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**SIMPLE ESTIMATORS FOR DYNAMIC PANEL  
DATA MODELS WITH ERRORS IN VARIABLES** R46

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Abstract:

Simple dynamic models with individual effects are considered in which lagged endogenous or exogenous variables are observed with error. The inconsistencies of estimators based on the elimination of individual effects are established. The results can be used to construct tractable consistent, and sometimes asymptotically efficient estimators. The emphasis is on simplicity of derivations and tractability of the resulting estimators, rather than on generality or newness of results.

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## 1. INTRODUCTION

The model considered in this paper is a rather simple dynamic error components model. Models of this type have been studied by a number of authors, including Nerlove (1967, 1971), Trognon (1978), Anderson and Hsiao (1981), and Sevestre and Trognon (1985). Our assumptions will be fairly conventional, except for the fact that lagged endogenous or exogenous variables are allowed to suffer from measurement error. A variant of this model, not including error components, has been studied extensively in the literature, cf. Aigner et al. (1984). A full treatment of ML estimation in this so-called dynamic shock-error model has been given by Ghosh (1989). Griliches and Hausman (1986) study another variant, namely a static panel data model with measurement error in the exogenous variables.

The analysis of parameter estimation in models of this type tends to lead to rather complicated, if not messy, algebra. Our emphasis is on the use of methods that simplify derivations. The estimators presented can be written down in a transparent way and are easy to compute. Given that we aim at simplicity and tractability, generality is sacrificed whenever thought necessary.

In the Handbook of econometrics, Hans Theil (1983) asked the question "Why are matrix methods useful?" and of course he himself gave a most convincing answer, as one would expect from somebody who contributed so much to econometrics, in terms of both content and method. The present piece is partly meant as another illustration of how useful matrix methods are.

The set-up of this essay is as follows. In section 2, we start by considering a dynamic model, assuming exact measurement. In this context, we derive plims of a broad class of inconsistent, parameter estimators and consider the implied consistent estimators in section 3. Variances of these estimators are the subject of section 4. Measurement error is introduced in section 5, where we briefly review some well-known results and give some new ones. Section 6 integrates the two themes, bringing measurement error and (simple) dynamics together. Section 7 concludes.

## 2. THE DYNAMIC MODEL

For the time being we entertain the simplest possible dynamic model for panel data. This is

$$(2.1) \quad y = \gamma y_{-1} + \iota_T \otimes \alpha + u,$$

where the symbols have the following meaning: let there be  $N$  households in the panel, each observed in  $T$  time periods. Then  $y$  is the  $NT$ -vector of observed values of some variable. The subscript  $-1$  indicates a one-period lag. By  $\iota_T$  we denote a  $T$ -vector of ones;  $\alpha$  is the  $N$ -vector of individual effects, and  $u$  ( $NT \times 1$ ) is the disturbance vector, assumed to be white noise with variance  $\sigma_u^2$ , independent of  $y$ . We do not specify whether  $\alpha$  is random or fixed. This issue is avoided since we consider throughout the paper estimators that eliminate these effects. This elimination is achieved by some matrix  $R$  ( $NT \times NT$ ) that has properties

$$(2.2) \quad R(\iota_T \otimes I_N) = R'(\iota_T \otimes I_N) = 0.$$

Below we will often impose more structure on  $R$ , frequently of the form

$$(2.3) \quad R = Q \otimes I_N$$

with  $Q = Q'$  a  $T \times T$ -matrix with  $Q\iota_T = 0$ . In view of the requirements of section 4 we will use a general  $R$  as much as possible, though.

