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BAYESIAN MULTIVARIATE EXOGENEITY ANALYSIS: AN APPLICATION TO A UK MONEY DEMAND EQUATION

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Bayesian Multivariate Exogeneity Analysis: An Application to a UK Money Demand Equation

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Abstract

We propose a general Bayesian Instrumental Variables technique for investigating the weak exogeneity of a *set* of variables relative to the coefficients of a single structural equation of interest. We use a so-called Recursive Extended Natural Conjugate prior distribution for the nuisance parameters, which consist of the coefficients of the auxiliary instrumental variables equations. Such priors can accommodate arbitrary linear restrictions so that a specific (parsimonious) set of instruments can be associated with each individual variable the exogeneity of which is under scrutiny. A (conditional) application of our methodology to a UK money demand equation leads to the conclusion that price and interest rate are jointly weakly exogenous throughout the sample period.

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1. Introduction

It is often the case that an econometrician's focus of interest is a specific "structural" equation, such as a demand for money equation in the context of the application we shall discuss below. If all current variables in that equation, except one, are assumed to be weakly exogenous, following the terminology in Engle et al. (1983), then standard least squares estimation techniques yield efficient estimates of its coefficients. However, violation of these (implicit) exogeneity assumptions may have severe consequences: inference on the coefficients of interest will be distorted and, often more importantly, shifts will be induced in the *estimated* coefficients as soon as the distribution of the erroneously assumed exogenous variables changes, even though the underlying *structural* coefficients may be invariant against such changes. See e.g. Hendry and Richard (1983, subsection 2.2) or Engle and Hendry (1989) for further discussion of this important issue. Hence the need for operational techniques whereby the exogeneity of key variables can formally be investigated.

A Bayesian "instrumental variables" approach to "testing" the exogeneity of a single variable has been proposed by Lubrano et al. (1986). Multivariate extensions of that approach are conceptually straightforward but the development of flexible and efficient computer programs for routine investigation of multivariate exogeneity assumptions is far more demanding. The object of the present paper is to report on the current status of this line of research. Our presentation builds upon recent analytical and numerical developments and yet aims at minimizing technical discussions. More technically oriented references are provided in the course of the paper. An application to a U.K. money demand equation serves illustrating the flexibility of the proposed techniques.

The paper is organized as follows: In section 2 we describe the class of models under consideration and survey the concepts and techniques which are currently applicable within this context, whereas in subsection 2.3 we derive the baseline formulas for a general Bayesian "instrumental variables" exogeneity analysis. In section 3 we analyse a U.K. money demand equation and present a (partial) application of the techniques under development. Conclusions and avenues of further research are discussed in section 4.

2. The Statistical Framework

2.1 The Model

Linear Dynamic Models have received much attention in the econometric literature. Useful surveys by Granger and Watson (1984), Hendry et al. (1984) and Geweke (1984) can be found in the Handbook of Econometrics. See also the papers by Richard (1984) and Florens et al. (1987) for presentations that are directly relevant to the object of our paper. The latter, in particular, discusses Limited Information analysis of dynamic models at a level of generality which sets in prospect a number of key modelling assumptions such as cuts, innovations, exogeneity or noncausality. We shall adopt here a mode of presentation which is more in line with conventional econometric formulations and largely focuses on computational issues.

Let the equation of interest be the following:

$$\beta' y_t + \gamma' x_t = u_t \qquad u_t \sim IN(0, \sigma^2) \qquad t : 1 \rightarrow T \tag{1}$$

where $y_t \epsilon \mathbb{R}^n$ is a vectors of "endogenous" variables (including at this stage of the discussion all variables whose potential exogeneity is under scrutiny), $x_t \epsilon \mathbb{R}^k$ is a vector of variables meant to characterize an information set I_t[1] which consists of current (weakly) exogenous variables and of the past history of all current variables in the model. Initial conditions are assumed to be known, though, at a higher level of generality they could be included in the list of unknown coefficients. β and γ are vectors of unknown coefficients and σ^2 is an unknown variance. Eventually $(\beta', \gamma', \sigma^2)$ has to be normalized and β and γ might be subject to exact (prior) restrictions as well, assumed to be linear, in the case of γ at least. For practical implementation such constraints ought to be explicited, if only in order to avoid conditionalization paradoxes, and (β, γ) would then be function of a vector of "free" coefficients. shall nevertheless keep using the notation associated with equation (1), as long as no ambiguity arises from such use. Throughout the rest of our discussion it is assumed that β, γ and σ^2 (or functions thereof) are the sole "parameters of interest".

Equation (1) is formally embedded in a sequential Linear Dynamic Model of the form

$$y_t | I_t \sim N_n(\eta_t, \Omega), \tag{2}$$

where η_t is a vector of conditional expectations and Ω an arbitrary symmetric positive definite matrix. Equation (1) is then reformulated as

$$\beta' \eta_t + \gamma' x_t = 0, \qquad t : 1 \to T \tag{3}$$

and formally defines an $(n-1)$-dimensional linear manifold for the vector η_t. Also $\sigma^2 = \beta' \Omega \beta$. As such, equation (3) includes $n-1$ "incidental" parameters essentially consisting of all but one components of η_t. Within an Instrumental Variables (IV) framework these incidental parameters are eliminated by means of $n-1$ additional linear relationships of the form

$$S' \eta_t = P' w_t, \tag{4}$$

where $w_t \epsilon \mathbb{R}^\ell$ is itself a selection of "instruments" from I_t[2], P is an $\ell \times (n-1)$ matrix of unknown coefficients and S is an $n \times (n-1)$ matrix of known constants, such as a selection matrix. Let v_t denote the "residual" associated with equation(4),

$$S' y_t - P' w_t = v_t, \ v_t | I_t \sim N_{n-1}(0, S' \Omega S). \tag{5}$$

Equations (4) and (5) incorporate the important assumption that v_t is an innovation relative to I_t - see footnote 2 - which is necessary to validate our exogeneity analysis in section 2.2. Let us now define

$$Q_\beta = (\beta \ : \ S). \tag{6}$$

It is assumed throughout our analysis that Q_β is non-singular (almost surely in β). With the introduction of the instrumental variables equations in (4), we have effectively eliminated all incidental parameters. Nevertheless, we still have to deal with a potentially large dimensional set of "nuisance" parameters, consisting of the "free" elements of P and of the variance and covariance matrices $S' \Omega S$ and $S' \Omega \beta$.

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[1] More specifically, it is assumed that, conditionally on x_t, u_t is independent of I_t, hence that u_t is an "innovation" relative to I_t.

[2] In practice, w_t might include variables that were not initially included in the information set I_t associated with equation (1). In such a case I_t has to be extended in such a way that it includes the additional "instruments" as well as their lagged values.

The matrix Ω, or bi-linear transformations thereof, can be handled by means of an Inverted-Wishart prior density, whether conditionally or unconditionally with respect to the exogeneity assumptions to be introduced below. Experience suggests that the class of Inverted-Wishart prior densities is flexible enough for most practical purposes. Furthermore, if the need arises it can easily be extended by means of well-known recursive factorizations, as documented e.g. in Richard and Steel (1988). Also, operational random number generators are available for Inverted-Wishart distributions and recursive generalizations thereof which can, therefore, be used as "importance functions" for Monte Carlo numerical integration. This issue is discussed further in subsection 2.3 below.

