View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Research Papers in Economics

Centre for Efficiency and Productivity Analysis

Working Paper Series
No. WP08/2011

Comparing Firm Performance Using Transitive Productivity Index Numbers in a Meta-frontier Framework

C.J. O'Donnell, S. Fallah-Fini and K, Triantis

Date:
September 2011

School of Economics
University of Queensland
St. Lucia, Qld. 4072
Australia

ISSN No. 1932 - 4398

COMPARING FIRM PERFORMANCE USING TRANSITIVE PRODUCTIVITY INDEX NUMBERS IN A META-FRONTIER FRAMEWORK

by

Christopher J. O'Donnell

Centre for Efficiency and Productivity Analysis, The University of Queensland, Brisbane 4072, Australia. E-mail: c.odonnell@economics.uq.edu.au

Saeideh Fallah-Fini (Corresponding Author)

Virginia Tech, System Performance Laboratory, Grado Department of Industrial and Systems Engineering, Blacksburg, VA, 24060, USA. Phone: (001) 617 894 0351. Fax: (001) 540 231 3322. E-mail: fallah@vt.edu

Konstantinos Triantis

Virginia Tech, System Performance Laboratory, Grado Department of Industrial and Systems Engineering, Northern Virginia, Center, Falls Church, VA, 22043, USA. E-mail: triantis@vt.edu

Abstract: The meta-frontier framework has been used extensively for evaluating the technical efficiency of heterogeneous production units that can be classified into different groups. This paper shows how the framework can also be used to make total factor productivity (TFP) comparisons within and across groups. The paper develops a new measure of the distance between a group frontier and the meta-frontier (the so-called 'technology gap'). It then shows how a spatially- and temporally-transitive TFP index can be decomposed into measures of global technical change (measuring movements in the metafrontier), local technical change (measuring movements in the group frontiers) and efficiency change (measuring movements towards or around the group frontiers). To illustrate the methodology, the paper examines the productive performance of road authorities responsible for maintaining interstate highways in the US state of Virginia.

Key Words: Meta-Frontier, Heterogeneity, Technology Gap, Productivity Indexes, Data Envelopment Analysis

1 Introduction

Productivity and efficiency are distinct but related concepts that refer to the ability of a production unit to transform a set of inputs into a set of outputs. Productivity is essentially a measure of output per unit of input, and efficiency is a measure of the distance between a given output-input combination and an optimal point on a production frontier. Frontiers can be estimated using parametric or non-parametric methods using cross-sectional or panel data on the inputs used and outputs produced by individual production units.

The most common approaches to estimating levels of productivity and efficiency assume that all production units are similar and have access to the same production technology. Under this assumption, a single frontier is usually estimated for purposes of evaluating the efficiency of all production units under analysis. However, there are many empirical contexts where the production units in the sample operate in slightly different production environments, which to say they have access to slightly different production possibilities sets. Such technological heterogeneity reflects differences in the social, physical and economic characteristics (e.g., geography, quality of the workforce, type of machinery) of the environments in which production takes place (O'Donnell, Rao and Battese (2008)). If the production units under analysis make choices from different production possibilities sets then the common approach of estimating a single technology frontier will yield efficiency and productivity estimates that do not accurately measure the capacity of production units to transform inputs into outputs.

The recently developed meta-frontier approach of Battese and Rao (2002), Battese, Rao and O'Donnell (2004) and O'Donnell et al. (2008) allows researchers to evaluate and compare the efficiency of production units that have access to different production possibilities sets. This framework has been used extensively in the literature to evaluate the efficiency of groups of firms in industries as wide-ranging as education (e.g., McMillan and Chan (2004), Worthington and Lee (2005)), finance (e.g., Kontolaimou and Tsekouras (2010)) and agriculture (e.g., Chen and Song (2008), O'Donnell et al. (2008)). The methodology involves estimating different frontiers for different groups of production units, then measuring the distances between these so-called group frontiers and a meta-frontier. The meta-frontier is a type of global frontier that envelops all the group frontiers. Rather than assessing the efficiency of individual production units with respect to the meta-frontier, the efficiency of a production unit is assessed relative to its own group frontier, and then the production environment faced by the group is assessed by measuring the distance between the group frontier and the meta-frontier. The distances between different group frontiers and the meta-frontier are known as technology gap ratios (TGRs) or meta-technology ratios (MTRs). MTRs measure the potential improvements in group performance that are possible when production units are given access to the production technologies of other groups.

A somewhat unsatisfactory feature of the standard meta-frontier methodology is that measures of distance between group frontiers and the meta-frontiers (the MTRs) depend on the way in which the frontiers are estimated (e.g.,

using an input or an output distance function representation of the technology) and are specific to the individual production units in the group (i.e., there are as many MTRs measuring the distance from the group frontier to the meta-frontier as there are production units in the group). What is often needed is a single measure of the distance between each group frontier and the meta-frontier (i.e., a single MTR) that does not depend on whether inputs or outputs are treated as fixed (i.e., on whether the group frontiers and the meta-frontier are estimated from an input-orientation or an output orientation). One of the objectives of this paper is to develop such a measure. We do this by identifying the maximum productivity that can be achieved by the production units in a particular group (TFPG*), and the maximum productivity that can be achieved by firms in any group (TFP*). The MTR for the group is simply the ratio of these two maximum productivity measures (MTR = TFPG*/TFP*).

A second feature of most meta-frontier analyses is that they focus on only one measure of firm performance, namely technical efficiency. Other measures of performance, such as total factor productivity (TFP) and scale efficiency are typically ignored. A second objective of this paper is to present these and other measures of productivity and efficiency within a meta-frontier framework. To do this, we follow O'Donnell (2008) and define measures of productivity and efficiency in terms of aggregate outputs and inputs. Among other things, the aggregate-quantity framework of O'Donnell (2008) allows us to decompose TFP indexes into measures of global technical change (i.e., changes in the position of the meta-frontier), local technical change (i.e., changes in the positions of the group frontiers), technical efficiency change (i.e., movements towards or away from the group frontiers), and scale-mix efficiency change (i.e., movements around the frontier surfaces to capture economies of scale and/or scope).

The methodology is applied to two groups of production units responsible for interstate highway maintenance in the US state of Virginia. The two groups operate under different types of contracts that limit the ways in which they can reach their production objectives: performance-based contracts (PBCs) set the minimum required conditions for the roads and traffic assets without directing the contractor to specific methods to achieve performance targets; the traditional contracting (TC) approach specifies in advance the tasks that are to be performed as well as the methods that should be used. To estimate and decompose measures of productivity and efficiency for these two groups, we estimate group frontiers and the meta-frontier using data envelopment analysis (DEA).

The structure of the paper is as follows. Section 2 explains the aggregate quantity-price framework developed by O'Donnell (2008) for measuring levels of (and changes in) productivity and efficiency. Section 3 explains how a spatially- and temporally-transitive productivity index can be decomposed within this aggregate quantity-price framework. Section 4 explains the meta-frontier framework and defines a new measure of distance between group frontiers and the meta-frontier. Section 5 shows how DEA can be used to estimate productivity indexes and measures of efficiency with respect to group frontiers and meta-frontiers. Section 6 describes the data and some relevant characteristics of interstate highway maintenance in Virginia. Section 7 reports and discusses the empirical results. Section 7 summarizes the contributions of the paper and identifies two opportunities for further research.

2 Measures of Productivity and Efficiency

The productivity and efficiency concepts used in this paper are defined within the aggregate quantity framework of O'Donnell (2008)[1]. This section summarizes this framework using language and notation that has been tailored to the analysis of panel data on *N* firms over *T* time periods.

Let $x_{it} \in \mathbb{R}_+^K$ and $q_{it} \in \mathbb{R}_+^J$ denote the non-negative input and output vectors of firm *i* in period *t*. The total factor productivity (TFP) of this firm is defined as (O'Donnell (2008)):

(1) $TFP_{it} = Q_{it} / X_{it}$ (TFP)

where $X_{it} = X(x_{it})$ and $Q_{it} = Q(q_{it})$ represent the scalar aggregate input and output corresponding to the vectors x_{it} and q_{it}, and $Q(.)$ and $X(.)$ are non-negative, non-decreasing and linearly-homogeneous aggregator functions. O'Donnell (2008) uses this definition to show how different measures of efficiency can be defined as ratios of measures of TFP. For example, let $\bar{X}_{it}$ denote the minimum aggregate input possible when using a scalar multiple of x_{it} to produce output q_{it}, and let $\hat{X}_{it}$ denote the minimum aggregate input possible using *any* input vector to produce q_{it}. Then measures of input-oriented technical and mix efficiency can be defined as (O'Donnell (2008)):

(2) $ITE_{it} = \dfrac{Q_{it} / X_{it}}{Q_{it} / \bar{X}_{it}} = \dfrac{\bar{X}_{it}}{X_{it}} \leq 1$ (Technical Efficiency) and

(3) $IME_{it} = \dfrac{Q_{it} / \bar{X}_{it}}{Q_{it} / \hat{X}_{it}} = \dfrac{\hat{X}_{it}}{\bar{X}_{it}} \leq 1$ (Pure Mix Efficiency)

To make these concepts more concrete, O'Donnell (2008) considers a simple case where firm *i* uses only two inputs (x_{1it}, x_{2it}) and where the input aggregator function is linear: $X(x_{it}) = \beta_1 x_{1it} + \beta_2 x_{2it}$. Figure 1 illustrates this case in input space. The curve passing through point B in Figure 1 is an isoquant representing all technically-efficient input combinations that can produce the same output q_{it}. The dashed line passing through point A is an iso-input line representing all possible combinations of inputs x_{1it} and x_{2it} that yield the same aggregate input X_{it} as at point A. If both the output vector and the input mix are held fixed then firm A can minimize the aggregate input by radially contracting its inputs to point B. Then the standard input-oriented measure of technical efficiency at point A can be defined in terms of the relative distances from points A and B to the origin: $ITE_{it} = \bar{X}_{it} / X_{it} = \|B\| / \|A\|$. It is clear from Figure 1 that if firm A can change the mix of its inputs then the aggregate input can further be reduced by moving around the isoquant to point U. The measure of efficiency that captures the reduction in aggregate input use associated with this change in input mix is the measure of input-oriented pure mix efficiency given by Equation (3).

