n+r-1) = 0$$

Solving recursively for a_n as a function of a_0 leads to:

$$a_n = \frac{\eta^n (n+r-1)! a_0}{\left[\left(\frac{1}{2}\sigma^2 (n+r-1) + \eta\overline{P}\right)(n+r) - (\lambda+\delta) + \lambda(1+u)^{n+r}\right]!}$$

Because a_0 appears in every term, it can be moved outside the summation. The factorial operator indexes every n inside the brackets and excludes n=0. It is shown in the Appendix using the ratio test that the terms of $V(P)$ converge to zero as n becomes large for all $u > -2$. The restriction on u is not a problem for this model, as we anticipate only positive (u>0) jumps. The general solution is of the form:

$V(P)=c_1V_1(P) + c_2V_2(P)$.

where the subscripts denote solutions associated with r_1 and r_2, and c_1 and c_2 are free coefficients, which can incorporate a_0. Thus substituting the definition of a_n into $V(P)$ leads to:

$$\begin{aligned} V(P) &= c_1 \sum_{n=1}^{\infty} P^{r_1} + \frac{\eta^n (n+r_1-1)! P^{n+r_1}}{\left[\left(\frac{1}{2}\sigma^2 (n+r_1-1) + \eta\overline{P}\right)(n+r_1) - (\lambda+\delta) + \lambda(1+u)^{n+r_1}\right]!} \\ &+ c_2 \sum_{n=1}^{\infty} P^{r_2} + \frac{\eta^n (n+r_2-1)!\ P^{n+r_2}}{\left[\left(\frac{1}{2}\sigma^2 (n+r_2-1) + \eta\overline{P}\right)(n+r_2) - (\lambda+\delta) + \lambda(1+u)^{n+r_2}\right]!} \end{aligned} \qquad (5)$$

A Particular Solution.

The particular solution to the non-homogeneous equation is an infinite series of the form:

$$\overline{V}(P) = \sum_{n=0}^{\infty} a_n P^n$$

$$\overline{V}'(P) = \sum_{n=1}^{\infty} a_n n P^{n-1}$$

$$\overline{V}''(P) = \sum_{n=2}^{\infty} a_n n(n-1) P^{n-2}$$

which is linearly independent of the general solutions to the homogeneous equation. Substituting into the non-homogeneous equation, combining powers of P, reindexing and rearranging as before results in:

$$\begin{aligned}&-(\lambda+\delta)a_0 + \lambda a_0 - CM + \left[\eta\overline{P}a_1 - (\lambda+\delta)a_1 + \lambda a_1(1+u) + M\right]P \\ &+\sum_{n=2}^{\infty}\left[\frac{1}{2}\sigma^2 a_n n(n-1) + \eta\overline{P}a_n n - \eta a_{n-1}(n-1) - (\delta+\lambda)a_n + \lambda a_n(1+u)^n\right]P^n \\ &= 0\end{aligned}$$

For the above equation to equal zero, each coefficient for each power of P must equal zero. For the first term (coefficient of $P^0=1$) solving for a_0 gives:

$$a_0 = \frac{-CM}{\delta}$$

The second term (coefficient of P) solved for a_1 gives:

$$a_1 = \frac{M}{\delta - \eta\overline{P} - \lambda u}$$

And the third, and recursive, term (coefficient of P^n) rearranged and solved for a_n as a function of a_1 gives:

$$a_n = \frac{\eta^{n-1}(n-1)!a_1}{\left[\frac{1}{2}\sigma^2 n(n-1) + \eta\overline{P}n - (\delta+\lambda) + \lambda(1+u)^n\right]!}$$

So the complete expression for $\overline{V}(P)$ is:

$$\overline{V}(P) = -\frac{MC}{\delta} + \frac{MP}{\delta - \eta\overline{P} - \lambda u} + \sum_{n=2}^{\infty} \frac{\eta^{n-1}(n-1)!\frac{M}{\delta - \eta\overline{P} - \lambda u}P^n}{\left[\left(\frac{1}{2}\sigma^2(n-1) + \eta\overline{P}\right)n - (\lambda+\delta) + \lambda(1+u)^n\right]!} \tag{6}$$

For $\overline{V}(P)$, the expected present value of the future profits from the generating plant, to be positive and increasing for $P>0$ and thus to make economic sense, the following regularity conditions must hold:

$$\delta > \eta\overline{P} + \lambda u. \tag{7}$$

and

$$\left(\frac{1}{2}\sigma^2(n-1) + \eta\overline{P}\right)n + \lambda(1+u)^n > \lambda + \delta \tag{8}$$

for $n>1$ which ensures that the infinite series terms are greater than zero. If they are not greater than zero, then for some (large) values of P the entire term may become negative.

