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THE EFFECTS OF INFLATION ON GROWTH AND FLUCTUATIONS IN DYNAMIC MACROECONOMIC MODELS

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ABSTRACT

Several empirical studies have found that inflation has adverse effects on economic growth. In the present theoretical paper we analyze the implications of such supply side effects for two prototype monetary models. Both an expectations augmented Phillips curve and a Barro and Gordon type of model are combined with an inflation sensitive growth rate in the natural level of output. We find that the steady state inflation rate is increased and the steady state output growth rate is reduced. Hence due to the supply side effects time consistent monetary policy has a stagflationary bias. This is a major difference with the standard Barro and Gordon model and strengthens the case for central bank independence. Also we find that the extent of cyclical fluctuations, measured by the asymptotic variance of inflation is increased. These results are robust with respect to expectations formation, i.e. they hold irrespective of whether inflation expectations are determined adaptively or rationally.

I INTRODUCTION[1]

As stated by Stanners (1993, p. 79) there can hardly be a more universally or firmly held belief than that low or zero inflation is an essential or at least very important condition for high and sustained growth, and that its attainment should be a main aim of government economic policy. For instance, in a recent paper Andrew Crockett, Executive Director of the Bank of England states that" it is generally accepted that price stability not only contributes to social equity, but is central to the goal of improving resource allocation and strengthening the basis of economic growth."[2] Thus policymakers typically believe that inflation has important adverse effects on long run economic performance.

Previous work on the relationship between inflation and growth is entirely empirical, with no theoretical discussion of cause and effect. This is not so in the present paper. We analyze the implications of the supply side effect of inflation for two prototype monetary models. Both an expectations augmented Phillips curve and a Barro and Gordon (1983) type of model are combined with an inflation sensitive growth rate in the natural level of output.

We find that the steady state inflation rate is increased, the steady state output growth rate is reduced, and that the extent of cyclical fluctuations, measured by the asymptotic variance of inflation is increased. These results hold irrespective of whether inflation expectations are determined adaptively or rationally.

The paper is organized into five remaining sections, followed by an appendix. Section 2 briefly outlines the empirical evidence supporting a negative relationship between inflation and output. Section 3 derives a relationship between output and inflation by combining an inflation sensitive growth rate in the natural level of output with an expectations augmented Phillips curve. The model is closed in different ways in sections 4 and 5. In section 4 a dynamic aggregate demand curve is used. In section 5 inflation and growth are determined as in Barro and Gordon (1983) type models. That is, time-consistent monetary policy is analyzed where the central bank minimizes a social loss function. It is necessary to make some assumptions about the determinants of inflationary expectations. We assume in turn, in both sections 4 and 5, a simple adaptive expectations process and rational expectations and derive and discuss the resulting model behavior. Our conclusions are given in section 6. Appendix provides the derivation of the asymptotic variances of output and inflation. Finally in appendix B we explain the implicit differentiation which is applied in section 5.

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1 An earlier version of this paper was written while David Smyth was a visitor at the CentER for Economic Research, Tilburg University. Eric Schaling gratefully acknowledges financial support from the Netherlands Organization for Scientific Research (NWO). The authors are grateful for helpful comments by Bas van Aarle, Harry Huizinga, two anonymous referees and seminar participants at CentER. Of course, the usual disclaimer applies.

2 See Crockett (1994).

II EMPIRICAL EVIDENCE ON THE SUPPLY SIDE EFFECTS OF INFLATION

Previous work provides support for a significant and substantial negative relationship between inflation or the change in the rate of inflation or both on output growth. The studies can be classified into two types, cross-country analyses and time series analyses for individual countries.

Using post-war data from forty-seven countries Kormendi and Meguire (1985) examined the cross-sectional relationship between output growth and a number of variables. They found a strong negative effect of change between the rate of inflation and output growth.

Kormendi and Meguire's work was extended by Grier and Tullock (1989) who used pooled cross-section and five-year average time series data for 113 countries, the Summers and Heston data set. Three inflation variables were used in the analysis, the rate of inflation, its first difference and the standard deviation of inflation. Grier and Tullock reported results for a number of country classifications and found that in some, but not all, of the cases there was a negative relationship between one or more of the inflation variables and economic growth.

In addition to these published studies there are some unpublished analyses. Kahn et al. (1992, p.17) report that "a preliminary empirical study by OECD staff that found a 10-percentage-point increase in inflation is associated with about a one-percentage-point slowing of productivity growth for a sample of 18 OECD countries over three periods (1960-73, 1973-79 and 1980-90) (..). These estimated effects are larger than those found in studies, such as Fischer (1992) and Corbo and Rojas (1992), that include developing countries."

All these cross-country studies include a number of variables other than the inflation variables. This is not so in Stanners (1993). Using mainly graphical analysis, Stanners compared growth rates and inflation across a variety of groupings for 14 countries for the period 1980 to 1988 and found no connection between growth rates and inflation. He reached the same conclusion with time series data for individual countries. However, Stanners' study is fatally flawed because no explanatory variables, other than inflation, are included in either his cross-section or his time series analyses.

Jarrett and Selody (1982) used a causality approach to Canadian data and found that a permanent one percentage rate increase in inflation reduced productivity by more than three-tenths of a percentage point.

Grimes (1991) applied regression analysis to data for 21 countries and concluded that "the costs of even a low inflation rate are estimated to be large given that it is the growth rate, not just the level of output, which is affected by inflation." (Grimes 1991, p.641).

Surprisingly, Fischer (1993), shows in linear equations that the statistical significance of the negative association between inflation and growth is stronger for inflation below 15 percent than for higher inflation rates.

Finally, Smyth (1994) quantified the effect of inflation on growth by embedding an inflation variable in an empirical growth relationship for the U.S. business sector. He found that for each one percentage point increase in inflation the annual growth rate is reduced by 0.19 percentage points.

As shown Levine and Renelt (1992) the growth equation is sensitive to regressor set specification. This means the impact of inflation on growth is sensitive to the introduction of other regressors. Thus according to the "Levine and Renelt Critique" inflation is not robustly correlated with growth.

Nevertheless policymakers typically believe that inflation has important adverse effects on long run economic performance. The question is then whether what the many know is merely difficult to prove, or rather is substantially exaggerated (Fischer (1984), p. 33).

We follow the first avenue of thought. In the next section we assume that there exists an inverse relationship between inflation and growth and combine this relationship with an expectations augmented Phillips curve.

III THE SUPPLY SIDE EFFECT OF INFLATION AND THE EXPECTATIONS AUGMENTED PHILLIPS CURVE

We assume that the growth rate in the natural level of output is governed by

$$\Delta\bar{y}_t = a_0 - a_1\Delta p_t + \delta_t \quad \delta_t \sim N(0, 1) \quad a_0 > 0 \quad 0 < a_1 < 1 \quad (1)$$

The inflation term results from the supply side effect of inflation. The higher the rate of inflation, the slower the rate of growth in the natural level of output. The variable δ_t is a productivity shock. It has mean zero, is serially uncorrelated, and following Fischer and Cooper (1973), p. 852 without loss of generality has variance unity.

