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UNEMPLOYMENT PERSISTENCE AND LOSS OF PRODUCTIVE CAPACITY: A KEYNESIAN APPROACH

by Theo van de Klundert and
Anton van Schaik

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Th. van de Klundert and A. van Schaik*)

Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands

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Abstract

Empirical evidence suggests that the European unemployment problem is somehow connected with a capital shortage. The paper introduces a Keynesian model to deal with this issue. In this model the rate of capacity utilization plays a central role in two simultaneously operating mechanisms thus exhibiting hysteresis: price-adjustment and capital accumulation. The implications for employment and unemployment are discussed, both for the long run and the development over time. It is demonstrated that equilibrium unemployment may be caused by negative aggregate demand shocks.

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1. Introduction

In the discussions on long-term unemployment in Europe the possibility of a capital shortage has been mentioned regularly. The main idea is that insufficient capital accumulation created a situation in which not enough jobs are available for the existing working force. Quite often also the existence of a capital shortage is rejected and different arguments are presented to substantiate this claim. For instance, Layard and Bean (1988) argue that the number of workers per machine can be varied on any shift and that the number of shifts can be varied too. Under these circumstances there can be no capital constraint. What then really matters are supply constraints originating in the labour market. In Layard and Nickell (1986), Nickell (1987) and other publications by the same authors this argument is elaborated. It is assumed that capital accumulation or decumulation has no effect on (equilibrium) unemployment, because changes in productivity are fully reflected in changes in real wages. In our view this argument may hold in a structural sense. Otherwise, unemployment would steadily rise or decline, which is clearly unrealistic. Nevertheless, deviations from the trend growth rate could have a lasting impact on unemployment if real wages are rigid to some extent.

The consequences of a reduction in the capital stock on impact of adverse demand and supply shocks deserve therefore proper attention. It should be noted in this connection that the problem has been analysed in a neoclassical setting (e.g. Bruno and Sachs, 1985; Van der Ploeg, 1987; Burda, 1988). In these studies reductions in the capital stock affect the demand for labour just as adverse supply shocks do. Demand shocks have no impact on employment unless they affect the real exchange rate for the case of an open economy (cf. Burda, 1988; Van de Klundert, 1988) or unless they affect the interest rate for a closed economy (crowding in). However, in a world characterized by price sluggishness demand shocks may have a more direct effect on the supply-side capacity. Blanchard (1988) takes this view into account, but gives no complete analysis. Nominal price inertia enters his model, because agents base price expectations on past realizations of prices. Production and investment decisions are still based on price signals. Therefore, once price expectations catch up with actual prices the neoclassical model is again fully applicable.

In our model sluggish price adjustment is associated with uncertainty about the outcome of the competitive process if the economy is hit by aggregative shocks. Firms may then behave rationally by adjusting quantities. In the short run this takes the form of a reduction in output on impact of an adverse demand shock. As the recession proceeds there will be an increase in company failures and some large firms may close plants, scrapping capital and making workers redundant. Once this has happened, once a factory has been demolished, it cannot suddenly begin production again if demand for its product were to increase. Hysteresis connected with capital decumulation induces a decline in employment. If there are sufficient substitution possibilities and if real wages are flexible enough full employment could of course be attained within a reasonable time span. However, a severe loss in capacity entails a loss in productivity, which may not be matched by real wage moderations. Following Blanchard (1988) there is an obvious loose end in the argument that a capital shortage leads to high unemployment. We agree that the argument is not complete. In addition something should be assumed about labour market clearing. But even then it may be important to give a proper analysis of the "physical capital story" (Blanchard and Summers, 1986). We think our paper may shed some new light on this issue.

Turning to the stylized facts it could be maintained that during the protracted recession of 1980-1982 demand-deficient unemployment developed into equilibrium unemployment, especially in Europe. Capacity utilization rates for those years reflect the impact of the recession. After a year of rapid recovery in 1984 the rates of capacity utilization in manufacturing in European countries returned to normal levels (OECD, 1988, Chart H). In this respect the time series for Germany (Table 1) provides us with a thoroughly illustrative example. It appears that after the recovery capacity utilization in Germany on the average has stayed at the peak level of 1979. Empirical evidence suggests that potential output was adjusted downward to actual output. (In 1985-1987 the average rate of output growth in Germany was about 2% a year, whereas in 1976-1979 it was about 4% a year.) Part of the loss of productive capacity may have taken place by excessive scrappage of idle capacity, which is not reflected in the official statistics. Nevertheless, decreasing investment ratios are a well-documented and widely observed fact, indicating the slowdown of capital growth in Europe.[1)]

Table 1 Rate of capacity utilisation in manufacturing (Germany)

1973	87.1	1976	80.2	1979	84.7	1982	76.3	1985	84.3
1974	82.5	1977	80.2	1980	82.4	1983	78.4	1986	84.7
1975	76.0	1978	81.2	1981	79.0	1984	80.6	1987	84.3

Source: OECD (1984) and recent issues of OECD Main Economic Indicators.

The paper proceeds as follows. Section 2 confronts the neoclassical model of imperfect competition, capital accumulation and unemployment persistence with our Keynesian approach. In section 3 an analytical solution of the dynamic system emerging out of the Keynesian model is presented. The system has one zero root, reflecting hysteresis and two negative roots, which stabilize the development over time. The long-run solutions give rise to a number of interesting observations. The paper closes with conclusions and suggestions for further research.

