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# A Note on the Decentralization of Pareto Optima in Economies with Public Projects and Nonessential Private Goods*

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**Abstract**

In the theory of economies with public goods one usually considers the case in which private goods are essential, i.e., each agent receives a fixed minimum level of utility if he consumes no private goods, irrespective of the public goods consumed. This note develops the second welfare theorem for economies with public projects and possibly inessential private goods. As a corollary we also derive conditions under which valuation equilibria exist.

# 1 Introduction

Mas-Colell (1980) departed from the earlier literature on economies with public goods in that he considered personalized prices for *access* to the public good ("valuation prices") rather than personalized prices for *units* consumed of the public good, as in the literature on Lindahl equilibrium. This departure in pricing permitted a reconsideration of how one describes public goods. With Lindahl prices one assumes that the quantity of each public good is represented by a real number, and the equilibrium establishes what quantity will be supplied. In valuation equilibrium the supplier chooses from a set of integral public projects, and there is no need to order projects by magnitude or any other criterion. The valuation prices faced by a consumer are different for different public projects, and equilibrium has the property that at these prices no consumer would prefer a different selection from the set of potential projects than the one offered. In addition, no other public project would be profitable.

Recently, some authors have extended the concept of valuation equilibrium in various ways. Diamantaras and Gilles (1994) extend the welfare theorems for valuation equilibrium to multiple private goods, as well as extending results on the related notion of cost-share equilibrium. Manning (1993) maintains the hypothesis of one private good, and shows that the welfare theorems persist even with a fixed number of multiple jurisdictions, where agents can move among jurisdictions if such a move would improve utility. Scotchmer and Wooders (1987) and Scotchmer (1994) introduce the notion of competitive equilibrium with "admission prices" in club economies with free entry, and show equivalence to the equal-treatment core. Admissions prices are like valuation prices in that they are personalized lump sum payments which depend on the public good. When there are multiple jurisdictions — a fixed number or an endogenous number — the valuation prices or admission prices must also depend on the coalition.

With one exception the theorems of these authors, including Mas-Colell, use an "essentiality" assumption for private goods, which consists at least of the statement that each agent receives a fixed minimum utility if he consumes no private goods, irrespective of the public goods. The essentiality condition is strong in that it excludes the case of transferable utility. However Scotchmer (1994) showed for club economies that the essentiality condition is not required for equivalence of the equal-treatment core and competitive equilibrium with admissions prices, and this suggests that the condition can be avoided more generally. In this paper we combine the ideas in that proof with the ideas in Diamantaras and Gilles (1994) their proof of the second welfare theorem, which used essentiality, to show that essentiality can be avoided, even with multiple private goods. Since essentiality is typically used for the second welfare theorem but not for the first one, we only present the second welfare theorem. (For the first welfare theorem with multiple private goods, see Diamantaras and Gilles (1994).)

The basic idea is as follows. Suppose that it is efficient to produce a public project $y$. Suppose there is another project $z$ that an agent, say $a$, would prefer to $y$ even if he were given no private goods. This would violate essentiality. Since $y$ is efficient, some agents must dislike $z$. Their compensations for accepting $z$ would have to be high enough so that, if they were compensated, covering the cost of $z$ would be socially infeasible. This implies that the valuation prices that make them indifferent between $z$ and $y$ are relatively low. We can find a valuation price for agent $a$ that exceeds the value of his endowment, so that he cannot afford $z$, but is still low enough so that $z$ is unprofitable.

## 2 Definitions and the main result

We study an economy in which $A$ is a finite set of economic agents, there are $\ell \in \mathbb{N}$ private commodities, and the commodity space is represented by $\mathbb{R}^\ell_+$. We denote by the function $w: A \to \mathbb{R}^\ell_+ \setminus \{0\}$ the *endowment* of private commodities of the agents in $A$ where $\overline{w} := \sum_{a \in A} w(a) \gg 0$.

There is a set $\mathcal{Y}$ of public projects, on which we do not impose any structure. Each public project has a cost in terms of each private good, and we capture this by the vector-valued function $c: \mathcal{Y} \to \mathbb{R}^\ell_+$. Of course, one can easily adopt some metric or Euclidean structure on the set $\mathcal{Y}$, as is customary when one discusses Samuelson conditions or Lindahl pricing.

