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AN "INFORMATIONALLY ROBUST EQUILIBRIUM"
FOR TWO-PERSON NONZERO-SUM GAMES

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AN "INFORMATIONALLY ROBUST EQUILIBRIUM" FOR TWO–PERSON NONZERO–SUM GAMES*

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1. INTRODUCTION AND SUMMARY

Recent years have seen a proliferation of work seeking to modify the notion of Nash equilibrium as originally defined (Nash, 1950). There have been two main strands. The first larger strand consists of attempts to refine the notion of Nash equilibrium so as to reduce the sometimes large number of equilibria. Among these is the notion of perfectness due to Selten (1975). "Trembling hand" perfectness can be viewed as an expression of slight uncertainty about whether one's opponents will actually choose their Nash strategies. This strand culminates, perhaps, in the various notions of stability in which the behavior of opponents and payoffs are effectively assumed to be possibly slightly uncertain. (See Kohlberg and Mertens, 1986.) The second much smaller strand relaxes the hypothesis underlying Nash equilibrium that each player's belief about an opponent be consistent with the opponent's behavior. Instead a hierarchical structure of beliefs is permitted which has the effect of enlarging the set of equilibria. (See Pearce, 1985, and Bernheim, 1985.) The present work considers the effect of uncertainty about the *informational structure* of the game while retaining the assumptions of known payoffs and rational behaviour in the usual precise Nash sense for all players.

Although the existence of uncertainty about the structure of a game seems as inherently plausible as uncertainty about payoffs or the behavior of opponents, it is much more open–ended. That is, the range of alternative perturbed structures for a given N–person game, with a complicated sequential structure, perhaps, may be rather large. In the interests of greater specificity, then, attention is limited to two–person nonzero–sum games with simultaneous moves. In this case, there would appear to be a natural candidate for the informationally perturbed game. This models the slight possibility that each player's choice of pure strategy be found out by the other before that second player's choice is made. The perturbed game here models the effect of slight uncertainty about the strategic sequence of moves. The associated refinement is denoted as an "informationally robust equilibrium", IRE for short.

The following section contains a formal treatment of this refinement procedure. It is shown indeed that an "informationally robust equilibrium" can be defined for a general class of two–person nonzero–sum games. Furthermore it is shown that the set of informationally robust equilibria must be non–empty and a subset of the set of Nash equilibria. The mathematical tractability of the notion of an informationally robust equilibrium is a theoretical point in its favor. Such tractability would also facilitate application of the concept in contexts where the informational perturbation seems appropriate.

The construction of an informationally robust equilibrium is loosely reminiscent of "trembling hand" perfectness, at least with a finite number of pure strategies. However, informational robustness does not imply perfectness. Indeed, a dominated strategy may be used in an informationally robust equilibrium. This happens because such a strategy may be attractive to a Stackelberg leader, despite being dominated.

It might be useful to informally analyze an example at this point. Example A is taken from van Damme (1983, p. 14) and is given in Figure 1.

INSERT FIGURE 1 HERE

The Nash equilibria, NE, here are (u,ℓ) and (d,r) but only (u,ℓ) is normal form perfect. Indeed, "u" is a dominant strategy for 1, as is "ℓ" for 2. However, 1 would choose "d" rather than "u", if 1 were a Stackelberg leader, given that 2 broke the resulting tie in 1's favor. Similarly, 2 would choose "r" rather than "ℓ" if the roles were reversed. Suppose there were instead only a small probability that either player's pure strategy would be revealed to the other prior to the second player's choice, with the most likely event being simultaneous choice. Player 1, for example, must then choose one strategy taking into account the possibility that it will be revealed to player 2 before 2's choice has to be made, as well as the likelihood of simultaneous choice. This will then turn (d,r) into a strict equilibrium of the perturbed game, roughly speaking. (Abstracting, that is, from choices made as followers. Ties should continue to be broken as before.) Hence, although "d" and "r" are dominated in the game with simultaneous moves, they are not dominated in a plausible informationally perturbed game.

Section 3 considers in greater detail the relationship between informational robustness and perfectness. It is shown that strict equilibria of a game with a finite number of pure strategies are both informationally robust and perfect. Example B then completes the demonstration that there is no logical relationship between the two refinements. It also shows that there may be no equilibrium which is both informationally robust and perfect.

Section 4 is a review of the rather scanty literature which relates directly to the present paper.