2. Capital accumulation and unemployment persistence: two views

In the neoclassical model structural unemployment is explained as a result of the controversy between employers and employees within an equilibrium context; product markets clear as a result of flexible prices. Therefore, structural or equilibrium unemployment is caused by push factors such as union militancy, mismatch on labour markets, the wedge between real consumers' and real producers' wages, etc. Actual unemployment may deviate from equilibrium unemployment in the short run, because of nominal wage or price inertia. Nominal inertia is then modelled in a complementary way by assuming that price or wage expectations of economic agents may deviate temporarily from their actual values. In a recent and useful review of economic theory behind the papers at the Chelwood Gate conferences on unemployment in Europe Blanchard (1988) critizes what he calls the false dichotomy between equilibrium and actual unemployment. The main reason for this is that the neoclassical theory at least in Chelwood Gate Mark I (Economica, 1986) does not fit the actual experience in Europe in the 1980s. During that period negative aggregate demand shocks induced an increase in actual unemployment followed by a rise in equilibrium unemployment.

According to Blanchard Chelwood Gate Mark II explores two channels which may explain that high actual unemployment induces a high level of equilibrium unemployment and which were already suggested at the first conference. The first channel is capital accumulation. A sustained period of unemployment may lead to capital decumulation and therefore to an increased equilibrium unemployment level. The second channel relates to different aspects of hysteresis on the labour market, implying that the natural rate of unemployment depends upon the actual rate (e.g. Sachs, 1987).

In this paper we focus on the problem of capital accumulation in relation to the European unemployment problem. We agree with Blanchard's critique on the dichotomy between actual and equilibrium unemployment. However, where Blanchard analyses the role of capital accumulation within a neoclassical model we depart from this view of the world. In our opinion the ultimate consequence of the dichotomy critique should be that behaviour of producers is modelled differently from the neoclassical paradigm. For instance if firms are confronted with a lack of effective demand this should influence their investments decisions or more generally it may even endanger the very existence of the firm. Blanchard (1988) suggests that the introduction of monopolistic competition into the neoclassical model takes care of the demand problem. This is not the route we want to follow. Monopolistic competition does not change the neoclassical model in an essential way. On the contrary, the formal results are the same except for a multiplicative factor determining the monopoly profit of the firm. The stories told sound of course different because firms are engaged in active price setting, whereas under perfect competition firms are price takers. But here again the difference is only superficial. Under perfect competition someone has to set prices and the dynamic evolution to a state of rest mirrors theories of imperfect competition as Arrow (1959) has pointed out long ago. Therefore stories of price setting and mark-up rules belong to the realm of perturbed economies whether or not competition is perfect or neatly imperfect as in the case of monopolistic competition.

To cope with the issue of demands shocks and capital decumulation we present a model which is more Keynesian in spirit. It is assumed that price adjustment is sluggish. The reasons for price intertia are not spelled out. Small menu cost (e.g. Blanchard and Kiyotaki, 1987; Ball and Romer, 1987) may be a cause, but we would prefer microeconomic theories focussing on informational aspects (e.g. Stiglitz, 1984; Van de Klundert and Peters, 1988). In an economy-wide recession firms may increase sales by lowering the price of output, but they may be highly uncertain whether this would entail a rise in revenue as competitors may lower their prices too. Under these circumstances it could be rational for risk-averse firms to stick to the prevailing price level.

To place our model in proper perspective it may be useful to compare the results with those of the neoclassical model. For this reason we first turn to the latter model.

The neoclassical model

As the model is rather well-known we may concentrate on the main issues, which are relevant for a comparison with the Keynesian model. Let us assume a Cobb-Douglas production function for convenience:

$$Y = \varepsilon L^{\lambda} K^{1-\lambda} \tag{1}$$

where Y is output, L is labour and K is capital. Under monopolistic competition firms face a downward sloping demand curve with elasticity $\eta(<-1)$, which is usually assumed constant.[2] Profit maximization by the representative firm gives then the short-run demand for labour as:

$$L = \frac{\eta+1}{\eta} \lambda \frac{Y}{W} \tag{2}$$

where W stands for the real wage rate. Substitution of (1) in (2) yields the implicit expression for the price firms charge given the nominal wage rate and the capital stock:

$$W = \frac{\eta+1}{\eta} \lambda\varepsilon \left[\frac{K}{L}\right]^{1-\lambda} \tag{3}$$

In the terminology of Layard and Nickell (1987) equation (3) gives the warranted real wage, i.e. the real wage rate implied by price setting of firms.

On the supply side of the labour market unions may maximize their utility by trading off employment for real wages (e.g. Oswald, 1985; Sachs, 1987). The target real wage resulting from a monopoly union model could then be written as:

$$W = \Omega\left[\frac{L}{N^S}\right]^{\nu} \tag{4}$$

where N^S denotes the labour force, which is set equal to one for convenience in the subsequent analysis. A change in the push factors mentioned above is reflected in a change in the parameter Ω. Equation (4) is referred to by Sachs (1987) as the wage-offer curve.[3)]

The equilibrium (un)employment rate may be found by combining equations (3) and (4):

$$L = \left[\frac{\eta+1}{\eta}\,\frac{\lambda\varepsilon}{\Omega}\,K^{1-\lambda}\right]^{\frac{1}{1+\nu-\lambda}} \tag{5}$$

The equilibrium rate of unemployment reconciles the income claims of workers and firms. Wage claims which are too high lead to a reduction in the demand for labour as the perhaps more familiar argument runs. An increase in push factors lowers equilibrium employment through a rise in Ω. Capital decumulation leads also to a lower level of employment as K falls.