Note that if $a_1 = 0$ for the *level* of the natural rate of output we have a simple stochastic trend specification discussed in Stock and Watson (1988). This specification says that over the long run the natural rate of output will grow at some average rate, labelled a_0. However, shocks to productivity, δ_t, can cause output growth to deviate from its mean. Moreover, these shocks are persistent with respect to the level of the natural rate: once perturbed by an δ_t shock, $\bar{y}$ will show no tendency to return to its trendline. Hence, the supply shocks are assumed to have permanent effects on the economy's productive capacity.

The actual level of output is given by

$$y_t = \bar{y}_t + h\,(\Delta p_t - E_{t-1}\Delta p_t) \qquad h > 0 \quad (2)$$

Equation (2) is an expectations augmented Phillips curve ("surprise aggregate supply function). An unanticipated rise in inflation of one percentage point increases the level of output by h percentage points.[3] Hence h is the slope of the Phillips curve.
Taking first differences of (2) and substituting for $\Delta\bar{y}_t$ from (1) yields

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[3] This relation can be derived from the nominal contracts approach of Fischer (1977).

$$\Delta y_t = \Delta \bar{y}_t + h(\Delta p_t - \Delta p_{t-1}) - h(E_{t-1}\Delta p_t - E_{t-2}\Delta p_{t-1})$$
$$= a_0 + (h - a_1)\Delta p_t - h\,\Delta p_{t-1} - h(E_{t-1}\Delta p_t - E_{t-2}\Delta p_{t-1}) + \delta_t \quad (3)$$

Equation (3) is a dynamic aggregate supply curve between growth in output, inflation, and expected inflation. To analyze the behaviour of output growth and inflation it is necessary to close the model by adding another equation between inflation and output growth and to specify how inflation expectations are formed. We shall add a Cambridge equation in section 4 and assume a Barro and Gordon (1983) type of macroeconomic policy regime in section 5. In both these sections we assume in turn that expectations are formed adaptively and rationally. We analyze both the steady state and cyclical properties of the models.

IV THE SUPPLY SIDE EFFECT OF INFLATION IN THE CAMBRIDGE MODEL

Aggregate demand is captured by a simple Cambridge equation (see Canzoneri and Henderson (1991), p. 12).

$$\Delta y_t = \Delta m_t - \Delta p_t \quad (4)$$

Where Δm is the growth rate of the nominal money supply.
Hence the rate of growth of aggregate demand increases with the rate of growth of real money balances.

4.1 Adaptive Expectations

We consider first an adaptive expectations hypothesis for expected inflation. The simplest adaptive expectations assumption is that expected inflation in year t is equal to the actual rate of inflation in year t-1.[4] Combining

$$E_{t-1}\Delta p_t = \Delta p_{t-1} \quad (5)$$

with equations (3) and (4) yields

$$\Delta p_t = \frac{\Delta m_t - a_0}{1 + h - a_1} + \frac{2h}{1 + h - a_1}\Delta p_{t-1} - \frac{h}{1 + h - a_1}\Delta p_{t-2} + \frac{\delta_t}{1 + h - a_1} \quad (6)$$

and

$$\Delta y_t = \frac{[(h - a_1)\Delta m_t - 2h\Delta m_{t-1} + h\Delta m_{t-2} + a_0]}{1 + h - a_1}$$

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4 If more complicated adaptive schemes are assumed, the resulting difference equations are of the third order or higher and accordingly the variance expressions are much more complicated.

$$+ \frac{2h}{1+h-a_1} \Delta y_{t-1} - \frac{h}{1+h-a_1} \Delta y_{t-2} + \frac{\delta_t}{1+h-a_1} \tag{7}$$

The average steady state values of inflation and output growth can be obtained by putting $\Delta m_t = \Delta m_{t-1} = \Delta m_{t-2} = \Delta m$, $\Delta p_t = \Delta p_{t-1} = \Delta p_{t-2} = \Delta p_t$, $\Delta y_t = \Delta y_{t-1} = \Delta y_{t-2} = \Delta y$ and $\delta_t = 0$. This yields

$$\Delta \tilde{p} = (\Delta m - a_0)/(1 - a_1) \tag{8}$$

and

$$\Delta \tilde{y} = a_0 - a_1 \Delta \tilde{p} \tag{9}$$

where superscript $\tilde{}$ denotes average values.[5]
Taking the first derivative of (8) with respect to a_1, we get

$$\frac{\partial \Delta \tilde{p}}{\partial a_1} = \frac{\Delta m - a_0}{1 - a_1} \tag{10}$$

From (8) and (10) we derive proposition 4.1.

PROPOSITION 4.1.: If the rate of growth of nominal money balances exceeds the average growth rate of the natural rate of output - i.e. if $\Delta m - a_0 > 0$ - the higher a_1 the higher the average steady state rate of inflation.

Differentiating (9) with respect to a_1 we find that

$$\frac{\partial \Delta \tilde{y}}{\partial a_1} = \frac{-(\Delta m - a_0)}{1 - a_1} \tag{11}$$

From (11) we derive proposition 4.2.

PROPOSITION 4.2.: If the rate of growth of nominal money balances exceeds the average rate of growth of the natural rate of output - i.e. if $\Delta m - a_0 > 0$ - the higher a_1 the lower the average steady state rate of output growth.

Thus with a positive steady-state inflation rate the supply side effect of inflation raise the average steady state rate of inflation and lower the average steady state rate of growth of output.
Now we will analyze the consequences of the supply side effect of inflation on economic fluctuations. As a measure of fluctuation we employ the steady-state or asymptotic variance, Var (.), which measures the variability of a variable around its long-run

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[5] The average or equilibrium value of a variable x is defined as the mathematical expectation of x at time t, the expectation being held at the end of t - 1. Thus, $\tilde{x} \equiv E_{t-1}x_t$.

equilibrium.
Equations (6) and (7) are second-order stochastic difference equations. As a deviation from its average value, for inflation we have

$$\Delta p_t + \beta\, \Delta p_{t-1} + \gamma\, \Delta p_{t-2} = \lambda_1\, \delta_t \tag{12}$$

where $\gamma = h/(1 + h - a_1)$
$\beta = -2\gamma$
$\lambda_1 = \gamma/h$

The stability conditions are [6]

$$\begin{aligned} 1 + \beta + \gamma &> 0 \\ 1 - \gamma &> 0 \\ 1 - \beta + \gamma &> 0 \end{aligned} \tag{13}$$

Provided the values of β and γ satisfy the stability conditions, the asymptotic variance of inflation is finite and given by[7]

$$\mathrm{Var}(\Delta p) = \frac{(1+\gamma)\lambda_1^2}{(1-\gamma)[(1+\gamma)^2 - \beta^2]} \tag{14}$$

