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Games with Permission Structures: The Conjunctive Approach*

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Abstract

This paper is devoted to the game theoretic analysis of decision situations, in which the players have veto power over the actions undertaken by certain other players. We give a full characterization of the dividends in these *games with a permission structure*. We find that the collection of these games forms a subspace of the vector space of all games with side payments on a specified player set.

Two applications of these results are provided. The first one deals with the projection of additive games on a permission structure. It is shown that the Shapley value of these projected games can be interpreted as an index that measures the power of the agents in the permission structure. The second application applies the derived results on games, where the organization structure can be analysed separately from the production capacities of the participating players.

1 Introduction

Recently, some authors have addressed the game theoretic analysis of (economic) decision processes in which one imposes asymmetric constraints on the behaviour of the decision takers. Several studies have enriched the game theoretic analysis of the consequences of adopting this type of constraints on economic behaviour. We mention the theory of cooperative games with arbitrary communication structures as described in e.g. Myerson (1977 and 1980), Owen (1986), Aumann and Myerson (1988) and Borm et al. (1990).

In this paper we introduce another type of asymmetry between players in a cooperative game with side payments. We describe an organization in which each player has veto power over the activities as performed by a specified collection of players. So, all players in the game are dominating a – possibly empty – collection of other players in the sense that they have veto power over the actions undertaken by these players.

To illustrate this type of asymmetry between players we discuss an example. Remind that a *cooperative game with transferable utilities*, or simply a *TU-game*, is a pair (N, v), where $N = \{1, \ldots, n\}$ is a finite set of players and $v: 2^N \to \mathbf{R}$ is a characteristic function, which assigns to every coalition $E \subset N$ an achievable payoff $v(E)$ such that $v(\emptyset) = 0$.

We consider the interaction between a potential seller and two potential buyers of some object by the use of a TU-game. The seller values the object at ten dollars, the first buyer values it at twenty dollars, and the second buyer values it at thirty dollars. Following Roth (1988) this situation can be modelled as a TU-game (N, v) with $N = \{1, 2, 3\}$ and v given by $v(\emptyset) = 0$, $v(1) = 10$, $v(2) = v(3) = 0$, $v(12) = 20$, $v(13) = 30$, $v(23) = 0$, and $v(N) = 30$. Applying the Shapley value, developed in Shapley (1953), as the appropriate standard in dividing these potential payoffs we derive that $\varphi_1(v) = 21\frac{2}{3}$, $\varphi_2(v) = 1\frac{2}{3}$, and $\varphi_3(v) = 6\frac{2}{3}$.

Next we introduce the additional information that the seller, player 1, only has the right to use the object, but that the property rights are in the hands of the first buyer, player 2. This implies that the seller has to get permission from the first buyer with respect to the sale of the object.* Instead of the game (N, v) as described above, we have to describe the new situation with the use of a modified game (N, w), where w is given by $w(\emptyset) = w(1) = w(2) = w(3) = 0$, $w(12) = 20$, $w(13) = w(23) = 0$, and $w(N) = 30$. In this modification we take account of the fact that player 2 has to be member of any payoff generating coalition. Again applying the Shapley value as the appropriate standard in dividing the payoffs gives us $\varphi_1(w) = \varphi_2(w) = 13\frac{1}{3}$ and $\varphi_3(w) = 3\frac{1}{3}$.

The example above describes the consequences of the separation between property rights and user rights. It is our purpose to separate the (potential) individual abilities as described by the game from the behaviouristic rules or the organization structure such as the separation of property rights from user rights. From the example we conclude that constraints imposed by an organization structure may influence payoffs considerably. This is the topic of this paper as well as the work by van den Brink and Gilles (1991) and Gilles and Owen (1991).

We refer to the interpretation of the dominance structure as considered in the example, in which a player has to get permission from *all* her superiors to pursue a certain goal, as the *Conjunctive approach.*† By assumption we exclude the possibility that players mutually have veto power over their actions.

The main part of this paper is devoted to the analysis of cooperative games with side payments in which the players are organized in a permission structure as described above. In our analysis we subsequently introduce such games with

*In other words, this means that player 2 can *veto* the sale of the object.

†Gilles and Owen (1991) analyse the consequences of another interpretation of the dominance structure within a hierarchical organization. In this *Disjunctive approach* it is assumed that every player has to get permission from *at least one* of her superiors.

permission structure and then apply the Conjunctive approach to give a description of the possibilities of the players in such a situation. We then modify the game accordingly. Our main result states that the collection of these modified TU-games is generated by a specific class of unanimity games, namely those on coalitions, which contain precisely all the players who have to give their permission to the actions of its members. These coalitions are called *autonomous* in the permission structure. With the use of this result we can give a description of the dividends of all coalitions in such games with permission structure.

Finally we discuss two applications of games with permission structure. The first application deals with additive games restricted to a permission structure. The Shapley value of such a restricted game can be interpreted as an index describing the (positional or social) power of the players in the permission structure. Our analysis shows that this provides an alternative for the power indices as developed by van den Brink and Gilles (1990). The power indices as described in that paper are based on a heuristic approach to social power in hierarchies, while the power indices resulting from restrictions of additive games to permission structures are essentially based on a game theoretic approach to social power.

The second application deals with an economic production situation, in which the productive players form the lowest level in an organization as described by a hierarchical permission structure. The managers in the higher levels are assumed to be unproductive, but are necessary for the organization of these productive players in productive units. We show that the managers can claim at least the average value of the productive players, whom they dominate.

An axiomatic approach to the Shapley value for games with permission structure is given by van den Brink and Gilles (1991).