In the long run firms can adjust both factors of production. The desired stock of capital follows from the first order condition for profit maximization with respect to K:

$$K = \frac{\eta+1}{\eta}\,(1-\lambda)\,\frac{Y}{R} \tag{6}$$

where R denotes the real user cost of capital, which are assumed constant throughout the analysis. Equation (6) may be applied to derive an investment function based on costs of adjustment (e.g. Brechling, 1975). The capital stock then gradually adapts to its desired level (K^d) given by equation (6). Investment ($\dot{K} = \frac{dK}{dt}$) is equal to:

$$\dot{K} = \psi(K^d - K) = \frac{\psi(1-\lambda)\varepsilon}{R} \frac{\eta+1}{\eta} \left[\frac{L}{K}\right]^{\lambda} K - \psi K \tag{7}$$

where the parameter ψ depends on the cost of adjustment. In the long run we have $K=K^d$ provided the system is stable. The investment function is introduced for the purpose of comparison with the corresponding investment equation in the Keynesian model. The dynamics of the neoclassical model are not relevant for our purpose. However, the long-run solution of the equilibrium (un)employment rate is of central importance for our topic, i.e. the persistence of unemployment in Europe.

Substitution of equations (2) and (6) in equation (1) yields the factor price frontier (FPF) of the present model:

$$\frac{1}{\varepsilon\lambda^{\lambda}(1-\lambda)^{1-\lambda}} W^{\lambda} R^{1-\lambda} = \frac{\eta+1}{\eta} \tag{8}$$

Combining the wage-offer curve (4) with the FPF (8) results in the long-run equilibrium employment rate:[4)]

$$L = \left[\frac{(\eta+1)\varepsilon\lambda^{\lambda}(1-\lambda)^{1-\lambda}}{\eta\Omega^{\lambda}R^{1-\lambda}} \right]^{\frac{1}{\nu\lambda}} \tag{9}$$

Following Blanchard (1988) the short-run and long-run solutions are illustrated graphically. In Figure 1 the curve S represents the wage-offer curve, equation (4), which could be conceived as a labour supply curve. The short-run labour demand curve D_{sr} follows from equation (3). In the long run labour demand is perfectly elastic as appears from equation (8). The corresponding long-run labour demand curve is indicated by $D_{\ell r}$ in Figure 1. Starting from a long-run equilibrium (point A) an increase in one of the push factors shifts the supply curve to the left (S'). In the short run the real wage rate rises and firms reduce labour demand. Profitability falls along with labour employed. Therefore, capital is adjusted downward and labour demand falls even further. As a consequence the real wage declines and the new long-run equilibrium is attained at point B. Meanwhile the short-run demand curve has shifted from D_{sr} to D'_{sr}. The result depends of course on the assumption of a fixed interest rate, which determines the user cost of capital R. As appears from the factor price frontier, equation (8), the long-run real wage rate remains in that case constant.

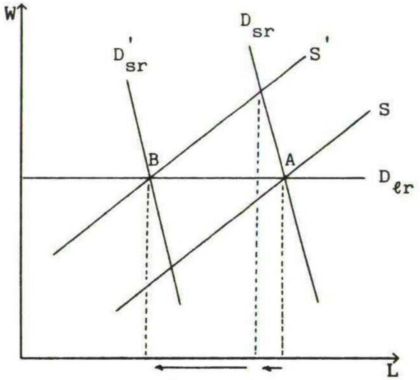


Figure 1

The discussion on the neoclassical model with real-wage rigidity reveals a number of important points:

a) Monopolistic competition and perfect competition lead to qualitatively the same results. In the latter case we have $\eta = \infty$ and the profit term $\frac{\eta+1}{\eta}$ in the formulas presented above vanishes.

b) Aggregate demand has no role to play whatsoever. The amount of money may determine the absolute price level, but this is immaterial to the present analysis and is therefore omitted.

c) Investment (capital accumulation) is entirely driven by relative factor scarcity. Capital and labour are complementary factors of production. A fall in labour employed leads to a decline in the profitability of capital. With the real user cost of capital fixed, capital will then be decumulated.

A Keynesian model

The central assumption of our Keynesian type of model is that (nominal) output prices and (nominal) wages are fixed in the short run. As a consequence it becomes necessary to distinguish between demand or actual output (X) and potential output or notional supply (Y) on the one hand and between actual employment (N) and notional labour demand or the availability of jobs (L) on the other hand, although these concepts are given a somewhat different content than in the standard literature on rationing (e.g. Malinvaud, 1977; Benassy, 1982). Firms expect to be in a situation of "Keynesian unemployment" most of the time. The possibility of "Repressed inflation" is not excluded, but may be less relevant as prices adjust fast in such a situation. With demand constrained in this way short-run profit maximisation is not an issue, but it may pay to produce at minimal costs. Suppose, that the production function relating notional output to notional labour demand and capital is given by equation (1). The first-order condition for a cost minimum can then be written as:

$$\frac{K}{L} = \frac{1-\lambda}{\lambda}\frac{W}{R} \tag{10}$$

It should be stressed that cost minimisation according to equation (10) is a long-run concern of firms. Factor substitution is considered for a situation where demand equals potential output (X=Y). In such an equilibrium situation firms want to produce at the lowest cost given factor remunerations. Cost minimisation therefore constitutes an element of <u>strategic</u> behaviour in the arrangements firms have to make.

