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THE SOURCES OF PRODUCTIVITY CHANGE IN THE MANUFACTURING SECTORS OF THE U.S. ECONOMY

by

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Abstract: The U.S. Bureau of Labor Statistics measures productivity change using an index formula that fails a transitivity test. This means the Bureau is likely to report productivity changes even when outputs and inputs in different (non-adjacent) periods are identical. I use alternative formulas that a) satisfy all economically-relevant tests from index theory and b) can be decomposed into measures of technical change and efficiency change. I find the main sources of productivity change are scale and mix efficiency change. This supports the view that most firms are technically efficient and rationally change their production plans in response to changes in prices.

KEYWORDS: Data Envelopment Analysis, Malmquist Index, Lowe Index, Färe-Primont Index, Geometric Young Index, Mix Efficiency

1. Introduction

Well-known drivers of productivity change include technical change and changes in measures of technical and scale efficiency (e.g., Nishimizu and Page (1982)). Technical change is essentially a measure of movements in the production frontier associated with changes in the stock of scientific knowledge and/or other characteristics of the production environment. Technical efficiency change is a measure of movements towards or away from the frontier, almost always associated with the adoption of new technologies and/or changes in the number of errors made during the production process. Scale efficiency change is a measure of movements around the frontier surface, often in response to changes in relative prices and/or other production incentives.

There are at least two reasons for wanting to identify the drivers of productivity change. First, all other things being equal, productivity growth that is driven by technical progress and/or increases in technical efficiency will always be associated with higher net returns. However, as I explain later in the paper, productivity growth that is driven by increases in scale efficiency will often be associated with lower net returns. Thus, identifying the technical change and efficiency change components of productivity change is critically important for determining whether productivity growth is associated with higher or lower net returns (and welfare). Second, different policies will generally have different effects on the different components of productivity change. For example, research and development (R&D) policies can be expected to have a larger effect on rates of technical change than on levels of scale efficiency. Similarly, policies designed to move firms[1] closer to the best-practice frontier (e.g., education and training programs) or increase levels of scale efficiency (e.g., changes in taxes and subsidies) are unlikely to increase productivity if firms are already fully technically efficient and operating at an optimal scale.

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[1] The term 'firms' is used generically in reference to decision-making units (e.g., individuals, industries, states, regions, countries).

In an influential[2] paper in the *American Economic Review*, Färe et al. (1994) use data envelopment analysis (DEA) to estimate and decompose the Malmquist productivity index of Caves, Christensen and Diewert (1982a, p. 1404). DEA estimation and decomposition of Malmquist indexes is now widespread in the productivity literature (Lovell (2003, p. 438)). This is unfortunate because, except in restrictive special cases, DEA estimates of Malmquist indexes are unreliable measures of productivity change. To demonstrate, later in this paper I provide an example where a firm is able to produce the same output using fewer inputs and yet the DEA estimate of the Malmquist index indicates that productivity is unchanged.

The widespread use of DEA to estimate Malmquist indexes can be attributed to three main factors. First, it can be computed without the need for price data – all that is needed is an estimate of the production technology. However, there are now at least two other indexes that can also be used to measure productivity change without the need for price data – a Hicks-Moorsteen index[3] proposed by Bjurek (1996) and a Färe-Primont index[4] proposed by O'Donnell (2011). Like the Malmquist index, these productivity indexes require an estimate of the production technology. In the simple example mentioned above, where a firm produces the same output using less input, DEA estimates of the Hicks-Moorsteen and Färe-Primont indexes quite sensibly indicate that productivity has increased.

Second, Färe et al. (1994) show that the Malmquist index can be decomposed into a measure of technical change and a measure of technical efficiency change. Indeed, until recently it seemed that the Malmquist index was the only productivity index that could be exhaustively decomposed into the measures of technical change and efficiency change that policy-makers need. However, O'Donnell (2008) has recently demonstrated that all theoretically-meaningful productivity indexes can be exhaustively decomposed into such measures.

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2 The paper has been cited more than 660 times since 1996 (Scopus).

3 The name 'Hicks-Moorsteen' has been used by Färe, Grosskopf and Roos (1996), Briec and Kerstens (2004) and Briec and Kerstens (2011). This usage derives from the fact that the index is the geometric average of two productivity indexes that Diewert (1992, p. 240) attributes to Hicks (1961) and Moorsteen (1961). However, Nemoto and Goto (2005) refer to the index as a Hicks-Moorsteen-Bjurek index.

4 I refer to the O'Donnell (2011) index as a 'Fare-Primont' index because it can be written as the ratio of two indexes defined by Färe and Primont (1995, p. 36, 38).

In the simple example mentioned above, the estimated increases in the Hicks-Moorsteen and Färe-Primont indexes can be fully attributed to increases in scale and mix efficiency (i.e., economies of scale and scope). Grifell-Tatjé and Lovell (1995) argue that, irrespective of how it is estimated, the Malmquist index ignores productivity changes associated with changes in scale. Later in this paper I provide evidence that DEA estimates of the Malmquist index may also fail to capture productivity changes associated with changes in scope (i.e., changes in output mix and input mix).

Finally, Lovell (2003, p. 438) attributes the popularity of the Malmquist index in part to the fact that DEA linear programs for computing and/or decomposing it have been incorporated into at least two software packages. The DEAP 2.1 software is especially popular because it is available free-of-charge. In this paper I develop DEA linear programs for computing and decomposing Hicks-Moorsteen and Färe-Primont indexes. These linear programs have recently been incorporated into an edition of the DPIN 3.0 software that is also available free-of-charge.

Within the large class of productivity indexes that can be broken into recognizable components, some indexes are more reliable than in others. For example, the Färe-Primont index can be used to make reliable multi-lateral and multi-temporal comparisons (i.e., comparisons involving many firms and time periods) but the Hicks-Moorsteen index can only be used to make reliable binary comparisons (i.e., comparisons involving only two firms or two time periods). This is because the Hicks-Moorsteen index fails the transitivity test of Fisher (1922). Transitivity means that a direct comparison of the productivity of two firms/periods will yield the same estimate of productivity change as an indirect comparison through a third firm/period. To illustrate the importance of transitivity, later in this paper I consider a simple case where a firm uses the same inputs to produce the same outputs in two different periods. A direct comparison of the two observations plausibly yields a Hicks-Moorsteen index value of one, indicating that productivity is unchanged, but an indirect comparison through a third observation yields a value of 1.18, indicating that productivity has increased by 18%. In contrast, direct and indirect comparisons using the Färe-Primont index yield index values of one.

If prices are available then the menu of available productivity indexes expands to include Törnqvist and Fisher indexes. The Törnqvist index is widely used in the growth accounting literature where it is better known as the Solow residual (see Timmer, O'Mahony and van Ark (2007, p. 65)). It is also used by the Bureau of Labor Statistics (BLS) to measure manufacturing sector productivity growth. The Fisher index is used by several statistical agencies, including the US Department of Agriculture (USDA) and the Australian Bureau of Agricultural and Resource Economics (ABARE), to measure farm sector productivity growth. Unfortunately, neither of these two indexes is transitive, so they can only be used to make binary comparisons. To make multi-lateral or multi-temporal comparisons, it is common to compute transitive versions of the Törnqvist and Fisher indexes using a geometric averaging procedure due to Elteto and Koves (1964) and Szulc (1964). However, although they may be transitive, these so-called Törnqvist-EKS and Fisher-EKS indexes[5] fail another fundamentally important property of index numbers – the identity axiom. The identity axiom says that if two firms produce the same outputs using the same inputs then the index should take the value one (i.e., indicate that the firms are equally productive). In this paper I provide an example where two firms choose exactly the same output-input combinations but the Fisher-EKS and Törnqvist-EKS indexes take values ranging from 0.97 to 1.17. In contrast, the Färe-Primont index satisfies both the identity axiom and the transitivity test and takes the value one. If prices are available then at least two other indexes also satisfy the identity axiom and the transitivity test and so can also be used for multi-lateral and multi-temporal comparisons. One of these is the Lowe productivity index proposed by O'Donnell (2010b), and the other is a Geometric Young index that has not yet received any attention in the productivity literature.

[5] The Fisher-EKS index is also known simply as the EKS index (e.g., Fox (2003)). The Tornquist-EKS index was first proposed by Caves, Christensen and Diewert (1982b, p. 78) and is also known as the CCD index or the generalized Theil-Törnqvist index (e.g., Pilat and Rao (1996, p. 119)).

In this paper I compute Färe-Primont, Lowe and Geometric Young productivity indexes for eighteen manufacturing sectors of the US economy for the period 1987 to 2008. I also decompose the Färe-Primont index into various technical change and efficiency change components. Until now, the Färe-Primont productivity index has only been computed and decomposed using Bayesian econometric methods (see O'Donnell (2011)). An advantage of the Bayesian approach is that it is possible to draw valid finite-sample inferences concerning rates of productivity growth and measures of technical change, technical efficiency change and scale-mix efficiency change. However, it is difficult to estimate levels of (and, for that matter, changes in) pure scale efficiency (i.e., the productivity gains associated with changes in scale alone) and pure mix efficiency (i.e., the gains associated with changes in scope alone). The Lowe productivity index has only ever been decomposed using DEA methodology (see O'Donnell (2010b)). In this paper I develop similar DEA methodology for computing and decomposing Färe-Primont indexes. The DEA approach has been chosen over the Bayesian approach, not just because it can be used to identify levels of pure scale and mix efficiency, but because it doesn't require any explicit assumptions about random variables representing statistical noise. Such assumptions are unnecessary because DEA implicitly assumes that all noise effects are zero. Because there are no noise effects, DEA side-steps an endogeneity problem[6] that often arises in the econometric estimation of multiple-input multiple-output technologies.

The structure of the paper is as follows. In Section 2 I present several productivity index number formulas, three of which – the Färe-Primont, Lowe and Geometric Young indexes – satisfy all economically-relevant axioms and tests from index number theory. These three indexes are members of a class of "multiplicatively-complete" productivity indexes. In Section 3 I outline the relationship between profitability change, productivity change and changes in relative prices. Among other things, I explain why falls in productivity are often associated with higher net returns. In Section 4 I explain that all multiplicatively-complete

[6] See Atkinson, Cornwell and Honerkamp (2003, p. 288).

productivity indexes can be decomposed into a measure of technical change and several measures of efficiency change. The efficiency measures include a measure of overall productive efficiency and several measures of technical, scale and mix (or scope) efficiency. In Section 5 I show how all of these components can be estimated using DEA methodology. Among other things, I reveal that DEA estimates of Hicks-Moorsteen and Färe-Primont MFP indexes can be viewed as Fisher and Lowe MFP indexes but with support (or shadow) prices used in place of observed market prices. In Section 6 I use BLS data to estimate levels of MFP and efficiency in US manufacturing. In Section 7 I summarize the paper and suggest directions for further research.

2. Measuring Multi-factor Productivity Change

The productivity of a single-output single-input firm is almost always defined as the output-input ratio[7]. O'Donnell (2008) generalizes this concept to the multiple-output multiple-input case by formally defining productivity to be the ratio of an aggregate output to an aggregate input. Consider a dataset containing observations on N firms over T time periods and let $q_{it} = (q_{1it},...,q_{Jit})'$ and $x_{it} = (x_{1it},...,x_{Kit})'$ denote the output and input vectors of firm i in period t. O'Donnell (2008) defines the multi-factor productivity[8] (MFP) of the firm as $MFP_{it} = Q_{it} / X_{it}$ where $Q_{it} \equiv Q(q_{it})$ is an aggregate output, $X_{it} \equiv X(x_{it})$ is an aggregate input, and $Q(.)$ and $X(.)$ are non-negative non-decreasing linearly-homogeneous aggregator functions. With this definition, the index that compares the MFP of firm i in period t with the MFP of firm h in period s is

$$(1) \qquad MFP_{hs,it} = \frac{MFP_{it}}{MFP_{hs}} = \frac{Q_{it} / X_{it}}{Q_{hs} / X_{hs}} = \frac{Q_{it} / Q_{hs}}{X_{it} / X_{hs}} = \frac{Q_{hs,it}}{X_{hs,it}}$$

where $Q_{hs,it} \equiv Q_{it} / Q_{hs}$ and $X_{hs,it} \equiv X_{it} / X_{hs}$ are output and input quantity indexes respective-

[7] It is also possible to define productivity as the output minus the input. However, this alternative measure is generally regarded as unsatisfactory because it is sensitive to units of measurement.

[8] O'Donnell (2008) uses the term total factor productivity (TFP) instead of multi-factor productivity (MFP). Statistical agencies such as the BLS often prefer the latter terminology in view the fact that multiple, but not all, factors of production are accounted for in the analysis.

ly. Equation (1) expresses MFP growth as a measure of output growth divided by a measure of input growth, which is how most economists define productivity change (e.g., Jorgenson and Griliches (1967)).

Productivity indexes that can be written in terms of aggregate quantities as in equation (1) are said to be multiplicatively-complete (O'Donnell (2008)). An example of a multiplicatively complete MFP index is the Hicks-Moorsteen index (e.g., Briec and Kerstens (2011, p. 768)):

(2) $$MFP_{hs,it} = \left[\frac{D_O(x_{hs}, q_{it}, s) D_O(x_{it}, q_{it}, t) D_I(x_{hs}, q_{hs}, s) D_I(x_{hs}, q_{it}, t)}{D_O(x_{hs}, q_{hs}, s) D_O(x_{it}, q_{hs}, t) D_I(x_{it}, q_{hs}, s) D_I(x_{it}, q_{it}, t)} \right]^{1/2}$$

where $D_O(x,q,t) = \max_{\lambda}\{\lambda > 0 : x$ can produce q/λ in period $t\}$ and $D_I(x,q,t) = \max_{\rho}\{\rho > 0 : x/\rho$ can produce q in period $t\}$ are the Shephard (1953) output and input distance functions representing the period-*t* production technology. The output and input aggregator functions underpinning the Hicks-Moorsteen index are $Q(q) = \left[D_O(x_{hs}, q, s) D_O(x_{it}, q, t)\right]^{1/2}$ and $X(x) = \left[D_I(x, q_{hs}, s) D_I(x, q_{it}, t)\right]^{1/2}$. One of the attractive features of the Hicks-Moorsteen index is that can be computed without having to collect price data. Thus, it can be used in non-competitive industries where input and output prices may be unavailable. However, it can only be computed by assuming (or estimating) a functional representation of the production technology. A related index that also requires knowledge of the production technology is the output-oriented[9] Malmquist MFP index (e.g., Caves et al. (1982a, p. 1404), Färe et al. (1994, p. 70)):

(3) $$MFP_{hs,it} = \left[\frac{D_O(x_{it}, q_{it}, s) D_O(x_{it}, q_{it}, t)}{D_O(x_{hs}, q_{hs}, s) D_O(x_{hs}, q_{hs}, t)} \right]^{1/2}.$$

Except in restrictive special cases, this index cannot be expressed in terms of aggregate quantities (i.e., it is not multiplicatively complete) nor as an output index divided by an input index (i.e., it is not a recognizable measure of MFP change). One special case is when the technology is input-homothetic and exhibits constant returns to scale (CRS). In this case, the

[9] An analogous input-oriented Malmquist productivity index is also available.

