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BANDS AND STATISTICAL PROPERTIES OF EMS EXCHANGE RATES

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BANDS AND STATISTICAL PROPERTIES OF EMS EXCHANGE RATES

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Abstract

So far, only little effort has been paid in the exchange rate literature to deal explicitly with the bands on the exchange rates in the European Monetary System (EMS). In this paper we use a Monte Carlo experiment to show that the target zone model, originally developed by Krugman [1990], can reproduce some stylized facts of EMS exchange rates. Among these facts we find that it is difficult to reject the unit root hypothesis. Furthermore, compared to the corresponding dollar exchange rates, the returns on Deutsche Mark rates vis-à-vis other currencies in the EMS show more non-normality and conditional heteroskedasticity, both of which the target zone model is able to generate endogenously.

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1. Introduction

In March 1979 the European Monetary System (EMS) was founded. The most important element in the system is the so-called Exchange Rate Mechanism, by which the bilateral exchange rates between participants are restricted to move within ±2.25% bands around a central parity (for Italian lira these bands were ±6% until they were narrowed to ±2.25% in January 1990). Whenever an exchange rate reaches a boundary of its band, the central banks of both countries involved are committed to intervene in the foreign exchange markets in order to prevent the exchange rate from going outside the band. If necessary, they can use credit facilities which are created for that purpose. However, several times in the history of the EMS the existing bands for the exchange rates were unsustainable, so that their central parities had to be changed, a so-called realignment. Since January 1987 no realignment has occurred, except for the narrowing of the bands for the lira in January 1990. In addition to the marginal interventions, there is a divergence indicator, which signals the divergence of each currency from its ECU-central rate. When an exchange rate crosses a treshold, the central bank of that country is expected to do what is necessary in order to reverse the movements of the currency. However, these actions are not compulsory, so that in reality the divergence indicator does not play an important role.

The EMS has been studied extensively in the economic literature (e.g. Giavazzi and Giovannini [1989]), but only little effort has been paid to model the effects of the bands on the exchange rates explicitly. However, recently Krugman [1990] proposed a nonlinear target zone model, in which the exchange rate was modelled as the sum of a fundamental and its own expected rate of change. In this framework, in which expectations are assumed to be rational, Krugman is able to show that the monetary authorities can push the exchange rate towards the centre of its target zone, by the mere, but fully credible, announcement that they will defend a similar band on the fundamental. This is the 'honeymoon effect'. Given that since January 1987 no major realignments have occurred in the EMS and that the Krugman model abstracts from realignments, this nonlinear model of bands may give a reasonable description of EMS exchange rates during recent years.

Indeed, we show in this paper, with the help of a Monte Carlo experiment, that the model is able to mimic some stylized facts of Deutsche Mark rates vis-à-vis other currencies in the EMS, even though sometimes heavy use was made of capital controls (esp. by France and Italy), which is theoretically excluded by the model. One of the stylized facts is that it is difficult to reject the unit root hypothesis for these exchange rates. Furthermore, their returns contain non-normality and sometimes conditional heteroskedasticity, especially when we compare them to the exchange rates between the U.S. dollar and currencies in the EMS. The main advantage of the target zone model is that it is nonlinear and thus able to generate the stylized facts of non-normality and conditional heteroskedasticity endogenously.

The fact that this model can give a reasonable description for EMS exchange rates, may have important implications. For example, Svensson [1989] shows that the existence of a band on the exchange rate in the Krugman model implies a similar band on the interest rate differential between the two countries concerned. Furthermore, narrowing the band on the exchange rate raises the asymptotic variability of the interest rate differential until it reaches a maximum after which the variability again declines. If this is undesirable, it implies that a gradual transition of the EMS to a European Monetary Union, is undesirable. Another thing to note is that, given the behavior of the fundamentals and the exchange rate regime which prevails, one can make assessments about the sustainability of the regime against speculative attacks (see Krugman and Rotemberg [1990]). This may be important if a breakdown of the exchange rate regime causes large political or economic costs.

The remainder of the paper is organized as follows. In Section 2 we summarize some stylized facts of Deutsche Mark rates in the EMS, sampled on a weekly frequency level since the realignment of January 1987, and we compare these rates with the corresponding dollar rates. The most striking fact is that tests for non-normality and conditional heteroskedasticity are more often significant for EMS exchange rates than for exchange rates between the dollar and EMS currencies. In Section 3 we investigate the theoretical implications for exchange rate behavior in the context of a simple continuous-time monetary model with a band on the exchange rate. The model is rewritten in the form used by Krugman [1990]. In Section 4 we decribe the results of a Monte Carlo experiment with the model. It is shown that there are values of the parameters for which the stylized facts of the Deutsche Mark rates can be reproduced. Section 5 summarizes and offers some potentially useful directions for future research.

2. Stylized facts of EMS exchange rates

In this section we summarize some stylized facts of weekly Deutsche Mark rates in the EMS for the period since the realignment in January 1987, and compare them with U.S. dollar rates vis-à-vis EMS currencies for the same period. The reason for this is that we want to compare the effects of the officially declared bands on EMS exchange rates with situations in which these bands are absent. However, before we do this, we summarize some empirical evidence found elsewhere in the literature on exchange rates, in order to compare this with the stylized facts of EMS rates later on.

First, it is difficult to reject the unit root hypothesis for dollar exchange rates. Using augmented Dickey-Fuller tests, Meese and Singleton [1982] cannot reject the hypothesis for weekly observations of the Swiss franc, the Canadian dollar and the Deutsche Mark for the period 1976-1981. Examples of other studies which cannot reject the unit root hypothesis are Baillie and Bollerslev [1989 a,b] and Baillie and McMahon [1989]. They use augmented Dickey-Fuller tests and Phillips-Perron [1988] tests for both daily and weekly dollar spot rates. The evidence for dollar rates suggests that it might be interesting to test the unit root hypothesis for EMS exchange rates as well, although if the bands on EMS rates are to be sustainable, these rates should be stationary. Still it might be difficult to reject the unit root hypothesis in this situation.

Second, empirical distributions of exchange rate returns are not normal. Exchange rates are often regarded as asset prices and it has been known since long that returns on assets are not normally distributed (e.g. Mandelbrot [1963]). Especially, they contain fat tails, which implies a kurtosis larger than 3. One explanation for this fact is given by models with autoregressive conditional heteroskedasticity (ARCH) - originally due to Engle [1982] - or one of its variations. Examples of studies, which apply ARCH models or generalisations of them to exchange rates, are Hsieh [1988], Baillie and McMahon [1989] and Baillie and Bollerslev [1989b]. Thus, the presence of non-normality and conditional heteroskedasticity often found for exchange rate returns or asset returns in general, makes it worthwhile to test whether EMS exchange rates exhibit these facts also.

