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ENDOGENOUSLY DETERMINED PRICE RIGIDITIES

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Endogenously Determined Price Rigidities * §

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Abstract

There exists an extensive literature about economies with price rigidities, where some constraints on the set of admissible prices are exogenously given. In this paper a general equilibrium model extended by a political system is described where the price rigidities are endogenously chosen by political candidates. Sufficient conditions for the existence of a mixed strategy and a pure strategy equilibrium are given. Finally an example is discussed, where in equilibrium both political candidates propose price rigidities excluding the Walrasian equilibrium price system. *Journal of Economic Literature Classification Numbers: C62, C72, D51, D72.*

1 Introduction

Often government behaviour is considered to be exogenous in economic modelling. However, there exists no reason why government behaviour should not be explained, while behaviour of other agents acting in the economy is endogenously determined. The influence of the government on for example minimum wages is substantial and the existence of minimum wages clearly influences economic behaviour of producers and consumers. Moreover the producers and consumers influence the level of the minimum wages by voting or by forming pressure groups. For example high levels of unemployment might increase the pressure on government to lower minimum wages. Hence in order to explain the existence of minimum wages and to give an analysis of the most important determinants of the level of minimum wages, government behaviour should be modelled endogenously. The existence of minimum wages is just one example of a price rigidity. Other examples are price controls to reduce inflation (see Cox (1980)), minimum prices for agricultural products, fixed exchange rates, price indexation, and the linkage between the wages of civil servants and the wages paid in industry. The existence of price regulations and price rigidities is a frequently occurring real world phenomenon. Nguyen and Whalley (1990, p. 667) make the same observation, stating: "Price controls have been employed by governments all over the world, during war and peace, in response to all manners of threats (both real and imaginary), and in all ages" and Levy (1991, p. 157) writes: "Price controls are pervasive in developing countries."

An important reason for the existence of price regulations is that they can be used to influence the redistribution of initial endowments. As Coughlin (1986) argues, redistribution has become one of the most important political issues of the last three decades. A drawback of price regulations is the misallocation of resources resulting in efficiency losses.

In this paper a formal model capturing the ideas above is given. A stylized model of the political system corresponding with a democracy is described. The government consists of two political parties or candidates who compete for the votes of the consumers in the economy. For the sake of simplicity it is assumed that there are no producers in the economy. The candidates have the possibility to propose price regulations in order to influence the redistribution of initial endowments of the consumers.

The private sector will be modelled by a general equilibrium model. However, in case a candidate proposes some regulation of the prices all the competitive equilibrium price systems might be excluded by the imposed price rigidities. Hence an equilibrium in the classical sense cannot always be achieved. In order to obtain an equilibrium situation rationing may occur. The inclusion of price rigidities and rationing into general equilibrium models has been introduced by Benassy (1975), Drèze (1975), and Younès (1975). In this paper the equilibrium concept of Drèze will be used in case price regulations are proposed.

In most of the existing literature about models with price rigidities some constraints on the set of admissible prices are exogenously given. Some exceptions worth mentioning are Hart (1982), Böhm, Maskin, Polemarchakis, and Postlewaite (1983), Madden (1983), and Silvestre (1988). In these papers (some of the) agents in the economy are price setters on some of the markets. In this way prices are endogenously determined and may be non-Walrasian and therefore possibly involve unemployment. However, it seems to be difficult to extend the results of these papers to general cases with more than three commodities.

In the model presented in this paper political candidates may impose price regulations on the markets. Both government behaviour and price rigidities are endogenously determined. The control of prices by government is also the subject of Nguyen and Whalley (1986) and Ginsburgh and van der Heyden (1988). In these papers government behaviour and price rigidities are not endogenously determined, but the attention is focused on non-rationing mechanisms to solve the mismatch between supply and demand resulting from price regulations. This mismatch is solved by endogenously determined equilibrium buying and selling prices in Nguyen and Whalley (1986) and by government sales and purchases in Ginsburgh and van der Heyden (1988).