[1] We use an input orientation to describe the aggregate quantity framework. The framework can also be easily described using an analogous output-orientation.

Output-oriented measures of technical and mix efficiency can also be defined and explained using similar diagrams in output space. Mix efficiency evaluates the change in productivity when restrictions on input and output mixes are relaxed (O'Donnell (2008)).

To further illustrate how measures of efficiency can be defined as ratios of measures of TFP, and to expand the discussion to general cases in which firms use many inputs to produce many outputs, O'Donnell (2008) maps technically-feasible input-output combinations into aggregate quantity space. Such a mapping is presented in Figure 2. Points A, B, and U in Figure 2 correspond to points A, B and U in Figure 1. The curve passing through points B and D in Figure 2 represents the frontier of a mix-restricted production possibilities set in which each point is an aggregate of input and output vectors that can be written as scalar multiples of x_{it} and q_{it}. The curve passing through points U and E is an unrestricted production frontier that envelops aggregates of all technically-feasible input and output vectors (i.e., the data points do not need to have the same output mix and input mix as firm A). In aggregate quantity space, the total factor productivity of each point can be viewed as the slope of the ray from the origin to that point (e.g., $TFP_{it} = Q_{it} / X_{it} = \text{Slope 0A}$). Consequently, the measures of technical and mix efficiency described above can be viewed as ratios of slopes of rays: for example, $ITE_{it} = (Q_{it} / X_{it}) / (Q_{it} / \bar{X}_{it})$ $= \text{Slope 0A/Slope 0B}$ and $IME_{it} = (Q_{it} / \bar{X}_{it}) / (Q_{it} / \hat{X}_{it}) = \text{Slope 0B/Slope 0U}$.

A third efficiency measure that can be defined terms of aggregate outputs and inputs (and can therefore be represented using slopes of rays in aggregate quantity space) is pure scale efficiency (O'Donnell (2008)):

$$(4) \qquad ISE_{it} = \frac{Q_{it} / \bar{X}_{it}}{\tilde{Q}_{it} / \tilde{X}_{it}} \leq 1 \qquad \text{(Pure Scale Efficiency)}$$

where $\tilde{Q}_{it}$ and $\tilde{X}_{it}$ are the aggregate output and input obtained when TFP is maximized subject to the constraint that the output and input vectors are scalar multiples of q_{it} and x_{it} respectively. For example, the input-oriented scale efficiency at point A in Figure 2 can be written as: $ISE_{it} = (Q_{it} / \bar{X}_{it}) / (\tilde{Q}_{it} / \tilde{X}_{it}) = \text{Slope 0B/Slope 0D}$. Pure scale efficiency is a measure of the difference between TFP at a technically efficient point (i.e., point B) and TFP at a point of optimal scale on the mix-restricted frontier (i.e., point D).

A fourth measure of efficiency introduced by O'Donnell (2010b) is scale-mix efficiency. Scale-mix efficiency is a measure of the improvement in productivity obtained by moving from a technically efficient point to a point of maximum productivity. The maximum TFP possible using the technology available in period *t* is denoted $TFP_t^* = Q_t^* / X_t^*$ and is represented in Figure 2 by the slope of the ray passing through point E: $TFP_t^* = Q_t^* / X_t^* = \text{Slope 0E}$. Mathematically, scale-mix efficiency is defined as (O'Donnell, 2010b):

$$(5) \qquad ISME_{it} = \frac{Q_{it} / \bar{X}_{it}}{TFP_t^*}. \qquad \text{(Scale-Mix Efficiency)}$$

Finally, O'Donnell (2008) defines the TFP efficiency (TFPE) of a firm as the ratio of observed TFP to the maximum TFP possible using the technology:

$$TFPE_{it} = \frac{TFP_{it}}{TFP_t^*}. \qquad \text{(TFP Efficiency)} \tag{6}$$

TFP efficiency measures the overall productive efficiency of a firm. In Figure 2, TFP efficiency is a measure of the increase in TFP as the firm moves all the way from point A to point E: $TFPE_{it}$ = Slope 0A/Slope 0E. An infinite number of decompositions of TFP efficiency are possible, and in Section 3 we focus on the decomposition implied by Equations (1), (2), (5) and (6):

$$TFPE_{it} = ITE_{it} \times ISME_{it}. \qquad \text{(TFP Efficiency)} \tag{7}$$

Further details regarding aggregate-quantity representations of production technologies and measures of efficiency are available in O'Donnell (2008) and O'Donnell (2010).

3 The Components of Productivity Change

If TFP is defined as in Equation (1) then the index number that compares the TFP of firm *i* in period *t* with the TFP of firm *h* in period *s* is

$$TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \frac{Q_{it} / X_{it}}{Q_{hs} / X_{hs}} = \frac{Q_{hs,it}}{X_{hs,it}} \tag{8}$$

where $Q_{hs,it} = Q_{it} / Q_{hs}$ is an output quantity index (a measure of output growth) and $X_{hs,it} = X_{it} / X_{hs}$ is an input quantity index (a measure of input growth). O'Donnell (2008) uses the term "multiplicatively-complete" to describe TFP indexes that can be expanded in the form of Equation (8).

Diffferent multiplicatively-complete TFP indexes can be constructed by choosing different aggregator functions. If the aggregator functions are fixed for all possible binary comparisons then the TFP index defined by (8) satisfies a commonsense transitivity test. The transitivity test says that a direct comparison of the TFP of two firms should yield the same estimate of TFP change as an indirect comparison through a third firm (i.e., $TFP_{hs,it} = TFP_{hs,kr} \times TFP_{kr,it}$). O'Donnell (2010b) constructs transitive indexes using the aggregator functions $Q(q) = p_0'q$ and $X(x) = w_0'x$ where p_0 and w_0 are representative price vectors. The output, input and TFP indexes obtained using these aggregator functions are

$$Q_{hs,it} = \frac{p_0'q_{it}}{p_0'q_{hs}} \tag{9}$$

(10) $$X_{hs,it} = \frac{w_0' x_{it}}{w_0' x_{hs}} \quad \text{and}$$

(11) $$TFP_{hs,it} = \left(\frac{p_0' q_{it}}{p_0' q_{hs}} \right) \Big/ \left(\frac{w_0' x_{it}}{w_0' x_{hs}} \right).$$

O'Donnell (2010b) refers to TFP index defined by Equation (11) as a Lowe index because the component output and input quantity indexes defined by Equations (9) and (10) have been attributed to Lowe (1823). The Lowe TFP index is temporally- and spatially-transitive and can be used to make multi-temporal (i.e., many period) and/or multi-lateral (i.e., many firm) comparisons of TFP and efficiency. As we shall see in Section 4 below, this means it can be used within a meta-frontier framework to measure the gaps between group frontiers and the meta-frontier. Section 5 will describe how the fixed vectors p_0 and w_0 can be chosen and/or calculated.

O'Donnell (2008) shows that all multiplicatively-complete TFP indexes can be decomposed into a measure of technical change and the measures of efficiency change given by Equations (2) to (5). To see this, it is useful to re-arrange Equation (6) as $TFP_{it} = TFP_t^* \times TFPE_{it}$. It follows that the TFP index defined by Equation (8) can be written as

(12) $$TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \left(\frac{TFP_t^*}{TFP_s^*} \right) \left(\frac{TFPE_{it}}{TFPE_{hs}} \right).$$

The term (TFP_t^* / TFP_s^*) in Equation (12) measures the change in the maximum TFP possible using the production technologies available in periods *s* and *t*. O'Donnell (2008) sees this as a natural measure of technical change. When this ratio takes a value greater (less) than one, the industry has experienced technical progress (regress). The second term on the right hand side of Equation (12) is a measure of overall efficiency change. O'Donnell (2010b) shows that this term can be further decomposed into various measures of technical, scale and mix efficiency change. For example, Equation (7) implies that Equation (12) can be decomposed more finely as

(13) $$TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \left(\frac{TFP_t^*}{TFP_s^*} \right) \left(\frac{ITE_{it}}{ITE_{hs}} \right) \left(\frac{ISME_{it}}{ISME_{hs}} \right).$$

Equation (13) reveals that a change in TFP can be decomposed into (i) a measure of technical change representing movements in the production frontier; (ii) a measure of technical efficiency change representing movements towards or away from the frontier; and (iii) a measure of scale-mix efficiency change representing movements around the frontier surface to capture economies of scale and scope. Several other input- and output-oriented decompositions of TFP change are discussed in O'Donnell (2008). In this paper, the focus is on the decomposition given by Equation (13).

4 Productivity Measurement in a Meta-frontier Framework

This section embeds measures of productivity change within a meta-frontier framework and shows how points of maximum TFP can be used to measure the gaps between group frontiers and the meta-frontier (i.e., meta-technology ratios).

Let $z \in \mathbb{R}^M$ be a vector of exogenous variables characterizing a particular production environment. The set of output-input combinations that can be produced in this environment (i.e., the production possibilities set) is formally defined as:

(14) $T(z) = \{(x, q)$: x can produce q in an environment characterized by $z\}$.