2 Games with permission structures

This section is devoted to an exposition and analysis of permission structures on sets of players. Before we are able to introduce the main instrument in the description and analysis of these permission structures, we have to make some notational conventions. Firstly we denote by $\mathbf{N} := \{1, 2, 3, \ldots\}$ the set of all natural numbers. Similarly we denote by $\mathbf{R}$ the set of all real numbers. If X is some finite set, then we denote by $\#X$ its cardinality. By $\mathcal{G}^N$ we denote the collection of all characteristic functions v

on the finite player set N, representing a TU-game (N, v). It is obvious that $\mathcal{G}^N$ is a $(2^n - 1)$-dimensional real vector space, where $n = \#N$.

A formal description of a domination structure on an arbitrary collection of players N is developed in the next definition.

Definition 2.1 *A* **permission structure** *on a finite playerset N is a mapping $S: N \to 2^N$, which is asymmetric, i.e., for every pair $i, j \in N$*

$$j \in S(i) \ \textit{implies that} \ i \notin S(j).$$

The collection of all permission structures on N is denoted as $\mathcal{S}^N$.

We remark that asymmetry of the permission structure S implies that it also satisfies irreflexity, i.e., for every player $i \in N$ it holds that $i \notin S(i)$. The players $j \in S(i)$ are called the *successors* of i. In our setting a player $i \in N$ is assumed to dominate his successors $j \in S(i)$, in which the notion of "domination" will formally be specified in the next section.

For every permission structure $S \in \mathcal{S}^N$ we can define a binary relation $R_S \subset N \times N$ given by

$$R_S := \{(i, j) \mid i \in N \text{ and } j \in S(i)\}.$$

It is clear that R_S is an asymmetric and irreflexive relation on N and describes the dominance relations induced by the permission structure S on N.

Let $S \in \mathcal{S}^N$ be a permission structure and R_S the belonging binary relation. Now we denote by $\text{tr}(R_S)$ the transitive closure of R_S.[‡] We introduce the mapping $\widehat{S}: N \to 2^N$ by

$$\widehat{S}(i) := \{j \in N \mid (i, j) \in \text{tr}(R_S)\},$$

assigning to every player $i \in N$ her *subordinates*. Similarly we denote by

$$\widehat{S}^{-1}(i) := \{j \in N \mid (j, i) \in \text{tr}(R_S)\}$$

the collection of the *superiors* of player $i \in N$ in the permission structure R on N.

For every coalition $E \subset N$ we define $S(E) := \cup_{i \in E} S(i)$. Analogously for every coalition $E \subset N$ we define the collections $\widehat{S}(E)$, and $\widehat{S}^{-1}(E)$.

[‡]The transitive closure $\text{tr}(R)$ of some binary relation $R \subset N \times N$ is given by $(i, j) \in \text{tr}(R)$ if and only if there exists a sequence $\{h_1, \ldots, h_m\} \subset N$ with $h_1 = i$, $(h_k, h_{k+1}) \in R$ for $1 \leq k \leq m - 1$, and $h_m = j$.

With the use of the concept of a permission structure as introduced above we define a game with permission structure.

Definition 2.2 *A* **game with permission structure** *is a triple (N, v, S), where N is a finite set of players, $v \in \mathcal{G}^N$ is a cooperative game with side payments on N, and $S \in \mathcal{S}^N$ is a permission structure on N.*

It is clear that the collection of all games with permission structure on a playerset N is precisely the collection $\mathcal{G}^N \times \mathcal{S}^N$.

3 The Conjunctive approach

If (N, v, S) is a game with permission structure, then we can interpret the situation described as follows. Essentially, we can think of $v \in \mathcal{G}^N$ as representing the economic possibilities open to every coalition in N. Thus $v(E)$ represents the amount of utility, which coalition $E \subset N$ could normally obtain were it not for the permission structure as imposed on the game. In the sequel we explicitly assume that the members of E cannot act without permission from *all* their predecessors. More precisely, if any $i \in E$ belongs to $S(N \setminus E)$, then she cannot act without permission of at least one player, who is not in E, and is therefore "lost" or "unproductive" to the coalition.[§] In this case coalition E can only count on the cooperation of those $i \in E$, who do *not* require outside permission for their acts. We refer to the interpretation as described above as the *Conjunctive approach* to games with permission structure. We remark that other interpretations are also possible, as is shown in Gilles and Owen (1991).

The reasoning as followed above leads to the introduction of a class of coalitions that are able to act without permission from players outside that coalition.

Definition 3.1 *Let $S \in \mathcal{S}^N$ be a permission structure on N. The coalition $E \subset N$ is* **autonomous** *in S if*

$$E \cap S(N \setminus E) = \emptyset.$$

The collection of all autonomous coalitions in the permission structure S is denoted by Φ_S.

[§] In the sale of an object as described in the introduction this is the case with player 1. He has to get permission from the property rights owner, player 2, before he is able to sell or execute the user rights.

According the Conjunctive approach the autonomous coalitions are essentially the only payoff generating coalitions within a game with permission structure. The proof of the following lemma is obvious.

Lemma 3.2 *Let $S \in \mathcal{S}^N$ be a permission structure on N and let $E \subset N$ be some coalition. Then E is autonomous if and only if $\widehat{S}^{-1}(E) \subset E$.*

Lemma 3.2 shows explicitly that indeed all superiors of the players in an autonomous coalition are also member of that coalition. With respect to the collection Φ_S of all autonomous coalitions we can say the following.