The treatment of P raises more problems. Conventional Bayesian "Limited Information" techniques, as surveyed e.g. in Drèze and Richard (1983), proceed under the assumptions that: (i) the set x_t of "predetermined" variables in equation (1) is a subset of the set of instrumental variables in (4), and; (ii) P is left completely unrestricted under either a Natural Conjugate (NC) Matricvariate Normal prior or a "Non-informative" limiting version thereof. Richard (1984) extends this LI analysis to the case where P is subjected to linear restrictions that preserve the matricvariate structure of the NC prior on P, i.e. restrictions that apply in exactly the same way to *all* rows or columns of P. In particular, components of x_t can be excluded from w_t at a reasonable cost of computation. Nevertheless, as initially noted by Rothenberg (1963), the matricvariate structure of NC priors on P remains far too restrictive for most practical purposes. In the context of the model under consideration, the variables in $S'y_t$ may depend on specific instrumental variables; Noncausality assumptions, in the sense of Granger (1969) may widely differ from one instrumental equation to another; the collection of *all* instrumental variables that are needed to characterize $S'y_t$ may be large relative to sample size, a.s.o. Regarding inference on exogeneity in particular, the use of a large unrestricted set of instruments in each equation of the system (4) will often generate "overfitting" and, hence, severely distort sample evidence.

In a recent paper, Richard and Steel (1988) have proposed a so-called Recursive Extended Natural Congugate (RENC) prior density for Seemingly Unrelated Regression Equations (SURE) models. This class of priors is flexible enough to accommodate an *arbitrary* matrix of second order prior moments for P and yet is numerically tractable for SURE models of moderate size, currently up to five or six equations. In particular, RENC prior distributions are fully compatible with arbitrary *exact* linear restrictions on P. It follows that under an RENC prior for P, the selection of instrumental variables can be specific to each component in $S'y_t$.

In addition to the set of IV equations in (5), to which the application of an RENC prior would be straightforward, our current model also includes the structural equation (1). A fully efficient application of the RENC framework ought to exploit the fact that, *conditionally* on β, the n equations in (1) and (5) jointly define a SURE system which can be written as

$$Q'_\beta y_t = \Pi' x_t + \epsilon_t \qquad \epsilon_t | I_t \sim N_n(0, Q'_\beta \Omega Q_\beta) \tag{7}$$

with

$$\Pi = (\gamma \; : \; P) \tag{8}$$

and where, for the ease of notation, x_t and w_t are now conflated with each other without loss of generality given that the RENC framework can accommodate arbitrary linear restrictions on Π - conditionally on β. In subsection 2.3 below, we shall specifically discuss how the RENC framework can be adapted to the system (7) with special emphasis on inference on the parameters $(\beta, \gamma, \sigma^2)$ and on the exogeneity status of $S'y_t$ or components thereof.

2.2 Weak Exogeneity

The concept of exogeneity we use is that of weak exogeneity, as defined by Engle et al. (1983) within a sampling theory framework. Bayesian extensions of this concept essentially require replacing the notion of "variation freeness" (among subsets of coefficients) by that of "prior independence". See e.g. Florens et al. (1989) or, in a bivariate framework, Lubrano et al. (1986). The key issue is whether or not the posterior density for the parameters of interest $(\beta, \gamma, \sigma^2)$ simplifies into the product of their prior density and the "marginalized likelihood" function associated with the (sequential) conditional distribution of $\beta' y_t$ given $S' y_t$ (and I_t).[3]

Straightforward (sequential) factorization of the joint density of $(Q'_\beta y_t | I_t)$, as characterized by equations (6) and (7), reveals that if:

(i)
$$Cov(\beta' y_t,\ S' y_t | I_t) = \beta' \Omega S = 0 \tag{9}$$

(ii) $(\beta, \gamma, \sigma^2)$ and $(P, S'\Omega S)$ are a priori independent,

then $(\beta, \gamma, \sigma^2)$ and $(P, S'\Omega S)$ are also a posteriori independent and the posterior density of $(\beta, \gamma, \sigma^2)$ is given by

$$D(\beta, \gamma, \sigma^2 | \{y_t\}) \propto \| Q_\beta \|^T \, D(\beta, \gamma, \sigma^2) \prod_{t=1}^{T} f_N(\beta' y_t + \gamma' x_t | 0, \sigma^2), \tag{10}$$

where $D(\cdot)$ and $f_N(\cdot|\mu, \sigma^2)$ respectively denote an arbitrary prior density and the density function of a univariate random variable with mean μ and variance σ^2. See e.g. Florens et al. (1987, subsection 3.3) for a general derivation of formula (10) or Lubrano et al. (1986, subsection 3.1) for a simpler - though easily generalizable-bivariate presentation.

In words, conditions (i) and (ii) are sufficient for the weak exogeneity of $S' y_t$ for the parameters of interest $(\beta, \gamma, \sigma^2)$ and condition (i) in particular wil be the focus of our analysis.

Within a conventional Limited Information framework, whereby the prior on P is Natural Conjugate, the (weak) exogeneity of $S' y_t$ entails that of any of its subvectors. This need not be the case under an RENC prior, as we now discuss. Let S and P be partitioned conformably with each other

$$S = (S_a \,:\, S_b) \qquad P = (P_a \,:\, P_b). \tag{11}$$

Under the assumption that $\beta' \Omega S_b = 0$, we can factorize the joint distribution of $Q'_\beta y_t | I_t$ as follows:

$$\left. \begin{matrix} \beta' y_t \\ S'_a y_t \end{matrix} \right| S'_b y_t, I_t \sim N\left(\begin{pmatrix} \gamma' x_t \\ \Delta'_{ba} S'_b y_t + P'_{a \cdot b} x_t \end{pmatrix}, \begin{pmatrix} \beta' \Omega \beta & \beta' \Omega S_a \\ S'_a \Omega \beta & \Sigma_{aa \cdot b} \end{pmatrix} \right) \tag{12}$$

$$S'_b y_t | I_t \sim N(P'_b x_t, \Sigma_{bb}) \tag{13}$$

[3] It is notationally - and often conceptually - convenient to discuss the exogeneity of $S' y_t$ in a way which does not critically depend on the normalization rule adopted for $(\beta, \gamma, \sigma^2)$. In practice, it will often be the case that a component of β, say the first one, is set equal to one and that the corresponding row in S is set equal to zero. In such a case $|Q_\beta| = 1$ and the marginalized likelihood used in (10) below represents the conditional distribution of the first component of y_t given all the others and I_t.

where $\Sigma_{ij} = S_i'\Omega S_j, i, j, \epsilon\{a, b\}$ and

$$\Delta_{ba} = \Sigma_{bb}^{-1}\Sigma_{ba} \qquad \Sigma_{aa\cdot b} = \Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba} \tag{14}$$

$$P_{a\cdot b} = P_a - P_b\Delta_{ba}. \tag{15}$$

It is well-known that under an Inverted-Wishart prior density for Ω and, hence, for Σ, the parameters $(\Delta_{ba}, \Sigma_{aa\cdot b})$ and Σ_{bb} are independent. This property remains valid under recursive generalizations of the Inverted-Wishart distribution. However, under a general RENC prior for P, the parameters $P_{a\cdot b}$ and P_b are typically *not* independent of each other in which case the condition that $S_b'\Omega\beta = 0$ is not sufficient *on its own* for weak exogeneity of $S_b'y_t$. An additional condition has to be introduced, namely that

(iii) $P_{a\cdot b}$ and P_b are a priori independent.

Condition (iii) is not easily verifiable and minimal sufficient conditions for it to hold may be highly specific to each individual model. A more operational but quite stronger condition would be

(iv .a) $\Delta_{ba} = 0$, and

(iv .b) P_a and P_b are a priori independent.

That condition (iv) is not necessary is best illustrated by the fact that condition (iii) holds under an NC prior for P, independently of whether or not condition (iv) applies.