The central issue is the behavior of the OLSE of  $\gamma$  in the model where  $R$  is used to eliminate the effects, that is, of

$$(2.4) \quad \begin{aligned} \hat{\gamma}(R) &= \frac{y' R y_{-1}}{y_{-1}' R y_{-1}} = \\ &= \gamma + \frac{u' R y_{-1} / N}{y_{-1}' R y_{-1} / N} \\ &= \gamma + \frac{\lambda_1(R)}{m_1(R)} \end{aligned}$$

with, in general,

$$(2.5) \quad \ell_t(R) \equiv \frac{1}{N} u' R y_{-t}$$

$$(2.6) \quad m_t(R) \equiv \frac{1}{N} y'_{-t} R y_{-t},$$

where the subscript  $-t$  indicates a  $t$ -period lag. We also need

$$r_t(R) \equiv \frac{1}{N} u' R u_{-t}.$$

The expectations of these variables will be of importance:

$$(2.7) \quad \lambda_t(R) \equiv E \ell_t(R)$$

$$(2.8) \quad \mu_t(R) \equiv E m_t(R)$$

$$(2.9) \quad \rho_t(R) \equiv E r_t(R).$$

Under general conditions there holds

$$(2.10) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma}(R) = \gamma + \frac{\lambda_1(R)}{\mu_1(R)}$$

and we now work this out. Throughout, we will only be concerned with plim's that have  $N$  go to infinity and take  $T$  fixed. This is motivated by the typical panel, which contains observations on many individuals at a few points in time.

### 3. PROBABILITY LIMITS

First an auxiliary result is needed, concerning the  $\rho_t(R)$ :

$$(3.1) \quad \begin{aligned} \rho_t(R) &= \frac{1}{N} E u' R u_{-t} \\ &= \frac{1}{N} \sigma_u^2 \text{tr} R (B'_t \otimes I_N) \end{aligned}$$

where  $B_t$  ( $T \times T$ ) is the  $t$ -period backward-shift operator:

$$(3.2) \quad B_t = \begin{bmatrix} 0 & \dots & 1 & \dots & 0 \\ & & & & \vdots \\ & & & & 1 \\ & & & & 0 \end{bmatrix}_{T-t},$$

for  $t = 0, \dots, T-1$ ;  $B_0 = I_T$  and  $B_T = 0$ . We can now elaborate  $\lambda_1(R)$ . Take model (1.1), lag it by  $t$ :

$$(3.3) \quad y_{-t} = \gamma y_{-(t+1)} + \iota_T \theta \alpha + u_{-t},$$

premultiply by  $u'R/N$  and take expectations to obtain

$$(3.4) \quad \lambda_t(R) = \gamma \lambda_{t+1}(R) + \rho_t(R).$$

So

$$(3.5) \quad \begin{bmatrix} 1 & -\gamma & & \\ & \ddots & \ddots & \\ & & 1 & -\gamma \\ & & & 1 \end{bmatrix} \begin{bmatrix} \lambda_1(R) \\ \vdots \\ \lambda_{T-1}(R) \end{bmatrix} = \begin{bmatrix} \rho_1(R) \\ \vdots \\ \rho_{T-1}(R) \end{bmatrix}.$$

Note that  $\lambda_T(R) = 0$ . Solving (3.5) gives for  $\lambda_1(R)$

$$(3.6) \quad \lambda_1(R) = (1, \gamma, \dots, \gamma^{T-2}) \begin{bmatrix} \rho_1(R) \\ \vdots \\ \rho_{T-1}(R) \end{bmatrix} \\ = \sum_{t=0}^{T-2} \gamma^t \rho_{t+1}(R) \\ = \frac{1}{N} \sigma_u^2 \text{tr} R \left( \begin{bmatrix} T-2 \\ \sum_{t=0} \gamma^t B'_{t+1} \end{bmatrix} \otimes I_N \right) \\ = \frac{1}{N} \sigma_u^2 \text{tr} R(L' \otimes I_N)$$

where  $L'$  ( $T \times T$ ) is implicitly defined as the matrix in braces:

$$(3.7) \quad L' = \begin{bmatrix} 0 & 1 & \gamma & \dots & \gamma^{T-2} \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \\ & & & & 1 \\ & & & & 0 \end{bmatrix}.$$