Hicks-Moorsteen and Malmquist MFP indexes both collapse to the same combined measure of technical change and technical efficiency change (Färe et al. (1996)). A second special case is when the technology exhibits CRS and there is no technical change. In this case it is easily shown that the Hicks-Moorsteen and Malmquist MFP indexes both collapse to a measure of technical efficiency change. These special cases suggest that the output-oriented Malmquist MFP index is an unreliable measure of productivity change unless the production technology exhibits CRS. In empirical practice, it is common to impose the CRS assumption even if the true technology exhibits variable returns to scale (VRS).

To illustrate the importance of multiplicative completeness, consider two firms that have access to the same single-input single-output production technology. Suppose that Firm A uses 6 units of input to produce 6 units of output, while Firm B uses 4 units of input to produce 3 units of output. If productivity is defined as the output-input ratio then the MFP of Firm A is $MFP_A = 6/6 = 1$, the MFP of Firm B is $MFP_B = 3/4 = 0.75$, and the index that compares the MFP of the two firms (using Firm A as the reference firm) is $MFP_{AB} = MFP_B / MFP_A = 0.75/1 = 0.75$. All multiplicatively-complete MFP indexes will take the value 0.75, indicating that Firm B is 25% less productive than Firm A. However, the Malmquist MFP index will take a value that depends on the assumed (or estimated) form of the production technology. Four cases are illustrated in panels a) to d) in Figure 1. Panel a) depicts a case where the technology (the solid curve) exhibits variable returns to scale (VRS) and both firms are fully technically efficient – in this case the Malmquist MFP index takes the value one, indicating that both firms are equally productive. Panel b) depicts a case where the technology exhibits decreasing returns to scale (DRS) and Firm B is only producing 60% of the output that is feasible using 4 units of input – in this case the Malmquist index takes a value 0.6. Panel c) depicts a case where the technology exhibits increasing returns to scale (IRS) and Firm A is only producing 75% of the output that is feasible using 6 units of input – in this case the Malmquist index takes a value 1.33. Finally, panel d) depicts a case where the technology exhibits CRS, Firm B is only producing 56.25% of the output that is feasible using 4 units of input and Firm A is only producing 75% of the output that is feasible using 6 units of input – in this case the Malmquist index takes a value 0.5625/0.75 = 0.75, indicating (correctly) that Firm B is 25% less productive than Firm A. This last panel illustrates that if the technology exhibits CRS and there is no technical change then the Malmquist and Hicks-Moorsteen indexes are both equal to a measure of technical efficiency change (i.e., the second special case mentioned above).

Different non-negative non-decreasing linearly-homogeneous aggregator functions yield different multiplicatively-complete MFP indexes. Examples of aggregator functions and associated productivity indexes that can be used for binary comparisons (i.e., comparisons involving only two observations) are presented in Table 1. In this table, $p_{it} = (p_{1it},...,p_{Jit})' \geq 0$ and $w_{it} = (w_{1it},...,w_{Kit})' \geq 0$ are vectors of output and input prices, and $r_{it} = (r_{1it},...,r_{Kit})' \geq 0$ and $s_{it} = (s_{1it},...,s_{Kit})' \geq 0$ are vectors of income and cost shares. O'Donnell (2008) refers to the aggregator functions in the first three rows of Table 1 as Laspeyres, Paasche and Fisher functions because they yield output and input quantity indexes that are well-known by those names. The aggregator functions in rows four and five are referred to in this paper as Malmquist-*hs* and Malmquist-*it* functions because they yield the firm-specific (and/or period-specific) Malmquist output and input quantity indexes of Caves et al. (1982a, p. 1399-1400). The aggregator functions in row six are the geometric averages of the Malmquist-*hs* and Malmquist-*it* functions and are referred to in this paper as Hicks-Moorsteen functions because they yield the Hicks-Moorsteen MFP index defined by equation (2). Finally, the aggregator functions in rows seven and eight yield quantity indexes that have received little, if any, attention in the productivity literature. In this paper I refer to them as Törnqvist functions because their geometric averages, given in the last row, yield well-known Törnqvist output, input and productivity indexes.

Index formulas are often selected according to whether or not they satisfy certain axioms and tests. In the case of the input quantity index $X_{hs,it} \equiv X(x_{it})/X(x_{hs}) \equiv X(x_{hs}, x_{it})$, for example, the economically-relevant[10] axioms and tests are[11]:

A.1 Monotonicity axiom: $X(x_{hs}, x_{it}) > X(x_{hs}, x_{gr})$ if $x_{it} \geq x_{gr}$ and $X(x_{hs}, x_{it}) > X(x_{gr}, x_{it})$ if $x_{gr} \geq x_{hs}$.

A.2 Linear homogeneity axiom: $X(x_{hs}, \lambda x_{it}) = \lambda X(x_{hs}, x_{it})$ for $\lambda > 0$.

A.3 Identity axiom: $X(x_{it}, x_{it}) = 1$.

A.4 Homogeneity of degree 0 axiom: $X(\lambda x_{hs}, \lambda x_{it}) = X(x_{hs}, x_{it})$ for $\lambda > 0$.

A.5 Commensurability axiom: $X(x_{hs}\Lambda, x_{it}\Lambda) = X(x_{hs}, x_{it})$ where Λ is a diagonal matrix with diagonal elements strictly greater than 0.

A.6 Proportionality axiom: $X(x_{hs}, \lambda x_{hs}) = \lambda$ for $\lambda > 0$.

T.1 Transitivity test: $X_{hs,it} = X_{hs,gr} X_{gr,it}$.

T.2 Time-space reversal test: $X_{hs,it} = 1/X_{it,hs}$

Axiom A.1 (monotonicity) says that the index increases with increases in any element of the comparison vector x_{it} and with decreases in any element of the base (or reference) vector x_{hs}. Axiom A.2 (linear homogeneity) says that a proportionate increase in the comparison vector will cause the value of the index to increase by the factor of proportionality. Axiom A.3 (identity) says that if the comparison and base vectors are identical then the index number is equal to one. Axiom A.4 (homogeneity of degree 0) says that multiplication of the comparison and reference vectors by the same constant will leave the index number unchanged. Axiom A.5 (commensurability) says the index number is robust to changes in units of measurement. Axiom A.6 (proportionality) says that if the reference vector is proportionate to the base vector then the index number is equal to the factor of proportionality. Test T.1 (transitivity) says the index number that directly compares the inputs of a comparison

[10] Other index number tests listed by Eichhorn (1976, p. 248-249) are mathematically convenient but are not directly relevant to the economic measurement of quantity change or productivity change.

[11] Let x_{jit} denote the *j*-th element of x_{it}. The notation $x_{hs} \geq x_{it}$ means that $x_{jhs} \geq x_{jit}$ for all $j = 1,...,J$ and there exists at least one value $j \in \{1,...,J\}$ where $x_{jhs} > x_{jit}$.

firm/period with the inputs of a base firm/period is identical to the index number computed when the comparison is made through an intermediate firm/period. Finally, Test T.2 (time and space reversal) says that the index comparing the inputs of a comparison firm/period with the inputs of a base firm/period is the inverse of the index obtained when the input vectors are interchanged. Output quantity indexes and MFP indexes must satisfy an analogous set of commonsense axioms and tests.

To illustrate the practical relevance of these axioms and tests, consider an industry in which firms use two inputs to produce a single output. Hypothetical price and quantity data for four firms in two periods are given in Table 2. Observe that firms 1 to 3 have chosen the same input-output combinations in period 2 as they chose in period 1. Thus, MFP indexes should indicate that these three firms were just as productive in period 2 as they had been in period 1 (i.e., the identity axiom should ensure $MFP_{i1,i2} = 1$ for $i = 1, 2, 3$). Also observe that firm 4 produced the same output in both periods, but used a smaller input vector in period 2 than it had used in period 1 (it used the same amount of input 2, but 20% less of input 1). With this reduction in input use, MFP indexes should indicate that firm 4 was more productive in period 2 than in period 1 (i.e., the monotonicity axiom should ensure that $MFP_{41,42} > 1$).

Table 3 reports Malmquist, Hicks-Moorsteen, Fisher and Törnqvist indexes measuring MFP change for each of the four firms. Both the Malmquist and Hicks-Moorsteen index values were obtained by assuming the technology can be represented by the CRS log-distance function[12] $\ln D_O(x,q,t) = \ln q + 0.6 - 0.2t \ln x_1 - (1-0.2t)\ln x_2 \leq 0$. The first three rows of Table 3 illustrate that the Malmquist, Hicks-Moorsteen, Fisher and Törnqvist indexes all satisfy the identity axiom (i.e., $MFP_{i1,i2} = 1$ for $i = 1, 2, 3$) and the fourth row illustrates that they satisfy the monotonicity axiom (i.e., $MFP_{41,42} > 1$). However, the last three rows demonstrate that all four indexes fail the transitivity test (i.e., $MFP_{11,12} \neq MFP_{11,41} \times MFP_{41,12}$). This has important implications for national statistical agencies such as the BLS. The BLS uses a chained

[12] The associated log-input distance function is $\ln D_I(x,q,t) = 0.2t \ln x_1 + (1-0.2t)\ln x_2 - 0.6 - \ln q \geq 0$.

Törnqvist formula to compute measures of MFP change for each of the major sectors of the US economy. A chained index that compares the MFP of sector i in period 1 with the MFP of sector i in period 3, for example, is computed as $MFP_{i1,i2} \times MFP_{i2,i3}$. The fact that the Törnqvist formula fails the transitivity test means that the BLS could easily measure increases or decreases in productivity even when input and output levels (i.e., levels of productivity) in non-adjacent periods are exactly the same.

All the MFP indexes listed in Table 2 fail the transitivity test. This means they are only suitable for making comparisons involving two observations (where there are no opportunities for chaining). Of course, most empirical applications involve comparisons across more than two firms and/or time periods. In these applications it is common to construct transitive MFP indexes using a geometric averaging procedure proposed by Elteto and Koves (1964) and Szulc (1964). To be specific, if $MFP_{hs,it}$ is any intransitive index then a transitive index can be computed as:

(4) $$MFP_{hs,it}^{EKS} = \prod_{g=1}^{N} \prod_{r=1}^{T} \left(MFP_{hs,gr} MFP_{gr,it} \right)^{\frac{1}{NT}}.$$

Unfortunately, this solution to the transitivity problem comes at the expense of the identity axiom. This is evident from the shaded cells in the first two rows of Table 3 – even though firms 1 and 2 chose the same input-output combinations in each period, the Fisher-EKS and Törnqvist-EKS indexes take values ranging from 0.97 to 1.17.

When computing index numbers, it is important to hold the aggregator functions Q(.) and X(.) fixed from one binary comparison to the next. Only then will all the economically-relevant axioms and tests from index number theory be satisfied. Unfortunately, this important requirement is rarely met in practice. For example, the Laspeyres quantity index $Q_{11,12}$ is implicitly computed using the aggregator function $Q(q) = p_{11}'q$ and the Laspeyres index $Q_{21,22}$ is implicitly computed using the (different) aggregator function $Q(q) = p_{21}'q$. This empirical practice is arguably no better or worse than using a Laspeyres index formula to make one binary comparison and using a Törnqvist formula to make the next. An alternative and more satisfactory approach is to use fixed-weight indexes of the type presented in Table 4. In this table, t_0 is a representative time period and p_0, w_0, q_0, x_0, r_0 and s_0 are fixed vectors of representative prices, quantities and shares. O'Donnell (2010b) refers to the MFP index in the first row of Table 4 as a Lowe MFP index because the component output and input quantity indexes have been attributed by Balk (2008, p. 6, 68) to Lowe (1823). Currently, the MFP index in the second row can only be traced back as far as O'Donnell (2011b). In this paper I refer to it as a Färe-Primont MFP index because the component output and input quantity indexes can be traced further back to Färe and Primont (1995, p. 36, 38). The MFP index in the third row of Table 4 does not appear to have received any attention in the productivity literature. In this paper I refer to it as a Geometric Young index because price analogues of the component output and input quantity indexes are known by that name (e.g., IMF (2004, p. 10)). All three indexes satisfy axioms A.1 to A.6 and tests T.1 and T.2 listed above. The fact that they satisfy the identity axiom and the transitivity test is evident from the results reported in the last three columns of Table 3 – observe that $MFP_{i1,i2} = 1$ for $i = 1, 2, 3$ (i.e., the identity axiom is satisfied) and $MFP_{11,12} = MFP_{11,41} \times MFP_{41,12}$ (i.e., the transitivity test is satisfied). The index numbers in these columns have been computed using sample means as representative prices, quantities and shares. The Färe-Primont indexes have been computed using the same CRS Cobb-Douglas distance functions that were used earlier in this section to estimate the Malmquist and Hicks-Moorsteen indexes.

Three final comments are in order regarding the fixed-weight MFP indexes defined in Table 4. The first concerns the choice of the representative vectors p_0, w_0, q_0, x_0, r_0 and s_0. Commonsense suggests that these vectors should be representative of the prices, quantities and shares faced by all firms/periods involved in the analysis (i.e., all observations that are to be compared). For this reason, O'Donnell (2010b) recommends using the sample mean vectors[13] $\bar{p}$, $\bar{w}$, $\bar{q}$, $\bar{x}$, $\bar{r}$ and $\bar{s}$ as representative vectors. These mean vectors may also be

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[13] Note that if p_0 and w_0 are set equal to the arithmetic means of observed output and input price vectors and there are only two observations in the dataset then the Lowe output and input quantity indexes are binary

representative of the data in a different (not necessarily larger) sample that may become available at a different point in time. Statistical tests (e.g., Wald) can be used to assess whether the mean of one sample is representative of the data in a second sample.

The second comment concerns the problem of choosing between different fixed-weight index number formulas. An idea that is implicit in the construction of most indexes, including the indexes in Table 4, is that aggregate quantities should be computed using a simple mathematical function that attaches different weights to different outputs and inputs, and that the weights should reflect the relative importance, or value, of the outputs and inputs to the decision-maker. Lowe indexes are constructed by choosing linear weighting functions and by choosing prices as measures of value, Färe-Primont indexes are constructed using non-linear weighting functions and normalized shadow (or support) prices[14] as measures of value, and Geometric Young indexes are constructed using log-linear weighting functions and income and cost shares as measures of value. Numerous alternative fixed-weight indexes can be constructed using other non-negative non-decreasing and linearly homogenous functional forms (e.g., generalized Leontief, generalized linear, constant elasticity of substitution) and other measures of value (e.g., elasticities of output response, marginal rates of transformation and substitution, even carbon footprints). The choice between different fixed-weight formulas is a subjective choice that may be less important in some empirical contexts than in others. For example, if the production technology is of the Cobb-Douglas form and if markets are perfectly competitive (so that elasticities of output response are equal to normalized income and cost shares) then the theoretical Färe-Primont and Geometric Young indexes will be identical.

Finally, it is important to recognize that the problem of measuring productivity change is quite distinct from the problem of explaining productivity change – as the simple examples in this section demonstrate, it is possible to measure the change in an output-input ratio without

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Marshall-Edgeworth indexes, named after Marshall (1887) and Edgeworth (1925).