In conclusion to this discussion, the choice of an RENC prior for P will often be motivated by the need to implement parsimony on P in situations where sample size is limited (relative to the total number of instrumental variables). Since, however, RENC prior distributions typically link together the parameters P_b and $P_{a\cdot b}$, it follows that the parameters of interest $(\beta, \gamma, \sigma^2)$ are no longer independent of the nuisance parameters (P_b, Σ_{bb}), even though $S_b'\Omega\beta = 0$. If, in addition, $S_a'\Omega\beta = 0$, then an additional factorization of the distribution in (12) yields the result that $S'y_t$ is *jointly* exogenous for $(\beta, \gamma, \sigma^2)$ although $S_b'y_t$ *on its own* is not. Note finally that the link between P_b and $(\beta, \gamma, \sigma^2)$ is "indirect" in the sense that they are independent of each other, conditionally on $P_{a\cdot b}$. It might be that except for pathological cases, a violation of condition (iii) would have no major impact on the posterior density of $(\beta, \gamma, \sigma^2)$ as long as $S_b'\Omega\beta = 0$.[4]

2.3 Implementation

It follows from the above discussion that the covariances $\beta'\Omega S$ are of primary concern as soon as the exogeneity of $S'y_t$ is under scrutiny. As usual within the algebra of Inverted-Wishart distributions, inference is conducted in terms of a set of regression coeffients associated with these covariances. Two such sets are available depending on whether we consider the regression of $\beta'y_t$ on $S'y_t$ (equivalently, of u_t on v_t) or that of $S'y_t$ on $\beta'y_t$ (equivalently, of v_t on u_t).

The regression of $\beta'y_t$ on $S'y_t$ is associated with the natural factorization of the density of $Q_\beta'y_t|I_t$ which leads to the very notion of weak exogeneity (see, in particular, equations (12) and

[4] Note, in particular, that the first equation in the conditional submodel (12) is the structural equation of interest itself. In the words of Engle et al. (1983), the condition $S_b'\Omega\beta$ is sufficient for the predeterminedness of $S_b'y_t$ in equation (1) but not for its weak exogeneity.

(13) which are relative to $S_b'y_t$). Hence it is hardly surprising that much of the early work on exogeneity testing focuses on that set of regression coefficients along the lines of the discussion in Wu (1973). Nevertheless, as discussed e.g. in Richard (1984) or Lubrano et al. (1986), the "reverse" regression of $S'y_t$ on $\beta'y_t$ leads to more operational factorizations of the likelihood function and, hence, to more tractable expressions for the posterior densities of interest. We shall accordingly conduct our analysis in terms of the coefficients of that "reverse" regression, which are given by

$$\lambda = S'\Omega\beta\,(\beta'\Omega\beta)^{-1}. \tag{16}$$

We note in passing that, as discussed in Lubrano et al. (1986), λ is a *bounded* function of β (while the regression coefficients of $\beta'y_t$ on $S'y_t$ are linear and, hence, unbounded in β). It follows that the existence of prior and posterior moments for λ does *not* require the existence of corresponding moments for β. This is not a purely casual issue. As discussed e.g. in Drèze and Richard (1983), we might wish to impose the requirement that the prior density of β be invariant with respect to the choice of a normalization rule in equation (1). Densities which satisfy that requirement, among which the Cauchy and certain types of so-called 1-1 poly-t densities, have no moments (as long as the origin belongs to their support).

Inference on λ may be conducted in several often complementary ways. A "direct" approach consists in selecting a prior density for λ which is centered around $\lambda = 0$, the hypothesis of concern, and in examining whether or not the posterior density of λ has significantly shifted away from the origin. This is the approach used by Lubrano et al. (1986) and which we shall also follow here. A straightforward alternative to that approach would consist in attaching a non-zero probability to the point hypothesis that $\lambda = 0$ and in computing the corresponding posterior odds. See e.g. Zellner (1971) or Leamer (1978) for the derivation of posterior odds for a variety of hypotheses of interest in the context of regression analysis. "Indirect" approaches are also available. In particular, Florens and Mouchart (1988) propose Bayesian extensions of the classical notion of specification tests, as defined by Hausman (1978). These extensions are based upon explicit comparisons of the marginal and conditional (relative to the hypothesis $\lambda = 0$) posterior densities for the coefficients of interest which, in the context of our analysis are β, γ and σ^2. Practical implementations of this conceptually attractive idea within our general instrumental variables framework belong to our research agenda.

Having discussed all the components of a general Bayesian IV approach to exogeneity testing, we now specifically discuss their software implementation. Let θ denote the set of all coefficients other than β. Loosely speaking, θ includes (the "free" elements of) γ, P and Ω - or whatever transformation of these coefficients is introduced in the course of the RENC analysis. A natural implementation of our analysis proceeds in two steps.

Step 1: Conditionally on β, we can apply the RENC techniques described in Richard and Steel (1988) to the SURE system (7).

The main input at this stage of the analysis consists of a prior density for θ, which, in all generality, can be conditional on β. Interestingly enough the decision on whether or not θ and β ought to be priori independent of each other has little implication on the overall cost of computing the posterior densities and, hence, may largely be considered on its own merits. This issue of independence is of special interest if we wish to specify a prior density for λ which is centered around the prior belief that $\lambda = 0$, since λ is a function of both θ and β. Lubrano et al. (1978, subsections 3.3 and 3.4) specifically discuss ways of selecting prior densities on (θ, β) subject to the condition that $E(\lambda) = 0$, when λ is a scalar. Multivariate extensions of their analysis are essentially straightforward. Alternatively, we can impose the stronger requirement that $E(\lambda|\beta) = 0, \beta$ - almost surely.

The main output of our conditional RENC analysis consists of:

(i) Conditional posterior densities and moments for the coefficients of interest (γ, σ^2) and also for λ;

(ii) A "marginalized" likelihood function for β, which is given by

$$L_{\bullet}(\beta; y) = \int_{\Theta} L(\beta, \theta; y)\, D(\theta|\beta)\, d\beta, \tag{17}$$

where $L(\beta, \theta; y)$ denotes the likelihood function associated with (7) and $D(\theta|\beta)$ the prior density of θ conditionally on β.

Step 2: A kernel of the posterior density of β is given by

$$D(\beta|y) \propto L_{\bullet}(\beta; y)\, D(\beta), \tag{18}$$

where $D(\beta)$ is the prior density of β. That posterior density is of interest on its own for inference on β, but is also needed for marginalizing *w.r.t.* β the conditional results obtained in step 1. For example, the unconditional posterior density of λ is

$$D(\lambda|y) = \int_{B} D(\lambda|\beta, y)\, D(\beta|y)\, d\beta, \tag{19}$$

where B denotes the support of the posterior density of β. The integral of the posterior kernel (18) is not known analytically. All integrations relative to β can be evaluated by means of Monte Carlo procedures with importance sampling. An obvious choice for the importance sampling distribution would be the posterior distribution of β, as derived under a Limited Information framework (i.e. under a NC prior for P and Ω), since the latter is a so-called poly-t distribution for which there now exist efficient random number generators. See Drèze and Richard (1983) or Bauwens and Richard (1985) for technical details.