When R has structure (2.3), (3.6) simplifies to:

$$(3.8) \quad \lambda_1(R) = \sigma_u^2 \text{tr } QL$$

and in particular for  $Q = A_T = I_T - \frac{1}{T} \iota_T \iota_T'$  the "within" transformation,

$$(3.9) \quad \begin{aligned} \lambda_1(R) &= \sigma_u^2 \text{tr } A_T L = \\ &= -\frac{1}{T} \sigma_u^2 \iota_T' L \iota_T \\ &= -\sigma_u^2 \frac{\varphi}{1-\gamma}, \end{aligned}$$

with

$$(3.10) \quad \varphi = 1 - \frac{1}{T} \frac{1-\gamma^T}{1-\gamma}.$$

Another interesting case is to eliminate the individual effect by differencing the data. Then for

$$(3.11) \quad D' = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \cdot & \cdot & \\ & & & \cdot & \cdot \\ & & & & -1 & 1 \end{bmatrix}^{T-1}$$

there holds  $Q = DD'$ , so

$$(3.12) \quad \begin{aligned} \lambda_1(R) &= \sigma_u^2 \text{tr } DD'L \\ &= \sigma_u^2 \text{tr } D'LD \\ &= -\sigma_u^2 (T-1). \end{aligned}$$

Hence  $\lambda_1(R)$  is independent of  $\gamma$ .



After  $\lambda_1(R)$  in the numerator of (2.10), we now consider  $\mu_1(R)$  in the denominator. We make a simplifying assumption of stationarity in the sense that  $\mu_t(R) = \mu(R)$ , independent of  $t$ . Then

$$\begin{aligned}
 (3.13) \quad \mu_1(R) &= \mu(R) \\
 &= \frac{1}{N} E y' R y \\
 &= \frac{1}{N} E (\gamma y_{-1} + u)' R (\gamma y_{-1} + u) \\
 &= \gamma^2 \mu(R) + \gamma \{ \lambda_1(R) + \lambda_1(R') \} + \frac{1}{N} \sigma_u^2 \text{tr } R \\
 &= \gamma^2 \mu(R) + \frac{1}{N} \sigma_u^2 \text{tr } R \{ (I_T + \gamma L + \gamma L') \otimes I_N \} \\
 &= \gamma^2 \mu(R) + \frac{1}{N} \sigma_u^2 \text{tr } R (S \otimes I_N)
 \end{aligned}$$

with  $S$  (implicitly defined) the usual AR(1) correlation matrix with parameter  $\gamma$ . From (3.13) it follows that

$$(3.14) \quad \mu(R) = \frac{1}{1-\gamma^2} \frac{1}{N} \sigma_u^2 \text{tr } R (S \otimes I_N).$$

For  $R = Q \otimes I_N$  this reduces to

$$(3.15) \quad \mu(R) = \frac{1}{1-\gamma^2} \sigma_u^2 \text{tr } Q S.$$

Combining (2.10), (3.8), and (3.15) we obtain

$$\begin{aligned}
 (3.16) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma} &= \gamma + (1-\gamma^2) \frac{\text{tr } QL}{\text{tr } QS} \\
 &= \gamma + (1-\gamma^2) \frac{\text{tr } QL}{\text{tr } Q + 2\gamma \text{tr } QL}.
 \end{aligned}$$

In the two particular cases considered above, we have for  $Q = A_T$  (so  $\text{tr } Q = T-1$ ),

$$(3.17) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma} = \gamma - (1+\gamma) \frac{\varphi}{T - \left[ 1 + 2\varphi \frac{\gamma}{1-\gamma} \right]}$$

and for  $Q = DD'$  (so  $\text{tr } DD' = \text{tr } D'D = 2(T-1)$ ),

$$(3.18) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma} = \gamma - (1-\gamma^2) \frac{T-1}{2(T-1)-2\gamma(T-1)}$$

$$= \frac{1}{2}(\gamma-1).$$

Both estimators are inconsistent. The inconsistency is introduced by the elimination of the effects. Both (3.17) and (3.18) can be used to arrive at a consistent estimator; write (3.17) or (3.18) as  $\text{plim } \hat{\gamma} = f(\gamma)$  and estimate  $\gamma$  by  $\hat{\gamma} = f^{-1}(\hat{\gamma})$ . This is trivial in case of (3.18) and requires numerical methods in the case of (3.17). Expression (3.17) has been derived before (along different lines, and in a somewhat different form) by Nickell (1981).