Exchange economies with price rigidities will be treated in Section 2. The voting behaviour of consumers and the public sector are described in Section 3. In Section 4 sufficient conditions for the existence of a mixed strategy and a pure strategy Nash equilibrium for the game defined in Section 3 are given. In order to prove existence it is shown that for every economy there exists an upper bound, being chosen independently of the price regulations imposed, such that if the price on a market is above this upper bound then in every possible constrained equilibrium no trade takes place on this market. This result is quite intuitive and has some interest in itself. In Section 5 an example with two commodities and Cobb-Douglas utility functions is presented in which both political candidates impose price regulations excluding the Walrasian equilibrium price system, and therefore the resulting equilibrium is characterized by rationing.

2 Economies with Price Rigidities

In order to allow political candidates to have the possibility to propose price regulations, a model dealing with price regulations in an exchange economy is given in this section. Moreover, some important properties of this model are provided. An exchange economy with price regulations is denoted by $\mathcal{E}_{(\underline{p},\bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})})$. In this economy there are m consumers indexed $i = 1,\ldots,m$, and $n+1$ commodities indexed $j = 0,\ldots,n$. Consumer i, $i = 1,\ldots,m$, is defined by a consumption set X^i, a preference relation $\succeq^i$ on X^i, and a vector of initial endowments w^i. The vector of total initial endowments will be denoted by $\bar{w}$, so $\bar{w} = \sum_{i=1}^{m} w^i$. The set of admissible prices is denoted by $P_{(\underline{p},\bar{p})}$. An exchange economy $(\{X^i, \succeq^i, w^i\}_{i=1}^{m})$ without a priori specified price regulations will be denoted by E. In the following, for $k \in \mathbf{N}$, I_k denotes the set of integers $\{1, \ldots, k\}$ and I_k^0 denotes the set of integers $\{0, 1, \ldots, k\}$. The following assumptions will be made with respect to an exchange economy with price regulations $\mathcal{E}_{(\underline{p},\bar{p})}$.

A1. The consumption set $X^i \subset \mathbf{R}_+^{n+1}$ is convex, closed, and $X^i + \mathbf{R}_+^{n+1} \subset X^i$, for every $i \in I_m$.

A2. The initial endowments w^i are an element of the interior of X^i, for every $i \in I_m$.

A3. The preference relation $\succeq^i$ on X^i is complete, transitive, continuous, strongly monotonic, and convex, for every $i \in I_m$.

A4. The set of admissible prices is equal to $P_{(\underline{p},\bar{p})} = \{p \in \mathbf{R}_+^{n+1} \,|\, p_0 = 1,\ \underline{p}_j \leq p_j \leq \bar{p}_j,\ \forall j \in I_n\}$, for some given $\underline{p} \in \mathbf{R}_+^n$ and $\bar{p} \in \mathbf{R}_+^n$ with $\underline{p}_j \leq \bar{p}_j$, for every $j \in I_n$.

If for a consumer $i \in I_m$, not $x^i \succeq^i y^i$ for two commodity bundles $x^i, y^i \in X^i$, then this will be denoted by $y^i \succ^i x^i$, and in case both $x^i \succeq^i y^i$ and $y^i \succeq^i x^i$ we denote this by $y^i \sim^i x^i$. The convexity assumption in A3 means that if $x^i, y^i \in X^i$ and if $x^i \succ^i y^i$, then $\lambda x^i + (1-\lambda) y^i \succ^i y^i$, $\forall \lambda \in (0,1)$. The strong monotonicity assumption implies that if $x^i, y^i \in X^i$, $x_j^i \leq y_j^i$, $\forall j \in I_n^0$, and $\exists j' \in I_n^0$ such that $x_{j'}^i < y_{j'}^i$, then $y^i \succ^i x^i$. As has been shown in Debreu (1959) Assumptions A1 and A2 imply that it is possible to represent the preferences of consumer $i \in I_m$ by a continuous utility function u^i from X^i into $[0,1)$. In the following it is assumed that some particular representation u^i is chosen.