For example, if $M = 1$ and $z = t$ then Equation (14) defines the set of output-input combinations that are possible in period t: $T(t) = \{(x, q)$: x can produce q in period $t\}$. The unrestricted production frontier depicted in Figure 2 represents the boundary of such a set in aggregate quantity space. In the terminology of Battese and Rao (2002), this unrestricted frontier is the "meta-frontier" in period t. In Section 2, the maximum TFP possible using this "meta-technology" was denoted TFP_t^*.

Even in period t, some firms may operate in more or less restrictive production environments than others. Suppose the set of firms that comprise the meta-technology set $T(t)$ can be divided into $G(>1)$ groups, where each group is characterized by its own production environment. Let $z_{gt} \in \mathbb{R}^M$ be the vector of exogenous factors characterizing the production environment of firms in group g in period t. The environmental constraints facing different groups will prevent firms in the groups from choosing from the full range of technologically feasible input-output combinations in the meta-technology set $T(t)$. We define the group-specific technology sets containing the input-output combinations available to firms in the g-th group in period t as:

(15) $T(z_{gt}) = \{(x,q)$: x can produce q in an environment characterized by $z_{gt}\}$.

In the terminology of Battese and Rao (2002), the boundaries of these restricted technology sets are "group-frontiers". In this paper we let $TFP_{gt}^* = Q_{gt}^* / X_{gt}^*$ denote the maximum TFP possible using the technology available to group g in period t. We make the following standard assumptions concerning the relationship between the period-t group frontiers and the period-t meta-frontier (e.g., O'Donnell et al. (2008, p. 235)):

R1. If $(x, y) \in T(z_{gt})$ for any g then $(x, y) \in T(t)$;

R2. If $(x, y) \in T(t)$ then $(x, y) \in T(z_{gt})$ for some g; and

R3. $T(t) = \{T(z_{1t}) \cup T(z_{2t}) ... \cup T(z_{Gt})\}$.

The measures of efficiency defined by Equations (2) to (6) in Section 2 are still valid within a meta-frontier framework, although their usefulness is limited by the fact that they only measure efficiency with respect to the meta-frontier, and for many firms the meta-frontier may be out of reach. If firm *i* is a member of group *g* then it may be more meaningful to measure efficiency with respect to the group-*g* frontier. In this paper we decompose the overall measure of productive efficiency given by Equation (6) into the following components:

$$(16) \qquad TFPE_{it} = \frac{TFP_{it}}{TFP_t^*} = \left(\frac{TFP_{it}}{TFP_{gt}^*}\right)\left(\frac{TFP_{gt}^*}{TFP_t^*}\right) = TFPEG_{it} \times MTR_{gt} \qquad \text{(TFP Efficiency)}$$

where $TFPEG_{it} \equiv TFP_{it} / TFP_{gt}^*$ compares the TFP of the firm to the maximum TFP that is possible in the group-*g* production environment,[2] and $MTR_{gt} \equiv TFP_{gt}^* / TFP_t^*$ is a meta-technology ratio comparing the maximum TFP possible in the group-*g* environment to the maximum TFP that is possible in *any* production environment. Meta-technology ratios that are less than one indicate the existence of a technology gap between the group frontier and the meta-frontier. For example, if $TFP_{gt}^* = 0.70$ and $TFP_t^* = 0.85$ then $MTR_{gt} = 0.7 / 0.85 = 0.82$. This indicates that the maximum productivity that can be achieved using the group-*g* production technology is only 82% of the maximum productivity that is feasible using the meta-technology.

These concepts are illustrated in Figure 3 where the curve labeled M-M' represents the meta-frontier and the curves labeled 1-1' and 2-2' represent two group-frontiers. This figure shows that the maximum TFP possible using a group-specific technology is always less than or equal to the maximum TFP possible using the meta-technology. Equivalently, the TFP efficiency of each observation with respect to its group frontier is always greater than or equal to the TFP efficiency of the same observation with respect to the meta-frontier (i.e., $TFPEG_{it} \geq TFPE_{it}$). Equivalently, the meta-frontier always envelops the group frontiers.

In the same way that the overall measure of efficiency TFPE was decomposed into various measures of efficiency in Section 2, the group-specific measure TFPEG can be decomposed into group-specific measures of technical, scale and mix efficiency. For example, the group-level analogue of Equation (7) is

$$(17) \qquad TFPEG_{it} = ITEG_{it} \times ISMEG_{it} \qquad \text{(TFP Efficiency with respect to the group frontier)}$$

---

[2] In the remainder of the paper we append the letter G to particular efficiency acronyms to indicate when measurements are taken with respect to a group frontier rather than the metafrontier. For example, TFPEG and ITEG denote measures of TFP efficiency and input-oriented technical efficiency with respect to a firm's group frontier, while TFPE and ITE denote measures of efficiency with respect to the metafrontier.

where $ITEG_{it}$ denotes input-oriented technical efficiency measured with respect to the group frontier, and $ISMEG_{it}$ denotes input-oriented scale-mix efficiency measured with respect to the group frontier. Group-level measures of input-oriented scale efficiency (ISEG) and input-oriented mix efficiency (IMEG) are also available, as are group-level measures of efficiency defined using an output-orientation.

Finally, consider the TFP index that compares the TFP of firm *i* in period *t* with the TFP of firm *h* in period *s*, and let firm *i* be a member of group *g* and firm *h* be a member of group *l*. Equations (12) and (17) imply that this TFP index can be decomposed as:

$$(18) \qquad TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{MTR_{gt}}{MTR_{ls}}\right)\left(\frac{ITEG_{it}}{ITEG_{hs}}\right)\left(\frac{ISMEG_{it}}{ISMEG_{hs}}\right).$$

Equation (18) reveals that a change in TFP can be decomposed into (i) a measure of global technical change representing movements in the meta-frontier; (ii) a measure of local technical change representing movements in the group frontiers relative to the meta-frontier (i.e., a measure of changes in the group-specific production environments); (iii) a measure of technical efficiency change representing movements towards or away from the group frontier(s); and (iv) a measure of scale-mix efficiency change representing movements around the group frontier surface(s) to capture economies of scale and scope. Finer decompositions involving input- and output-oriented measures of pure scale and mix efficiency of the type discussed in O'Donnell (2008) are also available.

5 Estimation Methods

Measuring and decomposing levels (or changes in) different types of efficiency necessarily involves estimating the boundary of the production possibilities set (i.e., the production frontier). In this paper we estimate the frontier and associated measures of efficiency using data envelopment analysis (DEA). DEA linear programs (LPs) for estimating technical and scale efficiency are well-known, and LPs for measuring levels of mix and scale-mix efficiencies have been developed by O'Donnell (2010) and O'Donnell (2010b). For example, LPs for measuring Farrell (1957) measures of output- and input-oriented technical efficiency with respect to the meta-frontier are:

$$(19) \qquad OTE_{it} = D_O(x_{it}, q_{it}, t) = \min_{\lambda,\theta}\left\{\lambda^{-1} : \lambda q_{it} \le Q\theta; X\theta \le x_{it}; \theta' \iota_{NT} = 1; \theta \ge 0\right\} \qquad \text{and}$$

$$(20) \qquad ITE_{it} = D_I(x_{it}, q_{it}, t)^{-1} = \min_{\rho,\theta}\left\{\rho : Q\theta \ge q_{it}; \rho x_{it} \ge X\theta; \theta' \iota_{NT} = 1; \theta \ge 0\right\}$$

where $D_O(x_{it}, q_{it}, t)$ is the Shephard (1953) output distance function representing the production meta-technology available in period *t*; $D_I(x_{it}, q_{it}, t)$ is the input distance function (an alternative representation of the meta-technology); Q is a $J \times NT$ matrix of observed outputs; X is a $K \times NT$ matrix of observed inputs; Z is an

$M \times NT$ matrix of environmental variables; θ is an $NT \times 1$ vector of LP decision variables; and ι_{NT} is an $NT \times 1$ unit vector. Both of these LPs allow the technology to exhibit variable returns to scale (VRS). If the technology is assumed to exhibit constant returns to scale (CRS) then the constraint $\theta' \iota_{NT} = 1$ must be deleted from each problem. If the technology is assumed to exhibit non-increasing returns to scale (NIRS) then $\theta' \iota_{NT} = 1$ must be replaced with $\theta' \iota_{NT} \leq 1$. Both problems also use all NT observations in the data set to define the frontier, implying the meta-technology does not exhibit technical change. If the meta-technology is permitted to exhibit both technical regress and technical progress then only observations in period t should be used to estimate the period-t frontier. If the technology is permitted to exhibit technical progress but not technical regress then observations up to and including period t should be used to estimate the period-t frontier. Finally, if technical efficiency is being measured with respect to a group frontier then only observations on firms within that group should be used to estimate the frontier.

The measure of pure scale efficiency given by Equation (4) is estimated as $ISE_{it} = ITE_{it}^{CRS} / ITE_{it}$ where ITE_{it} is the solution to LP (20) and ITE_{it}^{CRS} is the solution to LP (20) under the assumption the technology exhibits constant returns to scale. Output-oriented scale efficiency is estimated as $OSE_{it} = OTE_{it}^{CRS} / OTE_{it}$ where OTE_{it} and OTE_{it}^{CRS} are the solutions to LP (19) under VRS and CRS assumptions respectively. Measures of output- and input-oriented mix efficiency and maximum TFP depend on the aggregator functions used to construct productivity indexes. In the case of the transitive Lowe TFP index given by Equation (11) the measures of mix efficiency with respect to the meta-frontier are computed as

(21) $OME_{it} = \bar{Q}_{it} / \hat{Q}_{it}$ and

(22) $IME_{it} = \hat{X}_{it} / \bar{X}_{it}$

where $\bar{Q}_{it} = Q_{it} / OTE_{it} = p_0' q_{it} / OTE_{it}$; $\bar{X}_{it} = X_{it} \times ITE_{it} = w_0' x_{it} \times ITE_{it}$; and $\hat{Q}_{it}$ and $\hat{X}_{it}$ are the solutions to the following variants of LPs (19) and (20) (O'Donnell (2010b)):

(23) $\hat{Q}_{it} = \max_{\theta,q} \{ p_0' q : q \leq Q\theta; X\theta \leq x_{it}; \theta' \iota_{NT} = 1; \theta \geq 0 \}$ and

(24) $\hat{X}_{it} = \min_{\theta,x} \{ w_0' x : Q\theta \geq q_{it}; x \geq X\theta; \theta' \iota_{NT} = 1; \theta \geq 0 \}$.