Discussion of Regularity Conditions

The regularity conditions (7) and (8) ensure that $\overline{V}(P)$ is positive and increasing for all values of P. Clearly this is desirable from an economic point of view; it is difficult to think of conditions under which one would expect the present value of the plant to decrease for an increase in P. The first condition (7) is similar to the condition for GBM,

$\delta > \alpha$, where the interest rate is greater than the underlying growth rate of the price of the asset. Otherwise, investing in the asset is a risk-free 'money machine' where money can be borrowed at δ, invested at α, and produce unlimited profits. In this case, δ must be greater than an 'expected growth rate' each period, which is the movement $\eta \overline{P}$ toward the mean price level plus the expected rise due to a jump λu. If this constraint is violated, the expected growth in price each period is greater than the growth in asset value which would occur at the discount rate, again creating a 'money machine'.

The need for the second regularity condition is more complicated. If the first condition holds, $\overline{V}(P)$ may well be positive, especially for small values of P. However, as P rises the infinite series terms for $n>1$, which are made negative by the violation of the second condition, take on increased weight. This can cause $\overline{V}(P)$ to be downward sloping in P or even cause the value to turn negative. This violates economic logic. Looking at the second condition (8) for n=2 gives:

$$\sigma^2 + 2\eta\overline{P} + \lambda(1+u)^2 - \lambda > \delta$$

The left hand side is a modified variance term, with the model variance adjusted by the mean reverting term and jump terms. The second regularity condition is violated when the modified variance is too small. Thus to move away from violating the second regularity condition, we must increase the variance in price by increasing λ, u, or σ, or increase the expected movement in price toward the mean level by raising $\eta \overline{P}$. This is nearly the converse of the interpretation of the first condition, which is violated when there is too much price movement. An interpretation of the second regularity condition is that if these parameters are not sufficiently large, then the model is inappropriate. That the model is potentially unstable for small values of λ, u, and $\eta \overline{P}$ is not likely to be a problem in real world use. This is because for any price path which generated small values for these parameters, it would likely be difficult to distinguish from GBM using econometric techniques.

Combining the General and Particular Solutions

So the complete solution to the non-homogeneous functional differential equation is the sum of the general and particular solutions:

$V(P)=c_1V_1(P) + c_2V_2(P) + \overline{V}(P)$

as defined above in (5) and (6). So the full equation for *V(P)* is:

$$V(P) = c_1\sum_{n=1}^{\infty} P^{r_1} + \frac{\eta^n (n+r_1-1)!P^{n+r_1}}{\left[\left(\frac{1}{2}\sigma^2(n+r_1-1)+\eta\overline{P}\right)(n+r_1)-(\lambda+\delta)+\lambda(1+u)^{n+r_1}\right]!}$$
$$+c_2\sum_{n=1}^{\infty} P^{r_2} + \frac{\eta^n (n+r_2-1)!\ P^{n+r_2}}{\left[\left(\frac{1}{2}\sigma^2(n+r_2-1)+\eta\overline{P}\right)(n+r_2)-(\lambda+\delta)+\lambda(1+u)^{n+r_2}\right]!} \qquad (9)$$
$$-\frac{MC}{\delta}+\frac{MP}{\delta-\eta\overline{P}-\lambda u}+\sum_{n=2}^{\infty}\frac{\eta^{n-1}(n-1)!\frac{M}{\delta-\eta\overline{P}-\lambda u}P^n}{\left[\left(\frac{1}{2}\sigma^2(n-1)+\eta\overline{P}\right)n-(\lambda+\delta)+\lambda(1+u)^n\right]!}$$

Note that if there is neither mean reversion nor jumps (η=0, λ=0, u=0) the model becomes:

$$V(P) = c_1P^{r_1} + c_2P^{r_2} + \frac{(P-C)M}{\delta}$$

which is the same as the GBM model provided by Dixit and Pindyck (1994, p.187) if price and cost are both discounted at the same rate.