[14] This will become clear in Section 5.

needing to explain why firms might choose some output-input combinations over others (i.e., it is possible to measure productivity change without needing to explain why some firms might be more or less productive than others). This is important for national statistical agencies such as the BLS, because theoretically-plausible fixed-weight indexes are all-too-often abandoned by such agencies in favor intransitive changing-weight indexes on the grounds that they do not account for firm responses to changes in prices. For example, before 1995 the BLS used fixed-weight output indexes to compute productivity indexes for the business and non-farm business sectors of the US economy. However, in 1996 it abandoned these indexes on the grounds that "fixed weights do not take into account the effects [on quantities] of changing relative prices" (Dean, Harper and Sherwood (1998, p. 187)). This is unfortunate – firm responses to changes in relative prices are not directly relevant to the problem of *measuring* productivity change. However, as we shall see in the next section, firm responses to changes in relative prices (and other production incentives) are a key to *explaining* productivity change.

3. Profitability Change

Let R_{it} and C_{it} denote the total revenue and total cost of firm *i* in period *t*. Associated with the aggregate output and input quantities Q_{it} and X_{it} are the implicit aggregate prices $P_{it} = R_{it} / Q_{it}$ and $W_{it} = C_{it} / X_{it}$. Thus, profitability can be written $PROF_{it} = R_{it} / C_{it}$ $= P_{it}Q_{it} / W_{it}X_{it}$. Furthermore, the index that compares the profitability of firm *h* in period *s* with the profitability of firm *i* in period *t* can be written (O'Donnell (2010a, p. 531))

$$(5) \qquad PROF_{hs,it} = \frac{PROF_{it}}{PROF_{hs}} = \left(\frac{P_{it}Q_{it}}{W_{it}X_{it}}\right)\left(\frac{W_{hs}X_{hs}}{P_{hs}Q_{hs}}\right) = \left(\frac{P_{hs,it}}{W_{hs,it}}\right)\left(\frac{Q_{hs,it}}{X_{hs,it}}\right) = TT_{hs,it} \times MFP_{hs,it}$$

where $P_{hs,it} = P_{it} / P_{hs}$ is an output price index, $W_{hs,it} = W_{it} / W_{hs}$ is an input price index, and $TT_{hs,it} = P_{hs,it} / W_{hs,it}$ is a terms-of-trade index measuring output price change relative to input price change. Equation (5) reveals that profitability change can be deterministically decomposed into the product of a terms-of-trade index and an MFP index. This simple decomposition has several important implications for the measurement of productivity and profitability change. First, if profitability remains constant (e.g., in perfectly competitive industries) then productivity change can be measured as the inverse of the change in the terms-of-trade: $PROF_{hs,it} = 1 \Rightarrow MFP_{hs,it} = 1/TT_{hs,it}$. Second, if output prices change at the same rate as input prices then profitability change can be attributed entirely to productivity change: $TT_{hs,it} = 1 \Rightarrow PROF_{hs,it} = MFP_{hs,it}$. Finally, if the rate of growth in outputs is the same as the rate of growth in inputs then profitability change can be attributed entirely to price change: $MFP_{hs,it} = 1 \Rightarrow PROF_{hs,it} = TT_{hs,it}$.

O'Donnell (2010a) uses equation (5) to help explain the sources of productivity change in industries/sectors comprising rational profit-maximizing firms. Figure 2 depicts key components of this equation in two-dimensional aggregate quantity space. In this figure, the curve passing through points E, K and G is a production frontier that envelops all aggregate-input aggregate-output combinations that are technically feasible in period *t*. In aggregate quantity space, the MFP at any point is the slope of the ray from the origin to that point. For example, the MFP at point A is the slope of the ray passing through point A (i.e., $MFP_{\text{A}} = \text{slope 0A} = Q_{it} / X_{it} = MFP_{it}$) while the maximum productivity possible using the available technology is the slope of the ray passing through point E (i.e., $MFP_{\text{E}} = \text{slope 0E} =$ maximum MFP). The straight line passing through point K in Figure 1 is an isoprofit line with slope $-W_{it} / P_{it}$ (the inverse of the terms of trade) and intercept π_{it}^{*} / P_{it} (normalized maximum profit). Observe that, for this technology, the point that maximizes profit at aggregate prices P_{it} and W_{it} will coincide with the point of maximum productivity (point E) if and only if the maximum MFP possible using the technology equals the inverse of the terms of trade (i.e., the slope of the ray 0E equals the slope of the isoprofit line). This equality between the terms-of-trade and maximum productivity is a characteristic of perfectly competitive industries where, of course, normalized maximum profits are zero (i.e., the isoprofit line passes through the origin). Importantly, any rational efficient profit-maximizing firm will be drawn away from the point of maximum productivity (point E) in response to an improvement in the terms of trade, to a point such as K or G. The resulting inequality between the terms-of-trade and the level of maximum productivity is a characteristic of non-competitive industries and, in such cases, normalized maximum profits are strictly non-zero. Point G is the profit-maximizing solution in the limiting case where all inputs are relatively costless. For rational efficient firms, the economically-feasible region of efficient production is the region of decreasing returns to scale between points E and G. It is clear from Figure 2 that levels of profit and productivity will change as rational efficient profit-maximizing firms move optimally between points E and G in response to changes in the terms of trade.

This simple analysis extends to more general technologies and to industries where firms maximize any benefit function that is increasing in net returns (e.g., the expected utility of profits). Among other things, it provides a rationale for microeconomic reform programs designed to increase levels of competition in output markets – changes in the terms of trade that result from increased competition will tend to drive firms/industries towards points of maximum productivity. Of course, these considerations are only relevant to explaining changes in MFP, not to measuring them – productivity is a quantity concept and, as I demonstrated in Section 2, it is reasonably straightforward to measure productivity change using only quantity data without making any assumptions concerning market structure or the behavioral objectives of firms.

4. Technical Change and Efficiency Change

O'Donnell (2008) demonstrates that any multiplicatively-complete MFP index can be exhaustively decomposed into any number of measures of technical change and efficiency change. The simplest of these decompositions is given by equation (7) below and involves a plausible measure of technical change and a single measure of efficiency change. The technical change component is the change in the maximum productivity possible using the production technology (i.e., the change in the position of point E in Figure 2). The efficiency change component is the change in what O'Donnell (2008) refers to as MFP efficiency (MFPE). MFP efficiency is an overall measure of productive efficiency defined as the difference between observed MFP and the maximum MFP possible using the technology (i.e., the difference between MFP at points A and E in Figure 2). Mathematically, the MFP efficiency of firm *i* in period *t* is

(6) $$MFPE_{it} = MFP_{it} / MFP_{t}^{*}$$

where $MFP_{t}^{*} = Q_{t}^{*} / X_{t}^{*}$ denotes the maximum MFP possible using the technology available in period *t*. A similar equation holds for firm *h* in period *s*: $MFPE_{hs} = MFP_{hs} / MFP_{s}^{*}$. Thus, with some simple algebra the MFP index defined by equation (1) can be decomposed as

(7) $$MFP_{hs,it} = \frac{MFP_{it}}{MFP_{hs}} = \left(\frac{MFP_{t}^{*}}{MFP_{s}^{*}}\right)\left(\frac{MFPE_{it}}{MFPE_{hs}}\right).$$

This simple decomposition is useful whenever points of maximum productivity exist (e.g., for technologies of the type represented in Figure 2). If points of maximum productivity do not exist (e.g., if everywhere the technology exhibits increasing returns to scale) then alternative measures of technical change and overall efficiency change are still available – see, for example, O'Donnell (2010a, p. 538).

The efficiency change component on the right-hand side of equation (7) can be further decomposed into measures of technical efficiency change, scale efficiency change and mix efficiency change. Concepts and measures of technical efficiency and scale efficiency will be familiar to most economists and are widely used in the performance measurement literature – see, for example, Coelli et al. (2005). However, O'Donnell's (2008) concepts and measures of mix efficiency are relatively new. In the same way that scale efficiency is a measure of the potential productivity improvement associated with economies of scale, mix efficiency is a measure of the potential productivity improvement associated with economies of scope. Mix efficiency should not be confused with well-known concepts of allocative efficiency – mix efficiency is a productivity (i.e., quantity) concept while allocative efficiency is a cost, revenue or profit (i.e., value) concept.

Figure 3 illustrates relationships between input-oriented measures of technical, mix and allocative efficiency in the K = 2 input case. In this figure, the curve passing through points B, R and U is an isoquant that envelopes all input combinations that can produce a given output vector. Also in this figure, inputs have been aggregated using the simple linear aggregator function[15] $X_{it} = \alpha_1 x_{1it} + \alpha_2 x_{2it}$ where $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$. The dashed lines passing through points A, B, R and U are iso-aggregate-input lines with slopes $-\alpha_1 / \alpha_2$ and intercepts X_{it} / α_2, $\bar{X}_{it} / \alpha_2$, $\breve{X}_{it} / \alpha_2$ and $\hat{X}_{it} / \alpha_2$ respectively. The solid line that is tangent to the isoquant at point R is an isocost line with slope equal to the negative of the factor price ratio and intercept equal to normalized minimum cost. For the firm operating at point A, minimizing input use while holding the input mix fixed involves a move from point A to point B, and a decrease in the aggregate input from X_{it} to $\bar{X}_{it}$; minimizing cost without any restrictions on input mix involves a move to point R and a decrease in the aggregate input to $\breve{X}_{it}$; and minimizing aggregate input use without any restrictions on the input mix involves a move to point U and a further decrease in the aggregate input to $\hat{X}_{it}$. Associated measures of efficiency are:

(8) $$ITE_{it} = \bar{X}_{it} / X_{it},$$

(9) $$CAE_{it} = \breve{X}_{it} / \bar{X}_{it}$$ and

(10) $$IME_{it} = \hat{X}_{it} / \bar{X}_{it}.$$

The measure of efficiency given by equation (8) is an input-oriented measure of technical efficiency attributed to Farrell (1957), the measure given by (9) is a well-known measure of cost-allocative efficiency (see, for example, Coelli et al. (2005, p. 53)), and the measure given by (10) is the measure of input-oriented mix efficiency defined by O'Donnell (2008).

To further illustrate relationships between these and other measures of efficiency, Figure 4

[15] Any non-negative, non-decreasing linearly homogeneous functions could have been used as aggregator functions for purposes of this illustration, including any of the input aggregator functions listed in Tables 1 and 4.

maps the points A, B and U from Figure 3 into aggregate quantity space. In this figure, the curve passing through points B and D is a mix-restricted frontier enveloping all (aggregates of) technically-feasible output-input combinations that have the same output mix and input mix as the firm operating at point A. The curve passing through points U and E is an unrestricted frontier that envelops all (aggregates of) output-input combinations that are feasible when all restrictions on output mix and input mix are relaxed (this unrestricted frontier is the frontier depicted earlier in Figure 2). It is clear from Figure 4 that measures of efficiency can be viewed as measures of MFP change: for example, the Farrell (1957) input-oriented measure of technical efficiency defined by (8) is a measure of the increase in MFP as the firm moves from point A to point B (i.e., $ITE_{it} = MFP_{\mathrm{A}} / MFP_{\mathrm{B}} \equiv MFP_{\mathrm{BA}}$), while the O'Donnell (2008) input-oriented measure of mix efficiency defined by (10) is the increase in MFP as the firm moves from point B to point U (i.e., $IME_{it} = MFP_{\mathrm{B}} / MFP_{\mathrm{U}} \equiv MFP_{\mathrm{UB}}$). Three other measures of efficiency that are illustrated in Figure 4 are

(11) $$ISE_t = \frac{Q_t / \bar{X}_t}{\tilde{Q}_t / \tilde{X}_t}$$

(12) $$RME_{it} = \frac{\tilde{Q}_{it} / \tilde{X}_{it}}{MFP_{it}^{*}}$$ and

(13) $$ISME_{it} = \frac{Q_{it} / \bar{X}_{it}}{MFP_t^{*}}.$$

The measure of efficiency given by (11) is a common measure of input-oriented scale efficiency (see, for example, Balk (1998, p. 21)), the measure given by (12) is the measure of residual mix efficiency defined by O'Donnell (2008), and the measure given by (13) is the measure of input-oriented scale-mix efficiency defined by O'Donnell and Nguyen (2011). Residual mix efficiency is a measure of the increase in MFP as a firm moves from a point of maximum productivity on a mix-restricted frontier to a point of maximum productivity on the unrestricted frontier (e.g., in Figure 4, $RME_{it} = MFP_{\mathrm{D}} / MFP_{\mathrm{E}} \equiv MFP_{\mathrm{ED}}$), while input-oriented scale-mix efficiency measures the increase in MFP as a firm moves from a technically-efficient point on a mix-restricted frontier to a point of maximum productivity on the unrestricted frontier (e.g., in Figure 4, $ISME_{it} = MFP_{\text{B}} / MFP_{\text{E}} \equiv MFP_{\text{EB}}$). Further details concerning these and other input- and output-oriented measures of efficiency are available in O'Donnell (2008) and O'Donnell (2010b).

It is evident, both mathematically and from Figure 4, that the O'Donnell (2008) measure of MFP efficiency can be decomposed into several economically-meaningful components. For example, $MFPE_{it} = ITE_{it} \times ISME_{it} = ITE_{it} \times ISE_{it} \times RME_{it}$ or, in terms of Figure 4, $MFPE_{it} = MFP_{\text{BA}} \times MFP_{\text{EB}} = MFP_{\text{BA}} \times MFP_{\text{UB}} \times MFP_{\text{EU}}$. It follows that the MFP index given by equation (1) can be decomposed progressively more finely as

(14) $$MFP_{hs,it} = \left(\frac{MFP_t^*}{MFP_s^*}\right)\left(\frac{ITE_{it}}{ITE_{hs}}\right)\left(\frac{ISME_{it}}{ISME_{hs}}\right) = \left(\frac{MFP_t^*}{MFP_s^*}\right)\left(\frac{ITE_{it}}{ITE_{hs}}\right)\left(\frac{ISE_{it}}{ISE_{hs}}\right)\left(\frac{RME_{it}}{RME_{hs}}\right).$$

An analogous output-oriented decomposition is (O'Donnell (2008); O'Donnell (2010b))

(15) $$MFP_{hs,it} = \left(\frac{MFP_t^*}{MFP_s^*}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSME_{it}}{OSME_{hs}}\right) = \left(\frac{MFP_t^*}{MFP_s^*}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSE_{it}}{OSE_{hs}}\right)\left(\frac{RME_{it}}{RME_{hs}}\right)$$

where OTE_{it} is the Farrell (1957) measure of output-oriented technical efficiency, OSE_{it} is a common measure of output-oriented scale efficiency (see, for example, Balk (1998, p. 23)), and $OSME_{it}$ is the O'Donnell (2010b) measure of output-oriented scale-mix efficiency. In the next section I discuss linear programming methods for estimating these components.