Each agent $a \in A$ has preferences defined on $\mathbb{R}^\ell_+ \times \mathcal{Y}$, which are represented by a real-valued function $U_a\colon \mathbb{R}_+^\ell \times \mathcal{Y} \to \mathbb{R}$. The utility function $U_a$ is *strictly monotone* if for all $f, g \in \mathbb{R}_+^\ell$ and all $y \in \mathcal{Y}$ with $f > g$, $U_a(f,y) > U_a(g,y)$. The utility function $U_a$ is *quasi-concave* if for all $z \in \mathcal{Y}$ and for all $u \in \mathbb{R}$, the set $\{g \in \mathbb{R}_+^\ell | U_a(g,z) > u\}$ is convex. The utility function $U_a$ is *continuous* if for all $z \in \mathcal{Y}$ the function $U_a(\cdot, z)$ is continuous.

An *economy* is a collection $\mathbb{E} = \{A, (U_a)_{a\in A}, w, \mathcal{Y}, c\}$. An *allocation* for an economy $\mathbb{E}$ is a pair $(f,y)$ where $f\colon A \to \mathbb{R}_+^\ell$ and $y \in \mathcal{Y}$. An allocation $(f,y)$ is *feasible* if

$$\sum_{a\in A} f(a) + c(y) = \overline{w}.$$

Note that we do not assume free disposal in provision; however, our results hold under the free disposal assumption. We denote the set of feasible allocations by $\Phi$.

The following definition is standard.

**Definition 2.1** *A feasible allocation $(f,y) \in \Phi$ is* **Pareto efficient** *for the economy $\mathbb{E}$ if there is no allocation $(g,z) \in \Phi$ such that*

(i) *for every $a$ in $A$, $U_a(g(a),z) \geq U_a(f(a),y)$, and*

(ii) *there exists an agent $b \in A$ such that $U_b(g(b),z) > U_b(f(b),y)$.*

We will need the usual price space for the private goods:

$$S^{\ell-1} := \left\{ q \in \mathbb{R}_+^\ell \,\middle|\, \sum_{i=1}^{\ell} q_i = 1 \right\}.$$

The following definition is a natural extension of Mas-Colell's definition of valuation equilibrium to the case of multiple private goods. In this definition agents maximize utility taking into account price changes in the private sector of the economy resulting from changing the choice of the public project. Thus a complete contingent price system rather than a unique price vector is taken as given by the agents.

**Definition 2.2** *A feasible allocation $(f,y) \in \Phi$ is a* **valuation equilibrium** *for $\mathbb{E}$ if there exist a price system $p\colon \mathcal{Y} \to S^{\ell-1}$ and a valuation function $V\colon A \times \mathcal{Y} \to \mathbb{R}$, such that*

(i) *there is budget neutrality, i.e., $\sum_{a\in A} V(a,y) = p(y)\cdot c(y)$;*

(ii) *for every agent $a \in A$, $p(y)\cdot f(a) + V(a,y) = p(y)\cdot w(a)$ and for all $(g,z) \in \mathbb{R}_+^\ell \times \mathcal{Y}$, if $U_a(g,z) > U_a(f(a),y)$, then $p(z)\cdot g + V(a,z) > p(z)\cdot w(a)$;*

(iii) *$y$ maximizes the surplus $\sum_{a\in A} V(a,z) - p(z)\cdot c(z)$ for $z \in \mathcal{Y}$.*

In Diamantaras and Gilles (1994) the additional condition $V(a,z) \le p(z)\cdot w(a)$ for all $z \in \mathcal{Y}$ and all $a \in A$ is added to (ii). With this condition it might be impossible to support a Pareto optimum as an equilibrium if private goods are not essential. This is because a consumer might prefer an allocation $(g,z)$ to the Pareto efficient allocation $(f,y)$ even if the allocation $(g,z)$ gives him zero private goods. If the valuation price cannot exceed the value of his endowment he can afford such an allocation, and then condition (ii) of the definition of valuation equilibrium cannot be satisfied.