If the generator can be costlessly shut down when *P* falls below *C*, the value function can be separated into two parts, one of which solves the homogeneous equation and the other of which solves the non-homogeneous equation. Following the logic of Dixit and

Pyndick (1994), we would expect the value of the option to generate to go to zero as P goes to zero, and for the value of the option to shut down to go to zero as P becomes very large. Defining the roots of our implicit equation for r as $r_1>0$ and $r_2<0$, we can break up the value function for different values of P:

$$V(P) = \begin{cases} c_1 V_1(P) & for\ P \leq C \quad (r_1 > 0) \\ b_2 V_2(P) + \overline{V}(P) & for\ P > C \quad (r_2 < 0) \end{cases}$$

This is the value of the generator as a function of P with costless shutdown and restart. To determine c_1 and b_2 we use the value-matching and smooth-pasting conditions where $P=C$. The interpretation is that the value of the option to generate must equal the value of the option to shut down plus the value of output when $P=C$, and that the rate of change of these conditions (high order contact conditions) also be equal. These conditions are:

$$c_1 V_1(C) = b_2 V_2(C) + \overline{V}(C) \tag{10}$$

$$c_1 V_1'(C) = b_2 V_2'(C) + \overline{V}'(C) \tag{11}$$

The parameters c_1 and b_2 must be solved for numerically. Once they are determined, the value of the generator $V(P)$ can be found.

Evaluating the Model: Changing Volatility, Rate of Mean Reversion and Jump Size

Consider the following numerical examples with base parameter values summarized in Table 1. The calculated parameter values for the model are provided in Table 2. For model evaluation jump size u was varied from 0.4 to 1. As expected, increasing the size of jumps generally increased the value of a generating asset. This is shown in Figure 1. One interesting result of the model is that for a low jump size value (0.4), higher price levels produced asset values which rose above those produced by jump sizes of 0.6 and 0.8. Note that at this value, the second regularity constraint is close to being violated. Thus for this parameterization, the model is unstable for small jump sizes.

The effect of varying the rate of mean reversion η from 0 to 0.4 is shown in Figure 2. If the rate of mean reversion is zero, the model collapses to geometric Brownian motion with jumps. As expected, asset value increased with increasing rates of mean reversion. What was surprising was that the rate of mean reversion had such a significant effect on asset value. Doubling the rate of mean reversion (from 0.2 to 0.4, for example) approximately doubles asset value. This suggests that accurate determination of both whether a price process is mean reverting and the rate of mean reversion is important for asset valuation.

The standard deviation was varied from 0.05 to 0.4, with the effects shown in Figure 3. The results were interesting in that at low prices, a high standard deviation produced higher asset values, but at high prices, lower standard deviation produced higher values. One interpretation is that at low price levels, a high standard deviation is more likely to get you "back in the money", while at high price levels, is more likely to take you out of the money. However, the standard deviation of the stochastic process had a small effect on asset values relative to the other parameters. The relative insensitivity of $V(P)$ to the size of the standard deviation is potentially useful. It suggests that parameterizing the model with a high standard deviation will not strongly affect asset values, but can help in satisfying the regularity conditions.

III. Application to the Homer City Plant

The Homer City plant sale price is widely viewed by industry analysts, utility executives and regulators as being well above expectations. For example, while Homer City sold for approximately $955/KW of capacity, NMPC recently sold its Huntley and Dunkirk coal plants for $281/KW (net of two Huntley units slated for retirement). In part this is because Homer City is an especially desirable plant. It is a relatively new (the newest unit came on line in 1977) and efficient baseload coal plant near large coal supplies, with direct connections to two different regional electricity markets (Pennsylvania-New Jersey-Maryland and western New York). Is the Homer City value reasonable, or is it dramatically overvalued? Using the real options model developed in the previous section, with realistic parameter values, the Homer City price is close to the calculated plant value while the NMPC units appear undervalued. The model was parameterized for the Homer City plant as in Table 3.