5. Estimating and Decomposing MFP Indexes Using DEA

Estimating the components of MFP change involves estimating production frontiers of the type depicted in Figures 1 to 4. In this paper I estimate these frontiers using non-parametric DEA. DEA is non-parametric in the sense that it doesn't involve any error terms, so it doesn't involve any assumptions about the parameters (e.g., means and variances) of the distributions of those error terms. The term non-parametric should not be interpreted to mean that DEA is devoid of any assumptions concerning the functional form of the production frontier – DEA is underpinned by the assumption that the frontier is locally linear (O'Donnell (2010a)). The term 'locally linear' refers to the fact that if firm i in period t is technically efficient (i.e., on the frontier) then in the neighborhood of the point (q_{it}, x_{it}) (i.e., locally) the frontier takes the form $q'_{it}\alpha = \gamma + x'_{it}\beta$ (i.e., is linear). Alternative representations of this locally-linear technology include (local) output and input distance functions. For example, the (local) output distance function representing the technology available in period t is (O'Donnell (2010a, p. 542))

(16) $$D_O(x_{it}, q_{it}, t) = (q'_{it}\alpha)/(\gamma + x'_{it}\beta)$$

where α and β are non-negative. Restrictions can be imposed on γ to reflect different assumptions about returns to scale. For example, the restriction $\gamma = 0$ will ensure the technology exhibits local CRS, while the restriction $\gamma \geq 0$ will ensure the technology exhibits local non-increasing returns to scale (NIRS).

In practice it is common to break the dataset into sub-samples in such a way that all observations in each sub-sample are observations on firms that operate in the same production environment. Each sub-sample is then used to estimate a separate frontier. For example, if the period-s production environment is thought to differ from the period-t production environment then it is usual practice to use observations from period s to estimate a so-called period-s frontier, and to use observations from period t to estimate a separate period-t frontier. Of course, if all firms are thought to face the same production environment in all time periods (i.e., there is "no technical change") then all observations in the dataset are used to estimate a single frontier. In the remainder of this paper I use M_t to denote the number of observations used to estimate the frontier in period t.

O'Donnell (2010a) observes that the standard output-oriented DEA problem involves selecting values of the unknown parameters in (16) in order to minimize $OTE_{it}^{-1} = D_O(x_{it}, q_{it}, t)^{-1}$. If the technology is permitted to exhibit VRS then the only constraints that need to be satisfied are $\alpha \geq 0$, $\beta \geq 0$ and $D_O(x_{it}, q_{it}, t) \leq 1$ for all M_t observations. Unfortunately, this constrained optimization problem has an infinite number of solutions. The usual way forward is to identify a unique solution by setting $q_{it}'\alpha = 1$. With this additional constraint the DEA problem takes the form of a linear program (LP):

(17) $$D_O(x_{it}, q_{it}, t)^{-1} = OTE_{it}^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x_{it}'\beta : \gamma\iota + X'\beta \geq Q'\alpha; q_{it}'\alpha = 1; \alpha \geq 0; \beta \geq 0\}$$

where Q is a $J \times M_t$ matrix of observed outputs, X is a $K \times M_t$ matrix of observed inputs, and ι is an $M_t \times 1$ unit vector.

The output-oriented LP (17) is most often used in empirical contexts where inputs are regarded as fixed. An analogous input-oriented problem is used when outputs are regarded as fixed. In the input-oriented case, the production technology available in period t is represented by the (local) input distance function (O'Donnell (2010a, p. 542))

(18) $$D_I(x_{it}, q_{it}, t) = (x_{it}'\eta)/(q_{it}'\phi - \delta).$$

The input-oriented DEA problem is to maximize $ITE_{it} = D_I(x_{it}, q_{it}, t)^{-1}$ subject to the constraints $\phi \geq 0$, $\eta \geq 0$ and $D_I(x_{it}, q_{it}, t) \geq 1$ for all M_t observations. In this case, a unique solution is identified by setting $x_{it}'\eta = 1$. Thus, the input-oriented DEA problem is

(19) $$D_I(x_{it}, q_{it}, t)^{-1} = ITE_{it} = \max_{\phi,\delta,\eta}\{q_{it}'\phi - \delta : Q'\phi \leq \delta\iota + X'\beta; x_{it}'\eta = 1; \phi \geq 0; \eta \geq 0\}.$$

In the remainder of this section I explain how variants of problems (17) and (19) can be used to estimate aggregate quantities, levels of efficiency, and maximum MFP. These level measures can then be used to estimate the productivity indexes defined in Tables 1 and 4 and to decompose them into the measures of efficiency change identified above in Section 4.

5.1 Estimating Aggregate Outputs and Inputs

If prices are available then computing the aggregate outputs and inputs associated with Laspeyres, Paasche, Lowe, Fisher and Geometric Young indexes is straightforward. However, estimating Malmquist-*hs*, Malmquist-*it*, Hicks-Moorsteen and Färe-Primont aggregate quantities involves estimating (the reciprocals of) distances from different data points to the production frontier. In the Malmquist-*hs* case, for example, estimates of $Q_{it} = D_O(x_{hs}, q_{it}, s)$ and $X_{it} = D_O(x_{it}, q_{hs}, s)$ are obtained by solving the following variants of LPs (17) and (19):

(20) $$D_O(x_{hs}, q_{it}, s)^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_{hs}\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_{it}\alpha = 1; \alpha \geq 0; \beta \geq 0\}$$ and

(21) $$D_I(x_{it}, q_{hs}, s)^{-1} = \max_{\phi,\delta,\eta}\{q'_{hs}\phi - \delta : Q'\phi \leq \delta\iota + X'\eta; x'_{it}\eta = 1; \phi \geq 0; \eta \geq 0\}.$$

Estimates of Q_{it} and X_{it} for all $i = 1,\ldots,N$ and $t = 1,\ldots,T$ can then be computed as

(22) $$Q_{it} = (q'_{it}\alpha_{hs})/(\gamma_{hs} + x'_{hs}\beta_{hs})$$ and

(23) $$X_{it} = (x'_{it}\eta_{hs})/(q'_{hs}\phi_{hs} - \delta_{hs})$$

where α_{hs}, γ_{hs} and β_{hs} are the values of α, γ and β that solve (20) and ϕ_{hs}, δ_{hs} and η_{hs} are the values of ϕ, δ and η that solve (21). The subscripting on these parameters reflects the fact that the distance functions (16) and (18) are only locally linear, so the parameters may vary from one observation to the next. The same values α_{hs}, β_{hs}, γ_{hs}, ϕ_{hs}, δ_{hs} and η_{hs} are used to construct Q_{it} and X_{it} for all $i = 1,\ldots,N$ and $t = 1,\ldots,T$ in order to meet the requirement that the aggregator functions be held fixed (see Section 2) [16].

It is useful at this point to note that the first-order partial derivatives of output and input distance functions with respect to outputs and inputs can be interpreted as revenue- and cost-deflated output and input shadow prices (e.g., Färe and Grosskopf (1990, p. 124), Grosskopf, Margaritis and Valdmanis (1995, p. 578)). For example, consider the shadow prices obtained by evaluating the first-order partial derivatives of $D_O(x_{hs}, q_{it}, s) = (q'_{it}\alpha)/(\gamma + x'_{hs}\beta)$ and $D_I(x_{it}, q_{hs}, s) = (x'_{it}\eta)/(q'_{hs}\phi - \delta)$ at the parameter values that solve LPs (20) and (21):

(24) $$p^*_{hs} \equiv \partial D_O(x_{hs}, q_{it}, s)/\partial q_{it} = \alpha_{hs}/(\gamma_{hs} + x'_{hs}\beta_{hs})$$ and

(25) $$w^*_{hs} \equiv \partial D_I(x_{it}, q_{hs}, s)/\partial x_{it} = \eta_{hs}/(q'_{hs}\phi_{hs} - \delta_{hs}).$$

These two equations suggest that the Malmquist-*hs* aggregate quantities defined by (22) and (23) could be computed using the aggregator functions

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[16] Econometric estimation (i.e., stochastic frontier analysis) is less complicated because the distance function takes a parametric form and the parameters do not vary from one neighbourhood to the next (i.e., the aggregator function is fixed by design). For an example of such an aggregator function, see footnote 12.

(26) $Q(q) = q'p^*_{hs}$ (Malmquist-*hs*) and

(27) $X(x) = x'w^*_{hs}$. (Malmquist-*hs*)

Furthermore, comparing equations (26) and (27) with the Laspeyres aggregator functions in Table 1 suggests that DEA estimates of Malmquist-*hs* indexes can be computed as Laspeyres indexes but with the shadow prices defined by (24) and (25) used in place of observed prices. Indeed, this is the method I use to compute Malmquist-*hs* indexes in this paper. Similarly, DEA estimates of Malmquist-*it* and Färe-Primont indexes are computed as Paasche and Lowe MFP indexes but with appropriate estimated shadow prices used in place of observed and representative prices. Specifically, estimates of Malmquist-*it* and Färe-Primont indexes are obtained by first solving the following linear programs:

(28) $D_O(x_{it}, q_{hs}, t)^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_{it}\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_{hs}\alpha = 1; \alpha \geq 0; \beta \geq 0\}$

(29) $D_I(x_{hs}, q_{it}, t)^{-1} = \max_{\phi,\delta,\eta}\{q'_{it}\phi - \delta : Q'\phi \leq \delta\iota + X'\eta; x'_{hs}\eta = 1; \phi \geq 0; \eta \geq 0\}$

(30) $D_O(x_0, q_0, t_0)^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_0\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_0\alpha = 1; \alpha \geq 0; \beta \geq 0\}$ and

(31) $D_I(x_0, q_0, t_0)^{-1} = \max_{\phi,\delta,\eta}\{q'_0\phi - \delta : Q'\phi \leq \delta\iota + X'\eta; x'_0\eta = 1; \phi \geq 0; \eta \geq 0\}$

where t_0 defines the observations that are used to estimate the representative frontier. In a slight abuse of notation, let α_{it}, β_{it}, γ_{it}, ϕ_{it}, δ_{it} and η_{it} denote the solutions to LPs (28) and (29), and let α_0, β_0, γ_0, ϕ_0, δ_0 and η_0 denote the solutions to LPs (30) and (31). In this paper, Malmquist-*it* and Färe-Primont aggregate outputs are computed using the following aggregator functions:

(32) $Q(q) = q'p^*_{it}$ (Malmquist-*it*)

(33) $X(x) = x'w^*_{it}$ (Malmquist-*it*)

(34) $Q(q) = q'p^*_0$ (Färe-Primont) and

(35) $X(x) = x'w^*_0$ (Färe-Primont)

where

(36) $p^*_{it} \equiv \partial D_O(x_{it}, q_{hs}, t)/\partial q_{hs} = \alpha_{it}/(\gamma_{it} + x'_{it}\beta_{it})$

(37) $w^*_{it} \equiv \partial D_I(x_{hs}, q_{it}, t)/\partial x_{hs} = \eta_{it}/(q'_{it}\phi_{it} - \delta_{it})$

(38) $p_0^* \equiv \partial D_O(x_0, q_0, t_0)/\partial q_0 = \alpha_0 /(\gamma_0 + x_0'\beta_0)$ and

(39) $w_0^* \equiv \partial D_I(x_0, q_0, t_0)/\partial x_0 = \eta_0 /(q_0'\phi_0 - \delta_0)$.

Finally, Hicks-Moorsteen aggregate quantities are the geometric averages of the Malmquist-*hs* and Malmquist-*it* aggregate quantities (so Hicks-Moorsteen aggregates can be computed as Fisher aggregates but with shadow prices used instead of observed prices).

5.2 Estimating Levels of Efficiency and Maximum MFP

Irrespective of the aggregator functions chosen (i.e., irrespective of the MFP index chosen), estimates of output- and input-oriented technical efficiency can be obtained by solving LPs (17) and (19). In practice, it is common to solve the following dual problems:

(40) $OTE_{it} = D_O(x_{it}, q_{it}, t) = \min_{\lambda,\theta}\left\{\lambda^{-1} : \lambda q_{it} \le Q\theta; X\theta \le x_{it}; \theta'\iota = 1; \theta \ge 0\right\}$ and

(41) $ITE_{it} = D_I(x_{it}, q_{it}, t)^{-1} = \min_{\rho,\theta}\left\{\rho : Q\theta \ge q_{it}; \rho x_{it} \ge X\theta; \theta'\iota = 1; \theta \ge 0\right\}$

where θ is an $M_t \times 1$ vector. As they stand, these particular LPs allow the technology to exhibit variable returns to scale. To estimate levels of technical efficiency under a CRS assumption it is necessary to delete the constraint $\theta'\iota = 1$ from both LPs. Estimates of output- and input-oriented scale efficiency can then be computed as $OSE_{it} = OTE_{it}^{CRS} / OTE_{it}$ and $ISE_{it} = ITE_{it}^{CRS} / ITE_{it}$ where OTE_{it}^{CRS} and ITE_{it}^{CRS} denote technical efficiency estimates computed under the CRS assumption.

Estimating levels of output- and input-oriented mix efficiency is less straightforward. For example, estimating the input-oriented measure defined by equation (10) involves estimating $\bar{X}_{it} \equiv X_{it} \times ITE_{it}$ (the minimum aggregate input capable of producing q_{it} when the input mix is held fixed) and $\hat{X}_{it}$ (the minimum aggregate input capable of producing q_{it}). Estimating $\bar{X}_{it}$ is simple enough using the solution to the technical efficiency problem (41) and the estimate of X_{it} obtained in Section 5.1. Estimating $\hat{X}_{it}$ is slightly more difficult. To estimate this aggregate quantity it is convenient to first write LP (41) in the form

(42) $ITE_{it} = \bar{X}_{it} / X_{it} = \min_{\rho,\theta,x}\left\{X(x)/X(x_{it}) : Q\theta \ge q_{it}; x \ge X\theta; x = \rho x_{it}; \theta'\iota = 1; \theta \ge 0\right\}$.

The equivalence of (41) and (42) is easily established by noting that if $x = \rho x_{it}$ then linear homogeneity of the input aggregator function ensures that $X(x)/X(x_{it}) = X(\rho x_{it})/X(x_{it})$ $= \rho$. The formulation (42) is convenient because the constraint $x = \rho x_{it}$ makes it explicit that consideration is only being given to feasible input vectors that can be written as scalar multiples of x_{it} (i.e., the input mix is being held fixed). If the mix constraint is deleted then LP (42) becomes

(43) $$\hat{X}_{it}/X_{it} = \min_{\rho,\theta,x}\left\{X(x)/X(x_{it}) : Q\theta \geq q_{it}; x \geq X\theta; \theta'\iota = 1; \theta \geq 0\right\}$$ or

(44) $$\hat{X}_{it} = \min_{\theta,x}\left\{X(x) : Q\theta \geq q_{it}; x \geq X\theta; \theta'\iota_{NT} = 1; \theta \geq 0\right\}.$$

For any input aggregator function that is linear in inputs, problem (44) is a linear program that gives the minimum aggregate input that firm *i* in period *t* could use to produce its output vector. The output-oriented analogue of LP (44) is

(45) $$\hat{Q}_{it} = \max_{\theta,q}\left\{Q(q) : q \leq Q\theta; X\theta \leq x_{it}; \theta'\iota = 1; \theta \geq 0\right\}.$$

For any output aggregator function that is linear in outputs, problem (45) gives the maximum aggregate output that firm *i* in period *t* could produce using its input vector. The Laspeyres, Paasche and Lowe output and input aggregator functions are given in Tables 1 and 4, and the empirical versions of the Malmquist-*hs*, Malmquist-*it* and Färe-Primont output aggregator functions are given by equations (26), (27) and (32) to (35). All of these aggregator functions are linear in outputs or inputs. Geometric Young aggregator functions are nonlinear functions of outputs and inputs so LPs (44) and (45) cannot be used to estimate levels of pure mix efficiency associated with the Geometric Young productivity index.