Proposition 3.3 *Let $S \in \mathcal{S}^N$ be a permission structure on N. Then the collection Φ_S of autonomous coalitions satisfies the following properties:*

(i) $\emptyset \in \Phi_S$.

(ii) $N \in \Phi_S$.

(iii) *For all $E, F \in \Phi_S$ it holds that $E \cup F \in \Phi_S$ and $E \cap F \in \Phi_S$.*

PROOF

By Lemma 3.2 $E \in \Phi_S$ means that $\widehat{S}^{-1}(i) \subset E$ for every $i \in E$. It follows that $\emptyset \in \Phi_S$ (as no $i \in \emptyset$) and $N \in \Phi_S$ (as $\widehat{S}^{-1}(i) \subset N$ for all $i \in N$).

If $E, F \in \Phi_S$ and $i \in E \cup F$, then either $i \in E$ or $i \in F$. If $i \in E$, then $\widehat{S}^{-1}(i) \subset E \subset E \cup F$. Similarly, this holds for $i \in F$, and hence $E \cup F \in \Phi_S$.

If $E, F \in \Phi_S$ and $i \in E \cap F$, then $i \in E$ as well as $i \in F$. Thus, $\widehat{S}^{-1}(i) \subset E$ as well as $\widehat{S}^{-1}(i) \subset F$, and so $\widehat{S}^{-1}(i) \subset E \cap F$. Thus, $E \cap F \in \Phi_S$.

Q.E.D.

From the properties as mentioned in Proposition 3.3 it immediately follows that for any coalition $E \subset N$ there exists a largest autonomous subset and a smallest autonomous superset. This leads to the following definition.

Definition 3.4 *Let $S \in \mathcal{S}^N$ and let $E \subset N$. The* **sovereign part** *of E in S is the set*

$$\sigma(E) := \bigcup\{F \mid F \subset E, \quad F \in \Phi_S\}.$$

The **authorizing set** *of E in S is given by*

$$\alpha(E) := \bigcap\{F \mid E \subset F, \quad F \in \Phi_S\}.$$

In the framework of the Conjunctive approach it is clear that a coalition $E \subset N$ can maximally obtain the payoff generated by its sovereign part $\sigma(E)$. On the other hand the authorizing set $\alpha(E)$ of E is precisely the smallest coalition, which contains all members of E as well as their superiors. Hence, the authorizing set is the smallest coalition containing E, which can act autonomously.

Lemma 3.5 *Let $S \in \mathcal{S}^N$ and $E \subset N$. Then the following properties hold:*

(a) $\sigma(E) = E \setminus \widehat{S}(N \setminus E)$.

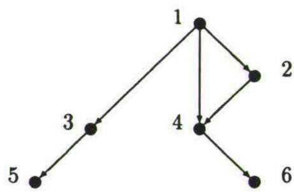
(b) $\alpha(E) = E \cup \widehat{S}^{-1}(E)$.

The proof of the lemma is left to the reader.

Example 3.6 Consider the player set $N = \{1,2,3,4,5,6\}$ and the permission structure $S: N \to 2^N$ given by

$$S(1) = \{2,3,4\},\ S(2) = \{4\},\ S(3) = \{5\},\ S(4) = \{6\},\ S(5) = \emptyset,\ S(6) = \emptyset.$$

This structure can be represented by the following directed graph.



Take $E = \{1,4,6\}$, then $S(N \setminus E) = \{4,5\}$. Clearly, since $E \cap S(N \setminus E) = \{4\} \neq \emptyset$, the coalition E is not autonomous. As $\widehat{S}(N \setminus E) = \{4,5,6\}$, the sovereign part of E is given by $\sigma(E) = E \setminus \widehat{S}(N \setminus E) = \{1\}$. Furthermore, the authorizing set of E is just $\alpha(E) = \{1,2,4,6\}$.

To complete the introductory analysis of the concepts of the sovereign part and the authorizing set of a coalition we prove the following properties.

Proposition 3.7 *Let $E, F \subset N$ be two coalitions. Then*

(i) $\sigma(E) \cup \sigma(F) \subset \sigma(E \cup F)$.

(ii) $\sigma(E) \cap \sigma(F) = \sigma(E \cap F)$.

(iii) $\alpha(E) \cup \alpha(F) = \alpha(E \cup F)$.

(iv) $\alpha(E \cap F) \subset \alpha(E) \cap \alpha(F)$.

Proof

From the definition we derive that for every $E \subset N$

$$\sigma(E) = \{i \in E \mid \widehat{S}^{-1}(i) \subset E\}.$$

Using this equality we prove the assertions of the proposition.

(i) Let $i \in \sigma(E) \cup \sigma(F)$. Then $\widehat{S}^{-1}(i) \subset E$ or $\widehat{S}^{-1}(i) \subset F$. Hence, $\widehat{S}^{-1}(i) \subset (E \cup F)$ and the assertion follows by definition.

(ii) Clearly $i \in \sigma(E \cap F)$ iff $\widehat{S}^{-1}(i) \in E \cap F$. This is equivalent to the statement that $i \in \sigma(E)$ as well as $i \in \sigma(F)$.

(iii) The assertion easily follows from the following equation:

$$\begin{aligned} \alpha(E \cup F) &= \bigcup_{i \in E \cup F} \widehat{S}^{-1}(i) \cup E \cup F \\ &= \bigcup_{i \in E} \widehat{S}^{-1}(i) \cup \bigcup_{j \in F} \widehat{S}^{-1}(j) \cup E \cup F \\ &= \alpha(E) \cup \alpha(F). \end{aligned}$$

(iv) For $i \in \alpha(E \cap F)$ it either holds that $i \in E \cap F$ or there is some $j \in E \cap F$ such that $j \in \widehat{S}(i)$.
If $i \in E \cap F$, then surely $i \in \alpha(E)$ as well as $i \in \alpha(F)$, i.e., $i \in \alpha(E) \cap \alpha(F)$.
If there is some $j \in E \cap F$ with $j \in \widehat{S}(i)$, then by the fact that $j \in E$ as well as $j \in F$ it is evident that $i \in \alpha(E)$ as well as $i \in \alpha(F)$.