The reverted-to price level is \$0.034/KWh, which is the average on-peak summer price level in PJM East from 1996 to 1998. The cost of production of \$0.038/KWh (double reported variable production costs in 1996, from *Load and Capacity Data* 1997) is assumed to include all fixed and variable costs not covered during off-peak hours. This would include amortized capital costs for repair and refurbishment. The plant is assumed to simply cover variable costs during the remainder of the week on average. The output per year in Kwh assumes a 90% capacity factor for on-peak hours for the plant's 1884 Mw. The model is well behaved with these parameter values, with both regularity conditions satisfied. Remember that since we are using a model of an infinitely lived plant, the calculated plant value is higher than what would be calculated for a finitely lived plant.

One interesting exercise is comparing asset values under different price path assumptions, i.e. mean reverting with jumps vs. mean reversion vs. GBM. Since the mean reverting with jumps model contains mean reversion and GBM as special cases, this is a straightforward process. To achieve mean reversion, λ was set to zero. To achieve GBM, η was also set to zero. Figure 4 shows the value of the generating plant for a range of prices for each price path. Adding simple mean reversion to GBM (with the current parameterization) increases the calculated Homer City plant value from \$931 million to \$1.279 billion when evaluated at $\overline{P}$. Using mean reversion with jumps increases plant value to \$1.836 billion. GBM produces the lowest values over the range of P, while mean reversion with jumps produces the highest.

Do these values make sense? It does make sense that mean reversion increases plant values, as the risk of low prices is lowered. Dixit and Pindyck (1994, p. 405) note that inclusion of mean reversion can easily affect asset values by 40%. That is consistent with these results. Adding jumps should also be expected to increase asset value, given that the jumps are only expected to be positive. Since in this case the jumps are to nearly double the current price level (1.75 times the current price level), large changes in asset value result. One mild surprise is that mean reversion always produces higher values than does GBM (with zero drift). This is surprising because one might expect that for high values of P and low values of $\overline{P}$ that GBM would produce higher values. This might be expected to occur because you expect that a mean reverting price process will revert toward the low $\overline{P}$, but that GBM will not necessarily produce lower prices.

Clearly model specification significantly affects plant value, with the differential increasing in electricity price, and the mean reverting with jumps plant value very close to the $1.8 billion purchase price of the plant. Remembering that the model overvalues assets by assuming an infinite life, this suggests that the actual sale price was high but not dramatically so. While other parameterizations would have generated different values, the current parameterization is a reasonable one, and produces a reasonable value. As important is that other price paths produce dramatically different asset values for common parameter sets, and these values are significantly lower than the observed sale price of Homer City.

For comparison Table 4 provides parameters for a joint model of the Dunkirk and Huntley plants, with the same mean reverting characteristics as for Homer City but with less frequent and smaller jumps because of Dunkirk and Huntley's location in western New York, not at the intersection of two regions. The cost of generation is raised by 50% over the Homer City plant to reflect the relative inefficiency of these plants, and the on-peak capacity factor is dropped to 40%. Capacity is adjusted appropriately. Note that the mean price level is raised to \$0.04/KWh while the expected number of jumps and jump size fall. The mean price level rises to reflect the reduced capacity factor, so while the plant is running less, the average price faced by the plant will be higher. The jumps decrease in frequency and size to reflect the less favorable location of the plant in western New York rather than between two regions, which might each experience price jumps. The resulting asset value of \$450.8 million using the mean reverting with jumps model is well over the purchase price of \$332 million (adjusted for retiring units). This result must be qualified by noting that with these plants, the buyer assumed a transition power contract with NMPC under which it is to sell electricity at guaranteed prices over the first four years of ownership, and that the model assumes an infinitely lived plant. Thus the plants will not face the (desirable) market price process in the short term, which might depress realized plant values. Still, the Dunkirk and Huntley plants would appear under-priced at \$332 million if the mean reverting with jumps model is appropriate.

IV. Conclusions

This paper has presented the derivation of a quasi-analytic solution to a real options model which is unique in the literature. The mean reverting with jumps specification is a flexible form which is potentially appropriate for a wide range of commodities and provides dramatically different valuation results when compared with standard models. The model is flexible enough to allow downward price jumps ($-2<u<0$), and the lack of restrictions on M allow a wide range of normalizations for P and C.