The sample period runs from January 1987 to August 1990, which yields 189 Wednesday observations. During this period eight countries participated in the exchange rate mechanism of the EMS. These countries are Germany, France, Italy, Belgium, Luxemburg, Denmark, Ireland and the Netherlands. Because Belgium and Luxemburg maintain a fixed parity, we exclude Luxemburg from our analysis. We use logarithms of exchange rates and changes in the logarithms as approximations to the exchange rate returns. Exchange rates between the Deutsche Mark and the other currencies in the EMS are plotted in the Figures 1 to 6 respectively (at the end of the paper), together with their ±2.25% bands (Italy ±6%). Although the EMS has been successful in maintaining the same exchange rate bands during the whole period we study, the French franc, the Belgium franc, the Danish kroner as well as the Irish pound spent some time near their lower boundaries, so that interventions were necessary to keep them in their bands. Recently, maintenance of the bands has been made easier by the Basle-Nyborg agreement in September 1987, which extends the credit facilities to the central banks. However, at the end of October 1987, there were large depreciations of the French franc and the lira, accompanied by rumours that the central parities of these currencies would be changed, which in fact did not happen. One reason that the credibility of the target zones for the French franc and the lira came into question, might have been the gradual abolition of capital controls (e.g. Giavazzi and Giovannini [1989] provide an analysis of capital controls in the EMS). The behavior of the Dutch guilder is different from that of the other currencies, because it always stays close to its central parity with the Deutsche Mark. This suggests that De Nederlandsche Bank has used much narrower bands in practice than the ±2.25% bands. In addition to the pictures of the EMS exchange rates, Figure 7 shows the dollar-Deutsche Mark rate, which fluctuates widely as one would expect in a free float.

Key statistics of the Deutsche Mark and the dollar rates are summarized in Tables 1 and 2 respectively (at the end of the paper). In none of the cases, the augmented Dickey-Fuller test (critical values can be found in Fuller [1976]) is able to reject the null hypothesis of a unit root in the exchange rates, although for bilateral EMS rates the results are usually less insignificant than for dollar rates. Note that in the presence of conditional heteroskedasticity or autocorrelation, a Phillips-Perron test is more appropriate to test the unit root hypothesis. However, the results for the Box-Pierce tests show that only for the Deutsche Mark-Belgium franc rate the null hypothesis of zero autocorrelation is rejected, and the ARCH tests reject the hypothesis of conditional homoskedasticity in only two cases. Thus the difficulty of rejecting the unit root hypothesis is a stylized fact not only of dollar exchange rates, as we already have seen above, but also of bilateral EMS exchange rates.

A comparison of the skewness, kurtosis and tests for non-normality reported in Tables 1 and 2, shows large differences between the returns on EMS rates and the returns on dollar rates. While for the dollar rates in none of the cases normality is rejected, we find for the EMS rates highly significant tests for non-normality, which is mainly due to the large excess kurtosis.

The fact that these differences are found for all EMS rates compared to the corresponding dollar rates, suggests that they might be explained by the presence of the bands on the EMS exchange rates. Differences between EMS and dollar returns are less clear if we compare the tests for conditional heteroskedasticity. Evidence for conditional heteroskedasticity is only found for the Deutsche Mark-guilder rate and for the Deutsche Mark-Irish pound rate.

3. A monetary model with exchange rate bands

As we saw in Section 2, stylized facts of EMS exchange rates are that unit roots are difficult to reject and that the returns show much more non-normality than dollar rates. In addition, for two series conditional heteroskedasticity is found, while this is not the case for dollar rates. Below, Krugman's [1990] target zone model in presented. This nonlinear model allows for bands on the exchange rate and is able to generate non-normality and conditional heteroskedasticity endogenously, a property which disappears if the bandwidth goes to infinity. Therefore, the model can in principle account for the differences between EMS and dollar rates just mentioned.

We start from a two-country monetary model in continuous time. Svensson [1989] shows that the crucial equation to be derived from the model, equation (4) below, is also consistent with a monetary model of a small open economy, which in some cases may be a more suitable description of the relationship between two partners in the EMS. The model we use reads as follows (we denote foreign variables by an asterisk and omit time indices for clarity):

$$m - p = \kappa y - \alpha i + \varepsilon , \tag{1}$$

$$m^* - p^* = \kappa y^* - \alpha i^* + \varepsilon^* , \tag{1*}$$

$$p^* + e - p = 0 \,, \tag{2}$$

$$E[de]/dt = i - i^* . \tag{3}$$

Here are y, m, p, i and e resp. national income, nominal money supply, the price level, the nominal interest rate and the nominal exchange rate (a rise in e is a depreciation of the domestic currency), all expressed in logarithms, except for i, which is given as a percentage. Equation 1 is the condition for money market equilibrium. It relates real money demand to income, interest rates and a stochastic term ε. Here α is the semi-elasticity of money demand for the interest rate. It is assumed that labour markets are perfectly flexible, so that employment is always at its natural level. The natural rate is normalized to zero for convenience, so that $y=y^*=0$. Equation 2 states that the real exchange rate between both countries is constant, which corresponds to purchasing power parity. Dynamics enter the model through equation 3, the uncovered interest parity. This is an arbitrage condition, which requires perfect capital mobility between both countries and perfect substitutability of interest earning securities in the two countries. Furthermore, expectations are assumed to be rational.

The model can be used to derive the exchange rate as a function of a fundamental f and its expected rate of depreciation:

$$e(t) = f(t) + \alpha\, E_t[de(t)]/dt \quad , \quad \alpha > 0 \,, \tag{4}$$

where $f(t) \equiv [m(t) - m^*(t)] - [\varepsilon(t) - \varepsilon^*(t)]$,
$E_t[.]$ = expectation conditional on the information available at t.

Equation (4) shows that α measures the degree of forward-lookingness in the model; the higher α, the stronger the anticipative effect of future changes in the current exchange rate level. This is also clear if (4) is integrated and bubbles are excluded (see Svensson [1989]):

$$e(t) = E_t \left[\int_{\tau=t}^{\infty} \{\exp[-(\tau-t)/\alpha]\} \, f(\tau)/\alpha \, d\tau \right] , \tag{5}$$

so that the exchange rate can be expressed as the expected present value of all future realisations of the fundamental, discounted by $1/\alpha$.

The fundamental consists of two parts, the difference between domestic and foreign money supply, $m(t)-m^*(t)$, which is under control of the monetary authorities, and an uncontrollable part $v(t) \equiv \varepsilon^*(t)-\varepsilon(t)$. This is the difference in cumulated money demand shocks and it is assumed to evolve as a Brownian motion with drift μ and instantaneous standard deviation σ:

$$dv(t) = \mu \, dt + \sigma \, dz(t) \; . \tag{6}$$

Here z is a Wiener process with $E[dz]=0$ and $E[(dz)^2]=dt$. Note that a positive value of μ means that the domestic currency is intrinsically weak vis-à-vis the foreign currency.