Q.E.D.

4 Conjunctive restrictions

In the definition of a game with permission structure (N, v, S) we introduced the potential payoffs, represented by the game $v \in \mathcal{G}^N$, independently from the permission structure $S \in \mathcal{S}^N$. Based on the Conjunctive approach, in this section we transform a game with permission structure into a single TU-game, which describes all possibilities open to the players in the permission structure S, given their potentials as described by the game v. The resulting TU-game is called the *Conjunctive restriction* of v on permission structure S.

For that purpose we introduce for an arbitrary permission structure $S \in \mathcal{S}^N$ the following collection of TU-games:

$$\mathcal{G}(N, S) := \{v \in \mathcal{G}^N \mid v(E) = v(\sigma(E)), \quad \text{for all } E \subset N\}.$$

The Conjunctive restriction of a game v on a permission structure S is now simply defined as the projection of v on the set $\mathcal{G}(N, S)$ in the real vector space $\mathcal{G}^N$:

Definition 4.1 *Let $v \in \mathcal{G}^N$ and let $S \in \mathcal{S}^N$. The game $w \in \mathcal{G}(N, S)$ is the* **Conjunctive restriction** *of v on S if it satisfies the property that for every coalition $E \subset N$*

$$w(E) = v(\sigma(E)).$$

Definition 4.1 introduces a mapping $\mathcal{R}_S : \mathcal{G}^N \to \mathcal{G}(N, S)$, which assigns to every game $v \in \mathcal{G}^N$ its (Conjunctive) restriction $\mathcal{R}_S(v) = w \in \mathcal{G}(N, S)$. It is evident that $\mathcal{R}_S$ is a linear mapping on $\mathcal{G}^N$. To study its properties we consider two alternative bases for the $(2^n - 1)$-dimensional real vector space $\mathcal{G}^N$.

The *standard basis* of $\mathcal{G}^N$ is given by the games $\{z_E \mid E \subset N, \ E \neq \emptyset\}$ defined by

$$z_E(F) = \begin{cases} 1 & \text{if} \quad E = F \\ 0 & \text{if} \quad E \neq F \end{cases}$$

It is easy to see that in terms of the standard basis the game $v \in \mathcal{G}^N$ can be expressed as

$$v = \sum_{\substack{E \subset N \\ E \neq \emptyset}} v(E) \cdot z_E. \tag{1}$$

The *unanimity basis* of $\mathcal{G}^N$ consists of the games $\{u_E \mid E \subset N,\ E \neq \emptyset\}$ given by

$$u_E(F) = \begin{cases} 1 & \text{if } E \subset F \\ 0 & \text{otherwise} \end{cases}$$

Following Harsanyi (1959) the game $v \in \mathcal{G}^N$ can be expressed as

$$v = \sum_{\substack{E \subset N \\ E \neq \emptyset}} \Delta_v(E) \cdot u_E, \tag{2}$$

where the quantity $\Delta_v(E)$ is referred to as the *dividend* of coalition E in game v. For every $E \subset N$ this dividend is given by

$$\Delta_v(E) := \sum_{F \subset E} (-1)^{\#E - \#F} v(F). \tag{3}$$

We remark that for every coalition $E \subset N$ its worth $v(E)$ and its dividend $\Delta_v(E)$ are related by both (3) and the equivalent system

$$v(E) = \sum_{F \subset E} \Delta_v(F).$$

To analyze the projection mapping $\mathcal{R}_S$ properly, we study its behaviour on the collection of all unanimity games u_E, where $E \subset N,\ E \neq \emptyset$.

Theorem 4.2 *Let $E \subset N,\ E \neq \emptyset$, be any coalition. Then*

$$\mathcal{R}_S(u_E) = u_{\alpha(E)}.$$

PROOF

Let $F = \alpha(E)$ and $w = \mathcal{R}_S(u_E)$. By Lemma 3.5 F is an autonomous coalition, i.e., $\sigma(F) = F$. Furthermore, let $G \subset N$ be any (non-empty) coalition.

First we look at the case that $F \subset G$. Then $E \subset F = \sigma(F) \subset \sigma(G)$, and so

$$w(G) = u_E(\sigma(G)) = 1.$$

Next suppose that F is not a subset of G, i.e., $F \setminus G \neq \emptyset$. Then there exists a player $j \in F$ with $j \notin G$. Since $j \in F$ we have either that $j \in E$ or $j \in \widehat{S}^{-1}(E)$.

If $j \in E$, then $E \setminus G \neq \emptyset$ and thus $E \setminus \sigma(G) \neq \emptyset$.

If $j \in \widehat{S}^{-1}(E)$, then there is some player $i \in E$ with $i \in \widehat{S}(j)$. As $j \notin G$, this means that $i \in \widehat{S}(N \setminus G)$, and so $i \notin \sigma(G)$. Again we arrive at the conclusion that $E \setminus \sigma(G) \neq \emptyset$.

In either case we may conclude that

$$w(G) = u_E(\sigma(G)) = 0.$$

This implies that

$$w(G) = \begin{cases} 1 & \text{if } \alpha(E) \subset G \\ 0 & \text{otherwise} \end{cases}$$

and so $w = u_{\alpha(E)}$.