Using the Cambridge equation (4), the variance of output growth is given by

$$\mathrm{Var}\ (\Delta y) = \mathrm{Var}\ (\Delta m - \Delta p) = \mathrm{Var}\ (\Delta p) \tag{15}$$

Hence in the Cambridge model the variance of growth is proportional to the variance of inflation.
If there is no supply side effect of inflation ($a_1 = 0$) then the stability conditions are always fulfilled. In the presence of a supply side effect stability is guaranteed if $\gamma < 1$ i.e. if $a_1 < 1$. Hence it would require a very large supply side effect from inflation for the model to become unstable.
We can expand the first derivative of Var (Δp) with respect to a_1 as

$$\frac{\partial\, \mathrm{Var}(\Delta p)}{\partial a_1} = \frac{\partial \gamma}{\partial a_1} \cdot \frac{\partial\, \mathrm{Var}(\Delta p)}{\partial \gamma} \tag{16}$$

Since the first term on the right hand side of (16) is always positive[8] the sign of the

6 See for instance Gandolfo (1983), p. 59.

7 See Bartlett (1962), p. 145 and Appendix A to this paper.

8 $\frac{\partial \gamma}{\partial a_1} = \frac{h}{[1 + h - a_1]^2} > 0$

derivative depends upon the sign of the second term. Taking the first derivative of (14) with respect to γ we find that it is given by

$$\frac{\partial \operatorname{Var}(\Delta p)}{\partial \gamma} = \frac{-\gamma/8(\gamma-1)(\gamma^2+3/4\gamma+1/4)}{[3\gamma^3-5\gamma^2+\gamma+1]^2 h^2} > 0 \tag{17}$$

From (15) and (17) we derive proposition (4.3)

PROPOSITION 4.3: The higher a_1 the higher the asymptotic variance of inflation.

To give a numerical example, with h being about 0.5 and a_1 equal to 0.2 we get Var (Δp) = Var (Δy) = 1[9] If there is no supply side effect ($a_1 = 0$), then Var (Δp) = Var (Δy) = 0.67. Thus the ratio of the variance with the supply side effect to the ratio without the supply side effect is 1.5. This example suggests that, in the Cambridge model with adaptive expectations, the supply side effect of inflation adds considerably to the magnitude of macroeconomic fluctuations. This completes the description of the results with adaptive expectations. We now move on to the case of rational expectations.

4.2 Rational Expectations

We now consider the case of rational expectations. Using the backward-shift (lag) operator, L, on equation (2) yields

$$y_{t-1} = \bar{y}_{t-1} + h\,(\Delta p_{t-1} - L\, E_{t-1}\Delta p_t) \tag{18}$$

Noting that in the case of rational expectations the lag operator operates on the time subscript of a variable (not on the time at which the expectation of that variable is held)[10]
we get L $E_{t-1}\Delta p_t = \Delta p_{t-1}$, using this fact, subtracting (18) from (2) and substituting for $\Delta\bar{y}_t$ from (1) yields

$$\begin{aligned} \Delta y_t &= \Delta\bar{y}_t + h\,(\Delta p_t - E_{t-1}\Delta p_t) \\ &= a_0 + (h - a_1)\Delta p_t - h\, E_{t-1}\Delta p_t + \delta_t \end{aligned} \tag{19}$$

Combining (19) with equation (4) yields

$$\Delta p_t = \frac{\Delta m_t - a_0}{1 - a_1 + h} + \frac{h}{1 - a_1 + h} E_{t-1}\Delta p_t - \frac{\delta_t}{1 - a_1 + h} \tag{20}$$

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9 For a_1 we have used the estimate in Smyth (1994). The value of h is taken from Smyth (1992).

10 For a lucid exposition on the use of operator algebra in rational expectations models see Blanchard and Fischer (1989), pp. 264-266.

Taking expectations conditional on information at the end of t - 1 of (20) we get

$$E_{t-1}\Delta p_t = \frac{E_{t-1}\Delta m_t - a_0}{1 - a_1} \tag{21}$$

Assuming that the rule for the evolution of the money supply is known, i.e. $E_{t-1}\Delta m_t = \Delta m_t$, substituting (21) in (20) we obtain the following closed form solutions for inflation

$$\Delta p_t = \frac{\Delta m_t - a_0}{1 - a_1} - \frac{1}{1 - a_1 + h}\delta_t \tag{22}$$

and output growth

$$\Delta y_t = \frac{1}{1 - a_1}[-a_1\Delta m_t + a_0] + \frac{1}{1 - a_1 + h}\delta_t \tag{23}$$

The steady state results on average inflation and output growth are the same as with adaptive expectations.
Now,

$$\beta = 0$$

$$\gamma = 0$$

$$\lambda_1^2 = \frac{1}{(1 + h - a_1)^2}$$

Setting $\beta = \gamma = 0$ in equation (14) yields

$$\text{Var}(\Delta p) = \lambda_1{}^2 = \frac{1}{[1 + h - a_1]^2} \tag{24}$$

Taking the first derivative of (24) with respect to a_1 we find that it is given by

$$\frac{\partial \text{Var}(\Delta p)}{\partial a_1} = \frac{2}{[1 + h - a_1]^3} \qquad \text{for } a_1 < 1 + h \tag{25}$$

From (25) we derive proposition (4.4).

PROPOSITION 4.4: If $a_1 < 1 + h$, the higher a_1 the higher the variances of growth and inflation.

To give a numerical example, substituting the same parameter values ($h = 0.5$ and $a_1 = 0.2$) into eq. (24) yields Var (Δp) = Var (Δy) = 0.59. If there is no supply side effect ($a_1 = 0$), then Var (Δp) = Var (Δy) = 0.44. Thus the ratio of the variance with the ratio of the variance with the supply side effect to the ratio without the supply side effect is about 1.3.

Like in the case of adaptive expectations, this example suggest that the supply side effect of inflation adds considerably to the magnitude of macroeconomic fluctuations. This completes the description of the results of the Cambridge model. We conclude that both with rational and adaptive expectations, under plausible assumptions the supply side effect of inflation increases the steady state inflation rate, lowers the long-run growth rate and increases the extent of cyclical fluctuations, measured by the (asymptotic) variances of growth and inflation.
In the next section we analyze the supply side effect of inflation in the context of a single-stage Phillips curve monetary policy game.