The monetary authorities prevent the exchange rate from going outside a pre-specified band and impose a lower bound $\underline{f}$ and an upper bound $\bar{f}$ on the movements of the fundamental. At the boundaries, the band is defended using infinitesimally small changes in the nominal money supply, i.e. the authorities set

$$dm(t) = dL(t) - dU(t) , \tag{7}$$

where

$$dL(t) = \begin{cases} -dv(t), \text{ if } f(t) = \underline{f} \text{ and } dv(t) < 0 \\ 0 \;, \text{ otherwise} \end{cases}$$

$$dU(t) = \begin{cases} dv(t) \;, \text{ if } f(t) = \bar{f} \text{ and } dv(t) > 0 \\ = 0 \;, \text{ otherwise} \end{cases}$$

When the exchange rate has a tendency to cross its upper bound, the domestic monetary authorities will keep it within the band by selling infinitesimal amounts of the foreign money and the foreign monetary authorities support the exchange rate by buying domestic money. The opposite occurs when the exchange rate has a tendency to cross its lower bound.

Applying Itô's lemma (e.g. Harrison [1985]) gives, after some algebra, the following solution of the exchange rate as a function of its fundamental (Krugman [1990], Svensson [1989]):

$$e(f) = f + \alpha\,\mu + A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f) \tag{8}$$

where λ_1 and λ_2 are the solutions of $(\alpha\sigma^2/2)\,\lambda^2 + \alpha\mu\,\lambda - 1 = 0$.

It has been assumed that the announced monetary policy is fully credible. Arbitrage with respect to this policy rule implies that e(f) should be flat at its boundaries. These are the so-called smooth pasting conditions. Using $e'(\underline{f}) = e'(\bar{f}) = 0$, one can derive

$$A_1 = \frac{\exp(\lambda_2 \bar{f}) - \exp(\lambda_2 \underline{f})}{\lambda_1 \exp(\lambda_1 \bar{f} + \lambda_2 \underline{f}) - \lambda_1 \exp(\lambda_1 \underline{f} + \lambda_2 \bar{f})}$$

$$A_2 = \frac{\exp(\lambda_1 \underline{f}) - \exp(\lambda_1 \bar{f})}{\lambda_2 \exp(\lambda_1 \bar{f} + \lambda_2 \underline{f}) - \lambda_2 \exp(\lambda_1 \underline{f} + \lambda_2 \bar{f})} \tag{9}$$

The band on the fundamental results in a similar and unique band on the exchange rate itself. This allows one to calculate $\underline{e}$ and $\bar{e}$ from $\underline{f}$ and $\bar{f}$. To illustrate the solution two special cases are considered. The first is when μ equals zero. Then the exchange rate curve has a symmetric S-shape as depicted in Figure 8. It shows that the band has a stabilizing effect on the exchange rate. The S-curve is always flatter than a 45-degree line, so that changes in the fundamental are less than fully reflected in the exchange rate. This stabilizing effect is stronger the larger the deviation of the exchange rate from its central rate and it results in a flat curve at the boundaries. Krugman [1990] provides the intuition for this. If at some point in time the exchange rate is precisely at its upper bound (for the exchange rate being at the lower bound a similar reasoning will apply), then one moment later two things can happen. Either, the exogenous part of the fundamental is hit by a positive shock and the authorities intervene, which keeps the exchange rate unchanged, or a negative shock takes the exchange rate back into the interior of the band. If the exchange rate curve is not horizontal, but kinked and, let's say, positively sloped for $f<\bar{f}$, then it is clear that the expected change in the exchange rate is strictly negative. Therefore, in anticipation of this, speculators will bid the exchange rate upwards when it is in the upper half of the band. A similar reasoning excludes the possibility of a negatively sloped curve at $\bar{f}$. The second case illustrated in Figure 8 is when the lower boundary and the upper boundary on the fundamental simultaneously go to minus infinity and plus infinity respectively. Then the exchange rate solution reduces to the free float solution:

$$e(f) = f + \alpha\mu \qquad (10)$$

Now, the curve of the exchange rate is a straight 45-degree line, and changes in the fundamental are matched by equal changes in the exchange rate.

Using Itô's lemma, we can show how exchange rate movements relate to movements in the fundamental, when it is in the interior of its band:

$$de(f) = [e'(f)\mu + e''(f)\ \sigma^2/2]\ dt + e'(f)\sigma\ dz(t) \qquad (11)$$

The instantaneous standard deviation of exchange rate changes, $e'(f)\sigma$, varies with the position of the fundamental in the band. If the fundamental is close to its boundaries, then $e'(f)$ is much smaller than when it is near its central parity. In the latter case the volatility of exchange rate returns is larger than in the first case, which implies that returns are conditionally heteroskedastic and that their unconditional distribution is not normal. Only when the bandwidth goes to infinity, returns become normally distributed and the conditional heteroskedasticity disappears. The target zone model is then able to account for the stylized facts of dollar rates vis-à-vis EMS currencies presented in Section 2.

4. Monte Carlo analysis

In Section 3 a target zone model was presented, in which monetary authorities were able to credibly impose a band on the fundamental. This implied a unique band on the exchange rate itself. Furthermore, the model generated conditional heteroskedasticity and non-normality endogenously. Given the stylized facts for EMS exchange rates presented in Section 2, the target zone model may give a reasonable description of the EMS rates.
We impose a band on the exchange rate which ranges from -0.0225 to 0.0225. This corresponds to the ±2.25% band on the EMS exchange rates. Given the parametervalues, we then calculate the band on the fundamental. In order to simulate the target zone model, we approximate it with a model in discrete time. The appendix contains more details on the approximation that has been used. Simulated values of the exchange rate are obtained from simulated values of the fundamental using (8) and (9).

The model implies that with probability zero the fundamental is exactly at $\underline{f}$ or $\bar{f}$. However, if the fundamental is simulated with a discretization at a weekly level, the probability that the fundamental will be on a boundary is non-negligible. Therefore, we have simulated the fundamental roughly at an hourly frequency level using appropriately adjusted parameters. Each time the fundamental crosses one of its boundaries, it is held at fixed at that boundary, until an appropriate increment takes it again into the interior of the band. Use of these 'in-between' simulations improves the simulation of the theoretical model, and the percentage of exchange rate simulations which are exactly on the boundaries can be made arbitrarily small.

One hundred series of 189 exchange rate observations are simulated. For each exchange rate value obtained the number of 'in-between' simulations is 50. We choose α=2 on a yearly basis, which amounts to an α of 100 on a weekly basis, given that there are approximately 50 weeks in a year (see the appendix). Intuitively, it is clear that the α parameter must be larger when the discretization of the continuous-time model is taken at a higher frequency level, because changes, which are expected to happen not so far in the future, exert more influence on the exchange rate now (see equation (4)). Table 3 presents the results of the Monte Carlo simulations for two different (μ,σ)-combinations, namely $(\mu,\sigma)=(-0.002,0.070)$ and $(\mu,\sigma)=(0,0.011)$. For each parametervector the mean, standard deviation, skewness and kurtosis, averaged over the one hundred series, are given.