What moral should be drawn from this discussion of two–person nonzero–sum games with simultaneous moves? In the case of games with more than two players or with non–simultaneous moves, it was already noted that the difficulty with the present procedure is not that there is no plausible informational perturbation but that there are many. No attempt is made here to analyze such general games. However, the moral of this paper is intended to be that the perturbation— informational, behavioral or whatever— which should be used for a particular game cannot be uniquely given in the abstract. To argue such an essentially negative view, it is enough then to focus on two–person nonzero–sum games. Then there is a salient and mathematically tractable informational perturbation to consider, as an alternative to the behavioral perturbation underlying perfectness. Section 3 proves, moreover, that the associated refinement is fundamentally distinct from perfectness. There may be no escape in the end from the need to tailor the perturbations considered to the context.

2. DEFINITION, EXISTENCE OF AN IRE

This section is devoted to a proof that a wide class of two–person nonzero–sum games, where moves are simultaneous, must possess an "informationally robust equilibrium", or IRE, for short. This class is large enough to include many games of economic interest which have a continuum of pure strategies, Cournot duopoly for example. The class is given as:

Definition 1: Simultaneous Move Game, G

A game with simultaneous moves is given as

$$G = ((S_i,U_i)|i=1,2)$$

where S_i are the pure strategy spaces, assumed compact metric spaces, and where

$$U_i\colon\ S_1 \times S_2 \longrightarrow R \qquad i=1,2$$

are continuous functions representing the payoffs.

As usual:

Definition 2: Mixed Strategies of G

The sets of mixed strategies for the game G are defined as

$$M_i = \{\text{set of probability measures on Borel sets of } S_i\},\ \ i=1,2$$

and the corresponding expected payoffs as

$$V_i(m_1,m_2) \equiv \int U_i dm, \quad i=1,2, \text{ for } m_i \in M_i, i=1,2$$

where $m = m_1 \times m_2$ is the unique product probability measure induced by m_1 and m_2 on $S_1 \times S_2$, (See Bartle, 1966, Ch. 10.)

Remarks. Billingsley (1968) shows that M_1 and M_2 are compact metrizable spaces with the topology of weak convergence. (See pp. 7–8 and Theorem 6 of the Appendix, p. 240. This topology can be induced by the "Prohorov" metric, as is shown pp. 237–238.) Parthasarathy (1967, p. 57, Lemma 1.1) shows that $V_i(m_1,m_2)$ are continuous functions on $M_1 \times M_2$, $i = 1, 2$.

This structure suffices to establish existence of a Nash equilibrium, or NE, in mixed strategies.

Theorem 1: Existence of a Nash Equilibrium for G

Any game G, as in Definition 1, has an NE in mixed strategies, as in Definition 2. That is, there exists $(m_1^*, m_2^*) \in M_1 \times M_2$ such that

$$V_1(m_1^*, m_2^*) \geq V_1(m_1, m_2^*), \forall m_1 \in M_1 \text{ and } V_2(m_1^*, m_2^*) \geq V_2(m_1^*, m_2), \forall m_2 \in M_2$$

PROOF Glicksburg (1952) proves a stronger version of this, where the pure strategy sets are merely compact Hausdorff spaces.

Now consider the definition of the perturbed game which models the presence of uncertainty concerning the order of moves.

Definition 3: Perturbed Game, $G(\epsilon_1,\epsilon_2)$

For each game G, as in Definition 1, a perturbed game, $G(\epsilon_1,\epsilon_2)$, say, is given as follows. The first move is nature's. She selects "1" with probability $\epsilon_1 \geq 0$, "2" with probability $\epsilon_2 \geq 0$ and "0" with probability $1-\epsilon_1 - \epsilon_2 \geq 0$. If "1" is selected, player 1's choice of pure strategy is revealed to player 2 prior to player 2's choice. Similarly, if "2" is chosen, then player 2's choice of pure strategy is revealed to player 1 prior to his choice. If "0" is chosen, no revelation takes place. In addition, player 1 cannot discriminate between states "0" and "1" and player 2 cannot discriminate between "0" and "2". The payoffs are then, say, for i = 1, 2,

$$U_i^{\epsilon}(s_1,s_2,f_1(s_2),f_2(s_1)) = \epsilon_1 U_i(s_1,f_2(s_1)) + (1-\epsilon_1-\epsilon_2)U_i(s_1,s_2) + \epsilon_2 U_i(f_1(s_2),s_2)$$

where $s_1 \in S_1$ is the choice made by player 1 in states "0" and "1" and $f_1: S_2 \to S_1$ is a function representing the strategy of player 1 for state "2" when the choice of $s_2 \in S_2$ is observed. Similar definitions apply for player 2.