For the Lowe TFP index given by Equation (11), the maximum TFP that is possible using the meta-technology available in period t is computed by simple enumeration. That is,

(25) $TFP_t^* = \max_i TFP_{it} = \max_i p_0' q_{it} / w_0' x_{it}$.

Similar problems involving subsets of data points (see the discussion above) are used to identify measures of efficiency with respect to group frontiers. In our empirical work we estimate these various frontiers and measures of efficiency using the DPIN 3.0 software available at http://www.uq.edu.au/economics/cepa/dpin.htm.

Finally, implementing this methodology involves selecting representative price vectors p_0 and w_0. If market prices are available then O'Donnell (2010b) suggests using an average of the prices faced by all the firms involved in the comparison (i.e., all the price observations in the data set). If market prices are unavailable then one possibility suggested by O'Donnell (2011) is to use the arithmetic average of the shadow prices associated with LPs (19) and (20)[3]. This paper follows this suggestion: p_0 is set equal to the average of the shadow prices associated with the output constraints in LP (19), and w_0 is set equal to the average of the shadow prices associated with the input constraints in LP (20).

6 Data

We use the concepts and methods discussed in Sections 4 and 5 to evaluate and compare the productivity of different highway maintenance contracting strategies in the US state of Virginia. In the past twenty years, the American Society of Civil Engineers (ASCE) has constantly rated the US road system as being in a poor condition (ASCE (2009a)). As pointed by the ASCE, US road authorities have been facing a huge gap between the actual level of capital investment and the amount that is needed to significantly improve the condition of the nation's road system (ASCE (2009b)). The badly deteriorated road system, major budgetary restrictions, as well as the significant growth in traffic have been placing road authorities under constant pressure. In particular, road authorities are being pressured to improve performance of the existing highway maintenance policies and practices to preserve a safe, reliable, and efficient road infrastructure that can support society's needs (TRB (2006)).

Some Departments of Transportation (DOTs) have responded to these pressures through the use of performance-based contracts. A performance-based contract (PBC) sets the minimum required conditions for the roads and traffic assets without directing the contractor to specific methods to achieve performance targets. This characteristic motivates the contractors to be innovative in project design (i.e., the tasks that should be performed) and selection of tools and materials. Moreover, PBCs are mid-term to long-term arrangements that tend to optimize the costs of maintenance over the project's lifecycle. In the TC approach, the tasks that are to be performed as well as the methods that should be used are specified in advance.

The different limitations, regulations, and maintenance practices associated with different contracting methods/strategies mean that the contractors or road authorities that are operating under different types of contracts are effectively operating in different operating/production environments. In mathematical terms, contractors that use different types of contracting methods potentially have access to (possibly non-intersecting) subsets of the production possibilities set. The methods described in Sections 2 to 5 allow us to measure and compare the performance of firms operating under the two contracting strategies (PBC and TC). The results of this comparison

[3] Technically, the Lowe TFP index then becomes an estimated Färe-Primont TFP index. For details, see O'Donnell (2011).

may help identify contracting strategies that lend themselves most easily to satisfactory maintenance of Virginia's Interstate.

The dataset used in this paper comprises observations on two output and two input variables: the outputs are AREA = lane-miles maintained (miles) and CCI = change in pavement condition (an index); the input variables are COST = real maintenance expenditures (an index) and LOAD = reciprocal of traffic load (ESAL[4]). The data relate to approximately 180 miles (across seven counties referred to as counties 1 to 7) of Virginia's Interstate highways that were maintained by the Virginia Department of Transportation (VDOT) using traditional maintenance practices (TC) during the fiscal years 2002 to 2006, and another 250 miles (across ten counties referred to as counties 8 to 17) of highways maintained using a performance-based maintenance strategy (PBC) over the fiscal years 2002 to 2005. The output variable AREA captures the extent of the workload each county has performed. The output variable CCI measures the change in the condition of maintained road sections with respect to load-related and non load-related stresses. The input variable COST measures the real cost of maintaining road sections using traditional or performance-based contracting. The BHWA-Highway and Street Construction Cost Index developed by the Bureau of Labor Statistics was used as a deflator to adjust the cost data of different years. Traffic volume and load are factors outside the control of the decision maker that lead to deteriorations in road conditions due to vehicle forces. Since higher values for traffic load lead to deteriorations in road conditions, the reciprocal of traffic load has been treated as an (uncontrollable) input variable (i.e., all other things being equal, increases in LOAD represent decreases in traffic load and will be associated with higher levels of the CCI output).

Given the nature of the expenditure data (only provided at the county level), counties were chosen as the decision making units (DMUs). The data covered only counties in Virginia where sections of the Interstate system were maintained using traditional or performance-based maintenance practices. The data set contained 25 observations on DMUs that use TC practices and 26 observations on DMUs that use PBC practices.

Finally, climate is an uncontrollable factor that can affect both maintenance efforts and the deterioration of the paved lanes. Data on some climate variables (e.g., minimum and maximum temperature, total rainfall, and total snowfall) were available from the National Climate Data Center NCDC (2010) for all the counties involved in the analysis for the relevant years. However, in view of the limited number of observations in the sample, we chose not to explicitly include climate variables in the analysis and instead we allow the effects of climate to be reflected in measures of technical change. Thus, we take a broad view of technical change and associate movements in production frontiers with environmental factors (such as climate condition, soil characteristics, etc.) as well as changes in technical knowledge (e.g., as a result of research and development expenditure). To allow for relatively

[4] Equivalent Single Axle Load

smooth rates of technical progress and regress, observations in a moving window of two periods were used to define the frontier for firms in period *t*.

7 Results

Measures of productivity and efficiency for all firms in the sample are reported in Table 1. The levels of TFP reported in this table are Lowe TFP indexes computed using average shadow prices as representative prices (as discussed in Section 5). The efficiency estimates reported in Table 1 are estimates of efficiency with respect to the meta-frontier. Measures of efficiency with respect to group frontiers will be presented later in this section. Estimates of efficiency levels with respect to the meta-frontier and the group frontiers have been obtained under the assumption the meta-technology and group technologies all exhibit variable returns to scale.

Measures of TFP (Change)

Columns 3 and 4 in Table 1 report the aggregate outputs (Q) and aggregate inputs (X) of all firms in the sample. These results allow us to compute levels of productivity (TFP) and subsequently make inter-temporal and inter-spatial comparisons of both productivity and efficiency. For example, the TFP of county 3 in period 1 is computed as $TFP_{31} = Q_{31} / X_{31}$ = 0.477/7.115 = 0.067, and the index number that compares the productivity of county 3 in period 1 with its productivity in period 4 is $TFP_{31,34} = TFP_{34} / TFP_{31}$ = 0.133/0.067 = 1.99. This value indicates that productivity in county 3 almost doubled over the four periods. By way of further example, the index number that compares the productivity of county 7 in period 2 (the county/period that maximized TFP in that period) with the productivity of county 3 in period 4 is $TFP_{34,72} = TFP_{72} / TFP_{34}$ = 0.324/0.133 = 2.436, indicating that county 7 was more than twice as productive in period 2 as county 3 was in period 4. Such index numbers are transitive, which means, for example, that the index that compares the productivity of county 7 in period 2 with the productivity of county 3 in period 1 can be computed either directly (i.e., $TFP_{31,72} = TFP_{72} / TFP_{31}$ = 0.324/0.067 = 4.836) or indirectly (i.e., $TFP_{31,72} = TFP_{31,34} \times TFP_{34,72}$ = 1.99 × 2.436 = 4.836).

Maximum TFP and Measures of Efficiency (Change) With Respect to the Meta-frontier

Column 6 in Table 1 (labeled TFP*) reports the maximum TFP possible in each period using the meta-technology. This column indicates that technical regress occurred between periods three and four. An examination of climate data for periods three and four reveals that most counties experienced a significant increase in snowfall in period four. For example, the snowfall in county 3 (from the TC group) was 11.8 inches in period 3 and 23 inches in period 4. The severe snowfalls experienced by all counties in period four are likely to have caused deteriorations in road conditions, in turn forcing road authorities to use more inputs to maintain the road in the same condition (i.e.,

causing maximum possible TFP to fall). Thus, the effects of climate are a possible reason for measured technical regress in period four.

Column 7 in Table 1 (labeled TFPE) reports levels of TFP efficiency (with respect to the meta-frontier) computed using Equation (6). The remaining columns of Table 1 report the technical, scale, mix and scale-mix efficiency components of TFPE. The interpretation of these estimates is straightforward. For example, the third row of Table 1 reveals that the productive efficiency of county 3 in period 1 was $TFPE_{31} = TFP_{31} / TFP_1^* = 0.067 / 0.324 = 0.207$. This overall measure of productive efficiency can be decomposed as $TFPE_{31} = OTE_{31} \times OSME_{31}$ = 0.819 × 0.253 = 0.207, indicating that most of the productivity shortfall was due to output-oriented scale and mix inefficiency. The output-oriented scale-mix efficiency component $(OSME_{31})$ can be further decomposed into a pure scale efficiency component $(OSE_{31} = 0.959)$ and a residual component that is neither pure output-oriented scale efficiency nor pure output-oriented mix efficiency. It can also be decomposed into a pure mix efficiency component $(OME_{31} = 0.975)$ and a (different) residual component. An input-oriented decomposition of TFPE is also possible: $TFPE_{31} = ITE_{31} \times ISME_{31} = 0.876 \times 0.237 = 0.207$. Further decomposition of the input-oriented scale-mix component reveals a 58% shortfall in productivity due to poor input mix $(IME_{31} = 0.420)$.