The results agree quite well with what is found for the real data. The percentage of rejections of the unit root hypothesis is relatively low, despite the fact that the target zone model implies a stationary exchange rate. Also, there is excess kurtosis, and normality is almost always rejected. Only the rejection percentage of conditional homoskedasticity is quite high for the first parameter combination.

In addition to these results, Figures 9 and 10 show two representative time paths of series simulated for the respective parameter combination. In both cases, the exchange rate stays for some time at the lower boundary and then turns back into the band. This is quite similar to what we have seen for the EMS exchange rates, except for the guilder and the Irish pound.

Summarizing, given the presence of a narrow band on the fundamental, there are regions in the parameter space of Krugman's [1990] model, for which it is able to reproduce a number of stylized facts of EMS exchange rates. The value of the forward-lookingness parameter α should be high enough to result in a clear bending of the exchange rate curve from the 45° line. This produces non-normality and conditional heteroskedasticity, if σ is also high enough, so that the the position of the exchange rate in the band is varied rather quick. Furthermore, we have shown that the data can be consistent with a non-zero drift in the fundamental, which models the situation in which one currency is intrinsically weaker than the other. If the bandwidth in the Krugman model goes to infinity, this corresponds to a situation in which the exchange rate moves in a free float. The model is then able to explain the stylized facts of non-normality and conditional homoskedasticity in the dollar rates as well.

6. Concluding remarks

In this paper some stylized facts of weekly bilateral EMS exchange rates for the period January 1987 to August 1990 were compared with dollar rates vis-à-vis currencies in the EMS. While for all exchange rate series the unit root hypothesis is difficult to reject, tests for non-normality and conditional heteroskedasticity are often significant for the EMS exchange rates, but in none of the cases for dollar exchange rates. One explanation for this phenomenon can be the presence of the bands on bilateral exchange rates in the EMS. Therefore, Krugman's [1990] target zone model is used to model the band on exchange rates. In this model, monetary authorities use marginal interventions at the boundaries of the fundamental in order to keep the exchange rate in its band. This policy is assumed to be fully credible. It is shown that by the presence of the band the model can generate conditional heteroskedasticity and non-normality endogenously. Only in the extreme case where the band is absent, the model predicts that exchange rate returns are normally distributed and conditionally homoskedastic.

Using a Monte Carlo experiment we are able to find parametervalues, for which the model can reproduce the stylized facts of Deutsche Mark rates vis-à-vis other currencies in the EMS, even though capital controls have been used in the period under consideration. Also the stylized facts of dollar exchange rates for EMS currencies, which are often assumed to be freely floating, can be mimicked by the Krugman model if the band on the exchange rate is infinitely wide.

Given that the Krugman model is only the simplest target zone model, extensions of the model may be useful to describe EMS exchange rates more accurately. Examples of extensions are the models developed by Bertola and Caballero [1990], who allow for repeated realignments of the central parity, and Lewis [1990], who allows for interventions in the band with some probability which increases with the deviation of the fundamental from its central rate. The empirical analysis of these models is left for future research.

Appendix: Simulation of the continuous-time target zone model

To simulate the target zone model, we approximate it by a discrete-time version. This means that, given the continuous-time parameter values, we have to choose proper values of the parameters used in the simulation procedure.

We denote the analogues of α, μ, σ in the discretized version of the theoretical model by α_d, μ_d and σ_d respectively. The sampling interval is divided into n parts (see Duffie and Singleton [1988]), and a process $\tilde{f}$ of 'in-between' simulations and a 'shadow process' $\tilde{\tilde{f}}$ are defined. For each in-between simulation, a value of the shadow process is obtained:

$$\tilde{\tilde{f}}[t+k/n] = \tilde{f}[t+(k-1)/n] + \mu_d/n + \varepsilon[t+k/n]\,\sigma_d/\sqrt{n} \quad (k=1,..,n)\ , \qquad \text{(A.1)}$$

where the ε's are independently simulated from a standardnormal distribution. The drift parameter and the standard deviation are adjusted for the number of in-between simulations, n. The in-between simulations are given by

$$\hat{f}[t+k/n] = \begin{cases} \underline{f} \ , & \text{if } \tilde{f} \le \underline{f} \\ \tilde{f}[t+k/n] \ , & \text{if } \tilde{f}[t+k/n] \in (\underline{f}, \bar{f}) \\ \bar{f} \ , & \text{if } \tilde{f} \ge \bar{f} \end{cases} \qquad \text{(A.2)}$$

After each run of n in-between simulations, a simulated value for the fundamental is obtained:

$$f[t+1] = \hat{f}[t+1]$$

It is easy to show that the unique solution of the exchange rate function, determined by equations (8) and (9) in the text, can be written in terms of the parametercombinations μ/σ^2 and $\alpha\mu$. In order to keep the exchange rate solution invariant with repect to the frequency level at which the exchange rate is simulated, μ_d/σ_d^2 and $\alpha_d\mu_d$ should be the same irrespective of the frequency level. The simulated value of the exchange rate at time t+1 is obtained through substitution of μ_d/σ_d^2 for μ/σ^2, $\alpha_d\mu_d$ for $\alpha\mu$, and the simulated value f[t+1] for f, into the solution of the exchange rate function.

We illustrate the simulation process in more detail for the simulation at a weekly frequency level in Section 4 with $(\alpha_d,\mu_d,\sigma_d)=(100,-0.002,0.070)$. Simulation of the model at a yearly frequency level, for instance, would require the parametervector to be adjusted to $(\alpha_d,\mu_d,\sigma_d) = (2,-0.1,0.7)$, keeping the relation between the exchange rate and the fundamental invariant with respect to the frequency level and also adjusting the drift and the standard deviation for the change in frequency level. In Section 4 we had fifty in-between simulations of the fundamental for each simulated value of the exchange rate. For each exchange rate value, we therefore used (A.1) and (A.2) fifty times with $\mu_d/n = -0.00004$ and $\sigma_d/\sqrt{n} = 0.0099$. Then, after such a run the exchange rate was calculated from (8) and (9), using $\mu_d/\sigma_d^2 = -0.41$ and $\alpha_d\mu_d = -0.2$.

References:

Baillie, R.T. and T. Bollerslev [1989a], 'Common Stochastic Trends in a System of Exchange Rates', **Journal of Finance**, Vol. XLIV, No.1.

Baillie, R.T. and T. Bollerslev [1989b], 'The Message in Daily Exchange Rates: A Conditional-Variance Tale', **Journal of Business and Economic Statistics**, Vol. 7, No. 3.

Baillie, R.T. and P.C. McMahon [1989], **'The Foreign Exchange Market - Theory and Econometric Evidence'**, Cambridge University Press.

Bertola and Caballero [1989], 'Target Zones and Realignments', mimeo **Princeton University/Columbia University**.

Duffie, D. and K.J. Singleton [1988], 'Simulated Moments Estimation of Diffusion Models of Asset Prices', mimeo **Stanford University**.