At the time this paper is being written step 1 is fully operational but we have yet to complete the software implementation of step 2. In fact, though the sequence we have just described is quite natural for expository purposes, there seem to exist numerically more efficient alternatives whose implementation, however, is somewhat more demanding. In short, the empirical findings in Richard and Steel (1988) unequivocally indicate that major efficiency gains are achieved when the bulk of the analysis - whether analytical or numerical - is conducted *conditionally on* Ω and the final step of computation consist of marginalization *w.r.t.* Ω under an Inverted-Wishart importance function. Heuristically, (marginal) inference on the structural coefficients β and γ is highly sensitive to a number of problems among which lack of "qualitative" identification (due e.g. to multicollinearity among included and omitted regressors). It follows that the posterior densities of these coefficients may be quite "ill-behaved" and severe skewness or even bimodality are not unheard of. In contrast, the posterior distribution of the coefficients in Ω, which measure the overall fit of the equations and not the identification of individual regression coefficients, is more robust against such problems (we have yet to find a bimodal posterior distribution for Ω!) and their evaluation by Monte Carlo procedures, based on Inverted- Wishart importance sampling distributions, seems to be remarkably efficient. Furthermore, conditionally on Ω, inference on β, γ and P is largely analytical or numerically easy to control. The RENC techniques described in Richard and Steel (1988) take full advantage of these findings. Their extension to our general instrumental variables framework necessitates inverting the order of integration with respect to β and Ω and generates additional technicalities relative to the simpler but less efficient procedure we have outlined in formulas (17) - (19).

3. A U.K. Money Demand Equation

3.1 Introduction

We now apply the techniques we have just described to a money demand equation for the UK. The background to our own study is found in Lubrano et al. (1986) (hereafter, LPR). Institutional details and a (single variable) exogeneity analysis of interest rate alone are found in LPR. In summary, LPR find that the impact coefficient of interest rate changes sign (from negative to positive) with the introduction in October 1971 of the set of measures known as Competition and Credit Control (CCC);[5] that the long term demand for money is stable across the change of regimes; that long term adjustments towards equilibrium are slow; that interest rate clearly is exogenous in the post-CCC regime and that its exogeneity cannot be rejected in the pre-CCC regime; and, finally, that money does not Granger (1969) cause interest rate in the pre-CCC regime.

Our current analysis extends that of LPR in several directions:

(i) First and foremost, we shall investigate the joint exogeneity of price and interest rate;

(ii) The data used in LPR cover the period 1961(iv) - 1981(ii) and, hence, net of initial conditions and forecasts include only 35 observations for each regime, which is not enough to conduct independent specification searches. Our data set covers the period 1955(i) - 1986(ii) with some minor revisions. The availability of this larger data set enables us to conduct independent specification searches for each regime individually. It ought to be mentioned here that CCC was gradually abolished and came to an end on 20th August 1981. This raises the possibility of an additional structural break around 1981(ii). All our post - 1971 equations have been estimated on the periods 1971(iv) - 1981(ii) and 1971(iv) - 1986(ii). The results being essentially the same over the two periods we shall only report those obtained for the longer period.

(iii) A growing part of the personal sector monetary aggregate M3, whose precise definition is found in Appendix A, is interest bearing. While LPR model disequilibrium feedbacks in terms of a log-level Error Correction Mechanism (ECM), Hendry (1987) suggests that ECM factors on interest bearing aggregates ought to be expressed in levels (leaving the short term components of the equation unchanged). Both versions will be used for the post-CCC regime as a substantial (and increasing) fraction of M3 is interest bearing during that period.

The variables used in our analysis are:

M: the M3 personal sector monetary aggregate;

Y: real personal disposable income;

P: the deflator of Y;

R: the Local Authorities short term interest rate;

U: the unemployment rate;

B: the level of real official reserves.

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[5] In short, the pre-CCC regime was one of direct control of short term interest rates, while the post-CCC regime is one in which control over the monetary aggregates is attempted through the free operation of market forces.

Definitions and data sources are given in Appendix A. The data are quarterly and seasonally unadjusted. The notation is that used in LPR: MP for $(M/P)_t$, MPY for $(M/PY)_t$, D for the difference operator (Δ when conventional notation is used), trailing i for the i-th lag operator, D_i for the i-th difference operator (Δ_i), DD for the squared difference operator (Δ^2), L for natural logarithms (ln), R for $100 + R_t$ and Q_i for the i-th quarter seasonal dummy. Periods A and B cover 1955(i) - 1971(iii) and 1971(iv) - 1986(ii) respectively, including initial conditions, whereas, for purposes of comparison, periods A1 and B1 will denote 1961(iv) - 1971(iii) and 1971(iv) - 1981(ii), respectively.

The specification search is conducted along the principles described in Hendry and Richard (1982,1983) and using the software PC-GIVE, documented in Hendry (1989). Tests for parameter constancy, forecast accuracy, mean innovation, autocorrelation, heteroskedasticity, autoregressive conditional heteroskedasticity, normality and the validity of linear restrictions are extensively used throughout the analysis. The test statistics are listed in Appendix B together with references (one degree of freedom test statistics are asymptotically chi-squared, two degrees of freedom test statistics are approximately F).

3.2 The Demand for Money

The outcome of our specification search for the demand for money in each subperiod is presented in Table 1 (constants and seasonal dummies are omitted, values in parentheses are asymptotic standard deviations, the test statistic are those listed in Appendix B together with degrees of freedom, one for χ^2 and two for F test statistics). Graphs of the actual and fitted values of DLM are found in Figures 1 and 2. While *LPR* used *DLMP* as the dependent variable we use instead *DLM*.[6]

On the overall our results agree with those obtained by *LPR* which, as already mentioned, are based on a substantially shorter sample period. The long term coefficients, which are reported in Table 2, are in complete agreement though clearly not estimated with much precision especially in period B. The short term dynamics differ in several respects, the most significant one being that the interest rate coefficient in period A has changed sign and is now positive, though not significantly so. In fact, the (impact) coefficient of interest rate is now positive and essentially constant throughout the sample period.[7] The long term elasticity of interest rates is negative as expected.

A second important change is that in period A, our ECM coefficient is more significant and adjustments towards long term equilibrium solutions are generally faster. Parameter constancy within period A is quite satisfactory judging by the statistics which are reported in Table 1 as well as by the one step-ahead Chow tests in Figure 3.

We note that in period B the ECM coefficient, whether in log or level form, remains quite small and not very significant with the consequence that long term elasticities are not well determined.

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[6] In doing so we draw a clear distinction between the endogenous variable DLM and the potentially exogenous one DLP. The two formulations are nevertheless equivalent since $DLMP = DLM - DLP$ and DLP is included in the list of regressors.

[7] The fact that the initial impact of an unexpected rise in interest rates is to increase the demand for money is by no means as counterintuitive as one might think. The private sector has substantial liabilities and the first effect of a rise in interest rate may well be an increase in the amount of money required for covering the interest charges. Note, in particular, that mortages in the UK have variable interest rates.

Clearly, more work is needed in order to obtain a more satisfactory specification of the money demand equation in period B, assuming one exists,[8] but this goes beyond the objectives of our present paper. In line with the conclusions we draw below it is comforting to know that additional investigations can proceed under the working assumption that price and interest rates are weakly exogenous.

3.3 The Instrumental Variables Equations

Though economic theory offers some guidelines for the selection of variables to be included in the price and interest rate equations, it tells us very little regarding their specific functional forms. Hence, beyond the initial selection of (current and lagged) variables entering the unrestricted versions of these two equations, we shall select their final parsimonious versions largely on purely statistical considerations (diagnostic tests). After all, these equations are of no interest to us and our sole preoccupation at this stage of our analysis is to estimate their innovation components for the purpose of drawing inference on the coefficient λ in equation (16).