For commodity demand we employ a simple quantity formula (cf. Blanchard, 1988):

$$X = \frac{A}{P} \tag{11}$$

where the parameter A is determined by monetary factors (i.e. the supply of money and the velocity of circulation). As capital is given in the short run firms will produce output (effective demand) with the minimal amount of labour necessary. This gives the equation for actual employment:[5)]

$$N = \left[\frac{X}{\varepsilon}\right]^{\frac{1}{\lambda}} K^{\frac{\lambda-1}{\lambda}} \tag{12}$$

The output gap is closed over time by two simultaneously operating mechanisms. First, it is assumed along traditional lines that prices decrease as a function of the rate of capacity utilization (Q = X/Y):[6)]

$$\frac{\dot{P}}{P} = \beta(Q-1) \tag{13}$$

Second, we assume that the stock of capital also changes with the degree of capacity utilisation:

$$\frac{\dot{K}}{K} = \alpha(Q-1) \tag{14}$$

Such an investment equation may seem restrictive but it is instructive to compare this relation with its neoclassical counterpart. For the case of perfect competition the neoclassical investment function (7) can be written as:

$$\dot{K} = \psi\left[\frac{(1-\lambda)}{R}\,\varepsilon\left[\frac{L}{K}\right]^{\lambda} - 1\right]K$$

Substitution of equations (1) and (10) into equation (14) taking account of the definition of Q yields the Keynesian investment function:

$$\dot{K} = \alpha\left[\frac{A}{KP}\,\frac{1}{\varepsilon}\left[\frac{(1-\lambda)}{\lambda}\,\frac{W}{R}\right]^{\lambda} - 1\right]K$$

The real user cost of capital is negatively related to investment in both equations. Changes in the real wage rate have a different effect. In the neoclassical model a rise in real wages leads to a fall in employment (L) and a decline in investment, whereas in the Keynesian model an increase in real wages pushes capital deepening. In the Keynesian model it is assumed that firms make their investment decision conditioned on a given level of output. The theory of investment has to fit in the effective demand framework (cf. Hall and Taylor, 1988). In the neoclassical theory there is no demand constraint and profitable investment projects may led to an expansion in output.

From a neoclassical perspective the present Keynesian model may perhaps seem ad-hoc. The relevant question is, however, whether the model has something to say about a world where uncertainty prevails, information may be asymmetrical distributed and competition takes different forms, which calls for stategic behaviour. If this is indeed the real world a model like the present one may be useful as a first step for understanding such a world. This does not imply that we should abandon the rationality postulate, which is at the heart of our science. But the assumption of representative agents with full information may stretch the postulate too far. Anyhow, as shown by Malinvaud (1980, 1989) and Kuipers (1988) investment functions which give the rate of capacity utilization a role to play can be derived from profit maximisation under uncertainty with respect to demand.[7)] Here we want to emphasize that a lack of effective demand may force firms to reduce their production capabilities or to close down in case of bankruptcy. Hudson (1988) provides empirical evidence of the association between company failures and the business cycle. It should be noted that bankrupties are not possible in the neoclassical model in which the Modigliani-Miller theorem holds. However, the real world seems more like a model which allows for equity rationing as discussed in Greenwald and Stiglitz (1987, 1988). In this model with circulating capital firms go bankrupt if what they promise to pay exceeds their income from producing commodities. With fixed capital it is optimal to liquidate an insolvent firm if the liquidation value exceeds expected net discounted revenue, adjusted to exclude any revenue in excess of current debts (Hudson, 1988).

Turning to the labour market we assume that unions set real wages with the intention not to endanger the existing employment opportunities. Because unions are uncertain about the effect of their policy the actual wage rate is adjusted to the target real wage rate with some delay. If in the meantime there turns out to be unemployment real wages are moderated accordingly. This leads to the dynamic wage equation:

$$\frac{\dot{W}}{W} = \vartheta\left[\frac{\Omega-W}{W}\right] - \gamma\left[\frac{N^s-N}{N^s}\right] \tag{15}$$

where Ω denotes the target real wage rate, which differs from its neoclassical counterpart given in equation (4). It should be noted that the target real wage rate will not be attained here in the long run unless $\gamma=0$. For $\dot{W}=0$ equation (15) yields:

$$\left[\frac{\Omega-W}{W}\right] = \frac{\gamma}{\vartheta}\left[\frac{N^s-N}{N^s}\right]$$

The gap between W and Ω may reflect a change in union preferences under the influence of actual developments in the labour market. The idea that wage aspirations may change over time is discussed in Alogoskoufis and Manning (1988) from a somewhat different angle.

The model has three state variables, i.e. K, P and W. Performing the appropriate substitutions the dynamic system can be summarised by the following three equations:

$$\dot{K} = \alpha \frac{A}{P}\frac{1}{\varepsilon}\left[\frac{1-\lambda}{\lambda}\frac{W}{R}\right]^{\lambda} - \alpha K \tag{16a}$$

$$\dot{P} = \beta \frac{A}{K}\frac{1}{\varepsilon}\left[\frac{1-\lambda}{\lambda}\frac{W}{R}\right]^{\lambda} - \beta P \tag{16b}$$

$$\dot{W} = \vartheta(\Omega-W) + \gamma\left[\left[\frac{A}{\varepsilon P K^{1-\lambda}}\right]^{\frac{1}{\lambda}}\frac{1}{N^s} - 1\right]W \tag{16c}$$

The implications of the model may be sketched by discussing the effects of a demand shock. A more formal analysis will be presented in the next section. Suppose that there is a long-run equilibrium initially. The long-run equilibrium is attained at point F on the Isoquant I_0 in Figure 2. The labour market is in equilibrium, $N=N^s$, so that the real wage rate equals its aspiration level (Ω). A decline in the parameter A shifts the isoquant downward to I_1. On impact of the shock firms operate at point G, laying off all the labour they do not need to produce the reduced level of output. The dynamics of the model induces a movement as shown by the arrow starting from point G. Excess capacity forces firms to lower prices, while in the meantime some firms may have to close down because they become insolvent. Real wages fall as there is an excess supply of labour and aspirations cannot longer be realized. In the long run the economy might settle at a new steady state