Engle, R.F. [1982], 'Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation', **Econometrica**, Vol. 50, pp. 987-1008.

Froot, K.A. and M. Obstfeld [1990], 'Stochastic Process Switching: Some Simple Solutions', mimeo **Harvard University/University of California in Berkeley**.

Fuller, W.A. [1976], **'Introduction to Statistical Time Series'**, John Wiley & Sons, New York.

Giavazzi, F. and A. Giovannini [1989], **'Limiting Exchange Rate Flexibility'**, The MIT Press, Cambridge, Massachusetts.

Harrison, J.M. [1985], **'Brownian Motion and Stochastic Flow Systems'**, John Wiley & Sons, New York.

Hsieh [1988], 'The Statistical Properties of Daily Foreign Exchange Rates: 1974 - 1983', **Journal of International Economics**, Vol.24, pp.129-145.

Krugman, P.R. [1990], 'Target Zones and Exchange Rate Dynamics', forthcoming in the **Quarterly Journal of Economics**.

Krugman, P.R. and J. Rotemberg [1990], 'Target Zones with Limited Reserves', presented at the **CEPR/NBER Conference**, University of Warwick.

Lewis, K.K. [1990], 'Can Managed Float Interventions Make Exchange Rate Behavior Resemble a Target Zone?', mimeo **NBER/New York University**.

Mandelbrot, M. [1963], 'The Variation of Certain Speculative Prices', **Journal of Business**, Vol. 36, pp. 392-417.

Meese, R.A. and K.J. Singleton [1982], 'On Unit Roots and the Empirical Modelling of Exchange Rates', **Journal of Finance**, Vol. XXXVII, No.4, pp. 1027-1035.

Phillips, P.C.B. and P. Perron [1988], 'Testing for a Unit Root in Time Series Regression', **Biometrica**, Vol. 75, pp. 335-346.

Svensson, L.E. [1989], 'Target Zones and Interest Rate Variability', mimeo **CEPR/NBER**.

Table 1: Summary statistics of Deutsche Mark exchange rates

	DM/Ffr	DM/lira	DM/Bfr
parity (e)	-1.210	-6.580	-3.027
ADF (e)	-1.99	-2.19	-0.24
mean (Δe) x10000	-0.36	-2.51	0.31
stdev (Δe) x1000	2.03	3.30	1.13
skewness (Δe)	-2.25	-2.11	0.32
kurtosis (Δe)	17.14	12.07	5.04
JB (Δe)	1768.11	804.12	37.71
ARCH(2) (Δe)	0.49	0.21	3.08
BP(10) (Δe)	13.75	7.66	19.39

	DM/Dkr	DM/IP	DM/Dfl
parity (e)	-1.339	0.985	-0.119
ADF (e)	-1.60	-2.66	-2.32
mean (Δe) x10000	-0.53	0.34	0.05
stdev (Δe) x1000	2.19	1.77	0.57
skewness (Δe)	-0.70	-0.12	-0.97
kurtosis (Δe)	5.55	6.59	5.83
JB (Δe)	68.97	105.62	95.70
ARCH(2) (Δe)	1.40	6.34	28.22
BP(10) (Δe)	4.31	9.26	7.07

Remarks:

1. Exchange rates have been calculated from closing rates of the guilder at the Amsterdam foreign exchange market, assuming perfect arbitrage. The data were kindly provided by De Nederlandsche Bank N.V..
2. Exchange rates are expressed as the number of Deutsche Marks needed to buy one unit of foreign currency.
3. The sample period runs from January 14, 1987 to August 22, 1990 (189 Wednesday observations).
4. The central parity is defined on the logarithm of the exchange rate.
5. Abbreviations: e = loglevel of exchange rate, Δe = first difference in loglevel, DM = Deutsche Mark, Ffr = French franc, Bfr = Belgium franc, Dkr = Danish kroner, IP = Irish pound, Dfl = Dutch guilder, ADF = augmented Dickey-Fuller test including a constant, a time trend and two lags of the differenced series, stdev = standard deviation, ARCH(2) = ARCH test of order 2 (under the null asymptotically chi-square distributed with 2 degrees of freedom), JB = Jarque-Bera test for normality (under the null asymptotically chi-square with 2 degrees of freedom) and BP(10) = Box-Pierce test of order 10 (under the null asymptotically chi-square with 10 degrees of freedom).
6. Significant test statistics at a 5% level are underlined.

Table 2: Summary statistics of dollar rates

	$/DM	$/FFr	$/lira	$/Bfr
ADF (e)	-1.17	-0.86	-1.15	-0.90
mean (Δe) x10000	9.52	9.16	7.01	9.82
stdev (Δe) x100	1.44	1.36	1.32	1.42
skewness (Δe)	0.16	0.11	0.14	0.14
kurtosis (Δe)	2.99	3.11	2.99	3.08
JB (Δe)	0.79	0.53	0.64	0.69
ARCH(2) (Δe)	2.05	1.46	3.17	2.19
BP(10) (Δe)	12.64	12.76	11.47	13.36

	$/Dkr	$/IP	$/Dfl
ADF (e)	-0.93	-0.98	-1.15
mean (Δe) x10000	8.98	9.85	9.57
stdev (Δe) x100	1.41	1.37	1.43
skewness (Δe)	0.14	0.064	0.14
kurtosis (Δe)	3.09	2.91	2.96
JB (Δe)	0.72	0.16	0.62
ARCH(2) (Δe)	2.21	0.64	2.38
BP(10) (Δe)	12.85	13.90	12.80

Remarks:

1. See remarks 1,3,5 and 6 Table 2.
2. Exchange rates are expressed as the number of dollars needed to buy one unit of foreign currency.

Table 3: Summary statistics and rejection percentages for simulated exchange rate series

(α,μ,σ)		(100,-0.002,0.071)	(100,0,0.011)
ADF (e)		25%	13%
mean (Δe)	x10000	0.01	0.04
stdev (Δe)	x1000	3.65	3.19
skewness (Δe)	x100	-4.13	-1.54
kurtosis (Δe)		5.05	4.32
JB (Δe)		94%	64%
ARCH(2) (Δe)		98%	39%
BP (Δe)		17%	14%

Remarks:

1. Results are based on 100 sets of 189 observations. Where applicable, the statistics are averages over the corresponding statistics for all time series separately.
2. See remark 5 table 1.
3. In all cases a 5% significance level is taken.