The results are reported in Table 3 for the price equation and in Table 4 for the interest equation. The only statistic that is not satisfactory is the test for Normality (η_{11}) in the interest rate equation for period A. A number of surges in interest rates [in particular for 1961(iii), 1964(iv), 1966(iii) and 1967(iv)] is not accounted for by the model. Given a structural explanation, dummies for these observations could be considered. None of the other diagnostics, however, seem to indicate any problems. The retained specifications make reasonable sense. They are quite parsimonious and yet the percentage point standard deviations range between 0.5% and 1.1%. Note in passing that the three equations constituting our final model differ completely from each other in their selection of regressors. Hence the relevance of our RENC approach. The application of conventional Bayesian Limited Information techniques would roughly require trebling the number of coefficients in P.

3.4 RENC prior densities

As already discussed in subsection 2.3, our empirical exogeneity analysis is conducted conditionally on β. We shall (partially) investigate the robustness of our findings by considering alternate choices of β. Conditionally on β, the system (7) constitutes a SURE system to which we can apply the RENC framework, selecting a prior density that suits the specific object of our analysis. The reader is referred to Richard and Steel (1988) for the technical details of an RENC implementation and we limit ourselves to discussing here the information content of the RENC prior densities we shall use. We mention in passing that RENC prior distributions are relative to a recursive reparameterization of $(\Pi, Q'_\beta \Omega Q_\beta)$ which can be selected in such a way that it includes λ, the key coefficient for our exogeneity analysis. It follows that RENC prior densities are ideally suited for the purpose of analysing the exogeneity structure of a single equation IV model.

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[8] The relative weakness of our specification from the perspective of economic theory should not overshadow the fact that it is statistically quite well behaved. It constitutes a parsimonious version of an unrestricted equation including five lagged values of all relevant variables (M, Y, P and R) and yet the standard deviation of its disturbance, expressed in percentage points, is of the order of 1,2% to 1,3%. It also passes a broad range of diagnostic test statistics and seems to be relatively invariant within regime B. Potential areas of investigation for future research are (i) the choice of the interest rate variable (LPR report unsuccessfully experimenting with other interest rates, including the own interest rate) or (ii) the explicit modelling of the finding that the fraction of $M3$ which bears interest has considerably changed over regime B.

We first briefly discuss the selection of our "baseline" RENC prior distribution. Its distinguishing feature, relative to a conventional NC prior, lies in the fact that it incorporates all the exclusion restrictions that have emerged from our specification search and, in particular, from our selection of instrumental variables.[9] For practical considerations our current software implementation only accommodates nondegenerate prior distributions though "diffuseness" can be achieved by the selection of a "large" value for the prior covariance matrix of the unconstrained elements of Π. In the case under consideration, prior variances are set equal to 100 and prior covariances to 50. These numbers can vary over a broad range with essentially no impact on the posterior densities.

As for all NC type prior distributions, the selection of the marginal prior distribution of the covariance matrix Ω requires attention. NC-type priors are given by the product of a conditional Normal density for the elements of Π given Ω and of a marginal density for Ω. In practice, however, prior assertions relative to Π typically are *unconditional* on Ω. It is, nevertheless, the *conditional* precision matrix of Π given Ω that essentially determines the weight of the prior information relative to the sample information. It is now well understood that the selection of a "non-informative" prior for Ω often generates unreasonably large relative precision matrices for Π given Ω and, hence, excessive weight for the prior on Π. See e.g. Richard (1973, p. 181) for a technical discussion of this issue within a univariate framework. In summary, we have to select a prior that will generate a "reasonable" weighted average between prior and sample information on Π and yet remains moderately informative in the absence of sharp prior information on Ω itself. Our current practice which is simple to implement and seems to achieve the compromise we are aiming for consists in selecting for the prior expectation of Ω a preassigned fraction of the *unconditional* sample covariance matrix of y_t, meant to reflect our crude beliefs relative to the overall "fit" of the model. See the discussion in Richard and Steel (1988, appendix D) for a more formal justification of this practice within a simpler univariate framework. Prior degrees of freedom, denoted by ν_0 below, are then selected in such a way that the implicit "hypothetical prior sample" size remains small relative to the actual sample size. Our practical experience suggests that posterior results typically are robust against ("moderate") deviations from our baseline prior and that the mix between prior and posterior information is reasonable. The results which follow are based on the prior expectation of Ω being set equal to 60% of the sample covariance matrix of y_t. Prior degrees of freedom are set equal to either 10 (largely uninformative) or 30 (moderately informative), the actual sample sizes being around 60.

Our baseline prior can easily be modified in such a way that it incorporates the prior belief

[9] We might equally relax these restrictions and replace them by probabilistic versions thereof. The burden of computation is not critically affected since, in either case, the resulting prior covariance structure is fundamentally incompatible with that of an NC prior density. We opted in favor of the "conditional" prior in order to avoid diluting sample information among an excessively large number of nuisance parameters.

We are fully aware of the fact that our procedure which consists of a preliminary ("pretest") specification search followed by a conditional Bayesian analysis is not fully consistent with a strict Bayesian perspective. A formal Bayesian analysis ought to proceed under a "grand" prior density covering all a priori acceptable specifications. It is, however, inapplicable in the current context where we have very little genuine prior information regarding, in particular, the (short term) lag structure of our equations so that the set of potentially acceptable specifications is enormous, while at the same time parsimony remains critical. We believe that our "pretest" procedure constitutes a reasonable compromise relative to a strictly Bayesian approach under a "grand" prior *and* a penalty function against non-parsimonious specifications.

that $S'y_t$ is weakly exogenous for $(\beta, \gamma, \sigma^2)$. Our so-called Weak Exogeneity (WE) prior deviates from our baseline prior in that :(i) $E(\lambda|\beta)$ is set equal to zero; and (ii) γ and P are assumed to be a priori independent.

3.5 Posterior distributions

Tables 5 to 8 regroup posterior means and standard deviations for γ and λ under various prior specifications, together with indicators of numerical accuracy. These four tables can be characterized as follows:

- Table 5 contains posterior results for period A under either the baseline prior or the WE prior, conditionally on the coefficients of DLR and DLP being set at their OLS estimated values, respectively 0.22 and 0.62;
- Table 6 contains the results of a similar analysis except that the coefficients of DLR and DLP are now set equal to -0.25 and 1.0 respectively. These values reflect the dogmatic prior belief that the coefficient of DLR ought to be negative and that price has no direct impact on the *real* quantity of money. More importantly in the context of our analysis, these values are meant to be instrumental in our assessment of the robustness of our findings relative to changes[10] in β;
- Tables 7 and 8 contain the results of a similar analysis for the *level* version of the demand for money equation in period B.

The tables include estimates of the *relative* error bounds ϵ_α associated with each individual posterior mean, measured in percentage points. As usual within the context of Monte Carlo numerical integration these measures are based on Central Limit Theorems. See e.g. Kloek and van Dijk (1978), Bauwens (1984) or, for a more formal presentation of existence conditions, Geweke (1989). These error bounds are known to be conservative[11] as it appears from the limited variation of the results between runs of computation corresponding to neighboring priors and, hence, to different simulations. We also report the relative error bound ϵ_{int} for the integrating constant of the posterior density. If anything, these error bounds are *amazingly* good given the low numbers of drawings on which they are based[12] and which are reported in the table for each run of computation.

Our tables also include posterior means for ρ, the correlation coefficient between the residuals of the price and interest rate equations, the role of which has been discussed in subsection 2.2.

Our main empirical findings can be summarized as follows:

(i) The posterior results are quite robust with respect to variations in the prior density (baseline versus WE, $\nu_0 = 10$ versus $\nu_0 = 30$) and to substantial changes in β;[13]

[10] Note that the changes we are considering are quite substantial and represent something in the order of two standard deviations, as estimated by OLS in Table 1.

[11] In particular, these bounds take no account of the typically high positive correlation between the estimates of the two components in the ratio of integrals that defines a posterior mean.