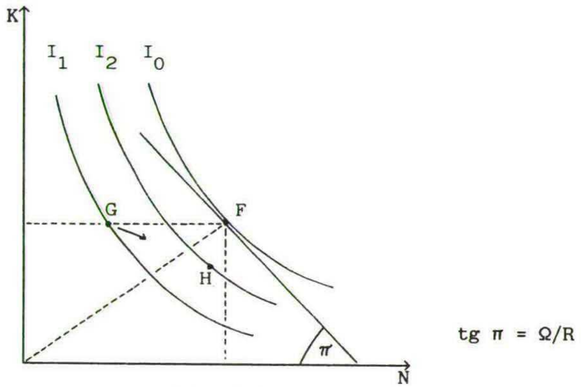


Figure 2

equilibrium, indicated by point H on the isoquant I_2 in Figure 2. Unemployment appears to be an equilibrium phenomenon resulting from a negative aggregate demand shock. There could of course be full employment if - given sufficient substitution possibilities - real wages decline with the right amount. Setting $\theta=0$ in equation (16c) would do the job. However, such an observation is a bit trivial. The point is that the loss of capital on impact of a negative demand pull makes everybody poorer. Workers are moderating their wage claims compared with the initial position ($W<\Omega$), but there may be barriers to their resilience. Unions may think that the government could do something to alleviate the problem by increasing demand or if that route is somehow blocked by implementing policies to retain production capacity (e.g. Hudson, 1988).

It should be noted that the model exhibits hysteresis. To make this more lucid let us assume for the time being that W is constant and consider the dynamics implied by equations (16a) and (16b). As appears from these equations the two loci $\dot{K}=0$ and $\dot{P}=0$ are identical and equal:

$$K = A \frac{1}{\varepsilon} \left[\frac{1-\lambda}{\lambda} \frac{W}{R}\right]^{\lambda} \frac{1}{P}$$

The corresponding phase diagram is given in Figure 3. Inspection of the differential equations shows that the system is stable, but the long-run equilibrium depends on the initial position of both variables. The implications of hysteresis for employment and unemployment in the long run will be discussed in the next section. In this section the log-linear version of the model will be used to present a formal solution of the dynamic system.

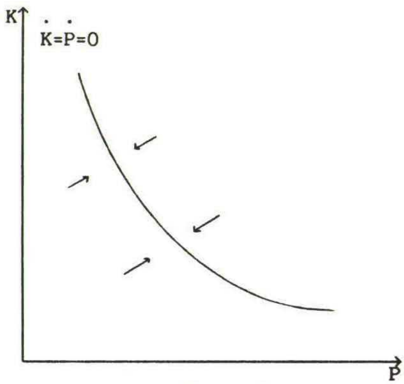


Figure 3

3. Hysteresis and umemployment in the Keynesian model

The model given in equations (1) and (10)-(15) can easily be transformed into its log-linear equivalent. The logarithm of a variable is denoted by a small letter. Coefficients are evaluated at an initial steady state with $N=N^S$ and $W=\Omega$. Ignoring irrelevant constants (including the user cost of capital) the linearized model is given by:

$$\dot{k} = \alpha[a - k - p + \lambda w] \tag{17a}$$

$$\dot{p} = \beta[a - k - p + \lambda w] \tag{17b}$$

$$\dot{w} = \vartheta(\omega - w) + \frac{\gamma}{\lambda}[a - (1-\lambda)k - p] - \gamma n^s \tag{17c}$$

$$y = k - \lambda w \tag{17d}$$

$$\ell = k - w \tag{17e}$$

$$n = \frac{1}{\lambda}[a - (1-\lambda)k - p] \tag{17f}$$

$$x = a - p \tag{17g}$$

It is convenient to rewrite the dynamic system in matrix form:

$$\begin{bmatrix} \dot{k} \\ \dot{p} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -\alpha & -\alpha & \alpha\lambda \\ -\beta & -\beta & \beta\lambda \\ -\gamma(1-\lambda)/\lambda & -\gamma/\lambda & -\vartheta \end{bmatrix} \begin{bmatrix} k \\ p \\ w \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 \\ \beta & 0 & 0 \\ \gamma/\lambda & \vartheta & -\gamma \end{bmatrix} \begin{bmatrix} a \\ \omega \\ n^s \end{bmatrix} \tag{18}$$

Determination of the characteristic equation of the state matrix on the RHS of (18) yields:

$$\sigma^3 + [\alpha+\beta+\vartheta]\sigma^2 + [(\alpha+\beta)\vartheta + \gamma(\alpha(1-\lambda) + \beta)]\sigma = 0$$

The roots of the system are therefore equal to:

$$\sigma_1 = 0$$

$$\sigma_{2,3} = \frac{-(\alpha+\beta+\vartheta) \pm \sqrt{(\alpha+\beta+\vartheta)^2 - 4[(\alpha+\beta)\vartheta + \gamma(\alpha(1-\lambda) + \beta)]}}{2} < 0$$