#### 4. VARIANCES

When a consistent estimator is derived by transforming  $\hat{\gamma}(R)$ , the next question is one of second-order properties. In order to say something about asymptotic distribution, the essential step is to derive the variance of  $\mathcal{L}_1(R)$ . We do so under the assumption of normality of  $u$ .

The method we use for easy computation is that of "repeated conditioning" as introduced by Merckens and Wansbeek (1989). To appreciate this method it is easiest to consider what it looks like in the present context:

$$(4.1) \quad E(\mathcal{L}_1(R))^2 = \frac{1}{N^2} E(u'Ry_{-1})^2$$

$$= \frac{1}{N^2} (E_{12}E_{34} + E_{13}E_{24} + E_{14}E_{23}) u'_{(1)} Ry_{-1(2)} u'_{(3)} Ry_{-1(4)}.$$

This means the following: the four random variables are labeled (in parentheses) 1-4, and the expectation operator is broken down in three terms of two subsequent operations each. For example,  $E_{12}$  denotes the expectation with respect to the random variables labeled 1 and 2, considering

everything else constant (even though variable 3 in this case is the same as variable 1!). The operator  $E_{12}E_{34}$  denotes the above operation, followed by taking the expectation w.r.t. variables 3 and 4. The order of both operations is immaterial. The method of repeated conditioning is not restricted to the case of four random variables, but extends to an arbitrary number.

We are now in a position to look at the variance. Since, trivially,

$$(4.2) \quad (E \ell_1(R))^2 = \frac{1}{N^2} E_{12}E_{34} u'_1 R_{y-1(2)} u'_3 R_{y-1(4)},$$

the variance of  $\ell_1(R)$  can be evaluated using (3.6) and (3.14) repeatedly:

$$\begin{aligned} (4.3) \quad \text{Var}(\ell_1(R)) &= E(\ell_1(R))^2 - (E\ell_1(R))^2 \\ &= \frac{1}{N^2} (E_{13}E_{24} + E_{14}E_{23}) u'_1 R_{y-1(2)} u'_3 R_{y-1(4)} \\ &= \frac{1}{N} E_{13} \left[ E_{24} \frac{1}{N} y'_{-1(2)} R' u'_1 u'_3 R_{y-1(4)} \right] \\ &\quad + \frac{1}{N} E_{14} \left[ E_{23} \frac{1}{N} u'_3 R_{y-1(4)} u'_1 R_{y-1(2)} \right] \\ &= \frac{1}{N} E \mu(R' u u' R) + \frac{1}{N} E \lambda_1(R_{y-1} u' R) \\ &= \frac{1}{N} \sigma_u^2 \left[ \mu(R' R) + E \frac{1}{N} \text{tr} R_{y-1} u' R (L' \otimes I_N) \right] \\ &= \frac{1}{N} \sigma_u^2 \left[ \mu(R' R) + E \frac{1}{N} u' R (L' \otimes I_N) R_{y-1} \right] \\ &= \frac{1}{N} \sigma_u^2 \left[ \mu(R' R) + E \lambda_1(R(L' \otimes I_N) R) \right] \\ &= \frac{1}{N^2} \sigma_u^4 \left[ \frac{1}{1-\gamma^2} \text{tr} R' R (S \otimes I_N) + \text{tr}(R(L' \otimes I_N))^2 \right]. \end{aligned}$$

For the case  $R = Q \otimes I_N$ , this simplifies to

$$(4.4) \quad \text{Var}(\ell_1(R)) = \frac{1}{N} \sigma_u^4 \text{tr} \left( \frac{1}{1-\gamma^2} Q^2 S + QLQL \right).$$

In the above derivation our calculus with the  $\lambda$ - and  $\mu$ -functions appears to pay off.