V A STATIC GAME BETWEEN WAGE-SETTERS AND THE CENTRAL BANK

5.1 The Social Welfare versus the Political Approach to Central Bank Behaviour

In section 4 we assumed that inflation was determined by a Cambridge equation. In this section following Barro and Gordon (1983) we think of the central bank as choosing inflation directly.[11] In order to investigate optimal monetary policy consider a central bank that is concerned with both price stability and low unemployment. We assume a quadratic loss function, that penalizes both inflation and unemployment. More specificly we use

$$L_t = \frac{1}{2}(\Delta p_t - \Delta p^*)^2 + \frac{d_2}{2}(\Delta y_t - k\,\Delta\bar{y}_t)^2 \qquad k > 1,\ d_2 > 0 \quad (26)$$

where Δp^* and $k\Delta\bar{y}_t$ are the inflation and growth targets of the central bank. The parameter d_2 measures the weight of output growth stabilization relative to inflation stabilization in the preferences of the central bank. Thus the target level of output growth exceeds the natural rate. The assumption that $k > 1$ is standard. (Canzoneri (1985)). It means that the central bank views the natural rate of growth $\Delta\bar{y}_t$ to be too small. Therefore it aims at a higher output goal. Given the incentive of the central bank to deviate from the natural rate there is a conflict between wage-setters and the central bank concerning the optimal rates of inflation and growth. We will return to this point later on.

The central bank chooses Δp_t and wage-setters "choose" $E_{t-1}\Delta p_t$. Normalizing Δp^* at zero yields

$$L_t = \frac{1}{2}(\Delta p_t)^2 + \frac{d_2}{2}(\Delta y_t - k\,\Delta\bar{y}_t)^2 \quad d_2 > 0 \qquad (27)$$

According to Cukierman (1992, p. 43) the recent literature on monetary policy games has given two competing interpretations to the loss function of the monetary policymaker in equation (27). One part of the literature regards this function as a social welfare function and the central bank as a benevolent social planner (Kydland and

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11 For a lucid exposition of monetary policy games with the money stock as policy instrument see Canzoneri and Henderson (1991).

Prescott (1977), Barro and Gordon (1983), Rogoff (1985) and Canzoneri (1985)). The other part views the central bank as a mediator between different interest groups that try to push monetary policy in various directions. On this view, the loss function (3.1) reflects a distributionally motivated political compromise mediated through the central bank between the advocates of employment stimulation and the advocates of price stability (Havrilesky (1987), Willet (1988) and Mayer (1990)). The coefficient d_2 then measures the relative political clout of the two groups (Cukierman and Meltzer (1986a), Cukierman (1986)).

The social welfare approach seems best suited to describe how a central bank *should* behave. However, (Cukierman (1992), p. 43) points out that it is a relatively weak paradigm for explaining the *actual* policies chosen by central banks. For as shown by Bade and Parkin (1988), Alesina (1988, 1989), Grilli, Masciandaro and Tabellini (1991), Alesina and Summers (1993) and Eijffinger and Schaling (1993) in most countries central banks are highly dependent on the government in general and the treasury or ministry of finance in particular. As a result, the policies implemented by central bankers are not independent from the general political proces in which distributional considerations are predominant. The impact of these considerations on the choice of policy varies with the degree of central bank independence. The greater the political independence given to the bank by law the smaller the impact of distributional and other political considerations on monetary policy.

More realisticly, we choose the political approach to central bank behaviour as the interpretation of equation (27). Hence we view the coefficient d_2 as a measure of the political dependence of the central bank. The lower d_2 the more independent the central bank[12]. That is, the degree of central bank independence equals d_2^{-1}

Following Alogoskoufis ((1993), p. 17) we start with a cooperative game, i.e. we assume that the natural rate of growth is efficient. In the context of the model, this can be represented by the assumption that $k = 1$. In the latter case the central bank has no incentive to try and increase the rate of growth above its natural rate. As there is no conflict between the output growth targets of wage setters and the central bank, the policy game can be seen as a cooperative one[13]. This state of affairs is summarized in the first column of table 1.

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12 An alternative approach is explicit modelling of the interaction of separate monetary and fiscal authorities. See e.g. Alesina and Tabellini (1987) and Cukierman (1992), pp. 351-355.

13 For a review of some important concepts of game theory see Blackburn and Christensen (1989).

Table 1

Policy Games and Output Growth Targets

Wage-Setters	Central Bank	
	$k = 1$ $\Delta y^* = \Delta\tilde{y}$	$k > 1$ $\Delta y^* > \Delta\tilde{y}$
Natural Rate	Efficient	Inefficient
Policy Game	Cooperative	Non-Cooperative

We now turn to the second column of table 1, i.e. the situation where the central bank views the natural rate of growth $\Delta\tilde{y}_t$ to be too small. In what follows we show that then the equilibrium inflation rate becomes proportional to the average rate of growth (a_0). In this case discretionary monetary policy is no longer a Pareto-equilibrium, due to the time-inconsistency of optimal monetary policy.

5.2 Optimal Monetary Policy Under Rational Expectations

Assuming that $k > 1$ the central bank has incentives to systematically create inflation in order to raise the rate of output growth above its natural rate. From the first order

conditions for a minimum of (27) i.e. $\frac{\partial L_t}{\partial \Delta p_t} = 0$ we get

$$\Delta p_t = -d_2(\Delta y_t - k\,\Delta\tilde{y}_t)\frac{\partial(\Delta y_t - k\,\Delta\tilde{y}_t)}{\partial \Delta p_t} \tag{28}$$

Substituting the equations for the growth rate of the natural level of output (1), and the "Phillips curve" (19) into the first-order condition we obtain the central bank's reaction function to private sector's expectations

$$\Delta p_t = \frac{-d_2\hat{h}}{1 + d_2\hat{h}^2}\,[(1-k)(a_0 + \delta_t) - h\,E_{t-1}\Delta p_t] \tag{29}$$

where $\hat{h} \equiv h - (1-k)a_1 > 0$. Note that if there is no supply side effect of inflation ($a_1 = 0$), then $\hat{h} = h$.

(29) gives the rate of inflation desired by the central bank under discretion.

Again it is necessary to specify how inflation expectations are formed. The Barro and Gordon (1983) model features consistent or rational expectations. Therefore first we analyze the rational expectations case. In section 5.2 we modify this assumption and turn to adaptive expectations.

Taking expectations conditional on information at t - 1 of (29) gives

$$E_{t-1}\Delta p_t = \frac{-d_2\hat{h}}{1 + d_2\hat{h}(\hat{h} - h)}\,[(1-k)a_0] \tag{30}$$

Equation (30) is the reaction function of wage setters. Upon substituting (30) in (29) we get

$$\Delta p_t = \frac{-d_2\hat{h}(1-k)}{1 + d_2\hat{h}(\hat{h}-h)} a_0 - \frac{d_2\hat{h}(1-k)}{1 + d_2\hat{h}^2} \delta_t \tag{31}$$

Figure 1 shows the central bank's reaction function for the non-stochastic Barro and Gordon (1983) case ($a_1 = \delta_t = 0$), i.e. the average inflation rate as a function of the expected inflation rate.[14]

[INSERT FIGURE 1]

The only point at which expectations are rational is at point N, which represents the non-cooperative Nash equilibrium. Note that the greater h (i.e. the greater the increase in output from unanticipated inflation) and the larger the distortion k - 1, the greater the inflation benefits of appointing independent central banks.