The model has a zero root, reflecting hysteresis implied by equations (17a) and (17b) and two negative roots, which are stabilizing. The dynamic system (18) can be simplified by noting that the first two differential equations generate a fixed ratio between k and p as a result of a shock in the exogenous variables. This can best be seen by considering a demand shock. A change in the exogenous variable a leads on impact to a change in k/p of magnitude α/β. The dynamic process following such a shock conserves the ratio $\frac{k}{p} = \frac{\alpha}{\beta}$ as appears upon inspection of the state matrix. Therefore, one of the first two differential equations can be replaced by the equation[8)]

$$p = \frac{\beta}{\alpha} k \tag{19}$$

Elimination of equation (17b) and substitution of (19) gives a reduced dynamic system in the capital stock and real wages:

$$\begin{bmatrix} \dot{k} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -(\alpha+\beta) & \alpha\lambda \\ -\frac{\gamma}{\lambda}(1-\lambda+\frac{\beta}{\alpha}) & -\vartheta \end{bmatrix} \begin{bmatrix} k \\ w \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 \\ \frac{\gamma}{\lambda} & \vartheta & -\gamma \end{bmatrix} \begin{bmatrix} a \\ \omega \\ n^s \end{bmatrix} \tag{20}$$

As can be easily checked the roots of the state matrix on the RHS of (20) are equal to σ_2 and σ_3 given above. As a result the dynamic system presented in (20) is stable. The rest point may be a stable node or the focus of a spiral, if the roots are conjugate complex. This result corresponds in a qualitatively sense with the outcome of the dynamic neoclassical model of Bruno and Sachs (1985, chapter 3). The long-run or steady state solution implying $\dot{k}=\dot{w}=0$ can be found from:

$$\begin{bmatrix} k^* \\ w^* \end{bmatrix} = -\begin{bmatrix} -(\alpha+\beta) & \alpha\lambda \\ -\frac{\gamma}{\lambda}(1-\lambda+\frac{\beta}{\alpha}) & -\vartheta \end{bmatrix}^{-1} \begin{bmatrix} \alpha & 0 & 0 \\ \frac{\gamma}{\lambda} & \vartheta & -\gamma \end{bmatrix} \begin{bmatrix} a \\ \omega \\ n^s \end{bmatrix} \tag{21}$$

Solving equations (21) yields:

$$k^* = \frac{\alpha}{\Delta} [(\gamma+\vartheta)a + \lambda(\vartheta\omega - \gamma n^s)] \tag{22a}$$

$$w^* = \frac{1}{\Delta} [\alpha\gamma a + (\alpha+\beta)(\vartheta\omega - \gamma n^s)] \tag{22b}$$

where the determinant of the state matrix $\Delta = (\alpha+\beta)\vartheta + \gamma(\alpha(1-\lambda) + \beta)$ is positive. From equations (19) and (22a) the long-run solution of p can be determined as:

$$p^{*} = \frac{\beta}{\Delta} [(\gamma+\vartheta)a + \lambda(\vartheta\omega - \gamma n^{S})] \tag{22c}$$

The solutions for the other endogenous variables can now be found by substituting (22 a, b and c) in (17 d, e, f and g). The results are:

$$y^{*} = x^{*} = \frac{1}{\Delta} [\alpha((1-\lambda)\gamma + \vartheta)a - \beta\lambda(\vartheta\omega - \gamma n^{S})] \tag{22d}$$

$$\ell^{*} = n^{*} = \frac{1}{\Delta} [\alpha\vartheta a - ((1-\lambda)\alpha + \beta)(\vartheta\omega - \gamma n^{S})] \tag{22e}$$

The long-run solutions give rise to a number of interesting observations:

a) A decline in the exogenous variable a (a negative demand shock) leads to a reduction in the stock of capital and a reduction in prices depending on the relative size of the relevant speed of adjustment. This is most clearly seen in the special case γ=0 and thus w^{*}=0 (no Phillips-curve effect). We then have $k^{*}=\frac{\alpha}{\alpha+\beta}$ a and $p^{*}=\frac{\beta}{\alpha+\beta}$ a. A relatively high value of the "accelerator", α, compared with the measure of price flexibility, β, leads to a relatively large capital decumulation while prices are much less affected. This looks plausible. If prices do not adjust fast profitability will not be restored quickly and a larger number of firms will face bankruptcy. In the case of price rigidity β=0 there is a one for one correspondence between the demand shock and the decline in the capital stock. If prices are fully flexible $\beta\to\infty$ the capital stock does not change at all and there is now an exact correspondence between a demand shock and the resulting price level.

b) A negative demand shock gives rise to unemployment unless ϑ=0. In the latter case wage aspirations do not intervene with the Phillips-curve. In such a case of zero wage aspirations real wages fall as long as there is unemployment. In contrast, positive wage aspirations ϑ>0 lead to a smaller moderation in real wages and a decline in employment n^{*} (and the number of jobs available ℓ^{*}).

c) A negative supply shock (i.e. an increase in ω or a decrease in n^{S}) induces a rise in real wages. The long-run demand curve for labour has therefore a negative slope in contrast with the neoclassical model where the curve is horizontal as shown in Figure 1. Rising real wages go along with an increase in the capital stock and a fall in employment. The fall in employment is caused by substitution of labour for capital and also by a reduction in output. Such a reduction in output is necessary to maintain the profitability of production as the cost of one factor of production (i.e. labour) increases. From a policy point of view it is interesting to note that a decline in employment in case of a negative supply shock can be prevented by a positive demand shock of sufficient strength. However, there may be a caveat here. From the mathematical point of view the model can be applied symmetrically. Positive and negative shocks exhibit the same quantitative effects in absolute terms. In reality speeds of adjustment may differ in both situations. In severe recessions α may be relatively large, whereas in booms β may be relatively large. Bankruptcies lead to selling out assets and an immediate loss of firm- or industry-specific physical capital. In contrast investment in new capacity may take more time as sometimes suggested in the cost of adjustment literature.

d) Anti-cyclical economic policy may not only take the form of stabilizing demand but may also be aimed at a reduction of the parameter α. Retaining supply side capacity is the main theme of Hudson (1988) in his thought-provoking contribution to the theory of the trade cycle. Whether this is a feasible policy remains to be seen as it requires empirical evidence which is not readily available.