Q.E.D.

With the use of the unanimity basis of $\mathcal{G}^N$ and the belonging dividends we now can express the linear mapping $\mathcal{R}_S$.

Corollary 4.3 *Let $v \in \mathcal{G}^N$ be any game. Then*

$$\mathcal{R}_S(v) = \sum_{F \in \Phi_S} \left\{ \sum_{\substack{E \subset N \\ \alpha(E) = F}} \Delta_v(E) \right\} \cdot u_F.$$

This gives the desired expression for the Conjunctive restriction belonging to an arbitrary game with permission structure. In the next section this expression is used frequently to analyze games with a permission structure.

The second main result adresses the properties of the mapping $\mathcal{R}_S$ as a projection mapping in the space of all TU-games $\mathcal{G}^N$.

Theorem 4.4 *The linear mapping $\mathcal{R}_S$ is a projection mapping of rank A on $\mathcal{G}^N$, where $A = \#\Phi_S - 1$ is the number of non-empty autonomous subsets in S. Its kernel is generated by the games $\{z_E \mid E \notin \Phi_S\}$. Its image is generated by the unanimity games $\{u_E \mid E \in \Phi_S\}$.*

PROOF

Suppose that the coalition $E \subset N$ is not autonomous. Let z_E be the standard basis game belonging to E and let $w = \mathcal{R}_S(z_E)$ be the restriction of z_E on S.
Now there is no coalition F such that $E = \sigma(F)$. Thus for any coalition $F \subset N$

$$w(F) = z_E(\sigma(F)) = 0.$$

We may conclude that w is the null game and so $z_E \in \text{Kernel}(\mathcal{R}_S)$.

Now suppose that $E \subset N$, $E \neq \emptyset$, is an autonomous coalition. By Theorem 4.2 it holds that $\mathcal{R}_S(u_E) = u_{\alpha(E)}$. With $E = \alpha(E)$ it immediately follows that $\mathcal{R}_S(u_E) = u_E$, and hence that $u_E \in \text{Image}(\mathcal{R}_S)$.

Now the $2^n - 1 - A$ games z_E, E not autonomous, all belong to the kernel of $\mathcal{R}_S$. Since these games are all linearly independent, the dimension of the kernel of $\mathcal{R}_S$ must be at least $2^n - 1 - A$.
On the other hand, the A games u_E, with E autonomous, all belong to the image of $\mathcal{R}_S$. These are also all linearly independent, and so the dimension of the image of $\mathcal{R}_S$ is at least A.
But the sum of these dimensions must be exactly $2^n - 1$. Thus

$$\dim(\text{Kernel}(\mathcal{R}_S)) = 2^n - 1 - A, \quad \text{and}$$

$$\dim(\text{Image}(\mathcal{R}_S)) = A.$$

The given sets of games clearly form bases for the kernel respectively the image of the linear mapping $\mathcal{R}_S$.

To see that $\mathcal{R}_S$ is a projection mapping we note that, if $v \in \text{Image}(\mathcal{R}_S)$, then v can be expressed as

$$v = \sum_{E \in \Phi_S} c_E \cdot u_E.$$

Hence, from Theorem 4.2 it immediately follows that $\mathcal{R}_S(v) = v$.

Q.E.D.

Based on the theorems as derived above and the properties as given in Proposition 3.7 we are able to prove some additional properties of the mapping $\mathcal{R}_S : \mathcal{G}^N \to \mathcal{G}(N, S)$. Before stating these properties we recall some well known game theoretic concepts.

Definition 4.5 *Let $v \in \mathcal{G}^N$ be a TU-game.*

(a) *v is* **monotone** *if for all coalitions $E, F \subset N$ with $E \subset F$ it holds that $v(E) \leq v(F)$.*

(b) *v is* **superadditive** *if for all coalitions $E, F \subset N$ with $E \cap F = \emptyset$ it holds that*

$$v(E \cup F) \geq v(E) + v(F).$$

(c) *v is* **convex** *if for all coalitions $E, F \subset N$ it holds that*

$$v(E \cup F) + v(E \cap F) \geq v(E) + v(F).$$

(d) *v is* **balanced** *if the Core of that game is not empty, i.e., there exists a function $x: N \to \mathbf{R}$ such that for every coalition $E \subset N$: $x(E) := \sum_{i \in E} x_i \geq v(E)$ and $x(N) = v(N)$.*

The next result states that most of the above properties are invariant with respect to taking the conjunctive restriction of a game on a permission structure.

Theorem 4.6 *Let $S \in \mathcal{S}^N$ be any permission structure.*

(i) *If $v \in \mathcal{G}^N$ is monotone, then its Conjunctive restriction $\mathcal{R}_S(v)$ is monotone also. Moreover, if v is balanced, then $\mathcal{R}_S(v)$ is balanced also.*

(ii) *For every superadditive game $v \in \mathcal{G}^N$ its Conjunctive restriction $\mathcal{R}_S(v)$ is superadditive also.*

(iii) *If $v \in \mathcal{G}^N$ is convex, then its Conjunctive restriction $\mathcal{R}_S(v)$ is convex also.*

(iv) *If S is such that there exists a player $i_o \in N$ with $\widehat{S}(i_o) = N \setminus \{i_o\}$, then the Conjunctive restriction $\mathcal{R}_S(v)$ of any monotone game $v \in \mathcal{G}^N$ is superadditive and balanced.*

PROOF

Take an arbitrary game $v \in \mathcal{G}^N$ and let $w := \mathcal{R}_S(v)$ be its Conjunctive restriction.