The results presented here suggest that the mean reverting with jumps real option model is a useful tool for electricity asset valuation, providing reasonable asset values for the given parameters. The model is relatively simple to parameterize and use despite the need for some numerical solutions. It is an inexpensive supplement to more detailed and time-consuming asset valuation methods, and incorporates the important notion of option value. Sensitivity testing of model parameters is straightforward. Unfortunately, while asset sales are occurring now, useful price data which would allow econometric parameter estimation are years away. But the price path is grounded in the realities of electricity markets and price histories of other commodities. Incorporating these realities is demonstrated to strongly influence calculated asset values. Once a number of asset sales have taken place in a region, it will be possible to calculate implied price parameters in a consistent and flexible framework.

The sale of electricity generating assets will have important effects on existing utilities and ratepayers. If the model developed here is appropriate, utilities should receive much more for generating assets than would be expected under traditional model assumptions. This will help to mitigate stranded costs and ultimately benefit ratepayers. The model also has implications for nuclear plants considering early retirement (e.g. Maine Yankee and Yankee Rowe in New England). If the real electricity price process is as favorable to investment as the mean reverting with jumps model suggests, early retirement should become less likely. High asset values will also induce higher than anticipated levels of capital investment in the electricity sector. This has favorable implications for electricity consumers in the form of highly competitive electricity markets and larger numbers of new, efficient plants.

APPENDIX

Lemma: $V(P)=\sum_{n=0}^{\infty}a_nP^{n+r}$, with a_n as defined in the text for the general solution, converges for all r and u>-2.

Proof: Using the ratio test, if $\frac{a_{n+1}P^{n+r+1}}{a_nP^{n+r}}\le c$ as $n\to\infty$, where c is a constant such that 0<c<1, then V(P) converges. Using the definition of a_n above:

$$\frac{a_{n+1}P^{n+r+1}}{a_nP^{n+r}}=$$

$$\frac{P^{n+r+1}\eta^{n+1}(r+n)!a_0\left[\left(\frac{1}{2}\sigma^2(n+r-1)+\eta\overline{P}\right)(n+r)-(\lambda+\delta)+\lambda(1+u)^{n+r}\right]!}{P^{n+r}\eta^{n}(r+n-1)!a_0\left[\left(\frac{1}{2}\sigma^2(n+r)+\eta\overline{P}\right)(n+r+1)-(\lambda+\delta)+\lambda(1+u)^{n+r+1}\right]!}$$

Simplifying:

$$\frac{a_{n+1}P^{n+r+1}}{a_nP^{n+r}}=\frac{P\eta(r+n)}{\left[\left(\frac{1}{2}\sigma^2(n+r)+\eta\overline{P}\right)(n+r+1)-(\lambda+\delta)+\lambda(1+u)^{n+r+1}\right]}$$

Clearly there exists an n^* large enough such that for all $n>n^*$, $\frac{a_{n+1}P^{n+r+1}}{a_nP^{n+r}}\le c$, where c is a constant such that 0<c<1. Thus $V(P)=\sum_{n=0}^{\infty}a_nP^{n+r}$ converges.∀

Lemma: $\overline{V}(P)=\sum_{n=0}^{\infty}a_nP^{n}$, with a_n as defined in the text for the particular solution, converges for u>-2.

Proof: Using the ratio test, if $\frac{a_{n+1}P^{n+1}}{a_n P^n} \leq c$ as $n \to \infty$, where c is a constant such that

$0<c<1$, then $\overline{V}(P)$ converges. Using the definition of a_n above:

$$\frac{a_{n+1}P^{n+1}}{a_n P^n} = \frac{P^{n+1}\eta^{n+1}(n)!a_1\left[\left(\frac{1}{2}\sigma^2(n-1)+\eta\overline{P}\right)(n)-(\lambda+\delta)+\lambda(1+u)^n\right]!}{P^n\eta^n(n-1)!a_1\left[\left(\frac{1}{2}\sigma^2(n)+\eta\overline{P}\right)(n+1)-(\lambda+\delta)+\lambda(1+u)^{n+1}\right]!}$$