We now consider the stochastic case with the supply side effect of inflation, i.e. equation (31). Again the average value of inflation can be obtained by putting $\delta_t = 0$, this yields

$$\Delta \bar{p}_t = \frac{-d_2\hat{h}(1-k)a_0}{1 + d_2\hat{h}[-(1-k)a_1]} \tag{32}$$

Hence, average inflation is above zero (the outcome in the cooperative case where k = 1).
Subtracting (30) from (31) we obtain the following expression for unanticipated inflation.

$$\Delta p_t - E_{t-1}\Delta p_t = \frac{-d_2\hat{h}(1-k)}{1 + d_2\hat{h}^2} \delta_t \tag{33}$$

Combining eq. (33) with (19) and (1) we get

$$\Delta p_t = \frac{1}{-a_1} [\Delta y_t + \frac{d_2\hat{h}[h(1-k)-\hat{h}] - 1}{1 + d_2\hat{h}^2} \delta_t - a_0] \tag{34}$$

Now, we derive the solution for output growth. Upon substituting (34) in (31) we get

$$\Delta y_t = \frac{1}{1 + d_2\hat{h}(\hat{h}-h)} a_0 + \frac{d_2\hat{h}hk + 1}{1 + d_2\hat{h}^2} \delta_t \tag{35}$$

The average value of output growth is

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[14] This figure is based on a similar one in Blanchard and Fischer (1989), p. 597.

$$\Delta\tilde{y}_t = a_0 - a_1 \Delta\tilde{p}_t \tag{9}$$

Equations (32) and (9) highlight the time inconsistency of optimal monetary policy. The monetary policy strategy of the central bank, i.e. equation (29) is time-consistent in the sense that at each point in time the inflation rate selected is best, given the current situation. However, as can be seen from equations (32) and (9), the resulting policy is socially sub-optimal. It is suboptimal since it results in an excessive level of inflation, i.e. it produces an inflationary bias.

Moreover, because of this inflationary bias the average rate of growth is reduced. Due to the presence of supply-side effects, time consistent monetary policy results in a lower steady-state growth rate than in the absence of supply side effects of inflation (a_1 = 0). This is a major difference with the Barro and Gordon (1983) Phillips curve monetary policy game. In the latter model time consistent monetary policy cannot reduce output beneath its natural rate. Hence, in the present model time consistent monetary policy creates a *stagflationary* bias. This state of affairs is summarized in Table 2.

Table 2

Stagflationary Bias of Time Consistent Monetary Policy

Policy Game	Inflation	Growth
(a) Cooperative $k = 1$	0	a_0
(b) Non-Cooperative $k > 1$	$\Delta\tilde{p}$	$a_0 - a_1\Delta\tilde{p}$
Bias = (b) ⁻/- (a)	$\Delta\tilde{p}$	$-a_1\Delta\tilde{p}$

We can expand the first derivative of the average inflation rate i.e. equation (32) with respect to a_1, as

$$\frac{\partial\Delta\tilde{p}_t}{\partial a_1} = \frac{\partial\Delta\tilde{p}_t}{\partial\hat{h}} \cdot \frac{\partial\hat{h}}{\partial a_1} = \frac{(1 - k)^2\, d_2 a_0}{[1 + d_2\hat{h}[-(1 - k)a_1]]^2} > 0 \tag{36}$$

From (36) we derive proposition (5.1)

PROPOSITION 5.1: The higher the supply side effect of inflation - i.e. the higher a_1 - the higher the average rate of inflation.

Differentiating (9) with respect to a_1 we find that

$$\frac{\partial\Delta\tilde{y}_t}{\partial a_1} = -\left[a_1 \frac{\partial\Delta\tilde{p}_t}{\partial a_1} + \Delta\tilde{p}\right] < 0 \tag{37}$$

From (37) we derive proposition (5.2).

PROPOSITION 5.2: The higher a_1 the lower the average rate of output growth.

Thus no matter what parameter values we assume, in the Barro and Gordon (1983) game-theoretic model of monetary policy the supply side effects of inflation raise the equilibrium steady state rate of inflation and lower the equilibrium steady state rate of growth of output.

This is different from the results in the Cambridge model. There it was shown that the supply side effect of inflation increases inflation and reduces the growth rate if the rate of growth of nominal money balances exceeds the average rate of growth of the natural rate of output i.e. if $\Delta m - a_0 > 0$.

Hence, being unconditional, results in the Barro Gordon model are more robust.

Using the notation of equation (12) we get

$$\beta = 0$$

$$\gamma = 0$$

$$\lambda_1 = \frac{-d_2 \hat{h}(1-k)}{1+d_2 \hat{h}}$$

Setting $\beta = \gamma = 0$ in equation (14) yields the variance of inflation

$$\text{Var}(\Delta p_t) = \lambda_1^2 = \frac{d_2^2 \hat{h}^2 (1-k)^2}{[1+d_2 \hat{h}^2]^2} \tag{38}$$

We can expand the first derivative of Var (Δp_t) with respect to a_1 as

$$\frac{\partial \text{ Var}(\Delta p_t)}{\partial a_1} = \frac{\partial \text{ Var}(\Delta p_t)}{\partial \lambda_1} \frac{\partial \hat{h}}{\partial a_1} \frac{\partial \lambda_1}{\partial \hat{h}} \tag{39}$$

Since the first two terms on the right hand side of (39) ar always positive[15], the sign of the derivative depends upon the sign of the last term.

Taking the first derivative of λ_1 with respect to $\hat{h}$ we find that it is given by

$$\frac{\partial \lambda_1}{\partial \hat{h}} = \frac{-d_2(1-k)[1-d_2\hat{h}^2]}{[1+d_2\hat{h}^2]^2} > 0 \qquad \text{for } d_2 < \frac{1}{\hat{h}^2} \tag{40}$$

From (40) we derive proposition 5.3.

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15 $\frac{\partial \text{ Var}(\Delta p_t)}{\partial \lambda_1} = 2\lambda_1 > 0, \quad \frac{\partial \hat{h}}{\partial a_1} = -(1-k) > 0$

PROPOSITION 5.3: If $d_2 < \frac{1}{\hat{h}^2}$, the higher a_1 the higher the variance of inflation.

To give a numerical example, assuming that $k = 2$ and the other parameters as before ($h = 0.5$ and $a_1 = 0.2$) the weight of ouput growth stabilization relative to inflation stabilization in the preferences of the central bank must be smaller than 2.
Identifying a fully independent central band with a value for d_2 of zero we can state the following. If the central bank is not too dependent, with rational expecations the relation between inflation and growth (a_1) will accentuate the variability of inflation.