This definition — along with the one in Diamantaras and Gilles (1994) — has the unusual characteristic of employing a price system for the private goods. This means that to a specific valuation equilibrium $(f,y)$ there corresponds not just one vector of prices of the private goods, but as many as the potential public projects. One can show that full Pareto efficiency may not be reachable by a definition that would imose a single price vector per equilibrium, but our use of price systems is motivated by considerations beyond necessity. In particular, it is akin to the notion of rational expectations equilibrium, although less strong in its implications. A price system, in our usage, embodies the predictions of what would happen to the prices of the private goods if the choice of the public project were to be altered. These predictions are assumed held in unison by all the agents, but they do not have to be "correct" in any sense. It is the last point that makes our definition conceptually less demanding than the idea of rational expectations equilibrium. A specific criticism of our price systems is that the agents do not act as price takers because a different choice of a public project yields a different price vector. However, a different choice of a public project can only be made collectively; no individual alone has much influence on it, at least in economies with more than a few agents. Further, the notion of agents as price takers in traditional equilibrium concepts, such as Lindahl equilibrium, is strained. Nevertheless, we use such equilibrium concepts with some confidence because we now know that it is possible to implement their allocations as equilibria of games. A similar application of implementation theory to our framework remains to be performed.

The valuation price $V(a,z)$ can be thought of as an access price for the right to consume public project $z$ and private goods at prices $p(z)$. Economies where such access prices arise naturally are club economies such as those discussed by Scotchmer (1994). In club economies the access price — or admission price — must depend on the coalition as well as on the public good since the public goods will be provided in a partition of the population where the elements of the partition can differ from each other in both public goods and the types of members. In competitive equilibrium the admission prices or valuation prices govern the partition of consumers into jurisdictions.

It is our purpose to investigate the equivalence of the set of Pareto efficient allocations and the set of valuation equilibria in an economy with public projects and multiple public projects. Diamantaras and Gilles (1994) show that if agents have monotone preferences any valuation equilibrium is Pareto efficient. For the reverse they, however, need certain essentiality conditions on private. The following theorem states that without essentiality conditions on the private goods any Pareto efficient allocation can be supported as a valuation equilibrium.

**Theorem 2.3** *For every agent $a \in A$ let the utility function $U_a$ be continuous, quasi-concave and strictly monotone. Then every Pareto efficient allocation in $\mathbb{E}$ can be supported as a valuation equilibrium.*

PROOF

Let $(f, y)$ be a Pareto efficient allocation in $\mathbb{E}$, and let $z \in \mathcal{Y}$ be arbitrary. We define

$$\begin{aligned}
A^\star(z) &:= \left\{a \in A \,\middle|\, U_a(g,z) < U_a(f(a),y) \text{ for every } g \in \mathbb{R}^\ell_+\right\}, \\
A^{\star\star}(z) &:= \{a \in A \,|\, U_a(0,z) > U_a(f(a),y)\}, \\
F(a,z) &:= \left\{g \in \mathbb{R}^\ell_{++} \,|\, U_a(g,z) > U_a(f(a),y)\right\}, \quad a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)], \\
\overline{F}(a,z) &:= \left\{g \in \mathbb{R}^\ell_{+} \,|\, U_a(g,z) \geq U_a(f(a),y)\right\}, \quad a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)], \\
F(a,z) &:= \mathbb{R}^\ell_{++}, \quad a \in A^{\star\star}(z), \\
\overline{F}(a,z) &:= \mathbb{R}^\ell_{+}, \quad a \in A^{\star\star}(z).
\end{aligned}$$

If we were to try to extend the definition of $F(\cdot, z)$ to $a \in A^\star(z)$, we would have $F(a,z) = \emptyset$ for $a \in A^\star(z)$. By the assumptions on $U_a$, for each $a \in A \setminus A^\star(z)$, $F(a,z)$ is nonempty, open, convex, and bounded from below. Let

$$\begin{aligned}
F(z) &:= \sum_{a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)]} F(a,z) + \{c(z) - \overline{w}\}, \\
\overline{F}(z) &:= \sum_{a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)]} \overline{F}(a,z) + \{c(z) - \overline{w}\}.
\end{aligned}$$

The set $F(z)$, when non-empty (i.e., when $[A^\star(z) \cup A^{\star\star}(z)] \neq A$), is open, convex, and bounded from below. The set $\overline{F}(z)$ has the same properties except that it is closed.