Simplifying:

$$\frac{a_{n+1}P^{n+1}}{a_n P^n} = \frac{P\eta n}{\left[\left(\frac{1}{2}\sigma^2(n)+\eta\overline{P}\right)(n+1)-(\lambda+\delta)+\lambda(1+u)^{n+1}\right]}$$

Clearly there exists an n^* large enough such that for all $n>n^*$, $\frac{a_{n+1}P^{n+1}}{a_n P^n} \leq c$, where c is a

constant such that $0<c<1$. Thus $\overline{V}(P) = \sum_{n=0}^{\infty} a_n P^n$ converges.∀

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Table 1. Parameter Values Used for Comparisons

Variable	Definition	Value
$P(0)$	Initial Price in $/Kwh	$0.021
$\overline{P}$	Mean Price of Electricity in $/Kwh	$0.025
η	Rate of mean reversion	0.2
u	Size of random jump	0.8
σ	Std. Deviation of electricity prices (yearly)	0.1
λ	Frequency of jumps	0.05
δ	Interest rate	0.06
C	Cost of production	$0.021
M	Output per year in Kwh	350,000,000

Table 2. Calculated Parameter Values

Parameter	Value
r1	1.221847296
r2	-4.61931422
b2	0.088818206
c1	43,171,481,446

Table 3. Homer City Parameter Values

Variable	Definition	Value
$P(0)$	Initial Price in $/Kwh	$0.034
$\overline{P}$	Mean Price of Electricity in $/Kwh	$0.034
η	Rate of mean reversion	0.10
u	Size of random jump	0.75
σ	Std. Deviation of electricity prices (yearly)	0.35
λ	Frequency of jumps	0.025
δ	Interest rate	0.09
C	Cost of production	$0.038
M	Output per year in Kwh	7,073,074,286

Table 4. Dunkirk and Huntley Parameter Values

Variable	Definition	Value
$P(0)$	Initial Price in $/Kwh	$0.040
$\overline{P}$	Mean Price of Electricity in $/Kwh	$0.040
η	Rate of mean reversion	0.10
u	Size of random jump	0.5
σ	Std. Deviation of electricity prices (yearly)	0.35
λ	Frequency of jumps	0.02
δ	Interest rate	0.09
C	Cost of production	$0.057
M	Output per year in Kwh	1,968,914,286