Now we will analyze the consequences of the supply side effect of inflation on output growth. As a deviation from its average value, the equation for output growth (35) may be written as

$$\Delta y_t = \lambda_1 \, \delta_t \tag{41}$$

where $\lambda_1 \equiv \frac{d_2 h\hat{h}k + 1}{1 + d_2 \hat{h}}$

Setting $\beta = \gamma = 0$ in equation (14) yields

$$\text{Var}\ (\Delta y_t) = \lambda_1^2 = \frac{[d_2 h\hat{h}k + 1]^2}{[1 + d_2 \hat{h}^2]^2} \tag{42}$$

We can expand the first derivative of Var (Δy_t) with respect to a_1 as

$$\frac{\partial \text{Var}(\Delta y_t)}{\partial a_1} = \frac{\partial \hat{h}}{\partial a_1} \, \frac{\partial \text{Var}(\Delta y_t)}{\partial \lambda_1} \, \frac{\partial \lambda_1}{\partial \hat{h}} \tag{43}$$

Since the first two terms on the right handside of (43) are always positive, the sign of the derivative depends upon the sign of the last term.
Taking the first derivative of λ_1 with respect to $\hat{h}$ we find that it is given by

$$\frac{\partial \lambda_1}{\partial \hat{h}} = \frac{d_2[hk - d_2 h\hat{h}^2 k - 2\hat{h}]}{[1 + d_2 \hat{h}^2]^2} > 0 \qquad \text{for } d_2 < \left[\frac{hk - 2\hat{h}}{hk}\right] \frac{1}{\hat{h}^2} \tag{44}$$

From (44) we derive proposition (5.4).

PROPOSITION 5.4: If $d_2 < \left[\frac{hk - 2\hat{h}}{hk}\right] \frac{1}{\hat{h}^2}$, the higher a_1 the higher the variance of ouput growth.

To give a numerical example, with the same parameter values as before the weight of output growth stabilization relative to inflation stabilization in the preferences of the central bank must be smaller that -0.8. Since ex hypothesi $d_2 > 0$ this numerical example suggests that with rational expectations the supply side effect of inflation may have a stabilizing effect on the real side of the economy. This completes the description of the results of the Phillips curve monetary policy game with rational expectations. We now move on to the case of adaptive expectations.

5.3 *Optimal Monetary Policy Under Adaptive Expectations*

Originally the Barro and Gordon (1983) model features consistent or rational expectations. In order to maintain symmetry with section 4 we now modify this assumption. In this sub-section we assume that expectations are formed adaptively.

Taking first differences of equation (2) and substituting $E_{t-1}\Delta p_t = \Delta p_{t-1}$ yields

$$\Delta y_t = \Delta y_t + h\Delta p_t - 2h\Delta p_{t-1} + h\Delta p_{t-2} \tag{45}$$

Substituting equations (1) and (45) into (28) we obtain a closed form solution for inflation under adaptive expectations

$$\Delta p_t = -\frac{d_2\hat{h}}{1 + d_2\hat{h}^2}\,[(1-k)(a_0 + \delta_t)\ [-2h]\Delta p_{t-1} + h\Delta p_{t-2}] \tag{46}$$

Combining equations (1) and (45) we get

$$\Delta p_t = \frac{1}{[h - a_1 - 2hL + hL^2]}\,[\Delta y_t - (a_0 + \delta_t)] \tag{47}$$

where L is the backward-shift (lag) operator.

Now, we derive the solution for ouput growth. Upon substituting (47) in (46) we get

$$\Delta y_t = \frac{1}{1 + d_2\hat{h}^2}a_0 - \frac{d_2\hat{h}[-2h]}{1 + d_2\hat{h}^2}\Delta y_{t-1} - \frac{d_2\hat{h}h}{1 + d_2\hat{h}^2}\Delta y_{t-2} + \frac{1 + d_2\hat{h}hk}{1 + d_2\hat{h}^2}\delta_t$$
$$\frac{-2d_2\hat{h}hk}{1 + d_2\hat{h}^2}\delta_{t-1} + \frac{d_2\hat{h}hk}{1 + d_2\hat{h}^2}\delta_{t-2} \tag{48}$$

The steady state results on average inflation and output growth are the same as with rational expectations. Using the notation of equation (12), for inflation we have

$$\gamma = \frac{d_2\hat{h}h}{1 + d_2\hat{h}^2}$$
$$\beta = -2\gamma$$
$$\lambda_1 = \frac{-(1-k)}{h}\gamma$$

If there is no supply side effect of inflation ($a_1 = 0$) then the stability conditions are always fulfilled. In the presence of a supply side effect stability is guaranteed if $\gamma < 1$ i.e. $d_2 > \frac{-1}{\hat{h}(\hat{h} - h)}$. Since ex hypothesi $d_2 > 0$ with the supply side effect of inflation

stability is also guaranteed.
Hence, the asymptotic variance of Δp_t is finite and given by

$$\mathrm{Var}(\Delta p) = \frac{(1+\gamma)\lambda_1^2}{(1-\gamma)[(1+\gamma)^2-\beta^2]} \tag{14}$$

We can expand the first derivative of Var (Δp) with respect to a_1 as

$$\frac{\partial\ \mathrm{Var}(\Delta p)}{\partial a_1} = \frac{\partial \hat{h}}{\partial a_1} \cdot \frac{\partial\, \mathrm{Var}(\Delta p)}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial \hat{h}} \tag{49}$$

The first term on the right hand side of (49) is always positive[16] Taking the first derivative of (14) with respect γ we find that it is given by

$$\frac{\partial\ \mathrm{Var}(\Delta p)}{\partial \gamma} = \frac{-\gamma/8(\gamma-1)(1-k)^2\ (\gamma^2+3/4\gamma+1/4)}{[3\gamma^3-5\gamma^2+\gamma+1]^2\ h^2} > 0^{17} \tag{50}$$

Hence, the sign of $\frac{\partial\ \mathrm{Var}(\Delta p)}{\partial a_1}$ depends upon the sign of $\frac{\partial \gamma}{\partial \hat{h}}$. Taking the first derivative of γ with respect to $\hat{h}$, we find that it is given by

$$\frac{\partial \gamma}{\partial \hat{h}} = \frac{d_2 h[1-d_2\hat{h}^2]}{[1+d_2\hat{h}^2]^2} > 0 \qquad \text{for } d_2 < \frac{1}{\hat{h}^2} \tag{51}$$

From (51) we derive proposition (5.5)

PROPOSITION 5.5: If $d_2 < \frac{1}{\hat{h}^2}$, the higher a_1 the higher the asymptotic variance of inflation.

This is the same condition as with rational expectations (see proposition 5.3). Thus, both with rational and adaptive expectations if the central bank is not too dependent the relation between inflation and growth (a_1) will accentuate the variability of inflation.

Now we will analyze the consequences of the supply side effect of inflation on output growth.