The construction of the perturbed game is represented diagramatically in Figure 2.

INSERT FIGURE 2 HERE

It is desired to establish now that $G(\epsilon_1,\epsilon_2)$ has an equilibrium in a suitable sense.

To this end:

Definition 4: Best Replies

Define the best–reply correspondences

$$R_2:S_1 \longrightarrow S_2,\ R_2(s_1) = \{s_2 \,|\, U_2(s_1,s_2) \geq U_2(s_1,s_2') \ \forall s_2' \in S_2\}$$

$$R_1:S_2 \longrightarrow S_1,\ R_1(s_2) = \{s_1 \,|\, U_1(s_1,s_2) \geq U_1(s_1',s_2) \ \forall s_1' \in S_1\}$$

Since S_1 and S_2 are compact, and U_1 and U_2 are continuous, it follows that R_1 and R_2 are nonempty and upper hemicontinuous. (See Berge, 1963, p. 116, "Maximum Theorem").

Now:

Definition 5: Subgame Perfect Equilibrium

The equilibrium concept for $G(\epsilon_1,\epsilon_2)$ is taken to be that of subgame perfect equilibrium or SPE. (This definition extends that given by Selten, 1975, for finite games.) Such an equilibrium is a 4–vector $(\overset{*}{m}_1,\overset{*}{m}_2,\overset{*}{f}_1,\overset{*}{f}_2)$, where $\overset{*}{m}_1 \in M_1$, $\overset{*}{m}_2 \in M_2$, $\overset{*}{f}_1:S_2 \to S_1$ and $\overset{*}{f}_2:S_1 \to S_2$. It is required first that $\overset{*}{f}_1$ and $\overset{*}{f}_2$ are measurable functions satisfying

$$\overset{*}{f}_1(s_2) \in R_1(s_2), \quad \overset{*}{f}_2(s_1) \in R_2(s_1), \quad \forall\, s_2 \in S_2, \forall\, s_1 \in S_1.$$

Expected utility, given $\overset{*}{f}_1$ and $\overset{*}{f}_2$, for each $m_1 \in M_1$ and $m_2 \in M_2$, is:

$$V_i^{\epsilon}(m_1,m_2,\overset{*}{f}_1,\overset{*}{f}_2) = \int U_i^{\epsilon}(s_1,s_2,\overset{*}{f}_1(s_2),\overset{*}{f}_2(s_1))dm, \text{ where } m = m_1{\times}m_2,$$

is the unique product measure induced by m_1 and m_2 on $S_1{\times}S_2$ and where U_i^{ϵ} are given in Definition 3, for $i = 1, 2$. (Since $\overset{*}{f}_1$ and $\overset{*}{f}_2$ are measurable, all these integrals exist.) It is secondly required that

$$V_1^{\epsilon}(\overset{*}{m}_1,\overset{*}{m}_2,\overset{*}{f}_1,\overset{*}{f}_2) \geq V_1^{\epsilon}(m_1,\overset{*}{m}_2,\overset{*}{f}_1,\overset{*}{f}_2), \text{ for all } m_1 \in M_1,$$

$$V_2^{\epsilon}(\overset{*}{m}_1,\overset{*}{m}_2,\overset{*}{f}_1,\overset{*}{f}_2) \geq V_2^{\epsilon}(\overset{*}{m}_1,m_2,\overset{*}{f}_1,\overset{*}{f}_2), \text{ for all } m_2 \in M_2.$$

That is, $(\overset{*}{m}_1,\overset{*}{m}_2)$ is a Nash equilibrium given $(\overset{*}{f}_1,\overset{*}{f}_2)$.