A more complete picture of productivity and efficiency levels in county 3 is provided in Figure 4. This figure plots levels of TFP and selected measures of efficiency over the four periods (time is measured on the horizontal axis). Observe that TFP increased steadily over the sample period, and this increase was mainly driven by an increase in TFPE. Also observe that the technical regress that occurred in period four (due to the severe snowfall and climate conditions discussed above) was more than offset by a 46% increase in TFPE. This 46% increase in TFP could be attributed to either a 64% increase in technical efficiency (i.e., $\Delta TFPE = \Delta OTE \times \Delta OSME$ = 1.64 × 0.89 = 1.46) or a 70% increase in scale-mix efficiency (i.e., $\Delta TFPE = \Delta ITE \times \Delta ISME$ = 0.86 × 1.70 = 1.46) depending on whether it is viewed from an output-oriented or an input-oriented perspective.

It is worth noting that road authorities in each county decide on the number of road sections (AREA) that should be maintained and the level of maintenance operations that should be performed. The level of maintenance operations determines the level of improvement in the quality of the road sections, something that is captured by the output variable CCI (change in pavement condition). For any fixed level of inputs, directing more inputs towards improving the condition of road sections (i.e., increasing CCI) will mean that fewer road sections can be maintained (i.e., AREA falls). Output-oriented mix efficiency is a measure of the extent to which road authorities have chosen the most productive CCI:AREA ratio. Input-oriented mix efficiency is a less relevant measure of performance for road authorities because LOAD is a largely non-discretionary (i.e., uncontrolled) input.

Maximum TFP and Measures of Efficiency (Change) With Respect to the Group Frontiers

Tables 2 and 3 report measures of productivity and efficiency for all counties in the TC and PBC groups respectively[5]. The aggregate inputs (X), aggregate outputs (Q), and measures of productivity (TFP) reported in these tables are identical to the estimates reported earlier in Table 1. However, the estimates of efficiency reported in Tables 2 and 3 differ from the estimates reported in Table 1 because the latter were computed with respect to the metafrontier, not the group frontiers.

A comparison of column 6 (labeled TFPG*) in Table 2 with column 6 (labeled TFP*) in Table 1 reveals that the maximum TFP possible in each period using the meta-technology was equal to the maximum TFP possible using the group-TC technology. This implies that the group-TC frontier coincides with the meta-frontier in the region of constant returns to scale. The remaining columns of Table 2 report the technical, scale, mix and scale-mix efficiency components of TFPEG (estimated with respect to the group-TC frontier).

A similar comparison of column 6 (labeled TFPG*) in Table 3 with column 6 (labeled TFP*) in Table 1 reveals that the maximum TFP possible in each period using the group-PBC technology is always less than the maximum TFP possible using the meta-technology. This means there is a gap between the group-PBC frontier and the meta-frontier in each period. To get a better idea of this gap, Figure 5 presents the estimated group frontiers and the estimated metafrontier for periods 1 and 2 (two periods are used because all frontiers are estimated using a window of size 2)[6]. The pairs of numbers in this figure are references to counties and time periods (e.g., "7, 2" is a reference to county 7 in period 2). The meta-technology ratio that measures the gap between the group-PBC frontier and the metafrontier is the difference between TFP at points "12, 2" and "7, 2" in Figure 5: $MTR_{21} = TFPG^{*}_{21} / TFP^{*}_{1} = TFP_{12,2} / TFP_{7,2}$ = 0.160/0.324 = 0.494. This estimated MTR indicates that the maximum TFP that could be achieved using the group-PBC technology in periods 1 and 2 was only half of the maximum TFP that was feasible using the meta-technology (unrestricted technology).

The group-frontier estimates reported in Tables 2 and 3 can be used to effect various decompositions of the TFP efficiency estimates reported in Table 1. Using Equation (16), for example, the overall measure of productive efficiency of county 9 in period 1 can be decomposed as $TFPE_{91} = TFPEG_{91} \times MTR_{21} = 0.836 \times 0.494 = 0.412$. This implies that the relatively poor productive performance of county 9 in period 1 was mainly due to the technology gap in period 1 (this is evident in Figure 5). The output-oriented analogue of Equation (17) can be used to effect an

[5] The TC and PBC groups are sometimes referred to as groups 1 and 2, respectively.

[6] The frontiers in Figure 5 can be viewed as DEA estimates of the theoretical frontiers depicted earlier in Figure 3. In Figure 5, the dotted line between points "2, 2" and "14, 2" forms part of the estimated metafrontier if and only if the meta-technology set is (assumed to be) convex.

even finer decomposition of the efficiency of this county: $TFPE_{91} = OTEG_{91} \times OSMEG_{91} \times MTR_{21}$ = 0.893 × 0.936 × 0.494 = 0.412. The output-oriented scale-mix efficiency component $(OSMEG_{91})$ can be further decomposed into a pure scale efficiency component $(OSEG_{91} = 1)$ and a residual component that is neither pure output-oriented scale nor pure output-oriented mix efficiency. It can also be decomposed into a pure mix efficiency component $(OMEG_{91} = 0.992)$ and a (different) residual component. Input-oriented decompositions are also available (e.g., $TFPEG_{91} = ITEG_{91} \times ISMEG_{91}$ = 0.922 × 0.906 = 0.836).

TFP indexes can be easily decomposed in ways that explicitly recognize the gaps that may exist between group frontiers and the meta-frontier. Using Equation (18), for example, the index that compares the TFP of county 4 (from the TC group) in period 2 with the TFP of county 9 (from the PBC group) in period 1 can be decomposed as:

$$(25) \qquad TFP_{91,42} \equiv \frac{TFP_{42}}{TFP_{91}} = \left(\frac{TFP_2^*}{TFP_1^*}\right)\left(\frac{MTR_{12}}{MTR_{21}}\right)\left(\frac{OTEG_{42}}{OTEG_{91}}\right)\left(\frac{OSMEG_{42}}{OSMEG_{91}}\right)$$

.

$$= \left(\frac{0.324}{0.324}\right)\left(\frac{1}{0.494}\right)\left(\frac{0.784}{0.893}\right)\left(\frac{0.462}{0.936}\right) = 1 \times 2.02 \times 0.877 \times 0.494 = 0.877$$

Note that there is no global technical change (i.e., no movement in meta-frontier) between periods 1 and 2 by design (i.e., $TFP_2^* / TFP_1^* = 1$ because of the use of a moving window of size 2). The ratio $MTR_{12} / MTR_{21} = 2.02$ indicates that the group-TC frontier in period 2 is closer to the meta-frontier than the group-PBC frontier in period 1, at least in the region of constant returns to scale (in fact, as we have seen in Figure 5, the group-TC frontier *is* the meta-frontier in the region of constant returns to scale).

In the standard meta-frontier approach of Battese and Rao (2002), Battese et al. (2004) and O'Donnell et al. (2008), gaps between the group frontiers and the meta-frontier are measured using ratios of input- or output-oriented technical efficiency scores. Specifically, the standard input- and output-oriented meta-technology ratios for firm *i* in period *t* are:

$$(26) \qquad IMTR_{it} \equiv \frac{ITE_{it}}{ITEG_{it}} \qquad \text{and}$$

$$(27) \qquad OMTR_{it} \equiv \frac{OTE_{it}}{OTEG_{it}}.$$

The input-oriented measure given by Equation (26) is an appropriate measure of the technology gap in situations where ITE is an appropriate measure of productivity change – that is, when the output vector and the input mix are fixed (see Section 2). Similarly, the output-oriented measure given by Equation (27) is an appropriate measure of the technology gap when the input vector and the output mix are fixed. If all inputs and outputs are variable (e.g., in

the 'long run') then the appropriate measure of the technology gap is the (i.e., observation-invariant) measure defined in Section 4 and discussed above. It is easily shown that:

$$(28) \qquad MTR_{gt} = \frac{OSME_{it}}{OSMEG_{it}} \times OMTR_{it} = \frac{ISME_{it}}{ISMEG_{it}} \times IMTR_{it}.$$

This implies that standard measures of the technology gap and the new measure developed in this paper will be identical whenever firms are fully scale and mix efficient (e.g., whenever single-input single-output firms have access to a constant-returns-to-scale technology).

Evaluating the "local" MTRs defined by Equations (26) and (27) is straightforward using the efficiency estimates reported in Tables 1 and 3. For example, the estimated output-oriented MTR for county 9 in period 1 is $OMTR_{91} = OTE_{91} / OTEG_{91} = 0.649 / 0.893 = 0.726$. This estimate indicates that this county is 27.4% less productive than it could have been had it been given access to the meta-technology (and had it also been required to keep its input vector and its output mix fixed). A single output-oriented measure of the technology gap is often obtained by taking the arithmetic average of Equation (27) over all firms in the group in period *t*: $\overline{OMTR}_{gt} = (1/N_{gt}) \sum_{i}^{N_{gt}} OMTR_{it}$ where N_{gt} denotes the number of firms in group *g* in period *t*. For example, the average output- and input-oriented MTRs for group-PBC in period 1 are $\overline{OMTR}_{21} = 0.906$ and $\overline{IMTR}_{21} = 0.931$. These estimates are higher than the "global" estimate reported earlier (i.e., $MTR_{21} = 0.494$) because they measure (shorter) distances to the boundaries of restricted production possibilities sets (the production possibilities sets are restricted in the sense that either the input level and the output mix must be held fixed, or the output level and the input mix must be held fixed).