The variance of output growth in terms of the parameters β, γ, λ_1, λ_2 and λ_3 may be obtained from (48)[18] it is

$$\mathrm{Var}(\Delta y) = \frac{[(1+\gamma)(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 2\gamma\lambda_1\lambda_3) - 2\beta\lambda_1\lambda_2]}{(1-\gamma)[(1+\gamma)^2 - \beta^2]} \tag{52}$$

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16 $\frac{\partial \hat{h}}{\partial a_1} = -(1-k) > 0$

17 It can be shown that with adaptive expectations the asymptotic variance in the Phillips curve monetary policy game equals $(1 - k)^2$ the asymptotic variance of inflation in the Cambridge model. Hence (50) is equal to $(1 - k)^2$. (17).

18 See Appendix A.

where

$$\lambda_1 \equiv \Lambda \qquad \Lambda \equiv \frac{1 + d_2\hat{h}hk}{d_2\hat{h}h}$$

$$\lambda_2 \equiv \frac{-2d_2\hat{h}hk}{1 + d_2\hat{h}^2}$$

$$\lambda_3 \equiv -\frac{1}{2}\lambda_2$$

We can expand the first derivative of Var (Δy) with respect to a_1 as

$$\frac{\partial \ \mathrm{Var}(\Delta y)}{\partial a_1} = \frac{\partial \hat{h}}{\partial a_1} \cdot \frac{\partial \gamma}{\partial \hat{h}} \cdot \frac{\partial \ \mathrm{Var}(\Delta y)}{\partial \gamma} \tag{53}$$

Since $\frac{\partial \hat{h}}{\partial a_1}$ is always positive, from (51) we know that the first two terms on the right hand side of (53) are positive if $d_2 < \frac{1}{\hat{h}^2}$.

However, the relevant derivative also depends on the sign of the last term. Determining the sign of the first derivative of (52) with respect to γ involves the solution fo a fifth-order equation.[19] Therefore we turn to implicit differentiation. Rewriting (52) in (equivalent) implicit form, totally differentiating this expression and approximating the last term on the right hand side of (53) by dVar $(\Delta y)/d\gamma$, we get

$$\frac{\partial \ \mathrm{Var}(\Delta y)}{\partial \gamma} = \frac{k[8k - \Lambda^2]}{4k - \Lambda^2} \tag{54}$$

[20]

where the reduced form parameter $\Lambda \equiv \frac{1 + d_2\hat{h}hk}{d_2\hat{h}h}$. (see equation (52)). From (51) and (54) we derive proposition (5.6).

PROPOSITION 5.6: If $d_2 < \frac{1}{\hat{h}^2}$ and $\frac{k[8k - \Lambda^2]}{4k - \Lambda^2} > 0$, the higher a_1 the higher the asymptotic variance of output growth.

In appendix B we show that for (54) to be positive the ratio between the transient and the stochastic response effects (Howrey (1967), p. 410) of the linear system (48) should be greater than a certain boundary value.

With the same parameter values as before both (51) and (54) are positive suggesting that in the adaptive expectations version of the monetary policy game the relation between inflation and growth (a_1) will accentuate the variability of growth.

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19 Note that with adaptive expectations in order to determine the sign of ∂ Var $(\Delta p)/\partial\gamma$ we had to solve third-order equations. See expressions (50) and (17).

20 For the derivation see Appendix B to this paper.

VI CONCLUSIONS

In the present paper we have analyzed the supply side effects of inflation for two major monetary models. We find that the steady state inflation rate is increased and the steady state output growth rate is reduced. These results hold both for the Cambridge model, and for the Barro and Gordon (1983) game-theoretic model. A further finding is the following. Due to the inflation sensitive growth rate in the natural level of output, in the game-theoretic model of monetary policy time consistent inflation policy causes a *stagflationary bias*. This is a major difference with the standard Barro and Gordon (1983) framework were monetary policy cannot reduce output beneath its natural rate.

The extent of cyclical fluctuations of ouput growth, measured by the asymptotic variances, is increased in the Cambridge model and in the Barro and Gordon (1983) model of monetary policy with adaptive expectations.

Finally, inflation variability, measured by the asymptotic variances of inflation is increased in both the Phillips' curve and the Barro and Gordon (1983) model. These results hold irrespective of whether inflation expectations are determined adaptively or rationally.

APPENDIX A DERIVATION OF THE ASYMPTOTIC VARIANCE OF OUTPUT GROWTH FROM EQUATION (48).

In this appendix we derive the asymptotic variance of output growth from

$$\Delta y_t + \beta \Delta y_{t-1} + \gamma \Delta y_{t-2} = \lambda_1 \delta_t + \lambda_2 \delta_{t-1} + \lambda_3 \delta_{t-2} \tag{A1}$$

The method by which the variance of output is obtained is that of Fischer and Cooper (1973, p. 874).

Multiply (A1) by Δy_t, Δy_{t-1}, Δy_{t-2}, δ_t, δ_{t-1} and δ_{t-2}, successively, and take expectations: denote by σ_0, σ_1 and σ_2 the (asymptotic) variance of output growth $E(\Delta y_t^2)$, the first autocovariance $E(\Delta y_t \Delta y_{t-1})$ and the second autocovariance $E(\Delta y_t \Delta y_{t-2})$ respectively.

Note that the autocovariance function is stationary so that

$$E(\Delta y_t \Delta y_{t-1}) = E(\Delta y_{t+1} \Delta y_t)$$

We get

$$\sigma_0 + \beta\sigma_1 + \gamma\sigma_2 = \lambda_1 E(\Delta y_t \delta_t) + \lambda_2 E(\delta_{t-1} \Delta y_t) + \lambda_3 E(\delta_{t-2} \Delta y_t) \tag{A2}$$

$$\beta\sigma_0 + (1 + \gamma)\sigma_1 = \lambda_2 E(\Delta y_{t-1} \delta_{t-1}) \tag{A3}$$

$$y\sigma_0 + \beta\sigma_1 + \sigma_2 = \lambda_3 E(\Delta y_{t-2} \delta_{t-2}) \tag{A4}$$

$$E(\delta_t \Delta y_t) = \lambda_1 \tag{A5}$$

$$E(\delta_{t-1} \Delta y_t) = \lambda_2 - \beta E(\delta_{t-1} \Delta y_{t-1}) \tag{A6}$$

$$E(\delta_{t-2} \Delta y_t) = \lambda_3 - \gamma\, E(\delta_{t-2} \Delta y_{t-2}) \tag{A7}$$

Solving this set of equations ((A2) - (A7)) for σ_0 - noting that $E(\delta_t \Delta y_t) = E(\delta_{t-1} \Delta y_{t-1}) = E(\delta_{t-2} \Delta y_{t-2})$ - one obtains equation (52) of the text. Note that equations (14), (24) and (42) are special cases of eq. (52). For the second order case equation (14) follows from (52) if $\lambda_2 = \lambda_3 = 0$. In the zeroth order case, equations (42) and (24) result if $\lambda_2 = \lambda_3 = \beta = \gamma = 0$.