The price set $P_{(\underline{p},\bar{p})}$ in the model allows for a minimum price, $\underline{p}_j$, and for a maximum price, $\bar{p}_j$, for commodity $j \in I_n$. Commodity 0 is a numeraire commodity with price equal to 1. For commodity $j \in I_n$ there may be total inflexibility of the price, i.e., $\underline{p}_j = \bar{p}_j$, or a more moderate form of price rigidity, $\underline{p}_j < \bar{p}_j$. Formally, on the market of each commodity some minimum and some maximum price is specified. However, in Theorem 2.4 it is shown that if the minimum price of a commodity is taken small enough or the maximum price large enough, this is equivalent to specifying no minimum price or no maximum price, respectively. Define the set R by

$$R = \{x \in \mathbf{R}_+^{2n} \mid \forall j \in I_n,\ x_j \leq x_{j+n}\}.$$

Then $P_{(\underline{p},\bar{p})}$ is a set of admissible prices satisfying Assumption A4 if and only if $(\underline{p}, \bar{p}) \in R$.

In the economy $\mathcal{E}_{(\underline{p},\bar{p})}$ a price system $p \in P_{(\underline{p},\bar{p})}$ at which for each commodity total supply is equal to its total demand does not necessarily exist. This is the case if no Walrasian equilibrium price system is an element of $P_{(\underline{p},\bar{p})}$. Then the price mechanism is not capable of equating demand and supply on all markets.

Roughly spoken, when there is excess demand on one of the markets and it is not possible to raise the price of this commodity, the market will be cleared by quantity adjustments. Not all the demand can be satisfied, so the excess demand of some consumers has to be restricted in such a way that demand equals supply. When there is an excess demand on a market and it is possible to raise the price, an upward price adjustment will result. In such a case a quantity adjustment is not desired. When there is an excess supply on a market and it is not possible to lower the price of this commodity, the market can also be cleared by quantity adjustments. Again the excess supply of some consumers has to be restricted in such a way that demand equals supply. If there is an excess supply on a market and it is possible to lower the price, then a quantity adjustment is not desired, because a downward price adjustment could be made in an attempt to increase the demand. These quantity adjustments will be called rationing in the following. Rationing should not affect excess supply and excess demand simultaneously to guarantee that markets are frictionless, and rationing should not force any consumer to exchange. Moreover, on the market of the numeraire commodity no rationing is allowed. These properties give the motivation for the equilibrium concept of Drèze (1975) for an exchange economy with price regulations. Before giving this equilibrium concept some additional notation will be introduced. It has to be remarked that alternative equilibrium concepts for exchange economies with price rigidities have been provided by Benassy (1975) and Younès (1975). In Silvestre (1982) conditions are given under which the approaches of Benassy, Drèze, and Younès are equivalent. Moreover it is shown that if no differentiability assumptions are made with respect to utility functions then Drèze's definition is the strictest one. This result and the mathematical convenience of Drèze's definition motivate the use of Drèze's equilibrium concept in this paper.

For the sake of simplicity it is assumed that there is uniform rationing, which means that the same upper bound on excess demand and on excess supply is specified for each consumer. Hence the rationing scheme of every consumer is given by a pair of vectors $(l, L) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+$. The maximal net amount a consumer may supply of commodity $j \in I_n$ is given by $-l_j$ and the maximal net amount a consumer may demand of commodity $j \in I_n$ is given by L_j. The constrained budget set of consumer $i \in I_m$ at rationing scheme $(l, L) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+$ and price system $p \in \mathbf{R}^{n+1}_+$ is equal to the set

$$\gamma^i(l, L, p) = \left\{ x^i \in X^i \,\middle|\, \forall j \in I_n,\ l_j \le x^i_j - w^i_j \le L_j,\ p \cdot x^i \le p \cdot w^i \right\}.$$