A useful auxiliary step in the construction of such an SPE is:

Definition 6: Tie–Breaking Rule

The tie–breaking rule is that, for example, player 2 breaks ties in favor of 1. That is, player 2 chooses among his best replies one which maximizes player 1's payoff. A similar requirement holds with the roles reversed. To be precise:

$$f_2^*(s_1) \in \underset{s_2 \in R_2(s_1)}{\arg\max} \; U_1(s_1,s_2) \text{ and } f_1^*(s_2) \in \underset{s_1 \in R_1(s_2)}{\arg\max} \; U_2(s_1,s_2),$$

respectively. Since S_1 and S_2 are compact metric spaces and R_1 and R_2 are upper hemicontinuous, such functions f_2^* and f_1^* exist as "measurable selections". (See Wagner, 1977, p. 880, Theorem 9.1(ii).) Hellwig *et al* (1990) discuss the role of such tie–breaking rules for games of perfect information.

This implies:

Lemma 1: Reduced Form Game, $g(\epsilon_1,\epsilon_2)$

Given a perturbed game as in Definition 3 and the use of the tie–breaking rule as in Definition 6, it follows that the perturbed game $G(\epsilon_1,\epsilon_2)$ is equivalent to a "reduced form game" with simultaneous moves given as

$$g(\epsilon_1,\epsilon_2) = ((S_i,u_i^\epsilon)\,|\,i=1,2)$$

where the S_i are as in Definition 1. That is, the payoff function U_i^ϵ, as in Definition 3, now depends only on (s_1,s_2), and is given as:

$$u_i^\epsilon : S_1 \times S_2 \to R, \; i = 1, 2.$$

Furthermore, u_i^ϵ is upper semicontinuous in (s_1,s_2) and continuous in s_j, $i \neq j$, i, j = 1,2.

PROOF Since player 2 breaks ties in favor of player 1, as in Definition 6, it follows that 1's payoff in state "1" is a function of s_1 only given by, say,

$$U_1^L(s_1) \equiv U_1(s_1,f_2^*(s_1)) = \underset{s_2 \in R_2(s_1)}{\mathrm{Max}} \; U_1(s_1,s_2)$$

where this is upper semicontinuous in s_1. (Berge, 1963, p. 116, Theorem 2.) On the other hand, 2's payoff in state "1" is also a function of s_1 only, given by, say,

$$U_2^F(s_1) \equiv U_2(s_1, f_2^*(s_1)) = \underset{s_2 \in S_2}{\text{Max}} \; U_2(s_1, s_2)$$

and this is a continuous function of s_1 (Berge, 1963, p. 116, "Maximum Theorem"). Similar observations hold for the analogous $U_2^L(s_2)$ and $U_1^F(s_2)$.

Now the payoffs, as in Definition 3, can be rewritten as, say,

$$u_1^\epsilon(s_1,s_2) = \epsilon_1 U_1^L(s_1) + (1-\epsilon_1-\epsilon_2)U_1(s_1,s_2) + \epsilon_2 U_1^F(s_2)$$

$$u_2^\epsilon(s_1,s_2) = \epsilon_1 U_2^F(s_1) + (1-\epsilon_1-\epsilon_2)U_2(s_1,s_2) + \epsilon_2 U_2^L(s_2)$$

and the stated continuity properties are immediate.

Now:

Theorem 2: ***Existence of an NE for $g(\epsilon_1,\epsilon_2)$ and an SPE for $G(\epsilon_1,\epsilon_2)$***

(a) The game $g(\epsilon_1,\epsilon_2)$, as in Lemma 1, has a NE in mixed strategies, as in Definition 2. That is, there exists $(m_1^*, m_1^*) \in M_1 \times M_2$ satisfying:

$$v_1^\epsilon(m_1^*, m_2^*) \geq v_1^\epsilon(m_1, m_2^*) \,,\, \forall m_1 \in M_1 \qquad v_2^\epsilon(m_1^*, m_2^*) \geq v_2^\epsilon(m_1^*, m_2) \,,\, \forall m_2 \in M_2.$$

where expected payoffs are now, for i= 1, 2, for all $m_1 \in M_1$ and $m_2 \in M_2$,

$$v_i^\epsilon(m_1,m_2) = \int u_i^\epsilon(s_1,s_2)dm.$$

Here, $m = m_1 \times m_2$ is the unique product measure induced by m_1 and m_2 on $S_1 \times S_2$ and the upper semicontinuity of u_i^ϵ implies its integrability.

(b) The game $G(\epsilon_1,\epsilon_2)$ has an SPE, as in Definition 5.

PROOF (a) See Robson (1990a). Lemma 1 establishes the properties of the payoffs needed to apply "Theorem 1". Note that Glicksburg (1952) cannot be applied here since the payoffs need not be continuous in pure strategies.