Figure 1: log(DM/Ffr)

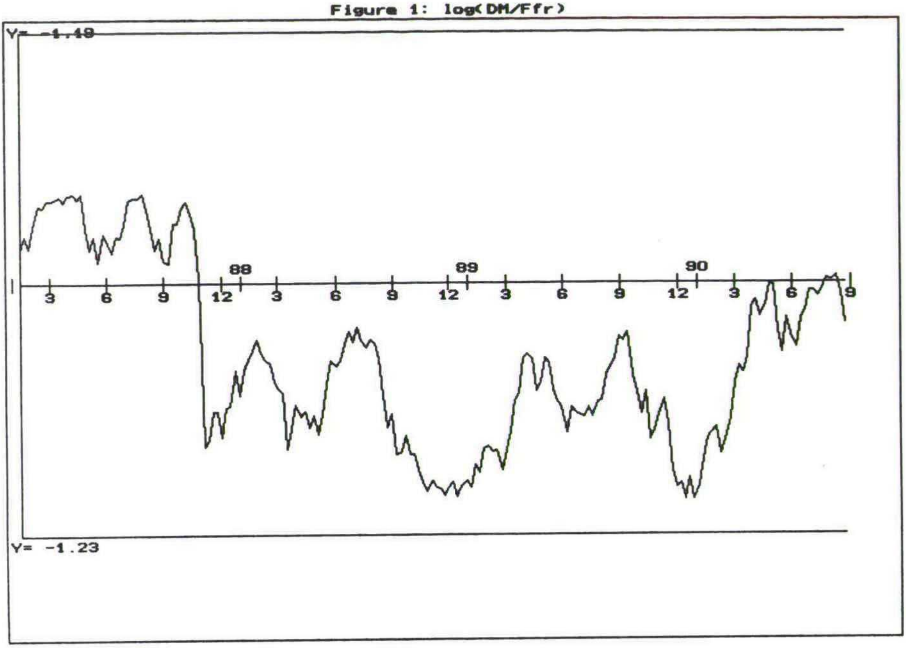


Figure 2: log(DM/lira)

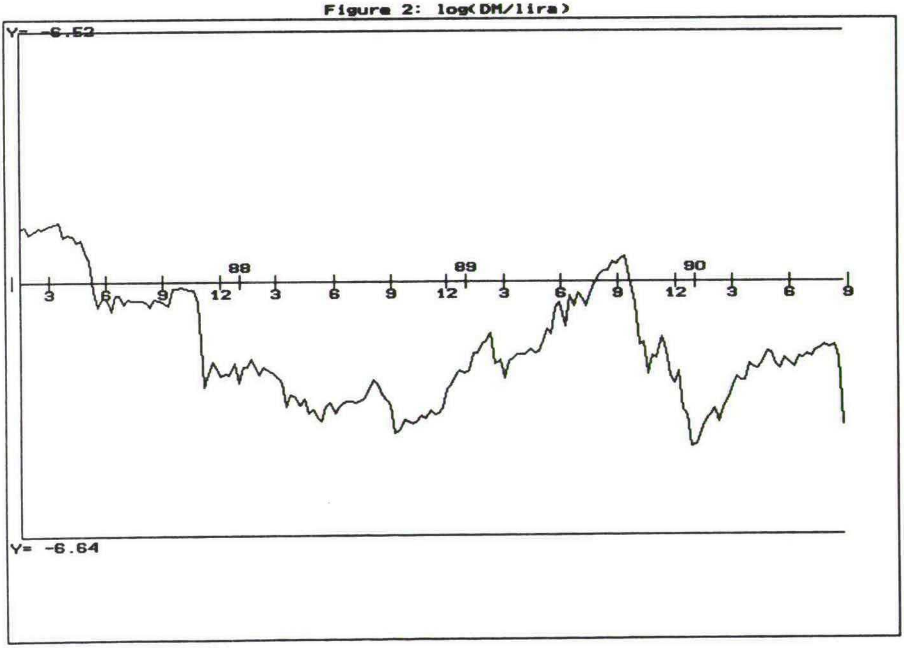


Figure 3: log(DM/Bfr)

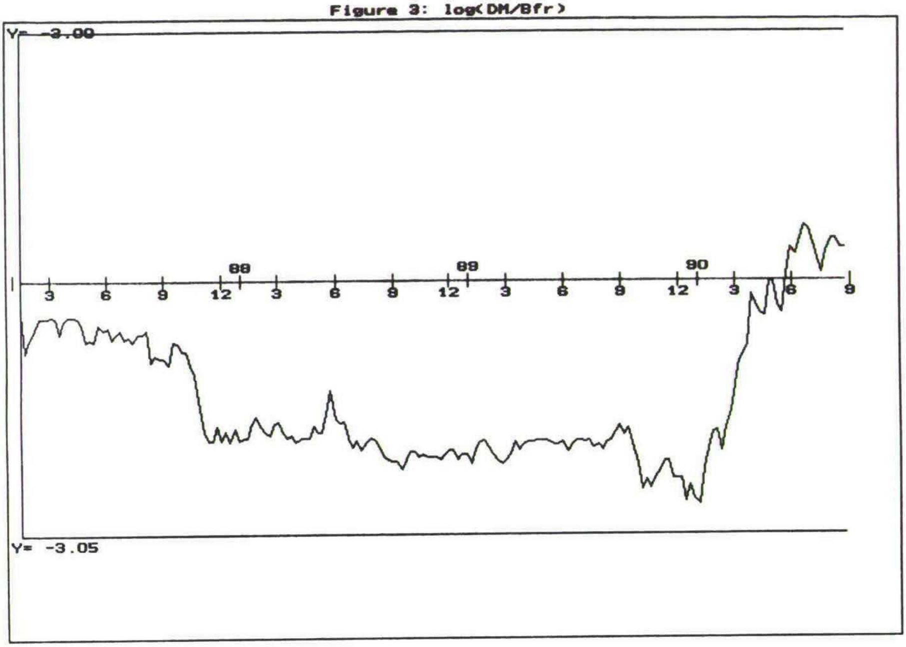


Figure 4: log(DM/Dkr)

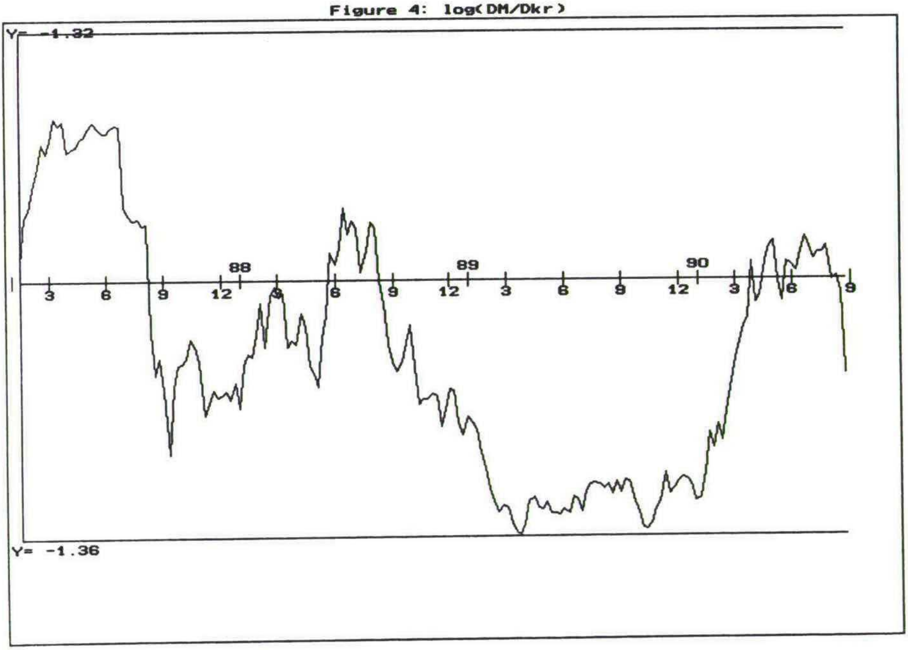


Figure 5: log(DM/IP)