The constrained budget set of consumer $i \in I_m$ is non-empty for all admissible l, L, and p, because $w^i \in \gamma^i(l, L, p)$. The demand of consumer $i \in I_m$ at rationing scheme $(l, L) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+$ and price system $p \in \mathbf{R}^{n+1}_+$ is denoted by $\delta^i(l, L, p)$ and is given by

$$\delta^i(l, L, p) = \left\{ x^i \in \gamma^i(l, L, p) \,\middle|\, u^i(x^i) = \max_{y^i \in \gamma^i(l,L,p)} u^i(y^i) \right\}.$$

Notice that the constrained budget set and the demand of consumer $i \in I_m$ also has been defined for price systems p with $p_0 \neq 1$. This will be convenient in some of the subsequent proofs. If $p_0 > 0$ then the budget set $\gamma^i(l, L, p)$ is compact using the definition and therefore $\delta^i(l, L, p)$ is non-empty valued if $p_0 > 0$, even if some of the other prices p_j, $j \in I_n$, equal zero. Sometimes it will be useful in proofs to consider the following constrained budget set. Let an economy E be given. Then for every consumer $i \in I_m$ the correspondence $\Gamma^i : -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \mathbf{R}^{n+1}_+ \to \mathbf{R}^{n+1}_+$ is defined by

$$\Gamma^i(l, L, p) = \left\{x^i \in \gamma^i(l, L, p) \mid x^i_0 - w^i_0 \leq \bar{w}_0\right\}, \ \forall (l, L, p) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \mathbf{R}^{n+1}_+.$$

The corresponding constrained demand correspondence of consumer $i \in I_m$, $\Delta^i : -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \mathbf{R}^{n+1}_+ \to \mathbf{R}^{n+1}_+$ has the advantage of being non-empty valued for every $(l, L, p) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \mathbf{R}^{n+1}_+$.

Given $(l, L, p) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \mathbf{R}^{n+1}_+$ consumer $i \in I_m$ is said to be constrained on his supply on market $k \in I_n$, or equivalently l_k is said to be binding for consumer $i \in I_m$, if $\tilde{x}^i \in \delta^i(\tilde{l}, L, p)$ and $x^i \in \delta^i(l, L, p)$ implies $\tilde{x}^i \succ^i x^i$, where $\tilde{l}$ is the rationing scheme with $\tilde{l}_j = l_j$, $\forall j \in I_n \setminus \{k\}$ and $\tilde{l}_k = l_k - \varepsilon$ for some arbitrary positive real number ε. Using the convexity of preferences it is not difficult to show that if consumer $i \in I_m$ is constrained on market k then $x^i \in \delta^i(l, L, p)$ implies $l_k = x^i_k - w^i_k$, and if consumer $i \in I_m$ is not constrained on market k then $x^i \in \delta^i(l, L, p)$ implies $x^i \in \delta^i(\tilde{l}, L, p)$. Similar remarks can be made with respect to demand rationing. The remarks given above are also true for the correspondence Δ^i.

Let be given an economy E. For every real number M a set of rationing schemes and prices, $\mathcal{L}_M$, is defined by

$$\begin{aligned} \mathcal{L}_M \ = \ & \{(l, L, p) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \{1\} \times \mathbf{R}^n_+ \mid \forall j \in I_n, \ l_j \geq -\bar{w}_j \text{ and } L_j \leq \bar{w}_j, \\ & \exists k \in I_n \text{ such that } p_k \geq M \text{ and } l_k = -\bar{w}_k, \text{ and} \\ & \text{if } l_j < 0 \text{ for some } j \in I_n \text{ then there is a } k \in I_n \text{ such that } p_k \geq p_j \text{ and } l_k = -\bar{w}_k\}. \end{aligned}$$