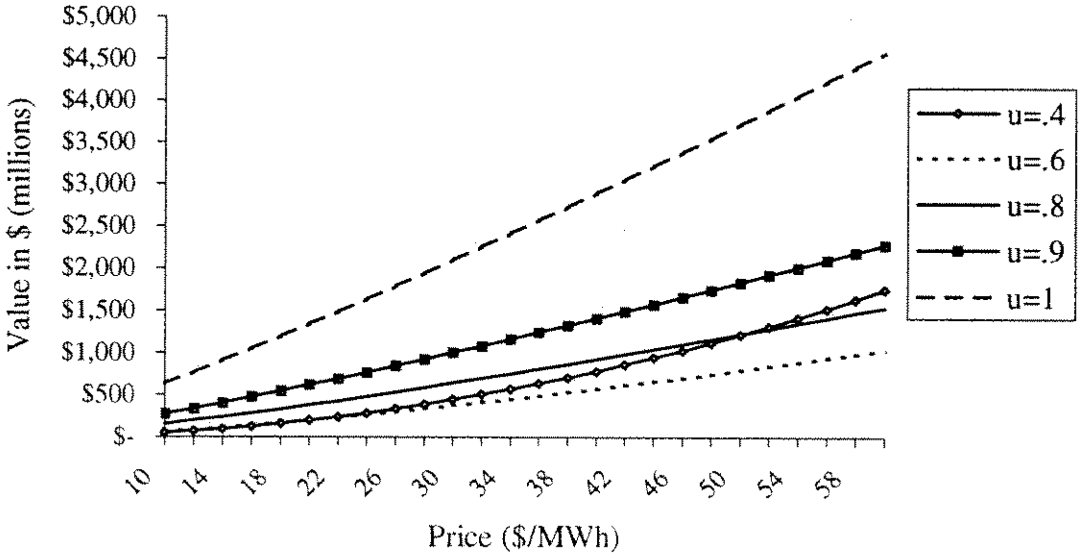

Figure 1. *V(P)* for a Range of Jump Sizes

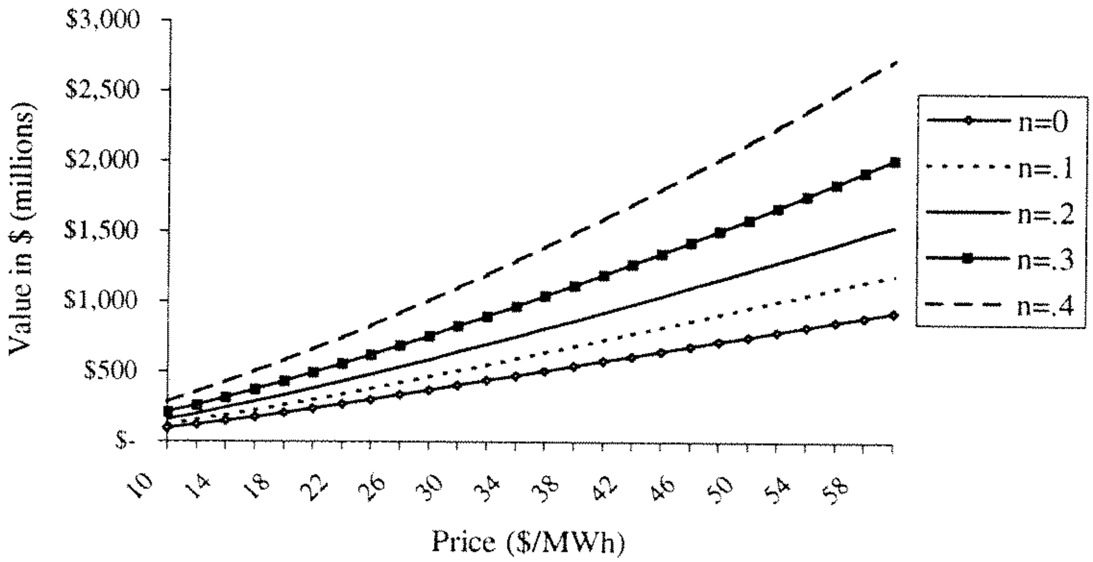

Figure 2. *V(P)* for a Range of Rates of Mean Reversion

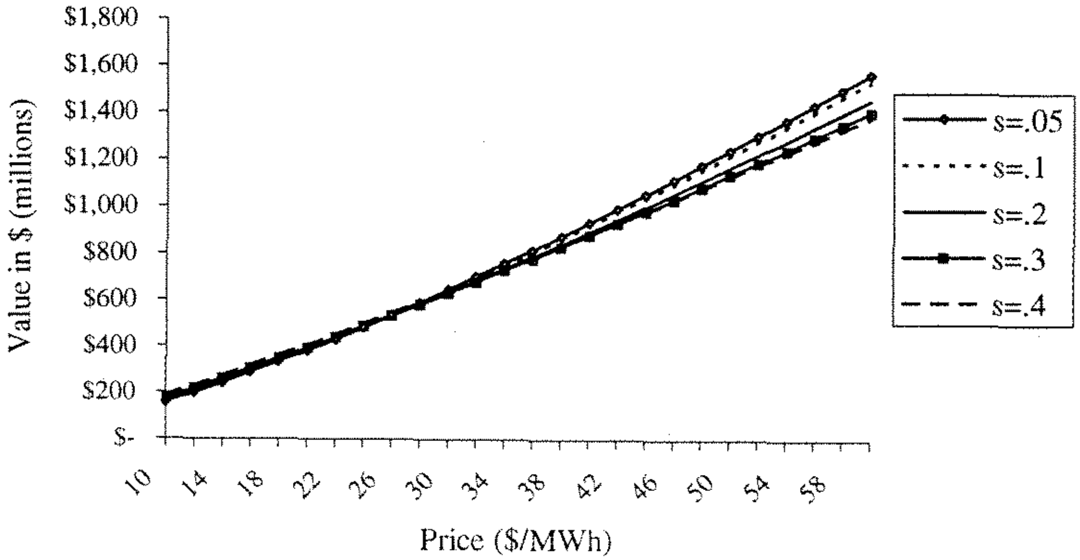

Figure 3. *V(P)* for a Range of Standard Deviations

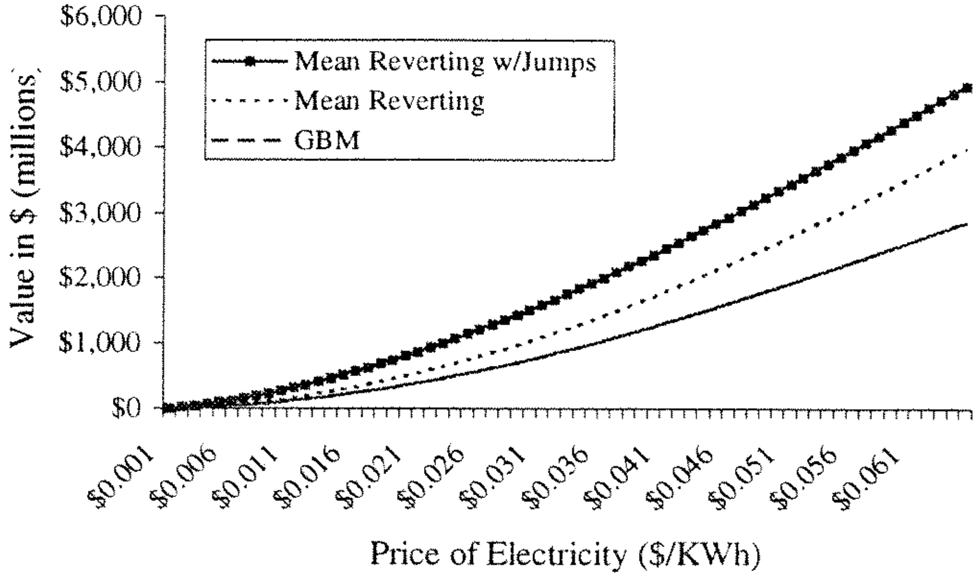

Figure 4. Homer City Plant Value Under Different Price Path Assumptions.

[1] In the United States, individual states have the power to deregulate at their own pace, and most are moving in that direction. California, for example, has a competitive wholesale market already in existence, with plans for retail competition. The Pennsylvania-New Jersey-Maryland region has had a functioning wholesale market since the spring of 1998. New York State plans to gradually introduce full competition by 2001.

[2] The total value of generating assets nationwide, assuming an average value of $450/KW, is $314 billion. There are currently 697,100 MWs of installed capacity in the United States (EIA 1997).

[3] While a generator can potentially operate in many electricity-related markets (real power, reactive power, spinning reserve, capacity), the primary market, especially for baseload plants such as the Homer City units, is the real power market. Real power, in cents/KWh, is the unit in which a typical homeowner is charged, though the costs for other aspects of electricity are bundled into the real power marginal rate.