(b) It is easy to check that a suitable SPE, as in Definition 5, is $(m_1^*, m_2^*, f_1^*, f_2^*)$ where (f_1^*, f_2^*) satisfy Definition 6, and where (m_1^*, m_2^*) is then the NE of $g(\epsilon_1,\epsilon_2)$ from (a).

Remark Definition 6 is not logically needed to prove that an SPE exists. It is simply convenient to show there is an SPE with this property.

Now:

Definition 7: Informationally Robust Equilibrium IRE

A strategy pair (m_1^*, m_2^*) is an "informationally robust equilibrium," or IRE, of G if and only if there exists some sequence of pairs $(\epsilon_1(n), \epsilon_2(n))$ and an associated sequence $(m_1^n, m_2^n, f_1^n, f_2^n)$, say, of SPEs of $G(\epsilon_1(n), \epsilon_2(n))$, as in Definition 5, such that

$$(\epsilon_1(n), \epsilon_2(n)) \to 0 \quad \text{and} \quad (m_1^n, m_2^n) \to (m_1^*, m_2^*), \text{ as } n \to \infty.$$

The main result is then:

Theorem 3: Existence of IRE as Refinement of NE

Any game G as in Definition 1 has a mixed strategy pair (m_1^*, m_2^*) which is an IRE, as in Definition 7. Furthermore, any IRE is also a NE of G, as defined in Theorem 1.

PROOF Consider any sequence $\epsilon(n) = (\epsilon_1(n), \epsilon_2(n)) \to 0$ as $n \to \infty$. By Theorem 2, the game $g(\epsilon_1(n), \epsilon_2(n))$, as in Lemma 1, has an NE, $(m_1^n, m_2^n) \in M_1 \times M_2$, say, for all n. This forms part of an SPE of $G(\epsilon_1(n), \epsilon_2(n))$, as in Definition 5. Since M_1 and M_2 are compact metrizable spaces, it follows without loss of generality that there exists a limit point of this sequence of NE, (m_1^*, m_2^*), say, which is then an IRE of G.

Suppose that (m_1^*, m_2^*) is an IRE but is not an NE of G. This means that, for example,

$$V_1(m_1', m_2^*) > V_1(m_1^*, m_2^*)$$

for some $m_1' \in M_1$, where V_1 is as in Definition 2. Suppose, indeed,

$$V_1(m_1', m_2^*) - V_1(m_1^*, m_2^*) = 5\Delta > 0$$

By Definition 7, there exists a sequence $(\epsilon_1(n), \epsilon_2(n))$ and a sequence $(m_1^n, m_2^n, f_1^n, f_2^n)$, say, of SPEs of $G((\epsilon_1(n), \epsilon_2(n))$, such that

$$(\epsilon_1(n), \epsilon_2(n)) \to 0 \quad \text{and} \quad (m_1^n, m_2^n) \xrightarrow{w} (m_1^*, m_2^*) \quad \text{as } n \to \infty.$$

Since V_1 is continuous, as noted following Definition 2, $\exists\, N_1 \ni n > N_1$ implies

$$|V_1(m_1^n,m_2^n) - V_1(m_1^*,m_2^*)| < \Delta \text{ and } |V_1(m_1',m_2^n) - V_1(m_1',m_2^*)| < \Delta$$

Hence

$$|V_1(m_1',m_2^n) - V_1(m_1^n,m_2^n)| =$$

$$|[V_1(m_1',m_2^n)-V_1(m_1',m_2^*)] + [V_1(m_1',m_2^*)-V_1(m_1^*,m_2^*)] + [V_1(m_1^*,m_2^*)-V_1(m_1^n,m_2^n)]|$$

$$> 5\Delta - \Delta - \Delta = 3\Delta > 0.$$

(Note the inequality $|A+B+C| \geq |B|-|A|-|C|$.) Now, from Definitions 2 and 5, it is not hard to show that:

$$|V_1^\epsilon(m_1,m_2,f_1,f_2) - V_1(m_1,m_2)| \leq (\epsilon_1 + \epsilon_2)(U_1^b - U_1^s)$$

where Definition 1 implies the existence of the upper and lower bounds for U_1 on $S_1 \times S_2$ given as U_1^b and U_1^s respectively. Hence $\exists\, N_2$ such that $n > N_2$ implies