## 5. MEASUREMENT ERROR IN STATIC PANEL DATA MODELS

We now introduce measurement error. To start with we do so in the context of a static model, and repeat the pertaining results in this area. These are from Griliches and Hausman (1986), in part elaborated by Wansbeek and Koning (1989). The model is

$$(5.1) \quad y = x\beta + \iota_T \otimes \alpha + u,$$

where the difference with (1.1) is the substitution of a (single strictly) exogenous variable  $x$  for  $y_{-1}$ . This  $x$  ( $NT \times 1$ ) is unobservable and instead we observe

$$(5.2) \quad x_* = x + v$$

with  $v$  white noise with variance  $\sigma_v^2$ . We start again from OLS in a model with effects eliminated by  $R = Q \otimes I_N$ ,  $Q = Q'$ . By entirely standard operations we arrive at

$$(5.3) \quad \begin{aligned} \text{plim}_{N \rightarrow \infty} \hat{\beta} &= \text{plim}_{N \rightarrow \infty} \frac{y'Rx_*}{y'Rx_*} \\ &= \beta(1 - \sigma_v^2 \psi) \end{aligned}$$

with

$$(5.4) \quad \psi = \frac{\text{tr } Q}{\text{tr } Q\Sigma_*}$$

and  $\Sigma_*$  is the  $T \times T$  covariance matrix of the  $x_n$ 's,  $x_n$  being the  $T$ -vector of  $x$ 's for household  $n$ .  $\Sigma_*$  is consistently estimable from the data and for all practical purposes we may assume it known. A consistent estimator for

$\beta$  is obtained by using two different Q's, hence two different  $\hat{\beta}$ 's,  $\hat{\beta}_1$  and  $\hat{\beta}_2$ , say and two different  $\psi$ 's,  $\psi_1$  and  $\psi_2$ , say. Then

$$(5.5) \quad \text{plim}_{N \rightarrow \infty} \hat{\beta}_i = \beta_i = \beta(1 - \sigma_v^2 \psi_i),$$

so by construction the estimators

$$(5.6) \quad \hat{\beta} = \frac{\psi_1 \hat{\beta}_2 - \psi_2 \hat{\beta}_1}{\psi_1 - \psi_2}$$

and

$$(5.7) \quad \hat{\sigma}_v^2 = \frac{\hat{\beta}_2 - \hat{\beta}_1}{\psi_1 \hat{\beta}_2 - \psi_2 \hat{\beta}_1}$$

are consistent. When we construct more than two ( $m$ , say) estimators by using  $m$  different Q's, we obtain  $m$  equations of the type (5.5), and we are faced with a situation of overdetermination since there still are two parameters. Optimal estimators are obtained by using the minimum distance method (e.g. Hsiao, 1986) based on a consistent estimator of the asymptotic covariance matrix of the  $\hat{\beta}_i$ 's. Wansbeek and Koning (1989) show that the  $(i,j)$ -th element of this matrix is

$$(5.8) \quad V_{ij} = \text{avar}(\hat{\beta}_i, \hat{\beta}_j) = \frac{1}{\text{tr } Q_i \Gamma_* \text{tr } Q_j \Gamma_*} \left\{ \beta^2 \sigma_v^4 \text{tr } Q_i Q_j + (\sigma_u^2 + \beta^2 \sigma_v^2) \text{tr } Q \Gamma_* Q \right\},$$

when the underlying distribution is normal. This result is easily derived by using the repeated conditioning method again.