[12] As described in Richard and Steel (1988) we follow an iterative procedure for the progressive refinement of the importance function from which values of Ω are drawn. Initial calibration is based on a mere 100 drawings and only very few (≤ 3) iterations were required for the results obtained.

[13] The posterior inference on exogeneity is even essentially unchanged if we adopt the LPR specification instead for the money demand function. For the sake of brevity, these results are not reported here.

(ii) Sample evidence is largely in favor of the joint exogeneity of price and interest rate. It is only for period B and conditional on the alternate values of β (Table 8) that we find posterior expectations of λ_{DLP} (i.e. the element of λ corresponding to prices) that are around two standard deviations away from zero. Note, however, that these results are conditional on a value of β that would be given little weight in the posterior distribution of β, judging at least from the estimated OLS standard deviations. Furthermore, they are indicative of a substantial negative correlation between the price coefficient and λ_{DLP}, mimicking results found by Lubrano et al. (1986) in their univariate analysis of the exogeneity of interest rate. It follows from these two remarks that, relative to the *conditional* results which are reported in Table 8, the *unconditional* posterior mean of λ_{DLP} is likely to be closer to zero and its unconditional standard deviation substantially larger.

4. Conclusions

Two types of conclusions emerge from our analysis. Firstly, at the level of the application we have just discussed, we feel quite confident in concluding that price and interest rate are *jointly* weakly exogenous in both periods, even though our conclusion is based solely on a partial (conditional) analysis. We expect our findings to be fully confirmed by later and computationally far more demanding unconditional investigations. In consequence, additional searches towards a satisfactory specification of a UK demand for money equation can safely proceed under that bivariate exogeneity assumption and, hence, be based upon standard single equation techniques.

At a higher level of generality we find that the Bayesian IV approach we analyse here is quite promising. The analysis conditional on β is already fully operational and exhibits excellent numerical accuracy. Though its current test implementation is highly inefficient and, in particular, uses non "streamlined" APL routines, a typical run of computation of an IBM/PC-AT only requires a few hours. Based on our experience, a fully optimized software implementation ought to reduce that time by a very substantial factor. Hence, a fully *unconditional* analysis, which is conceptually straightforward in the light of the developments we have discussed here, ought to be feasible in the near future and we are actively working on its implementation.

A final point is worth mentioning even though it mainly concerns the issue of computational statistics. The present analysis confirms our earlier finding that our (RENC) approach which is based on importance sampling on Ω requires an amazingly low number of drawings and, henceforth, can achieve very substantial efficiency gains relative to more conventional approaches based on importance sampling on Π and which, in our own experience, may require many more drawings or may even fail to converge if the problem is "ill-behaved". In addition, the dimensionality of most applications greatly favors drawings in the space of Ω, which has only six dimensions here, as opposed to the 25 or 29 (depending on the period) unrestricted coefficients in Π. This finding opens a promising avenue of research towards the development of operational Bayesian procedures that would be applicable to systems of simultaneous equations of moderate size.

Appendix A: The Data Sources

The variables are given by:

1. Personal sector M3 holdings. Cumulated from the flow of funds accounts in BESA (1955-1973) and FS (1974-1981(2)), and from the financial transactions accounts in the BEQB (1981(3)-1987(2)). Consists of changes in:

   notes and coins

   deposits with banking sector:

   - Sterling sight
   - Sterling time
   - foreign currency.

2. Local authority 3 month deposit rate at last working day. From BESA (1955-1974), FS (1975-1981(2)), and BEQB (1981(3)-1987(3)).

3. Real personal disposable income in millions of pounds and 1980 prices. Source: ETAS 1987 (1955-1986(2)).

4. Total personal disposable income in millions of pounds and current prices. Source: ETAS 1987 (1955-1986(2)).

5. Total unemployed including school leavers in thousands. Source: ETAS 1987 (1955-1986(3)).

6. Total working population in thousands. Source: ETAS 1987 (1955-1986(2)).

7. Level of official UK foreign reserves. Cumulated from flow of funds accounts in BESA (1955-1973) and FS (1974-1981), and from the financial transactions accounts in BEQB (1981(2)-1987(2)).

References:

ETAS	Economic Trends Annual Supplement
BESA	Bank of England Statistical Abstract
FS	Financial Statistics
BEQB	Bank of England Quarterly Bulletin

Appendix B: Statistics Used in Modelling

name	definition	reference
$\hat{\sigma}$	equation standard error	
R^2	coefficient of determination	
DW	Durbin-Watson statistic	
η_1	prediction test	Hendry (1980)
η_2	Chow's test for parameter constancy	Chow (1960)
η_3	t test for zero forecast innovation mean	Hendry (1989)
η_4	Box-Pierce test for autocorrelation	Box and Pierce (1970)
η_5	LM test for autocorrelation	Godfrey (1978)
η_6	F version of η_5	Harvey (1981)
η_7	ARCH test	Engle (1982)
η_8	heteroskedasticity test	White (1980)
η_9	test for functional misspecification and heteroskedasticity	White (1980)
η_{10}	RESET test	Ramsey (1969)
η_{11}	test for Normality	Jarque and Bera (1980)
η_{12}	joint F test of linear restrictions	Harvey (1981)

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Table 1: OLS Estimates for the UK Money Demand Equation
Dependent Variable: DLM

Period ECM	A log		B level	B log
LR3	-.40(.14)	DLY4	.30(.09)	.20(.09)
DLY	.27(.06)	DLP	.35(.14)	.30(.12)
DLP	.62(.18)	D2DLP2	-.40(.13)	-.30(.13)
LMPY1	-.17(.03)	DLR	.20(.12)	.17(.12)
D2LM2	-.48(.06)	MPY5	-.02(.01)	
$(D2LM2)^2$	4.12(.80)	LR5	-.09(.11)	-.12(.07)
DLR	.22(.21)	DLY1	.59(.14)	.59(.15)
DMPY3	.09(.04)	DLMY1	.40(.12)	.46(.11)
includes C		D4LR1	-.04(.11)	
		DLR4	.36(.12)	
		LMPY3		-.02(.02)
		includes C,Q1,Q2,Q3		
$\hat{\sigma}$	.01154	$\hat{\sigma}$	.01214	.01329
R^2	.80	R^2	.67	.59
DW	2.16	DW	2.19	2.05
η_1	5.52(8)	η_1	14.16(8)	16.96(8)
η_2	.67(8,46)	η_1	32.40(20)	43.20(20)
η_3	.29[t(39)]	η_2	1.53(8,37)	1.90(8,39)
η_4	.81(12,39)	η_2	1.28(20,25)	1.63(20,27)
η_5	13.07(6)			
η_6	2.09(6,48)	η_4	.53(12,21)	.11(5,49)
η_7	.17(6,47)	η_5	8.22(6)	11.31(6)
η_8	.99(15,38)	η_6	1.05(6,39)	1.62(6,41)
η_9	1.06(27,28)	η_7	.28(6,33)	.56(6,40)
η_{10}	1.42(2,52)	η_8	.52(23,21)	.41(19,27)
η_{11}	1.67(2)	η_9		.46(24,27)
η_{12}	0.85(19,34)	η_{10}	1.20(2,43)	.15(2,45)
		η_{11}	.89(2)	.12(2)
		η_{12}	.57(18,27)	1.37(15,32)