$$|V_1^{\epsilon(n)}(m_1,m_2,f_1,f_2) - V_1(m_1,m_2)| < \Delta,$$

uniformly in (m_1,m_2,f_1,f_2). Hence $n > \max(N_1,N_2)$ implies that

$$|V_1^{\epsilon(n)}(m_1',m_2^n,f_1^n,f_2^n) - V_1^{\epsilon(n)}(m_1^n,m_2^n,f_1^n,f_2^n)| =$$

$$|\{[V_1^{\epsilon(n)}(m_1',m_2^n,f_1^n,f_2^n)-V_1(m_1',m_2^n)] + [V_1(m_1',m_2^n)-V_1(m_1^n,m_2^n)] +$$

$$[V_1(m_1^n,m_2^n)-V_1^{\epsilon(n)}(m_1^n,m_2^n,f_1^n,f_2^n)]\}|$$

$$> 3\Delta - \Delta - \Delta = \Delta > 0.$$

(Recall $|A+B+C| \geq |B|-|A|-|C|$.) This contradicts the requirement that $(m_1^n,m_2^n,f_1^n,f_2^n)$ be an SPE for the game $G(\epsilon_1(n),\epsilon_2(n))$. QED.

3. INFORMATIONAL ROBUSTNESS AND PERFECTNESS

The first result is that the requirements of informational robustness and perfectness need not conflict.

Lemma 2. A Strict Equilibrium is an IRE

Suppose the game G, as in Definition 1, has a finite number of pure strategies for each player. Suppose G also has a strict equilibrium, which must then be in pure strategies. (That is, each player's Nash strategy is the unique best reply to the other's. This usage is as in Fudenberg, *et al.*, 1988.) This equilibrium is then "informationally robust".

PROOF Simply note that the strict equilibrium of G must be a (strict) equilibrium of $g(\epsilon_1,\epsilon_2)$, as in Lemma 1, if (ϵ_1,ϵ_2) is small enough.

Remark. Fudenberg *et al* (1988) observe that a strict equilibrium is indeed "hyperstable" as a singleton set, where hyperstable is as in Kohlberg and Mertens (1986).

The following 2×2 example is then sufficient to show that whether an equilibrium is informationally robust is logically independent of whether it is perfect. It indeed shows that a dominated strategy may be used in an IRE. (Example A of the introduction also showed this in an informal way.) Hence such an IRE is a member of no "stable set" as defined by Kohlberg and Mertens, violating their requirement of "admissibility". In addition, Example B shows that it is not possible to resolve this conflict by requiring both informational robustness and perfectness.

Example B is given in Figure 3.

INSERT FIGURE 3 HERE

Here, it is a dominant strategy for 2 to choose "r" and (u,r) is the unique normal form perfect equilibrium. However, there is a continuum of Nash equilibria in mixed strategies of the form

$$\{((1,0),(q,1-q))\,|\,q \in [0,\tfrac{1}{2}]\},$$

so that 1 plays "u" for sure and 2 plays "ℓ" with probability q. Hence the perfect equilibrium is at one end of this continuum. It will be shown that the unique IRE is at the other end of the continuum and thus is:

$$((1,0),(1/2,1/2)).$$

The only tie induced for the follower in either state "1" or state "2" occurs when player 1 chooses "u". However, player 1 is also then indifferent as to how this tie is broken. The reduced form game associated with Example B is then unique and is as in Figure 4.

INSERT FIGURE 4 HERE

This reduced form has unique equilibrium in mixed strategies, given by

$$p = \frac{(1-\epsilon_1-2\epsilon_2)}{(1-\epsilon_1-\epsilon_2)}, \text{ and } q = \frac{1-\epsilon_2}{2(1-\epsilon_1-\epsilon_2)}$$

where p is the probability that player 1 chooses "u" and q is the probability that 2 chooses "ℓ". Clearly,

$$(p,q) \to (1,1/2), \text{ as } (\epsilon_1,\epsilon_2) \to 0,$$

as required.

Example B has an NE which is an IRE but not perfect, namely ((1,0),(1/2,1/2)), which indeed uses the dominated strategy "ℓ". It also has an NE which is perfect but not an IRE, namely ((1,0),(0,1)), and a range of NE which are neither. Since Lemma 2 exhibits a class of NE which are both IREs and perfect, the logical independence of the two concepts follows. Finally, Example B clearly has no equilibrium which is both informationally robust and perfect.