(i) Suppose v is monotone. Take $E \subset F$ and let $G := F \setminus E$. Then

$$w(F) = v(\sigma(F)) = v(\sigma(E \cup G)) \geq v(\sigma(E) \cup \sigma(G)) \geq v(\sigma(E)) = w(E).$$

Suppose that v is balanced as well as monotone and let x be a Core imputation, i.e., $x(N) = v(N)$ and for every $E \subset N$: $x(E) \geq v(E)$. Then by monotonicity for every $E \subset N$ it holds that $v(E) \geq v(\sigma(E)) = w(E)$, and hence $x(E) \geq v(E) \geq w(E)$. Thus, x is a Core imputation of w also.

(ii) Suppose v is superadditive. Take $E, F \subset N$ such that $E \cap F = \emptyset$. Then

$$v(\sigma(E \cup F)) \geq v(\sigma(E) \cup \sigma(F)) \geq v(\sigma(E)) + v(\sigma(F)).$$

(iii) Suppose v is convex. Without loss of generality we may assume that $v(E) \geq 0$ for all coalitions $E \subset N$. Now take $E, F \subset N$. Then

$$w(E \cup F) = v(\sigma(E \cup F)) = v(\sigma(E) \cup \sigma(F) \cup H),$$

where $H = \sigma(E \cup F) \setminus [\sigma(E) \cup \sigma(F)]$. Since $H \cap \sigma(E) = H \cap \sigma(F) = \emptyset$ it follows by convexity of v that

$$v(\sigma(E \cup F)) \geq v(\sigma(E) \cup \sigma(F)) + v(H) \geq v(\sigma(E) \cup \sigma(F)).$$

Hence, with (ii) of Proposition 3.7,

$$\begin{aligned} w(E \cup F) + w(E \cap F) &= v(\sigma(E \cup F)) + v(\sigma(E \cap F)) \\ &\geq v(\sigma(E) \cup \sigma(F)) + v(\sigma(E) \cap \sigma(F)) \\ &\geq v(\sigma(E)) + v(\sigma(F)) = w(E) + w(F). \end{aligned}$$

(iv) Suppose that v is monotone. Since for every coalition $E \subset N$ it holds that $v(N) \geq v(E)$ as well as $v(N) = w(N) \geq w(E)$ and for every coalition $F \subset N \setminus \{i_o\}$ $w(F) = 0$ it follows immediately that the imputation x with $x_{i_o} = v(N)$ and $x_j = 0,\ j \neq i_o$ is in the Core of w.
To show superadditivity take $E, F \subset N$ with $E \cap F = \emptyset$. From the property of S it is clear that either $\sigma(E) = \emptyset$ or $\sigma(F) = \emptyset$ or $\sigma(E) = \sigma(F) = \emptyset$. Thus, we only have to establish that in case $\sigma(E) \neq \emptyset$ and $\sigma(F) = \emptyset$ it holds that

$$w(E \cup F) = v(\sigma(E \cup F)) \geq$$

$$\geq v(\sigma(E)) = v(\sigma(E)) + v(\sigma(F)) = w(E) + w(F).$$

Q.E.D.

5 Some applications

This section is devoted to two applications of our analysis of games with a permission structure. The first application discusses the collection of additive games and their restrictions to an acyclic permission structure. In this example we also derive an expression for the Shapley value of such a restriction. In this case the Shapley value gives a representation of the (weighted) hierarchical power of a player in the permission structure of the game. In the second application we discuss a specified class of games on a given *hierarchical* permission structure $S \in \mathcal{S}^N$, namely those of which the payoff generating players are in the lowest echelon or level in the hierarchy.

5.1 Additive games with permission structure

The valuation of a position in a permission structure depends, of course, on the abilities of the individual members, which are above and below that position. These abilities are represented by the original, unrestricted game $v \in \mathcal{G}^N$. By taking certain "standard" games for v, we can obtain insights into the "value" of a position in the structure as described by $S \in \mathcal{S}^N$. This analysis has to be performed with respect to the Conjunctive restriction $\mathcal{R}_S(v)$ of the original game v.

In this subsection we restrict ourselves to the analysis of *acyclic* permission structures with the use of additive games. A permission $S \in \mathcal{S}^N$ is acyclic if for every player $i \in N$ it holds that $i \notin \widehat{S}(i)$. Let $\lambda := (\lambda_1, \ldots, \lambda_n) \in \mathbf{R}^n_{++}$ be a strictly positive vector of weights. Next we introduce the game $v_\lambda \in \mathcal{G}^N$ as the additive game with weight vector λ given by

$$v_\lambda(E) := \sum_{i \in E} \lambda_i, \quad E \subset N.$$

Thus, it is assumed that the (original) individual abilities of player $i \in N$ are represented by the weight $\lambda_i > 0$. Since the player $i \in N$ has to give permission to her subordinates $j \in \widehat{S}(i)$, she can evidently claim a part of the payoff generated by these subordinates. This is exactly what is described by the restricted game $\mathcal{R}_S(v_\lambda)$. By analyzing these restricted games, we analyze the power structure within the permission structure.