8 Conclusion

The meta-frontier framework of Battese and Rao (2002), Battese et al. (2004) and O'Donnell et al. (2008) is commonly used to compare firm performance in empirical contexts where different subsets of firms have access to different production technologies. The measures of firm performance that are most widely used in the meta-frontier literature are output- and input-oriented measures of technical efficiency. In the output-oriented case, for example, it is standard practice to measure the technical efficiency of a firm with respect to both a group frontier (OTEG) and the meta-frontier (OTE). The ratio of these two technical efficiency scores is an output-oriented meta-technology ratio (OMTR = OTE/OTEG). This meta-technology ratio is first and foremost a measure of the 'technology gap' between the group frontier and the meta-frontier. However, it can also be viewed as a measure of the productivity gains that could be realized by a technically-efficient firm if it were to be given access to the meta-technology and forced to hold its input vector and its output mix fixed. An analogous input-oriented meta-technology ratio (IMTR

= ITE/ITEG) can be viewed as a measure of the productivity gains that could be realized if a technically-efficient firm were to be given access to the meta-technology and forced to hold its output vector and its input mix fixed.

In many empirical contexts it is unrealistic (and often unprofitable) to hold input or output vectors or mixes fixed. One of the contributions of this paper has been to develop an alternative measure of the technology gap that avoids such restrictions. Computing this measure involves estimating the maximum productivity that is possible using the group technology (TFP_g^*) and the maximum productivity that is possible using the meta-technology (TFP^*). The ratio of these two maximum productivity levels ($MTR = TFP_g^* / TFP^*$) is a natural measure of the gap between the group frontier and the meta-frontier. It can also be viewed as a measure of the productivity gains that could be realized by a technically efficient firm if it were to be given access to the meta-technology and if no restrictions were placed on input or output levels or mixes.

A second contribution of this paper has been to demonstrate how a temporally- and spatially-transitive measure of productivity change (ΔTFP) can be exhaustively decomposed into a global measure of technical change (ΔTFP^*), local measures of technical change (ΔMTR), measures of technical efficiency change (ΔOTEG or ΔITEG) and measures of scale and mix efficiency change (ΔOSMEG or ΔISMEG). Implementing this decomposition involves estimating the group frontiers and the meta-frontier. In this paper, data on road maintenance contractors was used to estimate separate DEA frontiers for contractors operating under performance based contracts (PBCs) and contractors operating under traditional contracts (TC). The results indicated that in every period the maximum productivity possible under TCs was higher than the maximum productivity possible under PBCs. Our results also indicated that the main driver of productivity change in the industry was efficiency change (i.e., ΔOTEG × ΔOSMEG = ΔITEG × ΔISMEG).

The productivity decomposition methodology developed in this paper can be applied in any empirical context where the standard meta-frontier methodology would normally be applied. Estimated meta-technology ratios of the type reported in this paper will be of particular interest to managers and policy-makers who have some capacity to change the production environment. In this context there are at least two opportunities for further research. First, there is a need to develop measures of reliability (e.g., standard errors) for meta-technology ratios and associated measures of productivity and efficiency change. Second, statistical methods (i.e., hypothesis tests) need to be developed and/or used to assess the validity of different assumptions concerning returns to scale, levels of technical efficiency, and the nature of technical change. It may be easier to pursue some of these research opportunities in a parametric rather than a non-parametric framework (i.e., using stochastic frontier analysis instead of DEA).

9 References

ASCE (2009a). "American Society of Civil Engineers: Reportcard for America's Infrastructure." Retrieved August 2011.

ASCE (2009b). "American Society of Civil Engineers: Facts About Roads." Retrieved August 2011.

Battese, G. E. and D. S. P. Rao (2002). "Technology Gap, Efficiency, and a Stochastic Metafrontier Function." *International Journal of Business and Economics* 1(2): 87-93.

Battese, G. E., D. S. P. Rao and C. J. O'Donnell (2004). "A Metafrontier Production Function for Estimation of Technical Efficiencies and Technology Gaps for Firms Operating Under Different Technologies." *Journal of Productivity Analysis* 21(1): 91-103.

Chen, Z. and S. Song (2008). "Efficiency and technology gap in China's agriculture: A regional meta-frontier analysis." *China Economic Review* 19(2): 287-296.

Kontolaimou, A. and K. Tsekouras (2010). "Are cooperatives the weakest link in European banking? A non-parametric metafrontier approach." *Journal of Banking & Finance* 34: 1946-1957.

Laurenceson, J. and C. O'Donnell (2011). "New Estimates and a Decomposition of Provincial Productivity Change in China." WP04/2011, University of Queensland.

Lowe, J. (1823). *The Present State of England in Regard to Agriculture, Trade and Finance.* . London, Longman, Hurst, Rees, Orme and Brown.

McMillan, M. and W. Chan (2004). "University efficiency: a comparison and consolidation of results from stochastic and non-stochastic methods." *University of Alberta Economics Working Paper Series*: No. 2005–04.

NCDC (2010). "National Climate Data Center Archive." http://www.ncdc.noaa.gov/oa/mpp/freedata.html Retrieved May 2009.

O'Donnell, C. (2008). "An aggregate quantity-price framework for measuring and decomposing productivity and profitability change." *Centre for Efficiency and Productivity Analysis Working Papers*: WP07/2008, University of Queensland.

O'Donnell, C. (2010b). "Nonparametric Estimates Of The Components Of Productivity And Profitability Change In U.S. Agriculture." *Centre for Efficiency and Productivity Analysis Working Papers*: WP02/2010, University of Queensland.

O'Donnell, C. J. (2010). "Measuring and Decomposing Agricultural Productivity and Profitability Change." *Australian Journal of Agricultural and Resource Economics* 54(4): 527-560.

O'Donnell, C. J. (2011). "The Sources of Productivity Change in the Manufacturing Sectors of the U.S. Economy." *Centre for Efficiency and Productivity Analysis Working Papers* WP07/2011. University of Queensland.

O'Donnell, C. J., D. S. P. Rao and G. E. Battese (2008). "Metafrontier Frameworks for the Study of Firm-level Efficiencies and Technology Ratios." *Empirical Economics* 34(2): 231-255.

TRB (2006). "Maintenance and Operations of Transportation Facilities 2005 Strategic Vision".

Worthington, A. and B. Lee (2005). "Efficiency, technology and productivity change in Australian universities, 1998–2003." *University of Wollongong School of Accounting and Finance Working Paper Series*: No. 05/01.

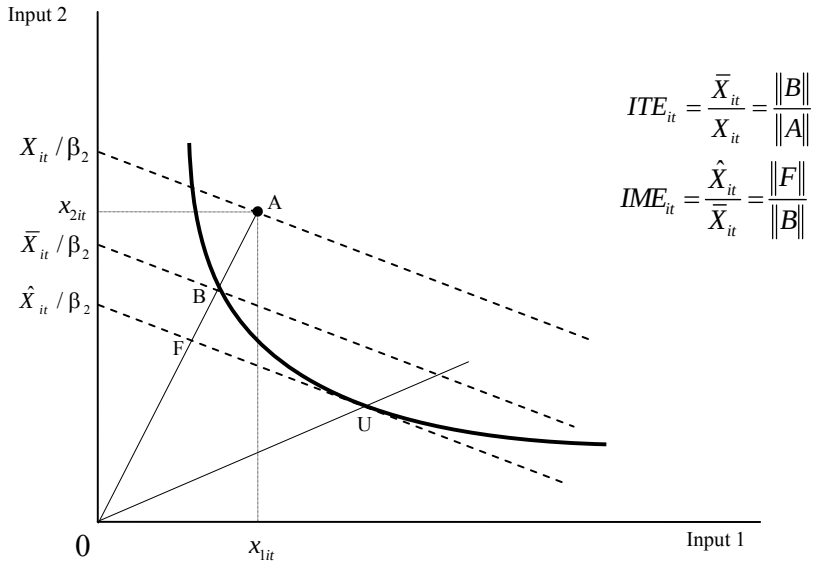

Figure 1: Input-Oriented Technical and Mix Efficiency for a Two-Input Firm
Source: O'Donnell (2008)

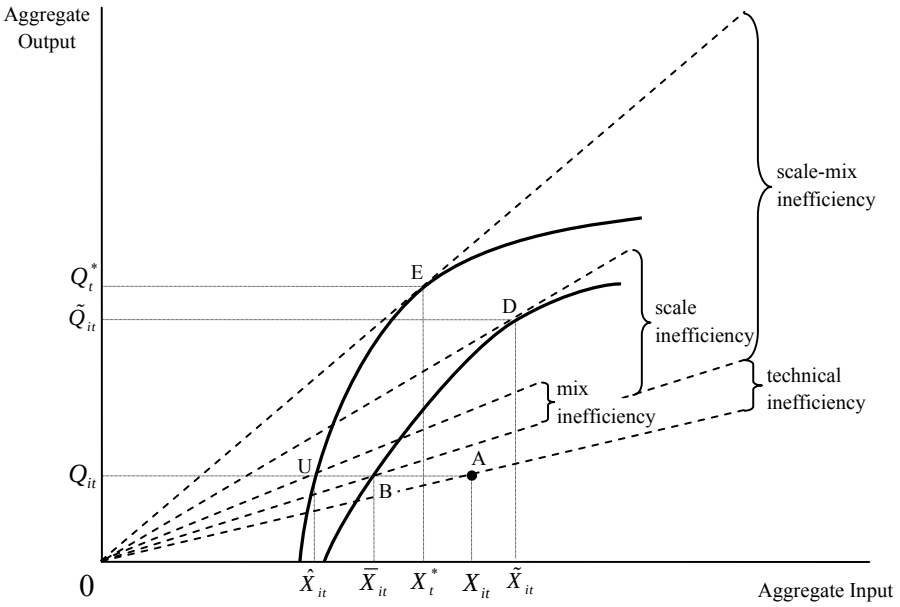

Figure 2: Input-Oriented Measures of Efficiency for a Multiple-Input Multiple-Output Firm
Source: Laurenceson and O'Donnell (2011)