Because the recession cones (Rockafellar (1970), page 61) of the sets $F(a,z)$ are all contained in $\mathbb{R}^{\ell}_{+}$, Corollary 9.1.1 of Rockafellar (1970, page 74) applies, hence $\overline{F}(z)$ is also the closure of $F(z)$, denoted $\mathrm{cl}F(z)$.

We now construct positive prices for private goods, $p(z) \in \mathrm{int}S^{\ell-1}$ for all $z \in \mathcal{Y}$. Together with the valuation prices constructed below, these will support the allocation $(f,y)$ as a valuation equilibrium.

Claim

Suppose that $A^{*}(z) = \emptyset$ and $A \setminus A^{**}(z) \neq \emptyset$. Then there exist $p(z) \in \mathrm{int}S^{\ell-1}$ and vectors $x(a,z) \in \overline{F}(a,z)$, $a \in A$, such that

(i) $p(z) \cdot x(a,z) = \inf\{p(z) \cdot x \mid x \in F(a,z)\}$ for all $a \in A$,

(ii) $\sum_{a\in A} x(a,z) + c(z) - \overline{w} \geq 0$, and

(iii) $x(a,z) = x(a,y) = f(a)$, for every $a \in A$, if $z = y$.

Proof of Claim

Because $(f,y)$ is efficient, we have $0 \notin F(z)$. By the strict monotonicity of preferences and $A^{*}(z) = \emptyset$, there exists $\kappa(z) \in \mathbb{R}_{++}$ such that $\kappa(z)e \in F(z)$, where $e := (1,1,\ldots,1) \in \mathbb{R}^{\ell}$. Hence there exists $\lambda(z) \in \mathbb{R}_{+}$ (possibly 0) such that $\lambda(z)e \in \mathrm{cl}F(z) \setminus F(z) = \overline{F}(z) \setminus F(z)$.

Now we choose values $x(a,z)$. For $z = y$ let $x(a,y)$ satisfy (iii). We have $0 = \lambda(y)e = \sum_{a\in A} f(a) + c(z) - \overline{w}$ because $(f,y)$ is an efficient allocation. For $z \neq y$ and all $a \in A \setminus A^{**}(z)$ we will choose $x(a,z) \in \overline{F}(a,z)$ such that

$$\lambda(z)e = \sum_{a\in A\setminus A^{**}(z)} x(a,z) + c(z) - \overline{w}.$$

By definition of $\lambda(z)e$ the vectors $x(a,z)$ cannot be in the interiors of $\overline{F}(a,z)$. For $a \in A^{**}(z)$, we set $x(a,z) = 0$.

Now we show that a supporting hyperplane for $F(z)$ at $\lambda(z)e$ must have positive coefficients (prices). For that purpose let $p(z) \in \mathbb{R}^{\ell} \setminus \{0\}$ be such that $p(z) \cdot v > p(z) \cdot \lambda(z)e$ for all $v \in F(z)$. That we can choose $p(z)$ to satisfy these properties follows from the standard supporting hyperplane theorem, e.g., Rockafellar (1970), Theorem 11.6, page 100, applied to $\overline{F}(z)$, since $\lambda(z)e \notin F(z)$, $F(z)$ is convex and open, and $F(z)$ is the interior of $\overline{F}(z)$.

Next let the vectors $\psi(a,z)$, for all $a \in A \setminus A^{**}(z)$, be defined by $\psi(a,z) = x(a,z) + (1/|A \setminus A^{**}(z)|)e^{1}$, where for any set $S$, $|S|$ denotes the number of elements of $S$, and where $e^1 = (1,0,\dots,0) \in \mathbb{R}^\ell$ is the first unit vector of $\mathbb{R}^\ell$. By strict monotonicity, $\psi(a,z) \in F(a,z)$ for all $a \in A \setminus A^{**}(z)$, hence

$$\psi(z) := \sum_{a \in A \setminus A^{**}(z)} \psi(a,z) + c(z) - \overline{w} \in F(z).$$

We then have that

$$p(z) \cdot \psi(z) - p(z) \cdot \lambda(z)e = p_1(z) > 0.$$

Repeating the argument for every coordinate $p_i(z)$ of $p(z)$ shows that $p(z) \gg 0$. We can now scale the vector $p(z)$ without loss of generality to achieve $p(z) \in \mathrm{int}S^{\ell-1}$. Condition (i) holds for all $z$ for which $A^\star(z)$ is empty, including $y$, because otherwise $p(z)$ could not be a supporting hyperplane to $F(z)$ at $\lambda(z)e$. Condition (ii) holds because $\lambda(z)e \geq 0$. Condition (iii) holds by construction. Thus the Claim is shown.