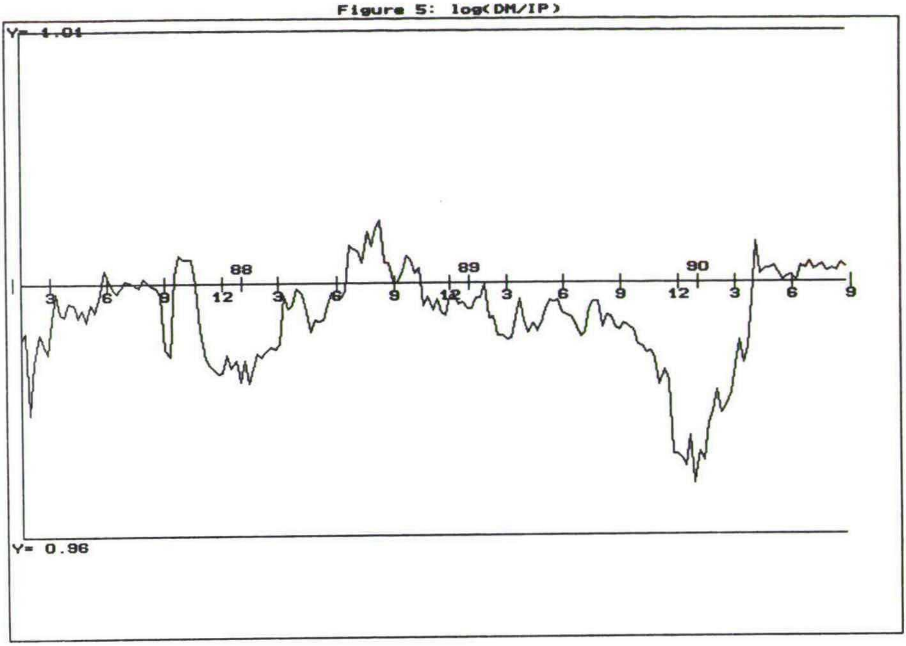


Figure 6: log(DM/Dfl)

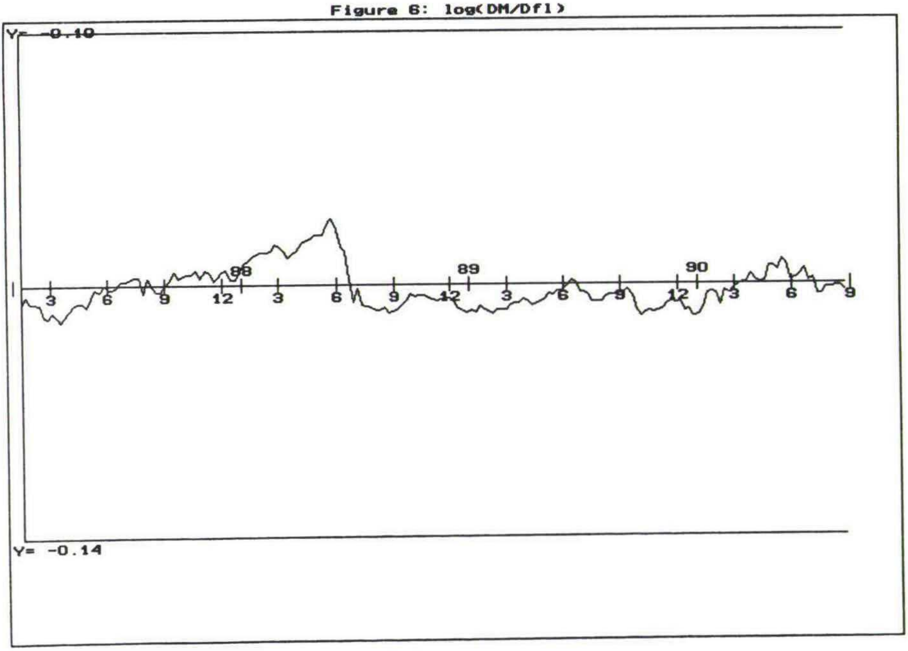


Figure 7: log($/DM)