Table 2: Long Run Solution Coefficient Values

Period Specif.	A1 LPR	A present	B1 LPR	B present levels	B present logs
α	-1.52	-2.36	-4.75		-6.71
	(2.27)	(1.14)	(4.73)		(7.98)
β	-6.53	-10.07	-16.98		-11.15
	(5.11)	(2.00)	(14.67)		(15.04)
γ	-5.54	-7.95	-19.21		-13.29
	(5.50)	(2.21)	(11.41)		(11.84)
δ		98.19			
		(27.27)			
ρ				-4.69	
				(9.31)	
μ				-6.10	
				(11.22)	
φ				-13.87	
				(13.20)	

generic equations:

$$\frac{M}{PY} \equiv V = R^{\alpha} exp(C + \beta g + \gamma f + \delta (f + g)^2)$$

$$V = C + \rho LR + \mu g + \varphi f$$

$$DLP = f , DLY = g , DLR = 0$$

Table 3: Reaction Function for Prices DLP			
Period	A		B
DLP2	.36(.08)	LP5	-.02(.003)
LR5	.09(.07)	LR5	.34(.05)
LM1	.012(.004)	D79(3)	.04(.01)
D2DLM2	-.16(.02)	DLMP1	-.31(.07)
DLR4	-.15(.10)	D3LR1	.30(.05)
LB3	-.013(.005)	DLY1	.16(.06)
D3LY2	.09(.02)		
includes C,Q3		includes C,Q1,Q2	
$\hat{\sigma}$	.004963		.009099
R^2	.76		.76
DW	1.83		1.94
η_1	11.36(8)		10.40(8)
η_1			50.00(20)
η_2	.92(8,45)		.99(8,42)
η_2			1.60(20,30)
η_3			-1.44[t(26)]
η_4	.33(6,50)		1.23(5,49)
η_5	1.53(3)		9.52(6)
η_6	.42(3,50)		1.41(6,44)
η_7	.39(3,49)		.24(3,46)
η_8	.45(15,37)		.43(13,36)
η_9	.60(27,27)		
η_{10}	.02(2,51)		.25(2,48)
η_{11}	2.50(2)		.69(2)
η_{12}	.29(24,29)		1.70(25,25)

Note: The dummy variable $D79(3)$ reflects an upward price shock due to the second oil crisis.

Table 4: Reaction Function for Interest Rates DLR			
Period	A		B
LR2	-.38(.07)	LR4	-.18(.06)
LU	-.009(.004)	DDLU2	-.08(.02)
LMY2	.06(.02)	DLM2	-.44(.15)
DLR5	.27(.10)	DLR2	-.30(.10)
D2LP3	-.17(.08)	DLM4	-.63(.16)
LB	-.02(.01)	DLB	-.03(.01)
LY	.01(.01)	D4LM1	.31(.08)
includes C,Q2		includes C	
$\hat{\sigma}$	.005369		.01143
R^2	.55		.61
DW	2.27		2.19
η_1	10.64(8)		4.00(8)
η_1			31.20(20)
η_2	.91(8,44)		.48(8,43)
η_2			.99(20,31)
η_3			−.82[t(19)]
η_4	.80(12,37)		.26(5,41)
η_5	3.75(6)		3.13(6)
η_6	.50(6,46)		.42(6,45)
η_7	.50(6,45)		1.91(6,39)
η_8	.83(15,36)		1.36(14,36)
η_9	.48(26,27)		
η_{10}	.61(2,50)		1.38(2,49)
η_{11}	8.29(2)		.40(2)
η_{12}	.26(25,27)		.82(26,25)

Table 5: Posterior Moments for Period A = 1955(i)-1971(iii); $\beta_{DLR} = 0.22$, $\beta_{DLP} = 0.62$

Prior		Baseline						WE					
ν_0		30			10			30			10		
#drawings		500			500			300			2000		
		mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α
γ	LR3	-.41	(.15)	6.8	-.40	(.13)	7.5	-.40	(.15)	9.8	-.40	(.13)	3.1
	DLY	.28	(.07)	4.5	.28	(.06)	5.0	.27	(.07)	6.6	.27	(.06)	2.1
	LMPY1	-.17	(.04)	3.9	-.17	(.03)	4.3	-.17	(.04)	5.5	-.17	(.03)	1.8
	D2LM2	-.46	(.06)	2.6	-.47	(.05)	2.8	-.47	(.07)	3.5	-.47	(.06)	1.1
	$(D2LM2)^2$	3.79	(.85)	4.3	3.86	(.75)	4.6	3.97	(.92)	5.9	3.94	(.79)	1.9
	DMPY3	.10	(.04)	8.7	.10	(.04)	9.0	.10	(.05)	11.4	.10	(.04)	3.7
λ	prior DLR	.018	(.060)		.018	(.12)		0	(.060)		0	(.12)	
	prior DLP	.21	(.063)		.21	(.12)		0	(.075)		0	(.15)	
	posterior DLR	.004	(.040)	182.5	-.009	(.053)	124.3	-.005	(.039)	205.3	-.013	(.052)	38.5
	posterior DLP	.17	(.04)	4.4	.14	(.05)	7.5	.051	(.041)	21.7	.084	(.050)	5.9
	ρ		.10			.18		.098			.18		
	# iterations		1			1		1			2		
	ϵ_{int}		7.4			14.1		11.4			3.5		

Table 6: Posterior Moments for Period A: 1955(i)-1971(iii); $\beta_{DLR} = -0.25$, $\beta_{DLP} = 1.00$

	Prior	Baseline						WE					
	ν_0	30			10			30			10		
	#drawings	500			500			2000			500		
		mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α
γ	LR3	-.54	(.15)	5.3	-.53	(.14)	4.8	-.52	(.16)	2.8	-.52	(.14)	5.2
	DLY	.26	(.07)	5.2	.25	(.06)	4.7	.24	(.07)	2.8	.24	(.06)	5.2
	LMPY1	-.17	(.04)	4.2	-.16	(.03)	3.7	-.16	(.04)	2.2	-.16	(.03)	4.0
	D2LM2	-.45	(.06)	2.7	-.45	(.06)	2.4	-.46	(.07)	1.3	-.46	(.06)	2.5
	$(D2LM2)^2$	3.69	(.89)	4.6	3.78	(.81)	4.0	3.91	(.95)	2.2	3.87	(.83)	4.2
	DMPY3	.09	(.05)	9.7	.09	(.04)	8.4	.10	(.05)	4.6	.09	(.04)	8.8
λ	prior DLR	.018	(.060)		.018	(.12)		0	(.060)		0	(.12)	
	prior DLP	.21	(.063)		.21	(.12)		0	(.075)		0	(.15)	
	posterior DLR	.042	(.042)	23.1	.057	(.052)	16.7	.029	(.040)	13.4	.050	(.052)	20.6
	posterior DLP	.14	(.040)	6.1	.097	(.047)	9.5	.026	(.043)	16.7	.050	(.051)	20.8
	ρ	.11			.20			.10			.19		
	# iterations	2			2			2			3		
	ϵ_{int}	7.5			5.1			3.0			8.0		

Table 7: Posterior Moments for Period B: 1971(iv)-1986(ii); $\beta_{DLR} = 0.20$, $\beta_{DLP} = 0.35$