Finally, for all aggregator functions (including Geometric Young functions), the maximum MFP in period *t* can be computed as $MFP_t^* = \max_i MFP_{it} = \max_i Q_{it}/X_{it}$. All other measures of efficiency defined in Section 4 can then be computed residually: $MFPE_{it} = MFP_{it}/MFP_t^*$, $OSME_{it} = MFPE_{it}/OTE_{it}$, $ISME_{it} = MFPE_{it}/ITE_{it}$ and $RME_{it} =$ $OSME_{it}/OSE_{it} = ISME_{it}/ISE_{it}$.

5.3 Zero Shadow Prices and Measures of MFP Change

In many DEA applications it is often the case that one or more (not all) estimated shadow prices are equal to zero. In such cases, variations in associated outputs and inputs will not be reflected in Malmquist, Hicks-Moorsteen or Färe-Primont estimates of output, input or productivity change (in effect, those outputs and inputs are estimated to be of no value to the firm). To illustrate, Table 5 presents DEA estimates of the components of $MFP_{41,42}$ computed using the hypothetical data presented earlier in Table 2. Parametric estimates of $MFP_{41,42}$ were previously reported in the fourth row of Table 3. To enable comparisons with the results reported in that table, the DEA estimates reported in Table 5 were computed under a CRS assumption. Observe from the first row in Table 5 that the Malmquist-*hs*, Hicks-Moorsteen and Färe-Primont indexes all indicate (correctly) that firm 4 was more productive in period 2 than in period 1 (i.e., $\Delta MFP = MFP_{41,42} > 1$). However, the estimated Malmquist-*it* index takes the value one. The estimated Malmquist productivity index defined by (3) is not reported in Table 5 but it also takes the value one (this estimate was obtained using the DEAP 2.1 software). These implausible findings are both due to the fact that the DEA estimate of the cost-deflated shadow price of input 1 is zero, so the 20% reduction in input 1 is not reflected in measures of input change (estimated cost-deflated shadow prices are reported in the bottom half of Table 5).

In this illustrative example, the fact that estimated Malmquist-*it* index is biased means that the estimated Hicks-Moorsteen index (the geometric average of the Malmquist-*hs* and Malmquist-*it* indexes) is also biased. In practice, if any estimated shadow prices are zero (and if this is regarded as implausible) then the constraints $\alpha \geq 0$ and $\eta \geq 0$ in problems (20), (21) and (28) to (31) can be replaced with the constraints $\alpha \geq a$ and $\eta \geq b$ where $a > 0$ and $b > 0$ are subjective measures of relative value[17]. In the case of Färe-Primont indexes, if any elements of α_0 and η_0 are equal to zero then a less subjective solution to the zero-shadow-

[17] In the DEA literature these types of restrictions are known as "weight restrictions" – see Allen et al. (1997).

price problem is to replace α_0 and η_0 with sample averages of the solutions to the output- and input-oriented technical efficiency problems (17) and (19).

Finally, observe from Table 5 that the (unbiased) Malmquist-*hs* and Färe-Primont indexes both indicate that firm 4 was 15% more productive in period 2 than in period 1. Input-oriented decompositions of both indexes indicate (correctly) that this improvement in productivity was due to a change in input mix (i.e., $MFPE_{41,42} = IME_{41,42} = 1.15$).

6. The Components of MFP Change in US Manufacturing

This section reports estimates of productivity change in the manufacturing sectors of the US economy over the period 1987 to 2008. The data were drawn from the sectoral MFP database compiled by the BLS (2010). This particular database contains observations on one output and five inputs (capital, labor, energy, materials and services) in eighteen sectors classified at the 3-digit level in the North American Industrial Classification System (NAICS). The output (Q) is the real value of total production (i.e., the real value of total "sales" plus changes in inventories) less any production that is consumed within the sector. Capital (K) is assumed to be proportional to the stock of physical assets (including equipment, structures, inventories and land). Stocks of depreciable assets are measured using the perpetual inventory method. Labor (L) is measured as hours worked. The BLS obtains its data on energy (E), materials (M) and services (S) inputs from the Bureau of Economic Analysis (BEA) input-output "use" tables. Output and input values are measured in billions of current dollars and prices are reported in the form of prices indexes with base 2005 = 100. More details concerning the construction of the dataset are provided by Gullickson (1995).

6.1 Revenues, Costs and Cost Shares

Average revenues, costs and cost shares in each of the eighteen sectors are reported in Table 6. To improve readability, the maximum values in each column are shaded green while the minimum values are shaded yellow (the same shading conventions will also be used in other tables presented below). The shaded entries in the first column, for example, reveal that the Food, Beverage and Tobacco Products sector was on average the largest sector by value ($451.1b) and the Apparel and Leather and Applied Products sector was the smallest ($53b). The shaded entries in the eighth row reveal that the Chemical Products sector spent significantly more on capital and energy than any other sector (an average of $84b on capital and $17.2b on energy). The shaded entries in the second last column reveal that materials purchases accounted for 72% of costs in the Petroleum and Coal Products sector but only accounted for 28% of costs in the Computer and Electronic Products sector. The sample average cost shares reported in the last row of Table 6 are used in this paper as representative shares for purposes of computing Geometric Young indexes: $s_0 = \bar{s} = (0.13, 0.28, 0.03, 0.42, 0.15)'$. Lowe indexes are computed using the sample average input price vector $w_0 = \bar{w} = (96.6, 76.6, 74.9, 89.9, 83.1)'$ while Färe-Primont indexes are computed using the estimated shadow input prices $w_0^* = (0.87, 0.66, 0.07, 0.37, 0.64)'$.

6.2 MFP Change

Estimates of MFP change are sometimes sensitive to the choice of index formula and, in the case of some index formulas (e.g., Hicks-Moorsteen and Färe-Primont), to different assumptions concerning the production technology and/or the nature of technical change. In this paper I seek to avoid any restrictive and empirically-untested assumptions about the technology and so I estimate Malmquist, Hicks-Moorsteen and Färe-Primont indexes using VRS LPs that allow for both technical progress and regress (i.e., only data from period *t* are used to estimate the production frontier in period *t*). These different index formulas nevertheless yield quite different estimates of productivity change in some sectors. For example, Figure 5 presents alternative MFP indexes for the Petroleum and Coal Products sector[18]. Associated

18 Activity in this sector is based around the transformation of crude petroleum and coal into usable products . The dominant activity is petroleum refining.

output and input quantity indexes are presented in Figure 6. For this sector, the chained Törnqvist MFP index (the BLS measure of MFP change) is highly correlated with the Färe-Primont and Lowe indexes (the correlation coefficients are 0.97 and 0.99 respectively) but poorly correlated with the Fisher-EKS index (the correlation coefficient is only 0.07). Large differences between the Törnqvist and Geometric Young indexes in the period 2003 to 2005 can be traced back to the treatment of changes in the energy, materials and services inputs[19]. For example, in 2003 the sector used 1% more capital, 1% less labor, 85% less energy, 15% less materials and 83% less services inputs than it had used in 2002 (see Figure 6). The associated Törnqvist input quantity index can be decomposed as $\Delta X = \Delta K \times \Delta L \times \Delta E \times \Delta M \times \Delta S$ = (1.00)(1.00)(0.98)(0.90)(0.96) = 0.85, indicating a 15% decrease in input use, while the Geometric Young input quantity index can be decomposed as $\Delta X = \Delta K \times \Delta L \times \Delta E \times \Delta M \times \Delta S$ = (1.00)(1.00)(0.95)(0.94)(0.77) = 0.68, indicating a 32% decrease in input use. It is evident that the difference between these two index values is largely due to the measure of change in the services input (the Törnqvist measure is ΔS = 0.96 while the Geometric Young measure is ΔS = 0.77). This can be traced back even further to the cost-share weights assigned to the services input – the binary Törnqvist index assigns the services input a weight of 2% (this is representative of the services cost share in the sector in 2002 and 2003) while the multi-lateral and multi-temporal Geometric Young index assigns a much larger weight of 15% (this is representative of the services cost share in all sectors in all time periods, as discussed in Section 6.1).

In the remainder of this section I focus on Färe-Primont, Lowe and Geometric Young estimates of MFP change. I largely ignore the Hicks-Moorsteen, Fisher and Törnqvist indexes because, for comparisons involving more than two sectors or more than two time periods, they are theoretically-implausible (see Section 2).

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[19] Large changes in these inputs coincided with changes in U.S. state and federal government legislation, including the 1998 Petroleum Refinery Initiative (a Clinton Administration initiative to ensure compliance with the Clean Air Act), various pieces of legislation in the early 2000s that led to the phasing out of methyl tertiary butyl ether (MTBE) as an oxygenate (and its replacement with ethanol), and the 2005 US Energy Policy Act which, among other things, mandated the end of a 2% oxygenate rule.

Index numbers that compare MFP in 2008 with MFP in 1987 are presented in Table 7. Interpretation of the entries in this table is straightforward. For example, the Färe-Primont estimate reported in the seventh row indicates that MFP in the Petroleum and Coal Products sector increased by 9.7% between 1987 and 2008 (ΔMFP = 1.097). All four indexes (BLS, Färe-Primont, Lowe and Geometric Young) indicate that some of the smallest increases in productivity occurred in the Food, Beverage and Tobacco Products sector (less than 4%) and the Nonmetallic Mineral Products sector (less than 7%). All four indexes also indicate that the highest rate of MFP growth occurred in the Computer and Electronics Products sector (more than 800%). A comparison of the four indexes suggests that the BLS (index) may be slightly understating rates of productivity growth in the Primary Metals Sector (NAICS code 331) and slightly overstating rates of productivity growth in the Textile and Textile Products Mills (313, 314), Plastics and Rubber Products (326), Nonmetallic Mineral Products (327) and Furniture and Related Products (337) sectors.

6.3 Technical Change and Efficiency Change

Färe-Primont estimates of the technical change and efficiency change components of MFP change over the period 1987 to 2008 are presented in Table 8. The estimated technical change component of any MFP index will depend on the assumptions that are made about the production technology. The Färe-Primont estimates reported in Table 8 are obtained under the assumption that the production technology exhibits VRS and that in any given period all sectors have access to the same production possibilities set. This second assumption means that all sectors must experience the same estimated rate of technical change – in Table 8, ΔMFP^* = 1.041, which equates to an average rate of technical progress of $\Delta \ln MFP^* = \ln(1.041)/(2008-1987) = 0.00189$ or 0.189% per annum. The production possibilities set is also permitted to both expand and contract, which means "technical progress" can take place in some periods and "technical regress" can take place in others.

Again, the interpretation of the entries in Table 8 is straightforward. For example, the estimates reported in the seventh row indicate that MFP in the Petroleum and Coal Products sector increased by 9.7% due to the combined effects of technical progress (4.1%) and efficiency improvement (5.4%) (i.e., $\Delta MFP = \Delta MFP^* \times \Delta MFPE = 1.041 \times 1.054 = 1.097$). The remaining entries in the seventh row reveal that all of this efficiency improvement was due to improvements in scale-mix efficiency (i.e., $\Delta MFPE = \Delta ITE \times \Delta ISME = 1 \times 1.054 = 1.054$). Observe that the Apparel and Leather and Applied Products sector experienced a 21.4% fall in productivity on the back of a 24.4% fall in efficiency (i.e., $\Delta MFP = \Delta MFP^* \times \Delta MFPE = 1.041 \times 0.756 = 0.786$). All of the 24.4% fall in efficiency in this sector was also due to changes in scale and input mix (i.e., $\Delta MFPE = \Delta ITE \times \Delta ISME = 1 \times 0.756 = 0.756$). In contrast, the Computer and Electronics sector experienced an eight-fold increase in productivity due to improvements in both technical efficiency (324.8%) and scale and mix efficiency (240.8%) (i.e., $\Delta MFP = \Delta MFP^* \times ITE \times \Delta ISME = 1.041 \times 3.248 \times 2.408 = 8.136$).

6.4 Levels of Productivity and Efficiency

If index numbers are properly constructed within the aggregate quantity framework of O'Donnell (2008) then it is possible to estimate *levels* of productivity and efficiency. Such estimates are both spatially- and temporally comparable and can provide important additional insights into the drivers of productivity and efficiency change. To illustrate, Figure 7 presents Färe-Primont estimates of levels of MFP in selected sectors. Among other things, this figure reveals that the eight-fold improvement in productivity in the Computer and Electronic Products sector reported earlier in Table 8 was enough to make the sector only slightly more productive than the Food, Beverage and Tobacco Products sector had been in 1987. Earlier in this section the Food, Beverage and Tobacco Products sector had been identified as a sector that had experienced a very slow rate of productivity growth. Figure 7 now reveals that this may simply have been due to the fact that the sector was already one of the most productive manufacturing sectors in the economy. If these relatively high levels of productivity are interpreted within the theoretical framework presented in Section 3 then it would seem that the Food, Beverage and Tobacco Products manufacturing sector was one of the most competitive sectors in the U.S. economy throughout the sample period. Conversely, the low levels of productivity in the Computer and Electronic Products sector in 1987 suggest that this sector was uncompetitive at that time.

Further insights into productivity change in the Computer and Electronic Products sector can be gleaned from the estimated levels of MFP and efficiency presented in Figure 8. This figure reveals that the sector experienced three reasonably distinct phases of productivity growth: between 1987 and 1993 productivity increased by 46.9% on the back of a 48.6% increase in scale and mix efficiency ($\Delta MFP = \Delta MFP^{*} \times ITE \times \Delta ISME$ = 0.974 × 1.015 × 1.486 = 1.469); between 1993 and 2001 productivity increased by a further 195% due mainly to a 206.6% increase in technical efficiency ($\Delta MFP = \Delta MFP^{*} \times ITE \times \Delta ISME$ = 1.013 × 3.066 × 0.95 = 2.95); and between 2001 and 2008 productivity increased by a further 87.8% due mainly to a 70.6% increase in scale-mix efficiency ($\Delta MFP = \Delta MFP^{*} \times ITE \times \Delta ISME$ = 1.055 × 1.044 × 1.706 = 1.878). In the 21-year period from 1987 to 2008 the Computer and Electronic Products sector changed from being the least productive manufacturing sector in the U.S. economy (with $MFP = MFP^{*} \times ITE \times ISME$ = 0.786 × 0.308 × 0.415 = 0.101) to being the most productive (with $MFP = MFP^{*} \times ITE \times ISME$ = 0.818 × 1 × 1 = 0.818).