APPENDIX B THE DERIVATION OF EQUATION (54).

In this appendix we turn to implicit differentiation of equation (52) in order to determine the sign of $\partial \mathrm{Var}\,(\Delta y)/\partial \gamma$.
Following Chiang (1984), pp. 204-209 we write (52) in the equivalent form

$$\sigma_0 - \frac{[(1+\gamma)(\lambda_1^2+\lambda_2^2+\lambda_3^2-2\gamma\lambda_1\lambda_3)-2\beta\lambda_1\lambda_2]}{(1-\gamma)[(1+\gamma)^2-\beta^2]} = 0 \tag{B.1}$$

where $\sigma_0 \equiv \mathrm{Var}(\Delta\ y)$
Now we no longer have an explicit function for Var (Δy). Rather the function (52) is only implicity defined by equation (B.1).
In general (B.1) can be denoted by

$$F(\sigma_0, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3) = 0 \tag{B.2}$$

Totally differentiating (B.2) yields

$$F_{\sigma_0} d\sigma_0 + F_\beta d\beta + F_\gamma d\gamma + F_{\lambda_1} d\lambda_1 + F_{\lambda_2} d\lambda_2 + F_{\lambda_3} d\lambda_3 = 0 \tag{B.3}$$

where F_i denotes the partial derivative of the function F with respect to argument i. Upon dividing through by $d\gamma$, noting that $F_{\sigma_0} = 1$ and solving for $\frac{d\sigma_0}{d\gamma}$ we get

$$\frac{d\sigma_0}{d\gamma} = -\frac{d\beta}{d\gamma} F_\beta - F_\gamma - \frac{d\lambda_1}{d\gamma} F_{\lambda_1} - \frac{d\lambda_2}{d\gamma} F_{\lambda_2} - \frac{d\lambda_3}{d\gamma} F_{\lambda_3} \tag{B.4}$$

From the definitions of β_1, λ_1, λ_2 and λ_3 it follows that

$$\frac{d\beta}{d\gamma} = -2, \quad \frac{d\lambda_1}{d\gamma} = \Lambda, \quad \frac{d\lambda_2}{d\gamma} = -2k \quad \text{and} \quad \frac{d\lambda_3}{d\gamma} = k$$

Substituting the latter results in (B.4) yields

$$\frac{d\sigma_0}{d\gamma} = 2F_\beta - F_\gamma - \Lambda F_{\lambda_1} + 2kF_{\lambda_2} - kF_{\lambda_3} \tag{B.5}$$

Noting that

$$F_\gamma = \frac{dF}{d\beta}\frac{d\beta}{d\gamma} = -2F_\beta \quad \text{and} \quad F_{\lambda_1} = \frac{\Lambda}{k} F_\beta \text{ we get}$$

$$\frac{d\sigma_0}{d\gamma} = F_\beta - \frac{k^2(F_{\lambda_3} - 2F_{\lambda_2})}{4k - \Lambda^2} \tag{B.6}$$

By definition

$$F_{\lambda_3} = \frac{dF}{d\lambda_3} = \frac{dF}{d\lambda_2}\frac{d\lambda_2}{d\lambda_3} = -2F_{\lambda_2} \text{ thus}$$

$$\frac{d\sigma_0}{d\gamma} = F_\beta + \frac{4k^2 F_{\lambda_2}}{4k - \Lambda^2} \tag{B.7}$$

From (B.7) it follows that

$$\frac{d\sigma_0}{d\gamma} > 0 \text{ for } \frac{F_\beta}{F_{\lambda_2}} > \frac{-4k^2}{4k - \Lambda^2} \tag{B.8}$$

Where broadly speaking - following Howrey (1967), p. 410 - F_{β}, - F_{λ_2} may be interpreted as respectively the transient and the stochastic response "effects" of the linear system (48). Thus a change in γ increases the asymptotic variance if the ratio between the transient and the stochastic response effect passes a lower bound defined by the strucutural parameters d_2, h, k and a_1. Noting that

$$\frac{F_\beta}{F_{\lambda_2}} = \frac{dF}{d\beta}\frac{d\lambda_2}{dF} = \frac{d\lambda_2}{d\beta} = \frac{d\lambda_2}{d\gamma}\frac{d\gamma}{d\beta} = -2k \,.\, -\frac{1}{2} = k,$$

i.e. that the latter ratio is equal to k, we get

$$\frac{d\sigma_0}{d\gamma} = \frac{k[8k - \Lambda^2]}{4k - \Lambda^2} \tag{B.9}$$

Approximating $\partial \text{Var}(\Delta y)/\partial\gamma$ by $\frac{d\sigma_0}{d\gamma}$ yields equation (54).

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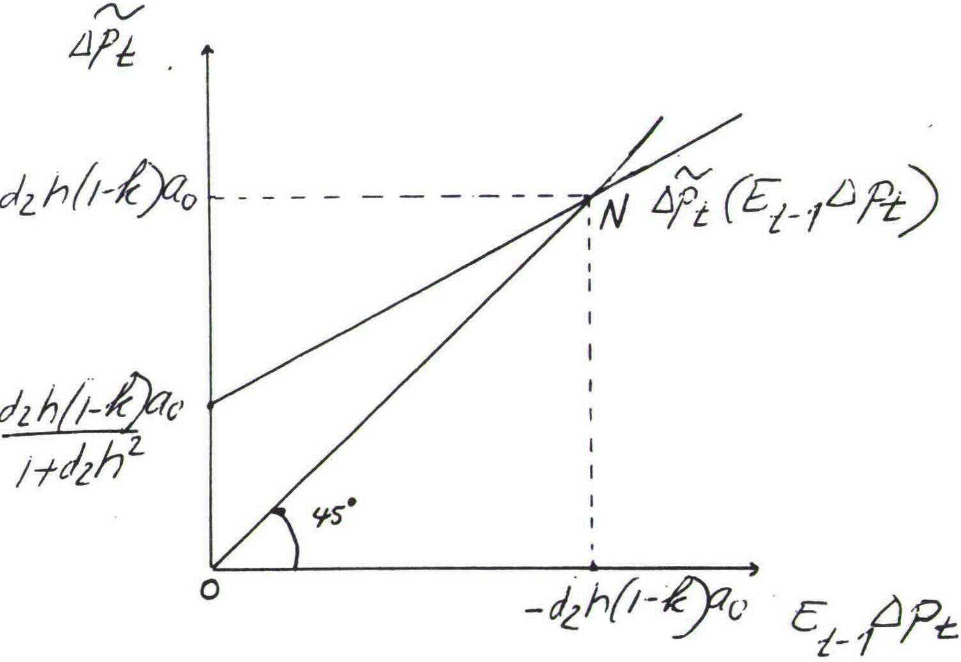
Figure 1

Actual and Expected Inflation with an Inefficient Natural Rate



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