	Prior	Baseline						WE					
	ν_0	30			10			30			10		
	#drawings	200			500			400			500		
		mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α
γ	DLY4	.29	(.08)	8.3	.30	(.08)	4.9	.29	(.08)	6.8	.29	(.08)	5.3
	D2DLP2	-.40	(.12)	8.7	-.40	(.11)	5.3	-.40	(.12)	7.3	-.40	(.11)	5.7
	DLR4	.33	(.12)	10.4	.37	(.11)	5.8	.36	(.12)	7.8	.36	(.11)	6.4
	MPY5	-.018	(.011)	17.6	-.019	(.010)	10.1	-.019	(.011)	13.9	-.019	(.010)	10.6
	LR5	-.09	(.08)	25.8	-.09	(.08)	16.7	-.09	(.08)	22.3	-.09	(.08)	17.4
	DLY1	.57	(.13)	6.4	.60	(.12)	3.7	.60	(.12)	5.0	.60	(.12)	3.9
	DLMY1	.39	(.11)	7.9	.40	(.10)	4.6	.40	(.10)	6.3	.41	(.10)	4.9
	D4LR1	-.04	(.08)	64.0	-.04	(.07)	35.1	-.04	(.08)	47.7	-.04	(.07)	35.7
λ	prior DLR	.18	(.19)		.18	(.37)		0	(.19)		0	(.38)	
	prior DLP	-.044	(.19)		-.044	(.38)		0	(.19)		0	(.38)	
	posterior DLR	.039	(.10)	79.2	-.097	(.13)	28.9	-.078	(.11)	41.4	-.12	(.13)	22.1
	posterior DLP	.085	(.10)	34.6	-.041	(.11)	53.3	-.022	(.09)	94.8	-.024	(.11)	95.8
	ρ		.02			.12		.02			.12		
	# iterations		1			2		2			1		
	ϵ_{int}		9.3			7.9		13.5			10.0		

Table 8: Posterior Moments for Period B: 1971(iv)-1986(ii); $\beta_{DLR} = -0.25$, $\beta_{DLP} = 1.00$

Prior		Baseline						WE					
ν_0		30			10			30			10		
#drawings		500			500			1500			500		
		mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α	mean	s.d	ϵ_α
γ	DLY4	.32	(.09)	5.7	.33	(.09)	5.0	.32	(.09)	4.5	.33	(.09)	5.5
	D2DLP2	-.47	(.14)	5.6	-.47	(.13)	5.0	-.47	(.14)	4.4	-.47	(.13)	5.6
	DLR4	.45	(.13)	5.8	.50	(.13)	4.9	.44	(.13)	4.7	.49	(.13)	5.5
	MPY5	-.063	(.013)	4.0	-.062	(.013)	3.7	-.064	(.013)	3.1	-.062	(.013)	4.1
	LR5	-.47	(.09)	3.9	-.46	(.09)	3.7	-.48	(.09)	3.0	-.46	(.09)	4.1
	DLY1	.78	(.14)	3.7	.79	(.14)	3.3	.78	(.14)	2.8	.79	(.14)	3.6
	DLMY1	.60	(.12)	4.0	.59	(.12)	3.7	.60	(.12)	3.1	.60	(.12)	4.0
	D4LR1	-.38	(.09)	4.7	-.37	(.09)	4.4	-.38	(.09)	3.0	-.37	(.09)	4.8
λ	prior DLR	.18	(.19)		.18	(.37)		0	(.19)		0	(.38)	
	prior DLP	-.044	(.19)		-.044	(.38)		0	(.19)		0	(.38)	
	posterior DLR	.074	(.091)	26.0	.043	(.089)	38.3	.031	(.088)	40.4	.035	(.097)	55.2
	posterior DLP	-.17	(.075)	8.9	-.22	(.076)	6.7	-.15	(.086)	9.2	-.20	(.077)	8.0
	ρ	.04			.13			.03			.13		
	# iterations	2			3			1			2		
	ϵ_{int}	9.5			5.2			10.4			8.6		

Figure 1: Actual and Fitted Values for Period A

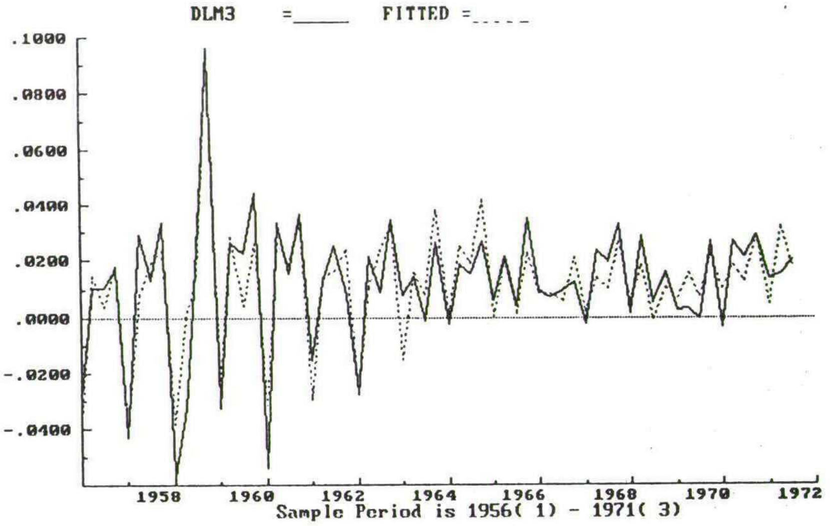


Figure 2: Actual and Fitted Values for Period B (levels ECM)

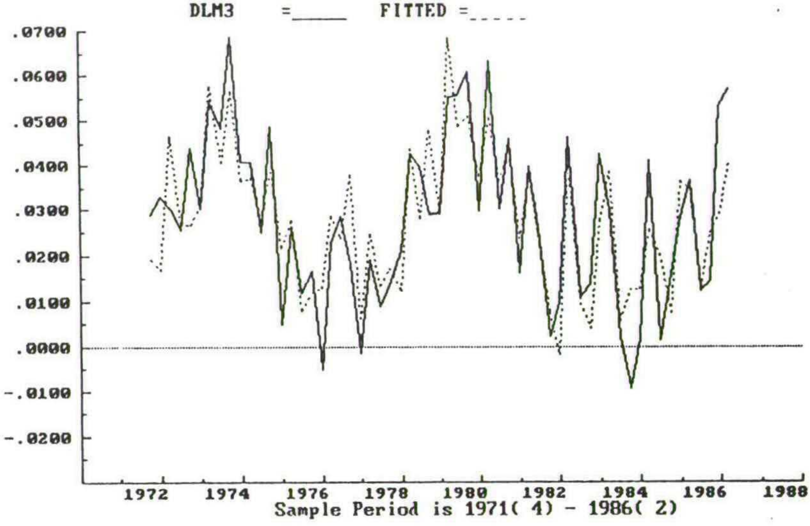
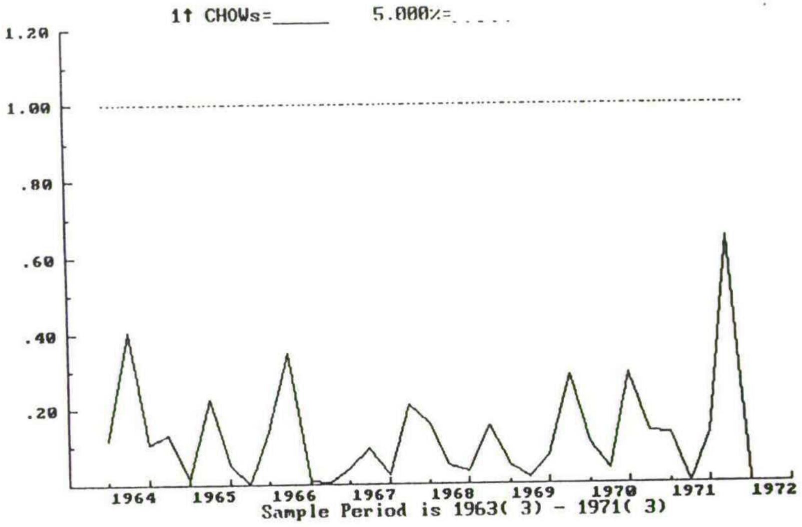


Figure 3: One Step-Ahead Chow Tests for Period A



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