As another example, Figure 9 presents estimated levels of MFP and efficiency in the Apparel and Leather and Applied Products manufacturing sector. This sector is relatively labor-intensive and has contracted significantly over the last two decades in the face of growing competition from low-wage foreign suppliers – between 1987 and 2006, output fell by 68% and input use fell by 70%. Figure 9 reveals that the sector remained fully technically efficient throughout this period. The largest change in productivity occurred in 2007 when productivity fell by 24.5% on the back of a 26% fall in scale-mix efficiency ($\Delta MFP = \Delta MFP^{*} \times ITE \times$

$\Delta ISME = 1.022 \times 1 \times 0.74 = 0.756$). Like many sectors, the main driver of productivity change in the Apparel and Leather and Applied Products manufacturing sector has been scale and mix efficiency change.

Finally, Table 9 reports estimated productivity and efficiency levels in each sector in 2008. This table reveals that in 2008 most sectors were highly technically efficient but somewhat scale-mix inefficient. Rational efficient firms will optimally adjust their scale and input mix (and, therefore, levels of scale-mix efficiency) in response to changes in production incentives (e.g., changes in relative prices). Thus, the relatively low levels of scale-mix efficiency reported in Table 9 may not require a government policy response.

6.5 Summary Snapshot

A final snapshot of productivity change in US manufacturing is provided in Figure 10. This figure plots estimated data points (i.e., estimated Färe-Primont aggregate output-input combinations) and estimated production frontiers for 1987 (blue) and 2008 (red). The labels in this figure are color-coded references to NAICS codes and industries (e.g., the blue label "311, 312 Food" identifies the aggregate output-input combination of the Food Beverage and Tobacco Products sector in 1987). Data points are labeled in this way if and only if they lie on the frontier (i.e., if and only if they are fully technically and mix efficient; they are not necessarily fully scale or MFP efficient). Each frontier is a DEA estimate of an unrestricted frontier of the type depicted in Figure 2. Only one frontier is depicted for 1987 (and only one for 2008) because in any given period all sectors were assumed to have access to the same production possibilities set. Observe from Figure 10 that:

- the 1987 and 2008 frontiers are very similar, even though no restrictions were placed on the nature of technical change (technical progress, technical regress and biased technical change were all permitted);
- the difference between the two frontiers in the region of the two points of maximum

productivity ("Food" in 1987 and "Computers" in 2008) is negligible (this difference corresponds to an average rate of technical progress of only 0.189% per annum);

- in the region of the data points represented in the sample[20], the estimated frontiers are typical of technologies that exhibit constant returns to scale, even though no restrictive assumptions were made about returns to scale;
- the Food, Beverage and Tobacco Products sector was the largest manufacturing sector in both 1987 and 2008;
- the Furniture and Related Products sector was the smallest sector in 1987 but the Apparel and Leather and Applied Products sector was the smallest sector in 2008; and
- the Computer and Electronics Products sector experienced the largest increase in productivity over the sample period, and by 2008 it had become the most productive sector in the economy (the dotted green line in Figure 10 traces out the path taken by this sector on its way to the 2008 frontier; MFP levels in 1993 and 2001 are marked on this dotted line because in earlier discussion these years were identified as break-points in three reasonably distinct phases of productivity growth);

7. Conclusion

Improvements in productivity are an important pre-condition for sustainable improvements in standards of living. The most basic requirement for effective policy-making in this area is the accurate measurement of productivity change. Unfortunately, most statistical agencies measure productivity change using index number formulas that fail commonsense axioms and tests. For example, the BLS measures US manufacturing sector productivity change using a chained Törnqvist formula that fails a transitivity test, while the USDA measures agricultural productivity change using a Fisher-EKS formula that fails an identity axiom. These failures mean that it is possible, even likely, that these agencies will report inter-temporal and/or

[20] At the points on the boundaries of these regions (e.g., points representing the "Food" and "Furniture" sectors in 1987) the frontiers become vertical or horizontal – this is an artifact of DEA methodology.

inter-spatial changes in productivity even when levels of inputs and outputs are exactly the same. In this paper I identify three alternative index formulas that satisfy all economically-relevant axioms and tests from index number theory, including the identity axiom and the transitivity test. These indexes are a Lowe index proposed by O'Donnell (2010b), a Färe-Primont index proposed by O'Donnell (2011), and a Geometric Young index that, as far as I know, has not previously been used to measure productivity change.

A second requirement for good policy-making is identification of the economic drivers of productivity change. In practice, this involves breaking theoretically-plausible MFP indexes into components that have unambiguous economic interpretations. O'Donnell (2008) shows how most, if not all, meaningful MFP indexes can be exhaustively decomposed into three intrinsically different components – a technical change component that measures movements in the production frontier, a technical efficiency change component that measures movements towards or away from the frontier, and a scale-mix efficiency change component that measures movements around the frontier surface. Identifying the scale-mix efficiency change component is especially important for policy-makers because productivity declines that are driven by falls in scale-mix efficiency are often associated with increases in net returns. Unlike some other decomposition methodologies, the O'Donnell (2008) methodology does not require strong assumptions concerning the structure of the production technology, and does not require *any* assumptions concerning either the degree of competition in product markets or the optimizing behavior of firms. In this paper I use the methodology (and quantity data, not price data) to compute and decompose Färe-Primont MFP indexes.

The choice of index number formula matters – the theoretically-plausible Färe-Primont index numbers reported in this paper and the theoretically-implausible chained Törnqvist index numbers reported by the BLS would lead policy-makers to draw qualitatively different conclusions about productivity change in several sectors. For example, the BLS finds that productivity in the Machinery (NAICS code 333) and Electrical Equipment, Appliances and

Components (335) sectors fell by 4.5%, and 9.4% respectively over the sample period, whereas I estimate that productivity in these sectors increased by 9.9% and 13.5% respectively.

Alongside the Färe-Primont MFP indexes I report coherent estimates of the components of productivity change. These estimates are coherent in the sense that they combine to form a recognizable MFP index. I estimate that US manufacturers experienced technical progress at an average annual rate of only 0.189%. I find that by the end of the sample period most sectors were highly technically efficient but somewhat scale-mix inefficient. Importantly, relatively low levels of scale-mix efficiency (and therefore productivity) do not necessarily warrant a government policy response. My empirical results support the view that most US firms are technically efficient and that they simply change the structure of their operations (i.e., scale and input mix and, therefore, levels of scale and mix efficiency) in response to changes in (expected) prices.

The MFP indexes and decomposition methods discussed in this paper can be applied in other empirical contexts. Estimates of the components of productivity change are likely to be of particular interest to policy-makers who have been asked to develop policy responses to measured productivity "slowdowns". In this context, there are at least two opportunities for further research. First, it would be useful to compute measures of reliability (e.g., standard errors) for the many estimated components of productivity change. If this is done using econometric methods (e.g., stochastic frontier analysis) then it would also be straightforward and useful to test assumptions concerning both the structure of the technology (e.g., constant returns to scale) and the nature of technical change (e.g., no technical regress). Second, it is common to estimate reduced-form relationships between MFP indexes and collections of variables that are known to influence economic activity (e.g., levels of R&D expenditure, education, prices). This paper presents researchers with interesting opportunities for estimating relationships between the individual components of productivity change and the variables that plausibly influence just those components (e.g., relationships between the technical change component and R&D expenditure, between the technical efficiency change component and education, and between the scale efficiency change component and prices). Estimated relationships of this type may provide interesting new insights into the costs and benefits of specific government policies and programs.

Table 1. Productivity Indexes for Binary Comparisons

	Output Aggregator Function	Input Aggregator Function	Productivity Index
Lasypeyres	$Q(q)=q'p_{hs}$	$X(x)=x'w_{hs}$	$MFP_{hs,it}=\frac{q'_{it}p_{hs}}{q'_{hs}p_{hs}}\frac{x'_{hs}w_{hs}}{x'_{it}w_{hs}}$
Paasche	$Q(q)=q'p_{it}$	$X(x)=x'w_{it}$	$MFP_{hs,it}=\frac{q'_{it}p_{it}}{q'_{hs}p_{it}}\frac{x'_{hs}w_{it}}{x'_{it}w_{it}}$
Fisher	$Q(q)=\left[q'p_{hs}p'_{it}q\right]^{1/2}$	$X(x)=\left[x'w_{hs}w'_{it}x\right]^{1/2}$	$MFP_{hs,it}=\left[\frac{q'_{it}p_{hs}}{q'_{hs}p_{hs}}\frac{q'_{it}p_{it}}{q'_{hs}p_{it}}\frac{x'_{hs}w_{hs}}{x'_{it}w_{hs}}\frac{x'_{hs}w_{it}}{x'_{it}w_{it}}\right]^{1/2}$
Malmquist-*hs*	$Q(q)=D_O(x_{hs},q,s)$	$X(x)=D_I(x,q_{hs},s)$	$MFP_{hs,it}=\frac{D_O(x_{hs},q_{it},s)}{D_O(x_{hs},q_{hs},s)}\frac{D_I(x_{hs},q_{hs},s)}{D_I(x_{it},q_{hs},s)}$
Malmquist-*it*	$Q(q)=D_O(x_{it},q,t)$	$X(x)=D_I(x,q_{it},t)$	$MFP_{hs,it}=\frac{D_O(x_{it},q_{it},t)}{D_O(x_{it},q_{hs},t)}\frac{D_I(x_{hs},q_{it},t)}{D_I(x_{it},q_{it},t)}$
Hicks-Moorsteen	$Q(q)=\left[D_O(x_{hs},q,s)D_O(x_{it},q,t)\right]^{1/2}$	$X(x)=\left[D_I(x,q_{hs},s)D_I(x,q_{it},t)\right]^{1/2}$	$MFP_{hs,it}=\left[\frac{D_O(x_{hs},q_{it},s)D_O(x_{it},q_{it},t)}{D_O(x_{hs},q_{hs},s)D_O(x_{it},q_{hs},t)}\frac{D_I(x_{hs},q_{hs},s)D_I(x_{hs},q_{it},t)}{D_I(x_{it},q_{hs},s)D_I(x_{it},q_{it},t)}\right]^{1/2}$
Törnqvist-*hs*	$Q(q)=\exp(r'_{hs}\ln q)$	$X(x)=\exp(s'_{hs}\ln x)$	$MFP_{hs,it}=\frac{\exp\left[r'_{hs}\ln q_{it}+s'_{hs}\ln x_{hs}\right]}{\exp\left[r'_{hs}\ln q_{hs}+s'_{hs}\ln x_{it}\right]}$
Törnqvist-*it*	$Q(q)=\exp(r'_{it}\ln q)$	$X(x)=\exp(s'_{it}\ln x)$	$MFP_{hs,it}=\frac{\exp\left[r'_{it}\ln q_{it}+s'_{it}\ln x_{hs}\right]}{\exp\left[r'_{it}\ln q_{hs}+s'_{it}\ln x_{it}\right]}$
Törnqvist	$Q(q)=\left[\exp(r'_{hs}\ln q)\exp(r'_{it}\ln q)\right]^{1/2}$	$X(x)=\left[\exp(s'_{hs}\ln x)\exp(s'_{it}\ln x)\right]^{1/2}$	$MFP_{hs,it}=\frac{\exp\left[0.5(r_{hs}+r_{it})'\ln q_{it}+0.5(s_{hs}+s_{it})'\ln x_{hs}\right]}{\exp\left[0.5(r_{hs}+r_{it})'\ln q_{hs}+0.5(s_{hs}+s_{it})'\ln x_{it}\right]}$

Table 2. Example Data

Observation	Firm	Period	q_{it}	x_{1it}	x_{2it}	p_{it}	w_{1it}	w_{2it}
1	1	1	30	60	120	3	6	12
2	2	1	20	60	30	2	6	3
3	3	1	20	20	60	2	2	6
4	4	1	10	50	20	1	5	2
5	1	2	30	60	120	2	2	6
6	2	2	20	60	30	2	2	6
7	3	2	20	20	60	2	2	6
8	4	2	10	40	20	2	2	6

Table 3. Selected MFP Indexes

Index	True	Malmquist	Hicks-Moorst.	Fisher	Törnqvist	Fisher-EKS	Törnqvist-EKS	Lowe	Fare-Primont	Geo. Young
$MFP_{11,12}$	1	1	1	1	1	0.973	0.983	1	1	1
$MFP_{21,22}$	1	1	1	1	1	1.163	1.166	1	1	1
$MFP_{31,32}$	1	1	1	1	1	1	1	1	1	1
$MFP_{41,42}$	> 1	1.069	1.069	1.153	1.151	1.369	1.321	1.041	1.069	1.087
$MFP_{11,41}$	?	1.450	1.450	0.830	0.851	0.908	0.907	1.057	1.234	1.093
$MFP_{41,12}$	?	0.810	0.810	1.125	1.122	1.072	1.083	0.946	0.810	0.915
$MFP_{11,41} \times MFP_{41,1}$	$MFP_{11,12} = 1$	1.175	1.175	0.934	0.955	1	1	1	1	1

Table 4. Productivity Indexes for Multi-lateral and Multi-temporal Comparisons

	Output Aggregator	Input Aggregator	Productivity Index
Lowe	$Q(q) = q'p_0$	$X(x) = x'w_0$	$MFP_{hs,it} = \frac{q'_{it}p_0}{q'_{hs}p_0}\frac{x'_{hs}w_0}{x'_{it}w_0}$
Färe-Primont	$Q(q) = D_O(x_0, q, t_0)$	$X(x) = D_I(x, q_0, t_0)$	$MFP_{hs,it} = \frac{D_O(x_0, q_{it}, t_0)}{D_O(x_0, q_{hs}, t_0)}\frac{D_I(x_{hs}, q_0, t_0}{D_I(x_{it}, q_0, t_0}$
Geometric Young	$Q(q) = \exp(r'_0 \ln q)$	$X(x) = \exp(s'_0 \ln x)$	$MFP_{hs,it} = \frac{\exp[r'_0 \ln q_{it} + s'_0 \ln x_{hs}]}{\exp[r'_0 \ln q_{hs} + s'_0 \ln x_{it}]}$

Table 5. Selected Indexes Comparing Firm 4 in Period 2 with Firm 4 in Period 1.

	True	Malmquist-*hs*	Malmquist-*it*	Hicks-Moorst.	Fare-Primont
$MFP_{41,42} = MFP_{42} / MFP_{41}$	> 1	1.15	1	1.07	1.15
$Q_{41,42} = Q_{42} / Q_{41}$	1	1	1	1	1
$X_{41,42} = X_{42} / X_{41}$	< 1	0.87	1	0.93	0.87
$MFP^*_{1,2} = MFP^*_2 / MFP^*_1$		1	1	1	1
$MFPE_{41,42} = MFPE_{42} / MFPE_{41}$	> 1	1.15	1	1.07	1.15
$OTE_{41,42} = OTE_{42} / OTE_{41}$		1	1	1	1
$OSE_{41,42} = OSE_{42} / OSE_{41}$		1	1	1	1
$OME_{41,42} = OME_{42} / OME_{41}$		1	1	1	1
$OSME_{41,42} = OSME_{42} / OSME_{41}$		1.15	1	1.07	1.15
$ITE_{41,42} = ITE_{42} / ITE_{41}$		1	1	1	1
$ISE_{41,42} = ISE_{42} / ISE_{41}$		1	1	1	1
$IME_{41,42} = IME_{42} / IME_{41}$		1.15	1	1.07	1.15
$ISME_{41,42} = ISME_{42} / ISME_{41}$		1.15	1	1.07	1.15
α_{it}			0.1	0.1	
η_{1it}			0	0	
η_{2it}			0.05	0.05	
α_{hs}		0.1		0.1	
η_{1hs}		0.015		0.015	
η_{2hs}		0.02		0.02	
α_0					0.05
η_{10}					0.01
η_{20}					0.01

Table 6. Average Values and Shares by Manufacturing Sector: 1987-2008.