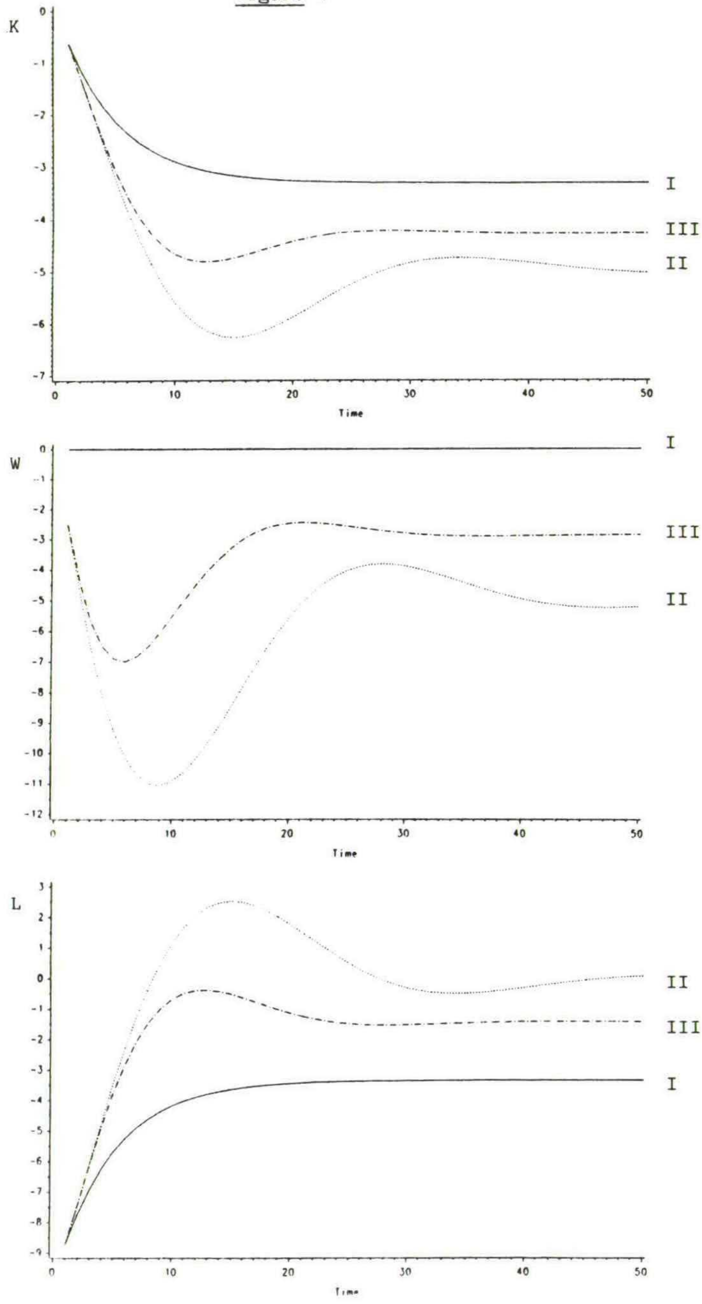
It may be instructive to conclude this section with a numerical example showing the development of the main variables over time. The parameter values applied in the simulations are presented in Table 2. Example I relates to the case of real wage rigidity: unions stick to the target real wage rate. In example II the Phillips-curve mechanism generates full employment in the long run. Case III gives the intermediate position with some learning on the side of unions.

The time paths[9)] of the capital, real wages and employment for each set of parameters in the case of a negative demand shock of 5% (a = -5) are given in Figure 4. Under real wage rigidity (Ex. I) employment falls substantially on impact of the shock, but recovers as prices decline and demand is positively affected. At the same time the stock of capital decreases, which has a negative effect on employment. A new equilibrium is attained when both effects cancel each other. As appears from Figure 4 the new long-run equilibrium is approximated closely after 20 periods (not to be interpreted as years). The working of the Phillips curve induces a fall in real wages and substitution of capital for labour (Ex. II). As a consequence the stock of capital is reduced even further than in the previous case. Full employment is restored eventually, but the system exhibits cyclical behaviour (there are complex roots in this case). The real wage undershoots its long-run equilibrium value, because the parameter γ is relatively high. Whether such a value is realistic or not is of course an empirical question. But the example illustrates that the real wage rate moves procyclical in our Keynesian model, which corresponds with empirical observations.[10)] The results in the intermediate case (Ex. III) lie neatly between the two other cases. Wage aspirations dampen the cycle, but also prevent the return to a situation of full employment in the long run. For both runs with $\gamma>0$ the real wage rate never recaptures its original value. This is the price to be paid for hysteresis, which takes the form of a permanent loss in the stock of capital.