[4] There are potentially interesting issues when considering stochastic price paths used in real option models as endogenously determined in an equilibrium context. While Lund (1993) argues that GBM can not be an equilibrium price path for exhaustible resources, Laughton (1998) notes that there are problems with mean reversion as an equilibrium price path. Because of a lack of storability, Laughton's arguments seem to be less relevant for electricity.

[5] On-peak prices are the high load 16 hours per day, generally 6am to 10pm, five days per week.

[6] For a discussion of why nodal prices are appropriate, see Schweppe et al. (1988).

[7] In a still regulated market, mean wholesale on-peak prices increased by 11.6% over the combined 1995, 1996, and 1998 average (data from Power Markets Weekly). In a volatile competitive market it is likely that this change would have been much greater. For example, Mount (1999) suggests that fairly small changes in capacity can produce large price changes in competitive markets.

[8] The size of the relevant production period can be arbitrarily small as the model can be shown to be homogeneous of degree zero in M. However, the effect of different estimation periods on the price process, and their effect on asset value, has yet to be explored.

[9] Note that similar versions of this equation, generated by related price paths (e.g. dropping the P coefficient of the mean reversion or jump components) do not seem to have convergent series solutions.

[10] For an introduction and overview to the solution using infinite series solutions, see Boyce and Diprima (1992).

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