Sector	NAICS Code	Revenues and Costs ($b)						Cost Shares				
		Q	K	L	E	M	S	K	L	E	M	S
1. Food, Beverage & Tobacco Prod.	311, 312	451.1	63.9	69.7	10.6	244.3	62.6	0.14	0.15	0.02	0.54	0.14
2. Textile & Textile Product Mills	313, 314	68.5	6.2	18.0	1.8	35.9	6.7	0.09	0.26	0.03	0.52	0.10
3. Apparel & Leather & Applied Prod.	315, 316	53.0	6.0	16.8	0.9	18.2	11.2	0.12	0.33	0.02	0.31	0.22
4. Wood Products	321	76.1	6.4	19.1	2.1	39.5	9.0	0.09	0.25	0.03	0.52	0.11
5. Paper Products	322	137.6	21.5	30.9	7.7	61.3	16.3	0.15	0.23	0.05	0.45	0.12
6. Printing & Related Support Activ.	323	88.1	6.3	29.9	1.6	34.8	15.5	0.07	0.34	0.02	0.40	0.17
7. Petroleum & Coal Products	324	243.9	48.7	12.4	2.0	175.6	5.2	0.18	0.06	0.01	0.72	0.03
8. Chemical Products	325	365.5	84.0	70.8	17.2	121.6	71.8	0.23	0.19	0.05	0.34	0.20
9. Plastics & Rubber Products	326	143.0	20.4	35.2	3.9	63.5	20.0	0.14	0.25	0.03	0.44	0.14
10. Nonmetallic Mineral Products	327	76.1	12.8	24.2	5.6	21.5	12.0	0.17	0.32	0.07	0.28	0.15
11. Primary Metals	331	124.6	14.6	31.6	10.2	50.0	18.2	0.11	0.26	0.08	0.40	0.14
12. Fabricated Metal Products	332	216.3	30.9	73.1	4.3	71.7	36.3	0.14	0.34	0.02	0.33	0.16
13. Machinery	333	225.7	26.3	74.4	2.3	86.0	36.7	0.12	0.34	0.01	0.37	0.16
14. Computer & Electronic Products	334	301.5	31.7	106.8	3.6	85.2	74.1	0.10	0.35	0.01	0.28	0.25
15. Electrical Equip, Appl & Comp.	335	95.2	15.4	27.1	1.1	39.3	12.4	0.16	0.29	0.01	0.41	0.13
16. Transportation Equipment	336	425.6	39.2	120.9	4.2	206.1	55.2	0.09	0.29	0.01	0.48	0.13
17. Furniture & Related Products	337	60.0	6.1	21.6	0.7	22.6	9.0	0.10	0.36	0.01	0.38	0.15
18. Miscellaneous Manufacturing	339	95.4	16.6	34.1	1.0	26.2	17.5	0.17	0.35	0.01	0.28	0.18
All Sectors		180.4	25.4	45.4	4.5	78.0	27.2	0.13	0.28	0.03	0.42	0.15

Table 7. Indexes comparing MFP in 2008 with MFP in 1987 (1987 = 1)

Sector	NAICS Code	BLS	Färe-Primont	Lowe	Geom. Young
1. Food, Beverage & Tobacco Prod.	311, 312	1.025	1.024	1.016	1.035
2. Textile & Textile Product Mills	313, 314	1.196	1.179	1.166	1.135
3. Apparel & Leather & Applied Prod.	315, 316	0.926	0.786	0.952	1.482
4. Wood Products	321	0.980	0.970	0.959	0.938
5. Paper Products	322	1.107	1.130	1.104	1.131
6. Printing & Related Support Activ.	323	1.109	1.059	1.111	1.082
7. Petroleum & Coal Products	324	1.165	1.097	1.118	1.286
8. Chemical Products	325	1.106	1.053	1.130	1.231
9. Plastics & Rubber Products	326	1.123	1.112	1.114	1.121
10. Nonmetallic Mineral Products	327	1.069	1.055	1.048	1.054
11. Primary Metals	331	1.126	1.219	1.138	1.195
12. Fabricated Metal Products	332	1.124	1.133	1.116	1.106
13. Machinery	333	0.955	1.099	0.988	0.894
14. Computer & Electronic Products	334	8.342	8.136	8.301	8.082
15. Electrical Equip, Appl & Comp.	335	0.906	1.135	1.017	0.912
16. Transportation Equipment	336	1.087	1.149	1.080	1.073
17. Furniture & Related Products	337	1.070	1.059	1.063	1.050
18. Miscellaneous Manufacturing	339	1.458	1.481	1.474	1.468

Table 8. MFP Change, Technical Change and Efficiency Change: 1987-2008 (%)

Sector	NAICS Code	ΔMFP	ΔMFP*	ΔMFPE	ΔITE	ΔISE	ΔIME	ΔISME
1. Food, Beverage & Tobacco Prod.	311, 312	1.024	1.041	0.985	1	1	1	0.985
2. Textile & Textile Product Mills	313, 314	1.179	1.041	1.133	1	1.135	1.188	1.133
3. Apparel & Leather & Applied Prod.	315, 316	0.786	1.041	0.756	1	1.110	1.275	0.756
4. Wood Products	321	0.970	1.041	0.932	1	1	0.964	0.932
5. Paper Products	322	1.130	1.041	1.085	1.04221	1.00422	1.049	1.041
6. Printing & Related Support Activ.	323	1.059	1.041	1.018	1	1	1.041	1.018
7. Petroleum & Coal Products	324	1.097	1.041	1.054	1	1	1.063	1.054
8. Chemical Products	325	1.053	1.041	1.012	1	1	1.021	1.012
9. Plastics & Rubber Products	326	1.112	1.041	1.068	1.05974	0.999	1.012	1.008
10. Nonmetallic Mineral Products	327	1.055	1.041	1.014	0.941	0.908	1.067	1.077
11. Primary Metals	331	1.219	1.041	1.172	1.029	1.001	1.145	1.138
12. Fabricated Metal Products	332	1.133	1.041	1.088	0.954	1.017	1.141	1.141
13. Machinery	333	1.099	1.041	1.056	0.952	0.996	1.108	1.109
14. Computer & Electronic Products	334	8.136	1.041	7.819	3.248	1.632	1.840	2.408
15. Electrical Equip, Appl & Comp.	335	1.135	1.041	1.091	1	1	1.103	1.091
16. Transportation Equipment	336	1.149	1.041	1.104	1	1	1.118	1.104
17. Furniture & Related Products	337	1.059	1.041	1.017	1	0.898	0.887	1.017
18. Miscellaneous Manufacturing	339	1.481	1.041	1.423	1.043	1.057	1.266	1.365

Table 9. Levels of Productivity and Efficiency: 2008

Sector	NAICS Code	MFP	MFP*	MFPE	ITE	ISE	IME	ISME
1. Food, Beverage & Tobacco Prod.	311, 312	0.805	0.818	0.985	1	1	1	0.985
2. Textile & Textile Product Mills	313, 314	0.716	0.818	0.875	1	1	1	0.875
3. Apparel & Leather & Applied Prod.	315, 316	0.459	0.818	0.561	1	1	1	0.561
4. Wood Products	321	0.755	0.818	0.923	1	1	0.964	0.923
5. Paper Products	322	0.802	0.818	0.981	1	1	0.993	0.981
6. Printing & Related Support Activ.	323	0.758	0.818	0.927	1	1	0.957	0.927
7. Petroleum & Coal Products	324	0.725	0.818	0.887	1	1	0.894	0.887
8. Chemical Products	325	0.750	0.818	0.917	1	1	0.926	0.917
9. Plastics & Rubber Products	326	0.703	0.818	0.860	0.885	0.991	0.981	0.971
10. Nonmetallic Mineral Products	327	0.678	0.818	0.829	0.941	0.908	0.912	0.881
11. Primary Metals	331	0.783	0.818	0.958	1	1	0.968	0.958
12. Fabricated Metal Products	332	0.723	0.818	0.884	0.920	0.982	0.963	0.961
13. Machinery	333	0.724	0.818	0.885	0.952	0.996	0.932	0.929
14. Computer & Electronic Products	334	0.818	0.818	1	1	1	1	1
15. Electrical Equip, Appl & Comp.	335	0.802	0.818	0.981	1	1	1	0.981
16. Transportation Equipment	336	0.766	0.818	0.937	1	1	0.949	0.937
17. Furniture & Related Products	337	0.673	0.818	0.823	1	0.847	0.887	0.823
18. Miscellaneous Manufacturing	339	0.693	0.818	0.848	1	0.869	0.859	0.848

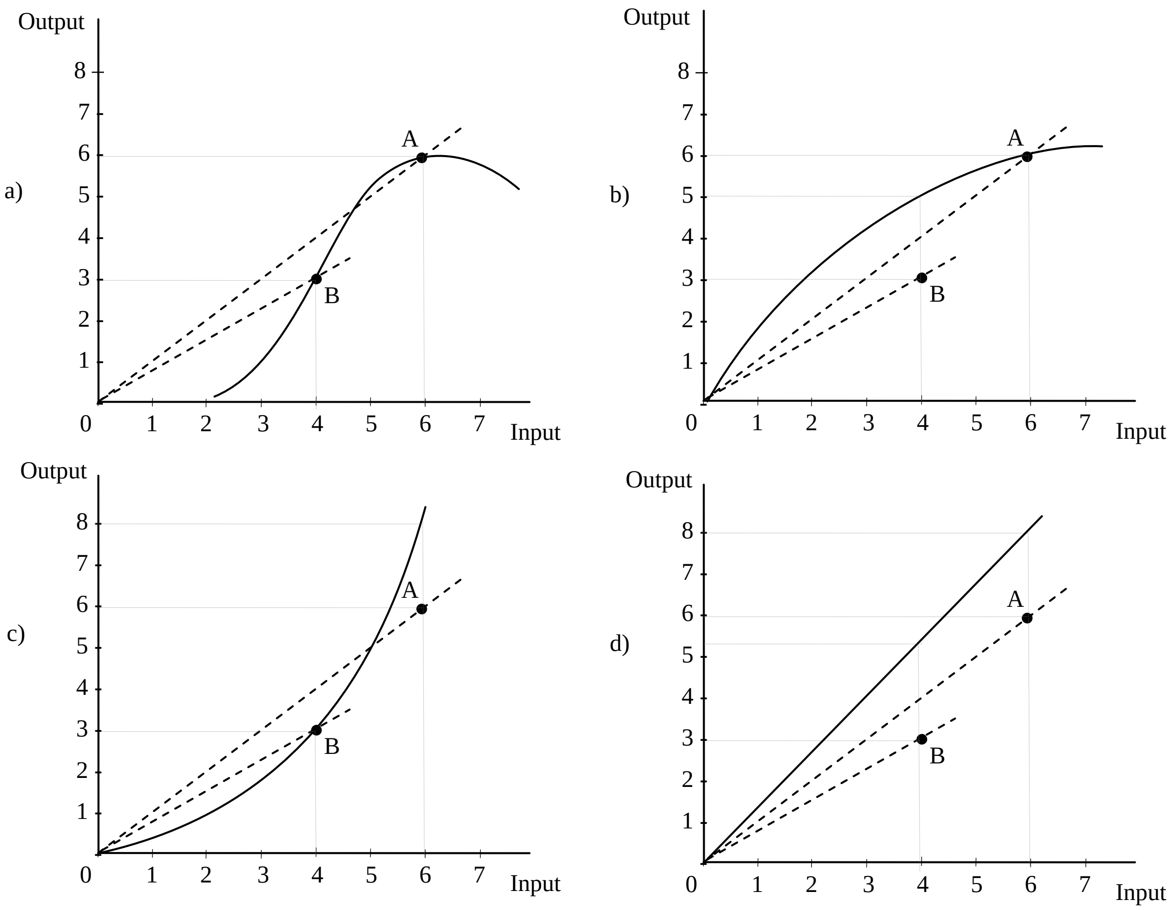

Figure 1. Alternative Single-Input Single-Output Production Frontiers

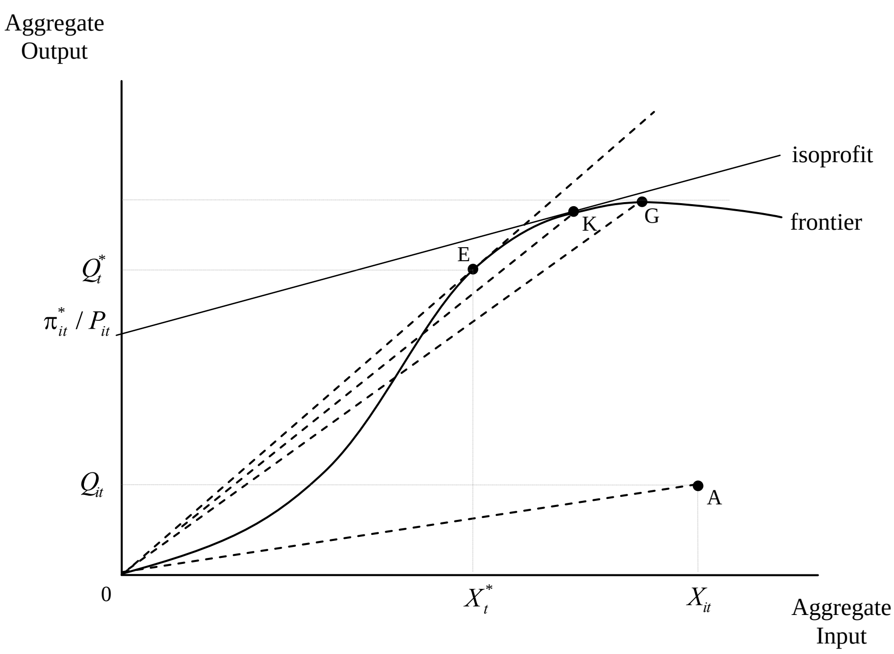

Figure 2. Productivity, Profitability and the Terms of Trade (*Source*: O'Donnell (2008)).