It is obvious that for every coalition $E \subset N$ it holds that

$$\Delta_{v_\lambda}(E) = \begin{cases} \lambda_i & \text{if } E = \{i\} \text{ for some } i \in N \\ 0 & \text{otherwise} \end{cases}$$

Let $w_\lambda := \mathcal{R}_S(v_\lambda)$. Then by Corollary 4.3 we can derive that for every coalition $E \subset N$, $E \neq \emptyset$,

$$\Delta_{w_\lambda}(E) = \sum_{\substack{i \in N \\ \alpha(\{i\})=E}} \lambda_i. \tag{4}$$

By definition of the authorizing set of a coalition and the acyclicity of S it is obvious that for all players $i, j \in N$ with $i \neq j$ it is not possible that $i \in \widehat{S}(j)$ as well as $j \in \widehat{S}(i)$. This implies that for all $i \neq j$: $\alpha(\{i\}) \neq \alpha(\{j\})$. From this we conclude that

$$\Delta_{w_\lambda}(E) = \begin{cases} \lambda_i & \text{if } E = \alpha(\{i\}) \text{ for some } i \in N \\ 0 & \text{otherwise} \end{cases}$$

The next step in our analysis is to give a complete description of the Shapley value of w_λ. A well known formula for the Shapley value, applied to the game w_λ is given by

$$\varphi_i(w_\lambda) = \sum_{\substack{E \subset N \\ i \in E}} \frac{\Delta_{w_\lambda}(E)}{\#E}, \quad i \in N. \tag{5}$$

Hence, substituting (4) in (5) yields for every player $i \in N$

$$\varphi_i(w_\lambda) = \sum_{\substack{j \in N \\ i \in \alpha(\{j\})}} \frac{\lambda_j}{\beta(j)+1} = \frac{\lambda_i}{\beta(i)+1} + \sum_{j \in \widehat{S}(i)} \frac{\lambda_j}{\beta(j)+1},$$

where $\beta(j) := \#\widehat{S}^{-1}(j)$ for every $j \in N$.

This expression of the Shapley value of the restriction of the additive game v_λ is clearly an index that measures the hierarchical power of players in the (acyclic) permission structure S. Taking the weights of the players into account this index only depends upon the organization structure as represented by S. The weight of some player $i \in N$ is equally spread over herself and her superiors.

Example 5.1 Consider the permission structure as given in Example 3.6. Clearly it is acyclic. We immediately see that $\beta(1) = 0$, $\beta(2) = 1$, $\beta(3) = 1$, $\beta(4) = 2$, $\beta(5) = 2$, and $\beta(6) = 3$. Now we assign to every player the unit weight, i.e., $\lambda = (1, \ldots, 1) \in \mathbf{R}^6_+$. The Shapley value of the Conjunctive restriction of the additive game v_λ is given by

$$\varphi(\mathcal{R}_S(v_\lambda)) = \frac{1}{12} \cdot (35, 13, 10, 7, 4, 3).$$

Comparing this power index with the Shapley value of the original additive game v_λ, which is given by $\varphi(v_\lambda) = (1, \ldots, 1) \in \mathbf{R}^6_+$, we conclude that a substantial shift in power has been resulting from the various positions of the players in the permission structure S. The leader $1 \in N$ clearly has gained a much higher payoff because of his leadership.

5.2 Games with unproductive superiors

In this subsection we consider *hierarchical* permission structures and apply this concept to analyze organizations in which the "productive" players are in the lowest level in the hierarchy.

We define a permission structure $S \in \mathcal{S}^N$ to be *hierarchical* if it is acyclic and for every pair $i, j \in N$ there exists a player $h \in N$ such that

$$\{i, j\} \subset [\widehat{S}(h) \cup \{h\}].$$

In van den Brink and Gilles (1990) it is shown that there exists a partition $L_1, \ldots, L_M$ of N such that

$$L_1 = \{i \in N \mid S(i) = \emptyset\}, \quad \text{and}$$

$$L_k = \left\{ i \in N \setminus \bigcup_{p=1}^{k-1} L_p \mid S(i) \subset \bigcup_{p=1}^{k-1} L_p \right\}, \quad 2 \leq k \leq M.$$

Moreover, it can be shown that L_M consists of a singleton only. The sets L_k are called the *echelons* or *levels* of the hierarchical permission structure S. Refering to Swamy and Thulasiraman (1981) we note that the belonging binary relation R_S describes an acyclic quasi-strongly connected directed graph in case S is hierarchical.

Let $E \subset N$ be some coalition. Then we indicate by

$$\rho(E) := \{i \in E \mid \widehat{S}(i) \cap E = \emptyset\}$$

the collection of *pending players* in E. With the definition of echelons in the permission structure S we derive that $\rho(N) = L_1$. With the use of the notion of pending players as defined above we can derive an alternative characterization of an autonomous coalition.

Lemma 5.2 *Let $E \subset N$ and let $F \subset E$. Then $E = \alpha(F)$ if and only if $\rho(E) \subset F$ and $E = \alpha(\rho(E))$.*

PROOF

If

Since $\rho(E) \subset F$ it is clear that $E = \alpha(\rho(E)) \subset \alpha(F) \subset \alpha(E) = \alpha(\rho(E)) = E$.

Only if

Suppose by contradiction that there is a player $i \in \rho(E)$ such that $i \notin F$. By definition $\widehat{S}(i) \cap E = \emptyset$. But $E = \alpha(F)$ implies that there exists a player $j \in F \subset E$ with $i \in \widehat{S}^{-1}(j)$, i.e., $j \in \widehat{S}(i)$. This is in contradiction with the supposition. Thus we conclude that $\rho(E) \subset F$ and furthermore $E = \alpha(E) = \alpha(\rho(E))$.

Q.E.D.