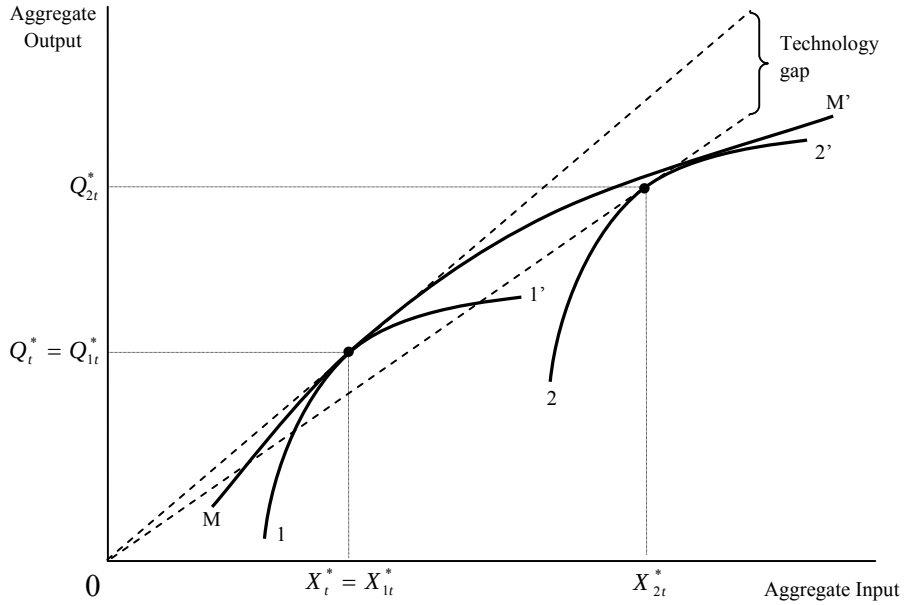

Figure 3: Productivity and Technology Gaps

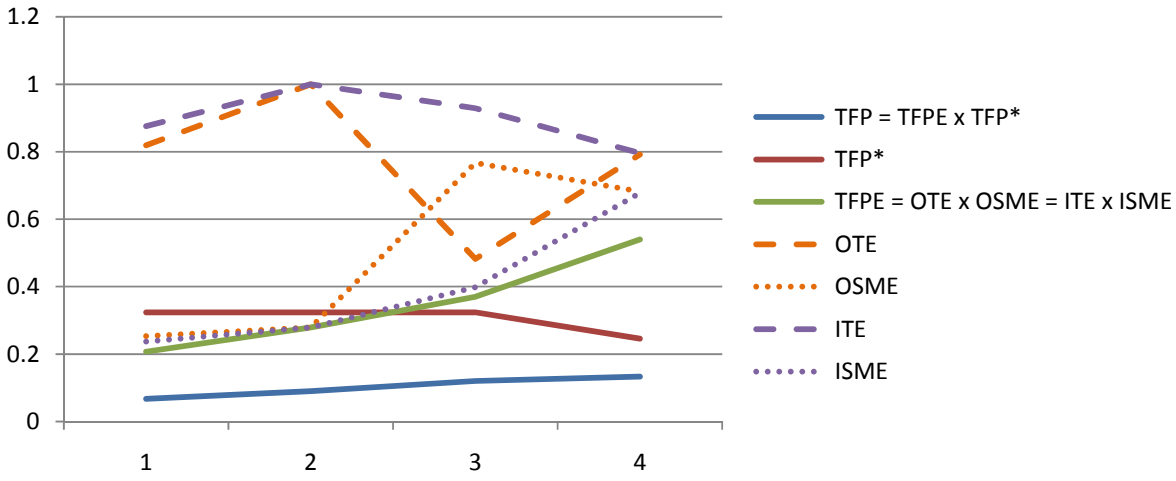

Figure 4: Changes in TFP, Maximum TFP and Measures of Efficiency: County 3

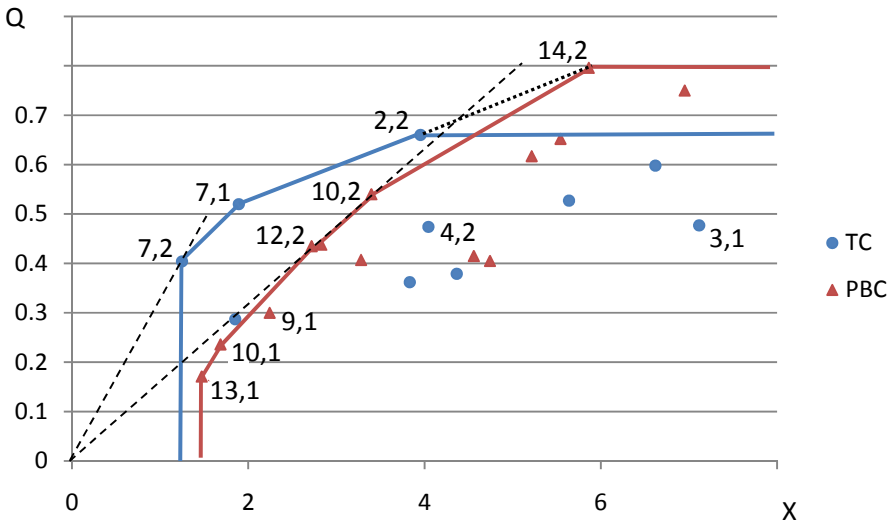

Figure 5: Group Frontiers and the Metafrontier in Periods 1 and 2

Table 1: Measures of Productivity and Efficiency With Respect to the Meta-frontier

County	Period	Q	X	TFP	TFP*	TFPE	OTE	OSE	OME	OSME	ITE	ISE	IME	ISME
2	1	0.527	5.639	0.093	0.324	0.289	0.746	0.623	0.930	0.387	0.478	0.972	0.989	0.604
3	1	**0.477**	**7.115**	**0.067**	**0.324**	**0.207**	**0.819**	**0.959**	**0.975**	**0.253**	**0.876**	0.896	**0.420**	**0.237**
4	1	0.362	3.832	0.094	0.324	0.292	0.577	0.881	0.961	0.506	0.524	0.970	0.893	0.557
5	1	0.287	1.852	0.155	0.324	0.478	1	0.641	1	0.478	1	0.641	0.736	0.478
6	1	0.379	4.365	0.087	0.324	0.268	0.704	0.747	0.854	0.381	0.571	0.922	0.618	0.470
7	1	0.520	1.893	0.275	0.324	0.849	1	1	1	0.849	1	1	1	0.849
8	1	0.750	6.958	0.108	0.324	0.333	1	0.606	0.942	0.333	1	0.606	1	0.333
9	1	0.300	2.252	0.133	0.324	**0.412**	0.649	0.822	0.917	0.635	0.619	0.862	0.896	0.665
10	1	0.236	1.678	0.141	0.324	0.435	1	0.847	0.929	0.435	1	0.847	0.744	0.435
12	1	0.617	5.218	0.118	0.324	0.366	1	0.934	1	0.366	1	0.934	1	0.366
13	1	0.171	1.482	0.115	0.324	0.356	1	0.693	1	0.356	1	0.693	0.842	0.356
16	1	0.206	1.846	0.112	0.324	0.345	0.628	0.904	0.885	0.549	0.788	0.720	0.858	0.438
17	1	0.174	1.605	0.109	0.324	0.335	0.888	0.707	0.960	0.378	0.954	0.658	0.816	0.351
2	2	0.660	3.954	0.167	0.324	0.516	1	1	1	0.516	1	1	1	0.516
3	2	0.598	6.619	0.090	0.324	0.279	1	1	1	0.279	1	1	0.586	0.279
4	2	0.474	4.044	0.117	0.324	0.362	0.709	0.809	0.998	0.510	0.574	0.999	0.993	0.630
5	2	0.297	2.578	0.115	0.324	0.356	0.649	0.996	0.858	0.549	0.943	0.685	0.618	0.377
7	2	0.404	1.248	**0.324**	0.324	1	1	1	1	1	1	1	1	1
8	2	0.405	4.740	0.085	0.324	0.264	0.620	0.808	0.911	0.426	0.504	0.994	0.966	0.524
9	2	0.407	3.289	0.124	0.324	0.382	0.709	0.734	0.931	0.539	0.537	0.968	0.950	0.711
10	2	0.540	3.391	0.159	0.324	0.492	0.975	0.676	0.888	0.505	0.710	0.928	0.904	0.693
12	2	0.435	2.723	0.160	0.324	0.494	0.761	0.881	0.990	0.649	0.683	0.981	0.957	0.723
13	2	0.438	2.826	0.155	0.324	0.479	0.797	0.812	0.941	0.601	0.661	0.980	0.925	0.725
14	2	0.796	5.871	0.136	0.324	0.419	1	0.684	1	0.419	1	0.684	1	0.419
15	2	0.415	4.559	0.091	0.324	0.281	0.625	0.812	0.943	0.450	0.510	0.994	0.969	0.551
16	2	0.652	5.550	0.118	0.324	0.363	0.912	0.619	0.995	0.399	0.781	0.722	0.884	0.465
17	2	0.479	4.326	0.111	0.324	0.342	0.738	0.748	0.954	0.463	0.620	0.891	0.956	0.552
1	3	0.325	3.181	0.102	0.324	0.315	0.595	0.804	0.936	0.530	0.531	0.901	0.823	0.594
2	3	0.797	6.903	0.115	0.324	0.357	1	0.666	1	0.357	1	0.666	1	0.357
3	3	0.204	1.703	0.120	0.324	0.370	0.482	0.991	0.980	0.767	0.929	0.515	0.789	0.398
4	3	0.370	2.409	0.154	0.324	0.475	0.852	0.950	0.867	0.558	0.883	0.917	0.738	0.538
5	3	0.329	2.368	0.139	0.324	0.429	0.720	0.927	0.989	0.597	0.850	0.785	0.714	0.505
7	3	0.428	1.740	0.246	0.324	0.761	1	1	1	0.761	1	1	1	0.761
8	3	0.356	4.012	0.089	0.324	0.274	0.555	0.829	0.983	0.493	0.487	0.946	0.842	0.562
9	3	0.312	2.364	0.132	0.324	0.408	0.644	0.782	0.962	0.633	0.552	0.912	0.964	0.739
10	3	0.208	1.766	0.118	0.324	0.364	0.809	0.847	0.681	0.450	0.877	0.781	0.806	0.415
11	3	0.285	1.422	0.201	0.324	0.620	1	1	1	0.620	1	1	1	0.620
12	3	0.432	2.985	0.145	0.324	0.447	0.960	0.799	0.825	0.466	0.928	0.826	0.931	0.482
13	3	0.346	2.224	0.155	0.324	0.480	0.724	0.993	0.966	0.663	0.774	0.929	0.852	0.621
14	3	0.603	4.010	0.151	0.324	0.465	0.935	0.776	0.992	0.497	0.873	0.831	0.961	0.532
15	3	0.218	1.992	0.110	0.324	0.339	0.674	0.919	0.987	0.503	0.747	0.829	0.839	0.453
16	3	0.186	1.734	0.108	0.324	0.332	0.531	0.970	0.940	0.626	0.776	0.664	0.928	0.428
17	3	0.322	3.224	0.100	0.324	0.309	0.596	0.812	0.942	0.518	0.511	0.947	0.947	0.605
1	4	0.156	1.970	0.079	0.246	0.321	0.482	0.923	0.995	0.665	0.664	0.670	0.765	0.483
2	4	0.380	4.316	0.088	0.246	0.358	0.681	0.793	0.998	0.525	0.546	0.989	0.682	0.655
3	4	0.335	2.522	**0.133**	0.246	0.540	0.792	0.829	0.969	0.682	0.796	0.824	0.751	0.678
4	4	0.263	2.511	0.105	0.246	0.426	0.789	0.988	0.784	0.540	0.814	0.958	0.718	0.523
5	4	0.130	0.920	0.142	0.246	0.575	1	0.816	1	0.575	1	0.816	1	0.575
7	4	0.409	2.015	0.203	0.246	0.824	1	1	1	0.824	1	1	0.839	0.824
1	5	0.383	2.432	0.158	0.203	0.776	1	1	0.949	0.776	1	1	1	0.776
2	5	0.399	2.701	0.148	0.203	0.727	1	1	1	0.727	1	1	0.808	0.727