Table 2 — Parameter values

I. Real wage rigidity	II. Phillips-curve	III. Intermediate case
$\lambda = 0.5$	$\lambda = 0.5$	$\lambda = 0.5$
$\alpha = 0.125$	$\alpha = 0.125$	$\alpha = 0.125$
$\beta = 0.0625$	$\beta = 0.0625$	$\beta = 0.0625$
$\gamma = 0$	$\gamma = 0.25$	$\gamma = 0.25$
$\vartheta = 0$	$\vartheta = 0$	$\vartheta = 0.125$

Figure 4


K
0
-1
-2
-3
-4
-5
-6
-7
0
10
20
30
40
50
Time
I
III
II
W
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
Time
I
III
II
L
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
Time
II
III
I

4. Conclusion

As indicated in the first section, empirical evidence gives support to a capital shortage interpretation of European unemployment, which during the recessions of 1974-1975 and 1980-1982 has increased by two substantial jumps. This paper has sought to explore how capital decumulation might arise as a consequence of severe negative demand shocks. The neoclassical model does not predict such a relationship, even not when monopolistic competition is taken into account. To cope with the problem we have therefore to resort to the assumption that the market for commodities exhibits price inertia. In such a Keynesian model quantity adjustments (of output and capacity) are of central importance.

The labour market is modelled by assuming that unions set real wages with the intention not to endanger the existing employment opportunities. In the long run there may be a gap between the actual and the target real wage rate reflecting a change in union preferences under the influence of actual developments in the labour market.

The implications of the model are sketched by discussing the effects of a negative demand shock. The model exhibits hysteresis: a demand shock leads to a reduction in the stock of capital, which is not restored by the reduction in prices unless in the limiting (and unrealistic) case that prices are assumed to be fully flexible. As a result, a negative demand shock gives rise to unemployment, unless the Phillips-curve can do its job. If so, full employment may be possible, but capital and output cannot return to their initial positions.

The present analysis can be extended in different directions. Insider effects and outsider effects (cf. Layard and Bean, 1988) may provide forms of hysteresis, which can be combined with state dependency of the capital stock. Moreover, the idea of fragile equilibria (cf. Summers, 1988) may be fruitful in analysing firm behaviour along the lines set in this paper.

Notes

*) We are indebted to F. van der Ploeg for useful comment on a first draft of the paper.

1) In European countries the ratio between investment and output has decreased in a considerable amount. The following figures show that after the recovery in 1984 investment ratios only slowly recover:

Investment ratios (EEC, Percentage changes)

1973	-0.4	1976	-2.1	1979	0.2	1982	-2.9	1985	0.1
1974	-4.0	1977	-1.8	1980	0.6	1983	-1.4	1986	0.8
1975	-5.1	1978	-0.7	1981	-5.2	1984	-0.8	1987	1.4

Source: OECD, 1988, Table R5 (Volume of gross fixed capital formation) and Table R1 (Real GNP).

2) As shown among others in Blanchard and Kiyotaki (1987) and Van de Klundert and Peters (1988) demand for the product of firm i can be written as:

$$Y_i = \frac{Y}{n}\left[\frac{P_i}{P}\right]^{\eta}, \quad \eta<1$$

where n is the number of firms in the economy and P is an average price index. It appears that firm demand depends upon aggregate demand Y. However, in a neoclassical model with flexible prices aggregate demand equals aggregate supply.

3) There are other interpretations possible of equation (4) (e.g. Layard and Nickell, 1986), but in a European context there should be at least some role for unions to play.

4) Burda (1988) presents a dynamic version of this model based on the familiar q-theory of investment.

5) Equation (12) follows from ex-post profit maximisation supposing that $(\delta X/\delta N) \geq W$ (cf. Malinvaud, 1989).

6) A possible effect of nominal wage changes on nominal price setting is not taken into account, because we ignore the leap-frogging of prices over wages and vice versa.

7) Malinvaud (1989) derives two conditions for (ex-ante) profit maximisation: (1) the capital cost of a unit of capacity must exactly be covered by the expected value of the (marginal) gross profit; (2) the (ex-ante) marginal rate of substitution between capital and labour, corrected for the expected rate of capacity utilization, should be equal to the factor price ratio. These two conditions determine the desired capital stock. An investment equation could be derived in the usual way by assuming that the actual capital stock gradually adjusts towards its desired level.

8) This procedure of "eliminating the zero root" can be generalised as zero roots reflect dependency of rows and columns in the state matrix. Now, the system of equations Ax = b, where the matrix A is of order n x n and of rank r, has a solution if the augmented matrix B = [A⋮b] is also of rank r. The solution is obtained from r equations and the remaining (n-r) equations can be ignored as derivable from them. The latter can be called "surplus" equations (e.g. Allen, 1956). The solution expresses r variables in terms of the (n-r) "surplus" variables. Substitution of the solution in the original system gives a reduced system with a singular matrix. Application to a system of differential equations yields a state matrix with non-zero roots, which can be handled in the usual way.

9) The roots are:

Ex. I: $\sigma_1 = 0,\ \sigma_2 = 0,\ \sigma_3 = -0.1875$

Ex. II: $\sigma_1 = 0,\ \sigma_{2,3} = \dfrac{-0.1875 \pm i\sqrt{0.125 - (0.1875)^2}}{2}$

Ex. III: $\sigma_1 = 0$, $\sigma_{2,3} = \dfrac{-0.3125 \pm i\sqrt{0.21875 - (0.3125)^2}}{2}$

The characteristic equation of the state matrix on the RHS of (20) can be written as: $\sigma^2+b_1\sigma+b_2=0$. It appears that $b_1>0$, $b_2>0$ and $b_1^2-4b_2<0$, so that the examples II and III exhibit oscillating and damped behaviour.

10) Procyclical real wages, which have been frequently observed are in contradiction with the results of the textbook Keynesian model based on nominal wage rigidity (cf. Stevenson, et. al., 1988). However, for a different view on the empirical pattern of real wages see Newell and Symons (1988).

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