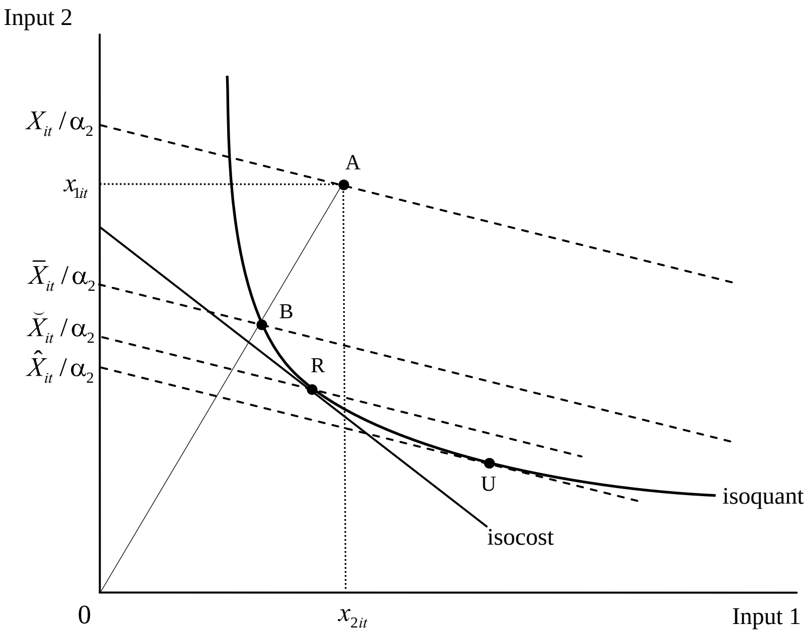

Figure 3. Input-Oriented Measures of Efficiency

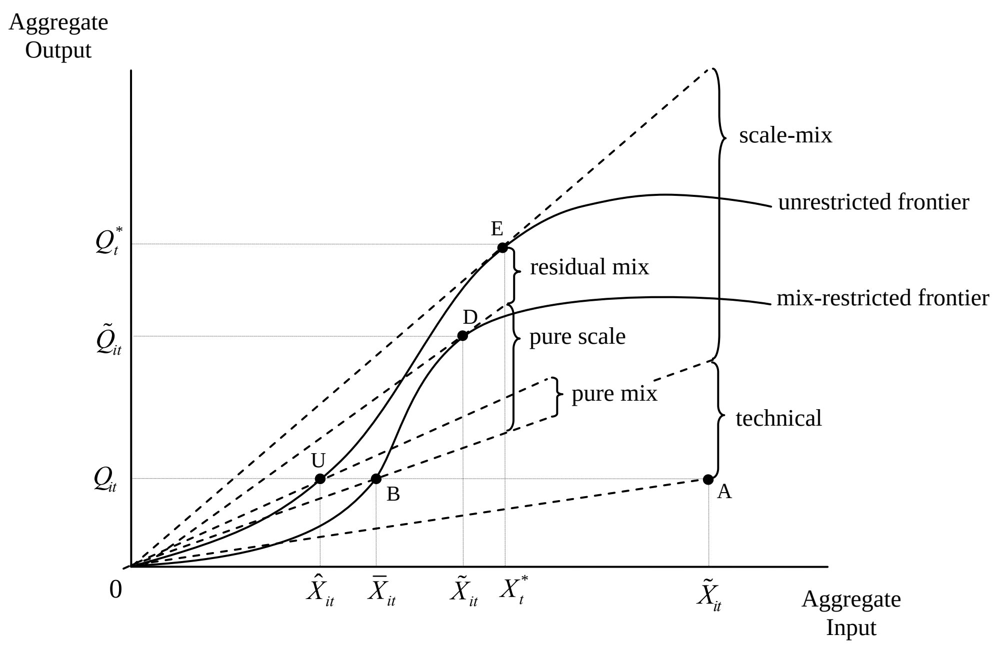

Figure 4. Selected Input-Oriented Measures of Efficiency in Aggregate Quantity Space

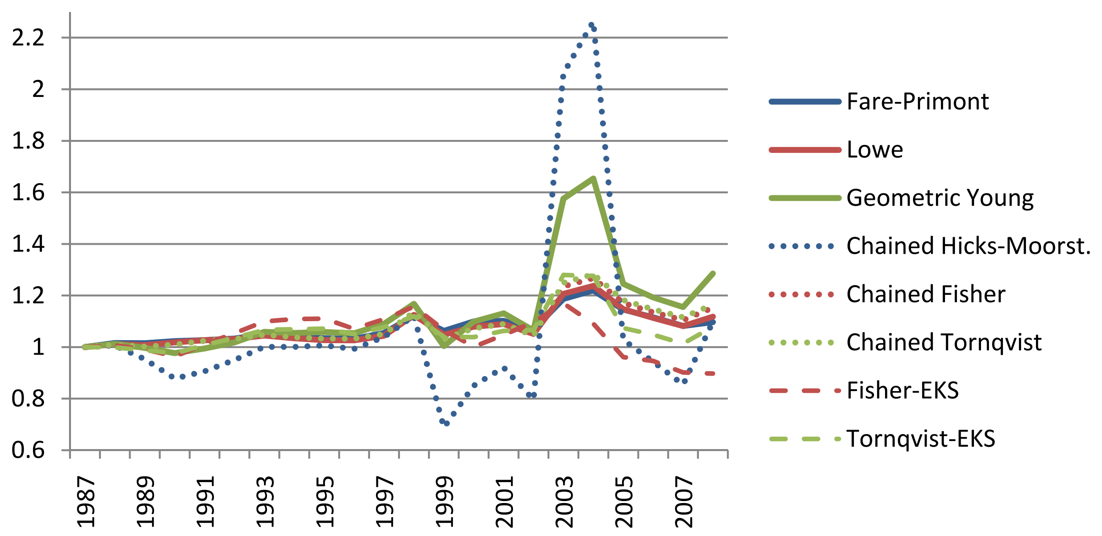

Figure 5. Productivity Change in the Petroleum and Coal Products Sector: 1987-2008.

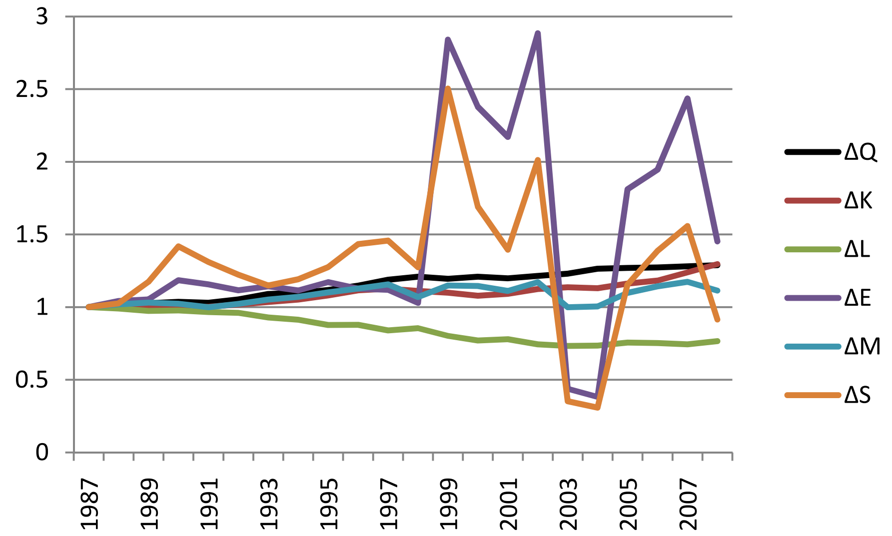

Figure 6. Output and Input Change in the Petroleum and Coal Products Sector: 1987-2008.

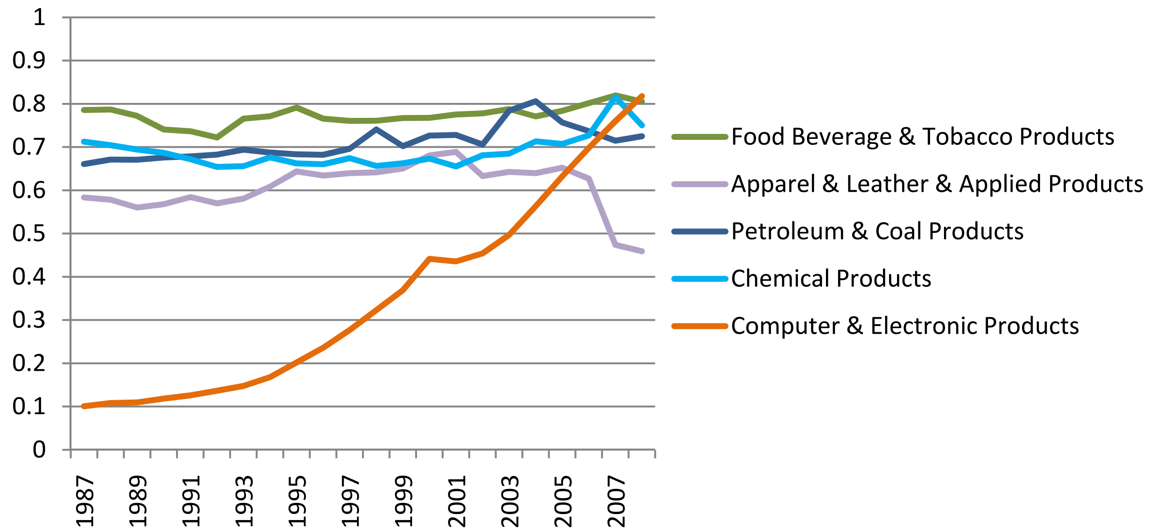

Figure 7. Productivity Change in Selected Sectors: 1987-2008.

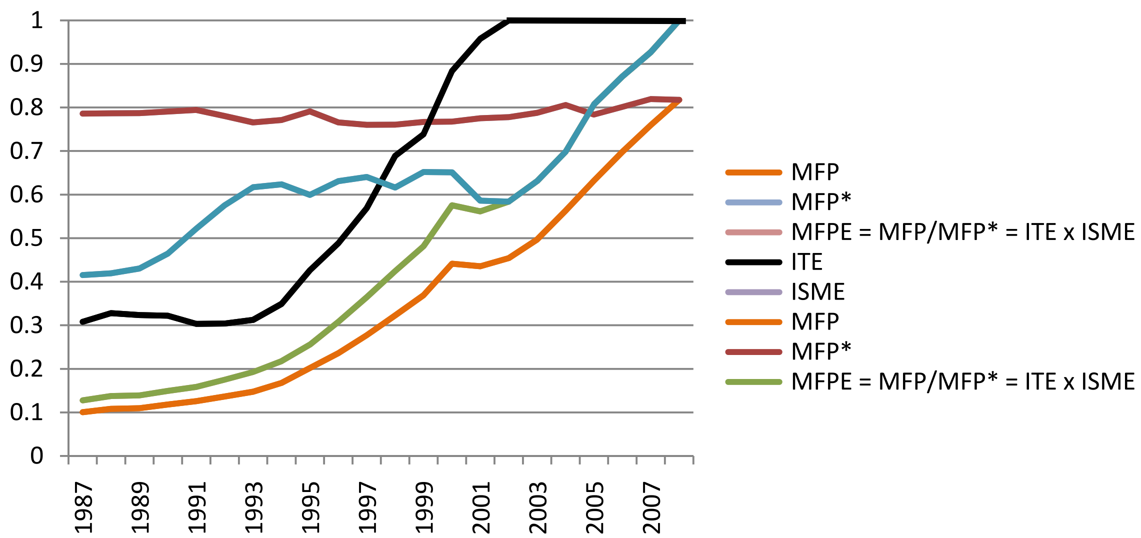

Figure 8. Levels of Productivity and Efficiency in the Computer and Electronic Products Sector: 1987-2008.

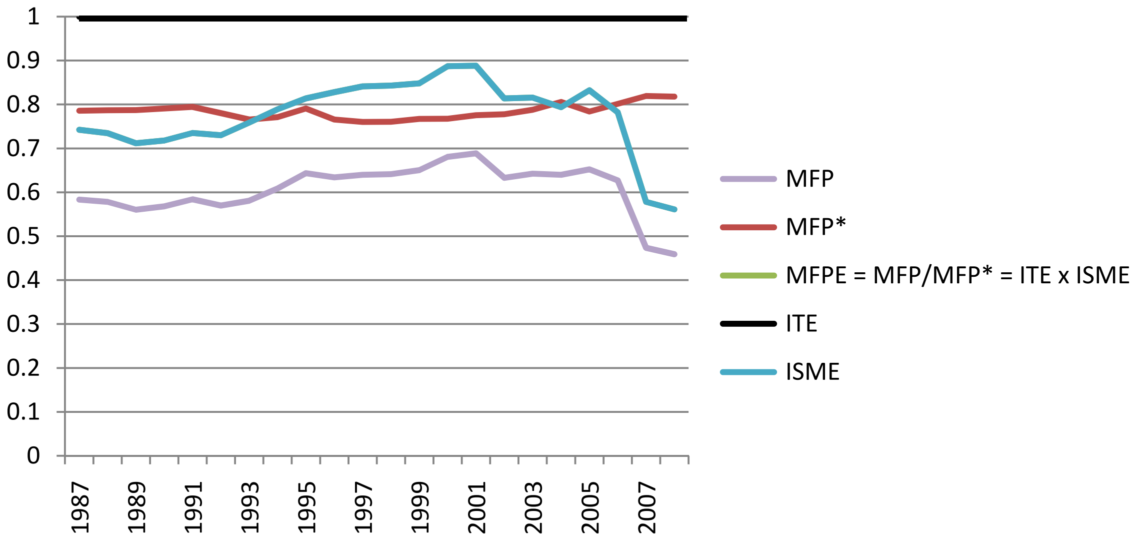

Figure 9. Levels of Productivity and Efficiency in the Apparel and Leather and Applied Products Sector: 1987-2008.

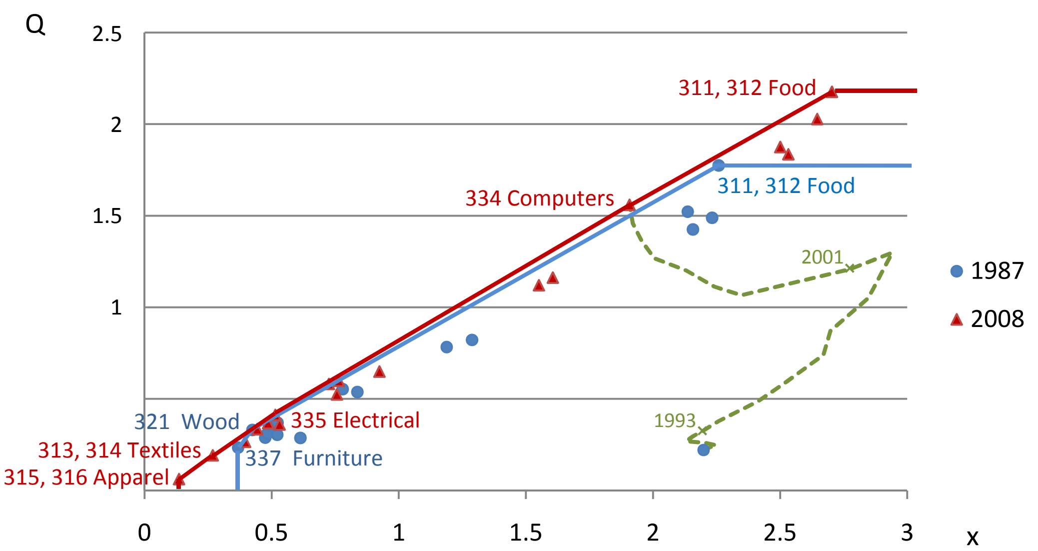

Figure 10. Estimated (Unrestricted) Production Frontiers: 1987 and 2008.

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