Table 2: Measures of Productivity and Efficiency With Respect to the Group Frontier – TC Counties

County	Period	Q	X	TFP	TFPG*	TFPEG	OTEG	OSEG	OMEG	OSMEG	ITEG	ISEG	IMEG	ISMEG
2	1	0.527	5.639	0.093	0.324	0.289	0.857	0.542	0.936	0.337	0.488	0.952	0.969	0.591
3	1	0.477	7.115	0.067	0.324	0.207	0.824	0.953	0.970	0.252	0.876	0.896	0.420	0.237
4	1	0.362	3.832	0.094	0.324	0.292	0.604	0.842	0.970	0.483	0.524	0.970	0.893	0.557
5	1	0.287	1.852	0.155	0.324	0.478	1	0.641	1	0.478	1	0.641	0.736	0.478
6	1	0.379	4.365	0.087	0.324	0.268	0.714	0.736	0.841	0.375	0.571	0.922	0.618	0.470
7	1	0.520	1.893	0.275	0.324	0.849	1	1	1	0.849	1	1	1	0.849
2	2	0.660	3.954	0.167	0.324	0.516	1	1	1	0.516	1	1	1	0.516
3	2	0.598	6.619	0.090	0.324	0.279	1	1	1	0.279	1	1	0.586	0.279
4	2	0.474	4.044	0.117	0.324	0.362	**0.784**	0.732	0.987	**0.462**	0.574	0.999	0.993	0.630
5	2	0.297	2.578	0.115	0.324	0.356	0.656	0.984	0.850	0.542	0.943	0.685	0.618	0.377
7	2	0.404	1.248	0.324	0.324	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	3	0.325	3.181	0.102	0.324	0.315	0.598	0.801	0.943	0.528	0.540	0.885	0.809	0.584
2	3	0.797	6.903	0.115	0.324	0.357	1	0.666	1	0.357	1	0.666	1	0.357
3	3	0.204	1.703	0.120	0.324	0.370	0.483	0.989	0.983	0.766	0.929	0.515	0.789	0.398
4	3	0.370	2.409	0.154	0.324	0.475	0.865	0.936	0.857	0.549	0.901	0.899	0.724	0.527
5	3	0.329	2.368	0.139	0.324	0.429	0.728	0.917	0.992	0.590	0.850	0.785	0.714	0.505
7	3	0.428	1.740	0.246	0.324	0.761	1	1	1	0.761	1	1	1	0.761
1	4	0.156	1.970	0.079	0.246	0.321	1	1	1	0.321	1	1	0.858	0.321
2	4	0.380	4.316	0.088	0.246	0.358	0.919	0.612	0.755	0.389	0.563	0.999	0.698	0.636
3	4	0.335	2.522	0.133	0.246	0.540	0.807	0.813	0.952	0.669	0.796	0.824	0.751	0.678
4	4	0.263	2.511	0.105	0.246	0.426	0.807	0.967	0.843	0.528	0.859	0.908	0.680	0.496
5	4	0.130	0.920	0.142	0.246	0.575	1	1	1	0.575	1	1	1	0.575
7	4	0.409	2.015	0.203	0.246	0.824	1	1	1	0.824	1	1	1	0.824
1	5	0.383	2.432	0.158	0.203	0.776	1	1	0.949	0.776	1	1	1	0.776
2	5	0.399	2.701	0.148	0.203	0.727	1	1	1	0.727	1	1	0.808	0.727

Table 3: Measures of Productivity and Efficiency With Respect to the Group Frontier – PBC Counties

County	Period	Q	X	TFP	TFPG*	TFPEG	OTEG	OSEG	OMEG	OSMEG	ITEG	ISEG	IMEG	ISMEG
8	1	0.750	6.952	0.108	0.160	0.674	1	0.895	0.942	0.674	1	0.895	1	0.674
9	1	0.300	2.243	0.133	0.160	**0.836**	**0.893**	**1**	**0.992**	**0.936**	**0.922**	0.968	0.983	**0.906**
10	1	0.236	1.685	0.140	0.160	0.876	1	1	1	0.876	1	1	1	0.876
12	1	0.617	5.216	0.118	0.160	0.740	1	1	1	0.740	1	1	1	0.740
13	1	0.171	1.472	0.116	0.160	0.726	1	0.967	1	0.726	1	0.967	1	0.726
16	1	0.206	1.850	0.111	0.160	0.697	0.911	0.982	0.864	0.765	0.918	0.974	0.969	0.758
17	1	0.174	1.611	0.108	0.160	0.676	0.957	0.970	0.900	0.706	0.962	0.965	0.970	0.702
8	2	0.405	4.743	0.085	0.160	0.533	0.793	0.999	0.805	0.672	0.792	1	0.876	0.673
9	2	0.407	3.280	0.124	0.160	0.775	0.793	1	0.991	0.978	0.799	0.992	0.989	0.970
10	2	0.540	3.397	0.159	0.160	0.993	1	1	1	0.993	1	1	1	0.993
12	2	0.435	2.719	0.160	0.160	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	2	0.438	2.829	0.155	0.160	0.968	1	0.993	0.983	0.968	1	0.993	0.979	0.968
14	2	0.796	5.864	0.136	0.160	0.849	1	1	1	0.849	1	1	1	0.849
15	2	0.415	4.559	0.091	0.160	0.568	0.792	0.999	0.832	0.717	0.791	1	0.900	0.718
16	2	0.652	5.543	0.118	0.160	0.736	0.994	0.851	0.919	0.740	0.990	0.855	0.860	0.743
17	2	0.479	4.320	0.111	0.160	0.693	0.818	0.997	0.973	0.847	0.823	0.991	0.991	0.842
8	3	0.356	4.016	0.089	0.202	0.439	0.642	1	0.923	0.684	0.644	0.996	0.952	0.681
9	3	0.312	2.366	0.132	0.202	0.653	0.792	1	0.997	0.825	0.994	0.797	0.840	0.657
10	3	0.208	1.762	0.118	0.202	0.586	1	1	0.637	0.586	1	1	1	0.586
11	3	0.285	1.413	0.202	0.202	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	3	0.432	2.979	0.145	0.202	0.719	1	1	0.952	0.719	1	1	1	0.719
13	3	0.346	2.217	0.156	0.202	0.772	1	1	0.914	0.772	1	1	1	0.772
14	3	0.603	4.014	0.150	0.202	0.745	1	1	1	0.745	1	1	1	0.745
15	3	0.218	1.988	0.110	0.202	0.544	0.754	0.996	0.926	0.722	0.792	0.949	0.952	0.688
16	3	0.186	1.726	0.108	0.202	0.535	0.690	0.995	0.839	0.775	0.992	0.693	0.930	0.539
17	3	0.322	3.223	0.100	0.202	0.496	0.669	0.991	0.944	0.741	0.806	0.823	0.903	0.615