For $z$ such that $A^\star(z) \neq \emptyset$ or $A \setminus A^{**}(z) = \emptyset$ we let $p(z) \in \mathrm{int}S^{\ell-1}$, and for all $a \in A \setminus A^\star(z)$, we choose $x(a,z)$ such that (i) holds. (It is obvious that such $x(a,z)$ exist, following a similar argument as used in the proof of the Claim.) For $a \in A^\star(z)$ let $x(a,z) = 0$.
We have thus defined a function $p\colon \mathcal{Y} \to \mathrm{int}S^{\ell-1}$. We now construct the valuation function. For this construction we will let

$$p(z)F(z) := p(z) \cdot \left[\sum_{a \in A} x(a,z) + c(z) - \overline{w}\right].$$

Define a parameter $\delta(z) \in \mathbb{R}_{++}$ when $A^{**}(z) \neq \emptyset$ as follows. If $p(z)F(z) > 0$, let $\delta(z) > 0$ be such that $\delta(z) < (|A^{**}(z)|)^{-1}p(z)F(z)$. If $p(z)F(z) \leq 0$, $A^\star(z) \neq \emptyset$. (Otherwise, since $A^{**}(z) \neq \emptyset$, there would be an allocation $(g,z)$ that Pareto dominates $(f,y)$.) In that case let $\delta(z) < (|A^{**}(z)|)^{-1} \sum_{a \in A^\star(z)} p(z) \cdot w(a)$. The latter bound is positive because $p(z) \gg 0$ and we have assumed that $w(a) \neq 0$ for all $a \in A$.
Let $V(\cdot,z)$ be defined by

$$V(a,z) := \begin{cases} p(z) \cdot w(a) - p(z) \cdot x(a,z), & \text{if } a \in A \setminus [A^\star(z) \cup A^{**}(z)], \\ p(z) \cdot w(a) - p(z) \cdot x(a,z) + \delta(z), & \text{if } a \in A^{**}(z), \\ \min\{0, (|A^\star(z)|)^{-1}p(z)F(z)\}, & \text{if } a \in A^\star(z). \end{cases}$$

We now check the conditions for a valuation equilibrium.

CONDITION (I)
By definition, $A^\star(y) = A^{**}(y) = \emptyset$, so $V(a,y) = p(y) \cdot [w(a) - f(a)]$. Hence

$$\sum_{a \in A} V(a,y) - p(y) \cdot c(y) = p(y) \cdot \left[ \sum_{a \in A} w(a) - \sum_{a \in A} f(a) - c(y) \right] = 0,$$

by the feasibility of the allocation $(f, y)$.

CONDITION (III)

Note that $x(a,z) = 0$ if $a \in A^{\star}(z)$ or $a \in A^{\star\star}(z)$. We have

$$\begin{aligned}
\sum_{a \in A} V(a,z) - p(z) \cdot c(z) &= \\
&= p(z) \cdot \sum_{a \in A \setminus A^{\star}(z)} w(a) - p(z) \cdot \sum_{a \in A} x(a,z) + |A^{\star\star}(z)| \delta(z) \\
&\quad + \min\{0, p(z) F(z)\} - p(z) \cdot c(z) \\
&= p(z) \cdot \sum_{a \in A} [w(a) - x(a,z)] - p(z) \cdot \sum_{a \in A^{\star}(z)} w(a) + |A^{\star\star}(z)| \delta(z) \\
&\quad + \min\{0, p(z) F(z)\} - p(z) \cdot c(z) \\
&= -p(z) F(z) - p(z) \cdot \sum_{a \in A^{\star}(z)} w(a) + |A^{\star\star}(z)| \delta(z) + \min\{0, p(z) F(z)\} < 0,
\end{aligned}$$

by the definition of $\delta(z)$.