Corollary 5.3 *$E \subset N$ is an autonomous coalition if and only if $E = \alpha(\rho(E))$.*

With these notions and results we can restate the expressions for the dividends of the Conjunctive restriction of a game in terms of the dividends of the original game as derived in Section 4. Let $v \in \mathcal{G}^N$ and let $w = \mathcal{R}_S(v)$. Then we derive that for all $E \subset N$ with $E = \alpha(\rho(E))$:

$$\Delta_w(E) = \sum_{\substack{F \subset N \\ \rho(E) \subset F}} \Delta_v(F).$$

This again can be rewritten as

$$\Delta_w(E) = \sum_{F \subset \widehat{S}^{-1}(\rho(E))} \Delta_v(F \cup \rho(E)).$$

Now we turn to the description of a situation with unproductive superiors.

Let $P = \{1, \ldots, p\}$ and $Q = \{p+1, \ldots, p+q\}$. Define $N := P \cup Q$. (Hence, it holds that $n = p + q$.) Now we take a hierarchical permission structure $S \in \mathcal{S}^N$ such that

$$Q = \rho(N) \; (= L_1). \tag{6}$$

From (6) it follows that for every $i \in Q$: $S(i) = \emptyset$. Hence, the collection Q is the lowest echelon in the hierarchy as described by the permission structure S. It is our purpose to describe a situation in which the players in Q are (potentially) "productive", while the players in P are (potentially) "unproductive". However, from their positions in the hierarchy the unproductive players or managers in P can claim certain portions of the payoffs generated by the productive players or workers in Q.

We construct such a game with permission structure as follows. Let $u \in \mathcal{G}^Q$ be any game on the player set Q. Now we define the game $v \in \mathcal{G}^N$ by

$$v(E) := u(E \cap Q), \quad E \subset N.$$

It is clear that (N, v, S) as constructed above indeed describes a situation with managers $i \in P$ and workers $i \in Q$. The allocation of payoffs in this particular situation can be analyzed with the use of the Shapley value of the Conjunctive restriction of v on S.

Thus, we define $w = \mathcal{R}_S(v)$ as the relevant description of the productive situation. Now by the results as proved in Section 4 we derive that

$$\Delta_w(E) = \begin{cases} \Delta_u(E \cap Q) & \text{if } E = \alpha(E \cap Q) \\ 0 & \text{otherwise} \end{cases}$$

We note that the requirement that $E = \alpha(E \cap Q)$ is equivalent to the condition that $E = \alpha(\rho(E))$ and $\rho(E) \subset Q$. With use of this formulation we can analyze the positions of the players in the production game w by means of the Shapley value.

For the productive workers in (N, v, S) we can deduce the following. Let $i \in Q$, then

$$\varphi_i(w) = \sum_{\substack{E \subset N \\ i \in E}} \frac{\Delta_w(E)}{\#E} = \sum_{\substack{F \subset Q \\ i \in F}} \frac{\Delta_u(F)}{\#\alpha(F)}.$$

Evidently, for every player $i \in F \subset Q$ it holds that $\#\alpha(F) \geq \#F + \#\widehat{S}^{-1}(i) = \#F + \beta(i)$. Hence,

$$\frac{\#F}{\#\alpha(F)} \leq \frac{\#F}{\#F + \beta(i)} \leq \frac{q}{q + \beta(i)}.$$

This leads to the conclusion that for every $i \in Q$

$$\varphi_i(w) \leq \frac{q}{q + \beta(i)} \cdot \varphi_i(u).$$

We remark that the bound is exact if and only if i is the unique productive player in the game w, i.e., $Q = \{i\}$.

We give a similar analysis for the "unproductive" managers in the collection P. For every $i \in P$ define $Q(i) := \{F \subset Q \mid F \cap \widehat{S}(i) \neq \emptyset\}$. Then the expected payoff, represented by the Shapley value, is for every $i \in P$ given by

$$\varphi_i(w) = \sum_{F \in Q(i)} \frac{\Delta_u(F)}{\#\alpha(F)}.$$

Therefore for $i \in P$ – by defining $q_i := \#\rho(\widehat{S}(i))$, where $\rho(\widehat{S}(i)) = Q \cap \widehat{S}(i)$ – it follows that

$$\varphi_i(w) \geq \max_{j \in \rho(\widehat{S}(i))} \varphi_j(w) \geq \frac{1}{q_i} \cdot \sum_{j \in \rho(\widehat{S}(i))} \varphi_j(w).$$

Example 5.4 Again take the permission structure as described in Example 3.6. It clearly is hierarchical with echelons $L_1 = \{5,6\}$, $L_2 = \{3,4\}$, $L_3 = \{2\}$, and $L_4 = \{1\}$. Take $P := N \setminus L_1 = \{1,2,3,4\}$ and $Q := L_1 = \{5,6\}$. Now let the game $u \in \mathcal{G}^Q$ be given by

$$u(\emptyset) = 0 \quad ;$$

$$u(\{5\}) = u(\{6\}) = 1 \quad ;$$

$$u(Q) = 5.$$

Evidently it holds that the dividends are given by

$$\Delta_u(\{5\}) = \Delta_u(\{6\}) = 1 \quad ;$$

$$\Delta_u(Q) = 3.$$

As before define the game $v \in \mathcal{G}^N$ as $v(E) := u(E \cap Q)$, for every $E \subset N$. Applying the formulas as derived above we can compute that

$$\varphi(w) = \frac{1}{12} \cdot (13, 9, 10, 9, 10, 9),$$

where $w := \mathcal{R}_S(v)$. This shows that the upper bound as given above for players in Q indeed gives a good indication for the payoff that is actually reached under the Conjunctive approach to the description of a production organization. Moreover, it shows that the lower bound for certain players in P can be exact as is the case for players 2 and 3.

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