The minimum distance estimators have a closed-form solution. Let

$$(5.9) \quad \psi = (\psi_1, \dots, \psi_m)'$$

$$(5.10) \quad t = (\hat{\beta}_1, \dots, \hat{\beta}_m)'$$

then

$$(5.11) \quad \hat{\beta} = \frac{\frac{\psi' t}{\psi' \psi} - \frac{t' t}{t' \psi}}{\frac{\psi' t}{\psi' \psi} - \frac{t' t}{t' \psi}}$$

$$(5.12) \quad \hat{\sigma}_v^2 = \frac{\frac{\psi' t}{\psi' \psi} - \frac{t' t}{t' \psi}}{\frac{\psi' \psi}{\psi' t} - \frac{t' \psi}{t' t}}$$

where all inproducts are in the metric of  $V$ , see (5.8).

## 6. MEASUREMENT ERROR IN A DYNAMIC MODEL

We again start from (2.1) and introduce measurement error in  $y$ :  $y_* = y + v$  with  $v$  again white-noise measurement error. The equation for the observable vector  $y_*$  then becomes:

$$(6.1) \quad y_* = \gamma y_{*-1} + t_T \theta \alpha + u + v - \gamma v_{-1}$$

with OLSE

$$(6.2) \quad \hat{\gamma}(R) = \frac{y_*' R y_{*-1}}{y_{*-1}' R y_{*-1}} \\ = \gamma + \frac{(u+v-\gamma v_{-1})' R (y_{-1}+v_{-1})/N}{(y_{-1}+v_{-1})' R (y_{-1}+v_{-1})/N}.$$

The expectations involving  $v$  are

$$(6.3) \quad E v' R y_{-1} = E v_{-1}' R y_{-1} = E u' R v_{-1} = 0$$

$$(6.4) \quad \frac{1}{N} E v' R v_{-1} = \frac{1}{N} \sigma_v^2 \text{tr} R (B_1' \theta I_N)$$

$$(6.5) \quad \frac{1}{N} E v_{-1}' R v_{-1} = \frac{1}{N} \sigma_v^2 \text{tr} R.$$

Then for  $R = Q \theta I$  there holds

$$(6.6) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma}(R) = \gamma + \frac{\sigma_u^2 \text{tr QL} + \sigma_v^2 \text{tr QB}_1 - \sigma_v^2 \gamma \text{tr Q}}{\frac{1}{1-\gamma^2} \sigma_u^2 \text{tr QS} + \sigma_v^2 \text{tr Q}}$$

For  $Q = A_T$  this becomes

$$(6.7) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma}(R) = \gamma - (1+\gamma) \frac{\varphi + \vartheta(\gamma + 1/T)}{T - (1+2\gamma) \frac{\varphi}{1-\gamma} + \vartheta(1+\gamma)}$$

with

$$(6.8) \quad \vartheta = (1-\gamma)(T-1)\sigma_v^2/\sigma_u^2$$

and for differencing this becomes

$$(6.9) \quad \text{plim}_{N \rightarrow \infty} \hat{\gamma}(R) = \gamma - \frac{1}{2}(1+\gamma) \frac{1 + (1+2\gamma)\sigma_v^2/\sigma_u^2}{1 + 2(1+\gamma)\sigma_v^2/\sigma_u^2}.$$

Once again, the plims can be used to construct a simple consistent estimator for the parameters in the model. One still needs only two different estimators to do so, although the number of parameters involved is now three. But  $\sigma_u^2$  and  $\sigma_v^2$  enter only via their ratio, cf. (6.8) and (6.9).

## 7. CONCLUDING REMARKS

As suggested in the introduction, the main aim of this note is to offer simplicity. We have exploited some convenient matrix tricks as well as a useful repeated conditioning rule for the evaluation of higher order moments of normally distributed random variables. Given this apparatus, the derivation of estimators for parameters in slightly more complicated models (e.g., with exogenous variables added to (6.1)) is rather straightforward. Generally, one can attain higher efficiency in estimation by employing full information methods, like ML. Even then, the availability of consistent starting values allows one to attain the same efficiency by two-step methods. Hence the derivations given here, also serve a purpose in that context.

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