CONDITION (II)

We have $p(z) \gg 0$ by construction. For $a \in A^{\star}(z)$, there is no $(g, z)$ preferred to $(f(a), y)$, so the condition is satisfied trivially. For $a \in A^{\star\star}(z)$, $V(a,z) > p(z) \cdot w(a) = p(z) \cdot w(a) - p(z) \cdot x(a,z)$. Suppose there exists $g \in \mathbb{R}^{\ell}_{+}$ such that $U_a(g,z) > U_a(f(a),y)$. Then $p(z) \cdot g \geq 0$ and so, since $x(a,z) = 0$, $p(z) \cdot g + V(a,z) \geq p(z) \cdot x(a,z) + V(a,z) > p(z) \cdot w(a)$, which proves that the required condition holds.

For $a \in A \setminus [A^{\star}(z) \cup A^{\star\star}(z)]$, suppose that there exists $g \in \mathbb{R}^{\ell}_{+}$ such that $U_a(g,z) > U_a(f(a),y)$. By $a \notin A^{\star\star}(z)$, $U_a(0,z) \leq U_a(f(a),y) < U_a(g,z)$, hence $g > 0$. Suppose, without loss of generality, that $g_1 > 0$. Let $e^1$ denote the first unit vector in $\mathbb{R}^{\ell}$. By the continuity of $U_a(\cdot, z)$, there exists $\epsilon > 0$ such that $U_a(g - \epsilon e^1, z) > U_a(f(a),y)$, implying that $p(z) \cdot g > p(z) \cdot (g - \epsilon e^1) \geq p(z) \cdot x(a,z)$, where the first inequality follows because $p(z) \gg 0$ and the second one by the construction of $x(a,z)$. But then we have $p(z) \cdot g + V(a,z) > p(z) \cdot x(a,z) + V(a,z) = p(z) \cdot w(a)$, which proves that the required condition holds. □

The second welfare theorem for valuation equilibrium differs from the second welfare theorem for competitive equilibrium in exchange economies in that we do not need to choose different endowments in order to support different Pareto optima as equilibria. Since the valuation prices can serve the purpose of transfering endowments among agents, the same endowments can be used for different Pareto optima. Thus there is little distinction between proving the second welfare theorem and proving that such equilibria exist. This is summarized in the following corollary.

**Corollary 2.4** *Consider an economy such that for every agent $a \in A$ the utility function $U_a$ is continuous, quasi-concave and strictly monotone on $\mathbb{R}^\ell_+$, and $w(a) > 0$ for all $a \in A$. Then there exists a valuation equilibrium.*

PROOF

By the previous theorem every Pareto efficient allocation can be supported as a valuation equilibrium with endowments $w(a) > 0$ fixed in advance. Thus it suffices to show that there exists a Pareto efficient allocation. But this follows because utility functions are continuous, the aggregate endowment is finite, and the number of agents and potential public projects are finite. □

# References

DIAMANTARAS, D. AND R.P. GILLES (1994), "The Pure Theory of Public Goods: Efficiency, Decentralization, and the Core," *Working Paper E94-01*, Department of Economics, Virginia Tech, Blacksburg.

MANNING, J. (1993), "Efficiency in Economies with Jurisdictions and Public Projects", mimeo, University of New South Wales, Kensington, Australia.

MAS-COLELL, A. (1980), "Efficiency and Decentralization in the Pure Theory of Public Goods", *Quarterly Journal of Economics* 94, 625–641.

ROCKAFELLAR, R.T. (1970), *Convex Analysis*, Princeton University Press, Princeton.

SCOTCHMER, S. (1994), "Concurrence et Biens Publics", *Annales d'Economie et de Statistique* 33, 158–186, reprinted as "Public Goods and the Invisible Hand", J. Quigley and E. Smolensky, eds, *Modern Public Finance,* Harvard University Press (Cambridge) 1994.

SCOTCHMER, S. AND M. WOODERS (1987), "Competitive Equilibrium and the Core in Club Economies with Nonanonymous Crowding", mimeo, University of California, Berkeley.

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