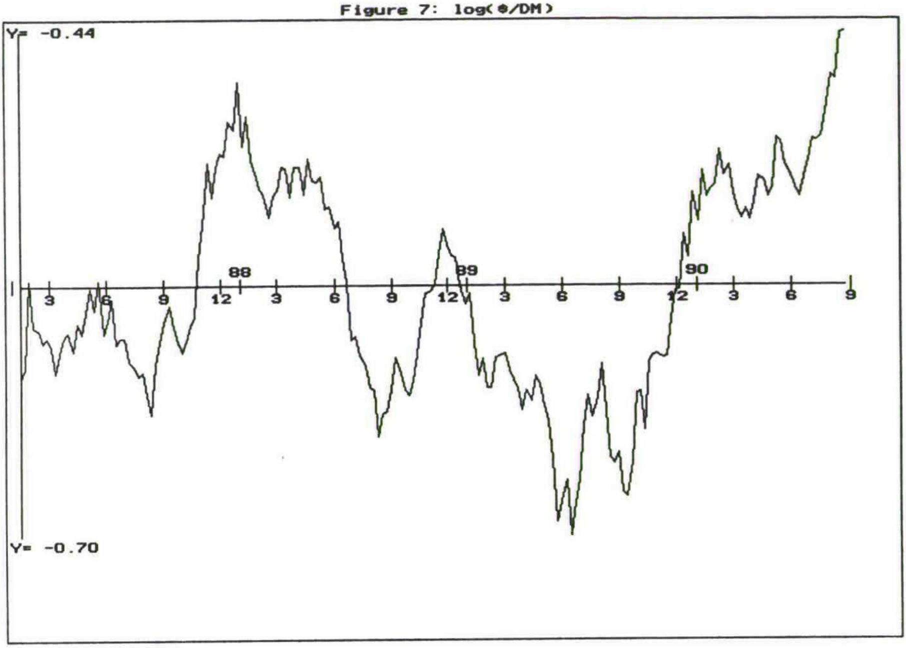
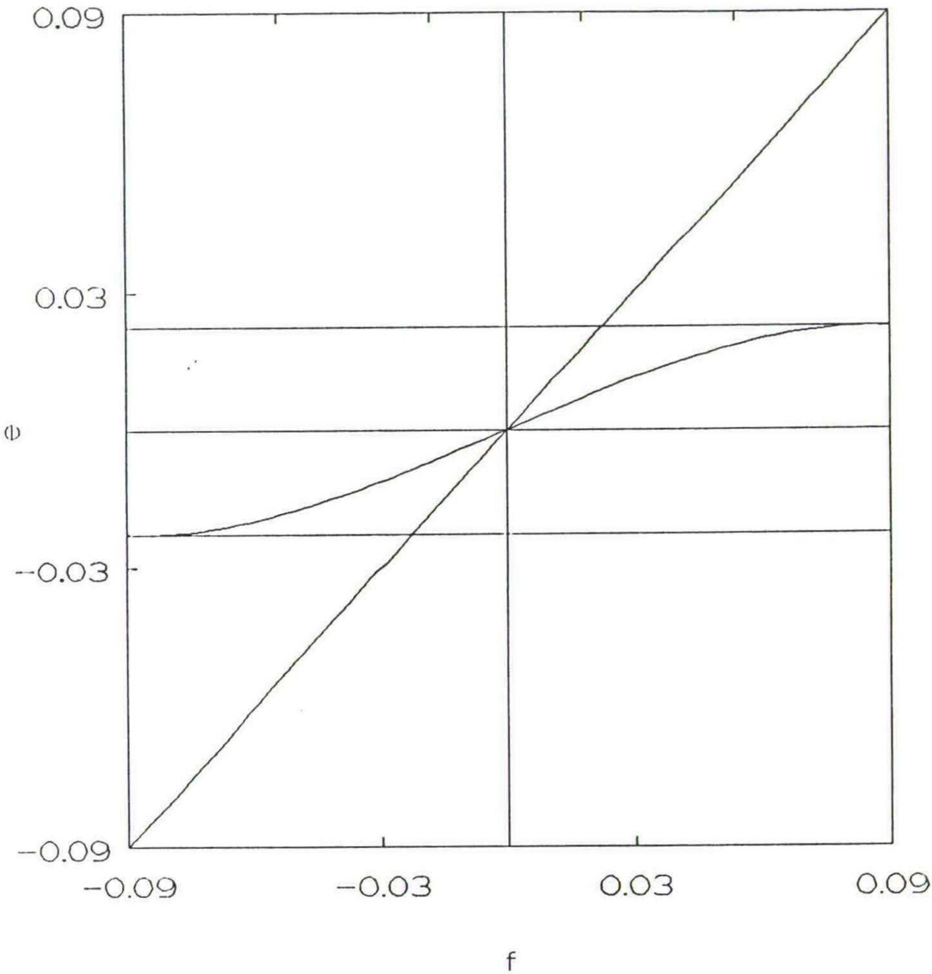


Figure 8: Exchange rate curves



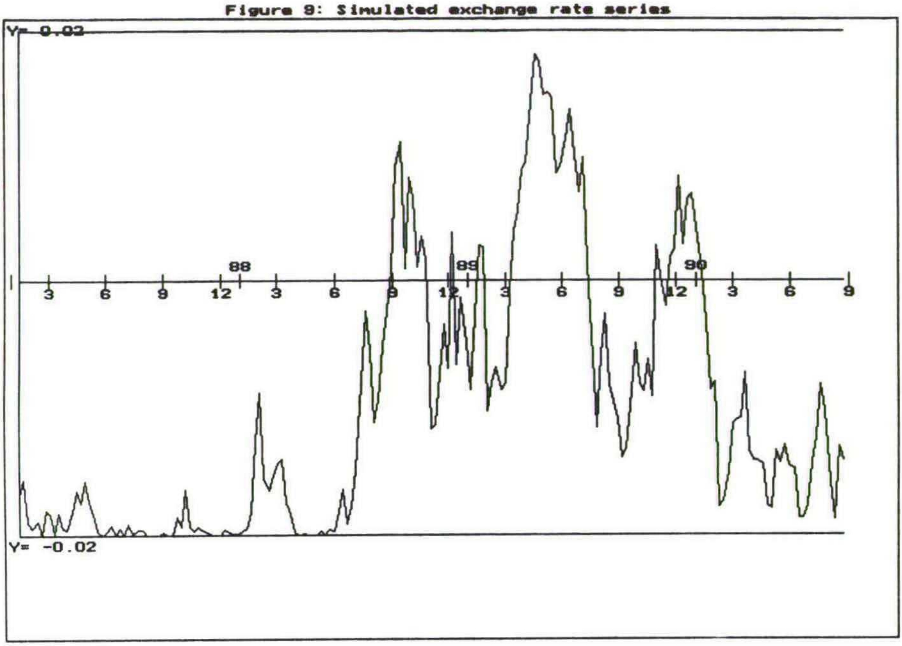


Figure 9: Simulated exchange rate series
Y= 0.02
88
89
90
3
6
9
12
3
6
9
12
3
6
9
12
3
6
9
Y= -0.02

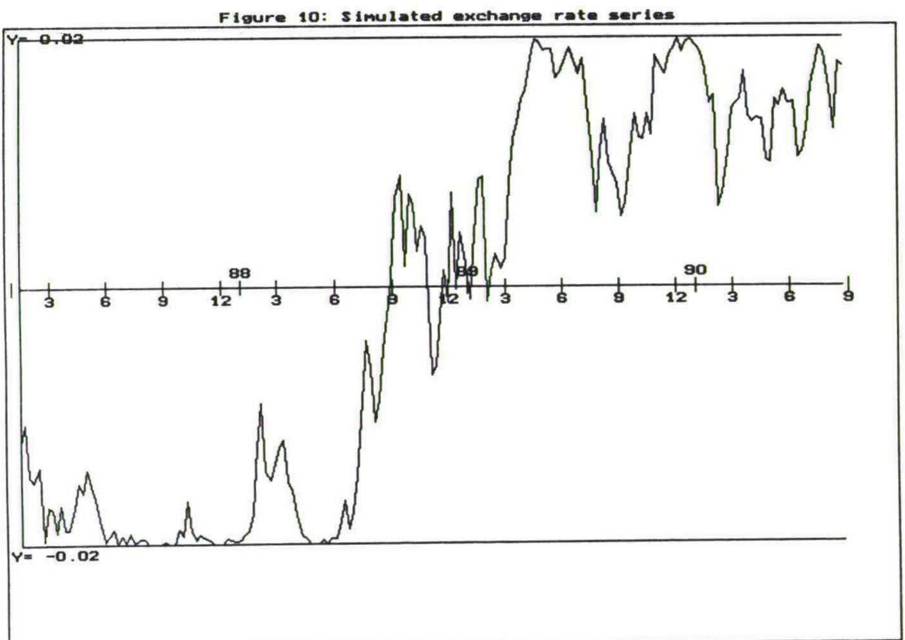


Figure 10: Simulated exchange rate series
Y= 0.02
88
89
90
3
6
9
12
3
6
9
12
3
6
9
12
3
6
9
Y= -0.02

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