4. RELATED LITERATURE

There is little other work which considers alterations in the informational structure of a game. Two exceptions are noted here.

Matsui (1989) considers a supergame in which there is a slight possibility that one player's entire strategy will be disclosed to the other before the game itself is played. This is shown to imply that only Pareto efficient outcomes can be sustained as equilibria. There is no analogous result here. (Consider the simplest 2×2 co–ordination game, for example, with two strict Nash equilibria, where one Pareto dominates the other. Both are then IREs.) Supergames are at once more special and more complicated than need be the case here.

More closely related is an independent informal paper by Rosenthal (1989). He considers two–person nonzero–sum games, given in normal form. He then investigates the question of which Nash equilibria are "commitment–robust" in the sense of being invariant under either of the transformations of the game which designate one player as Stackelberg leader and the other as follower. It is the mixed strategy of the leader rather than its realization which is revealed to the follower in these transformed games. (This is reminiscent of the approach of von Neumann and Morgenstern, 1947, to two–person zero–sum games.) Rosenthal points out that a Stackelberg leader may wish to use a strategy which is dominated in the original simultaneous move version of the game. He does not consider any argument involving perturbations of the original game analogous to that here. (The original version of the present paper also considered the possibility of mixed strategies being revealed, but considered perturbed games otherwise similar to those here. See Robson, 1990b, also.) Finally, whereas there is a quite general existence result concerning IREs here, Rosenthal notes that a commitment–robust equilibrium need not exist.

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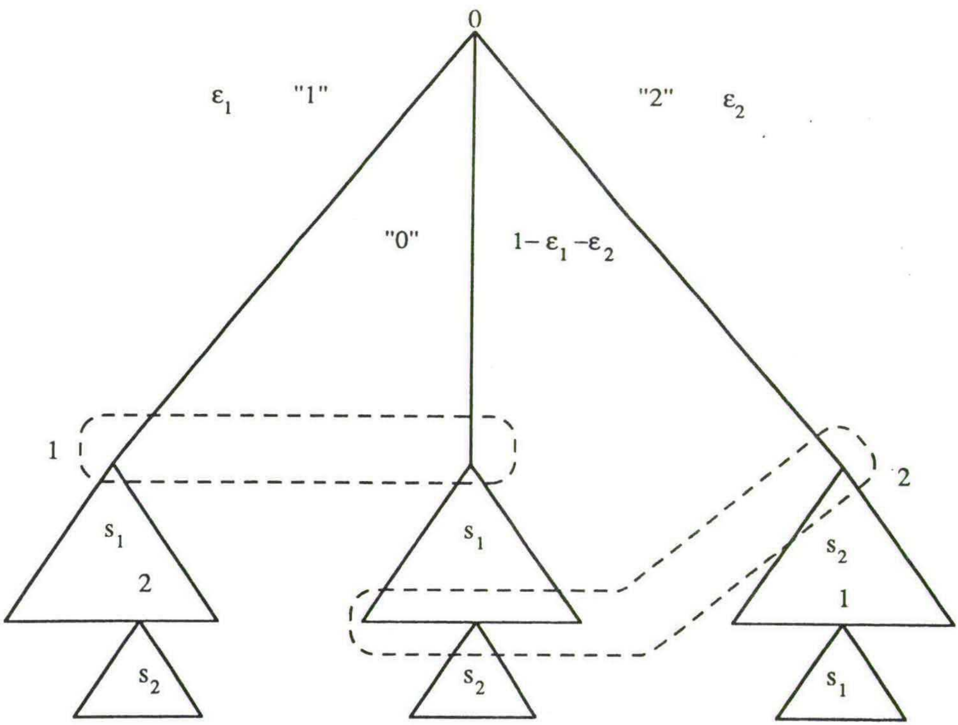

Figure 2. The Extensive Form of the Perturbed Game

1 \ 2	ℓ	r
u	(1, 1)	(10, 0)
d	(0, 10)	(10, 10)

Figure 1 Example A

1 \ 2	ℓ	r
u	(1, 0)	(1, 0)
d	(2, 1)	(0, 2)

Figure 3 Example B

1 \ 2	ℓ	r
u	$(1+\epsilon_2, \epsilon_2)$	$(1, 0)$
d	$(2-2\epsilon_1, 1+\epsilon_1)$	$(\epsilon_2, 2-2\epsilon_2)$

Figure 4 Reduced Form of Perturbed Game, Example B

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