If $(l, L, p) \in \mathcal{L}_M$ then there is at least one commodity $k \in I_n$ with price greater than or equal to M and with $l_k = -\bar{w}_k$ which guarantees that no consumer is constrained on his supply on market k. Moreover, if it is possible to supply a positive amount of some commodity $j \in I_n$, i.e., $l_j < 0$, then there exists a commodity $k \in I_n$ with price at least as high as p_j and with $l_k = -\bar{w}_k$. Finally, the rationing schemes in $\mathcal{L}_M$ are bounded, for every commodity $j \in I_n$ it holds that $-\bar{w}_j \leq l_j \leq 0$ and $0 \leq L_j \leq \bar{w}_j$. It should be noticed that $M^1 > M^2$ implies $\mathcal{L}_{M^1} \subset \mathcal{L}_{M^2}$. The following lemma states that if M is large enough then for every consumer $i \in I_m$ the demand of commodity 0 according to Δ^i at prices and rationing schemes in the set $\mathcal{L}_M$ exceeds the total initial endowment of commodity 0. Lemma 2.1 is closely related to theorems providing boundary conditions on demand functions without taking into account the possibility of rationing, see for example Proposition

3.1.4 of Hildenbrand and Kirman (1988). Lemma 2.1 can therefore be considered as an extension of those theorems to the case where rationing is allowed. Lemma 2.1 will play an important role in the proofs of Theorem 2.4, Theorem 4.2, and Lemma 4.3.

Lemma 2.1

Let an economy E satisfying Assumptions A1, A2, and A3 be given. Then there exists an $M \in \mathbf{R}$ such that if $(l, L, p) \in \mathcal{L}_M$ then for every $i \in I_m$, $x^i \in \Delta^i(l, L, p)$ implies $x^i_0 > \bar{w}_0$.

Proof

Suppose Lemma 2.1 is false. Then there exists a consumer $i \in I_m$ and a sequence $(l^r, L^r, p^r, x^{i^r})_{r \in \mathbf{N}}$ in the set $\mathcal{L}_r \times X^i$ such that $x^{i^r} \in \Delta^i(l^r, L^r, p^r)$ and $x^{i^r}_0 \leq \bar{w}_0$. Consider the bounded sequence

$$\left(l^r, L^r, \frac{p^r}{\|p^r\|}, x^{i^r}\right)_{r \in \mathbf{N}}, \tag{1}$$

which without loss of generality can be assumed to converge to an element $(l, L, 0, p, x^i) \in -\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \{0\} \times \mathbf{R}^n_+ \times X^i$. Notice that since $x^{i^r} \in \Gamma^i(l^r, L^r, p^r)$ and $(l^r, L^r, p^r) \in \mathcal{L}_r$, x^{i^r} remains in a bounded set. Moreover, $(l^r, L^r, p^r) \in \mathcal{L}_r$ implies that $\|p^r\| \to \infty$ if $r \to \infty$. By the homogeneity of degree zero in prices of the demand correspondence it holds that for every $r \in \mathbf{N}$, $x^{i^r} \in \Delta^i(l^r, L^r, \frac{p^r}{\|p^r\|})$.
Suppose $p \cdot l > 0$, then by the lemma in Drèze (1975, p. 304) it holds that Δ^i is upper semi-continuous at $(l, L, 0, p)$ and therefore $x^i \in \Delta^i(l, L, 0, p)$. By the strict monotonicity of preferences it holds that $x^i_0 - w^i_0 = \bar{w}_0$. This contradicts $x^{i^r}_0 \leq \bar{w}_0$, $\forall r \in \mathbf{N}$. Consequently $p \cdot l = 0$.
Suppose there exists a subsequence $(l^{r^s}, L^{r^s}, \frac{p^{r^s}}{\|p^{r^s}\|}, x^{i^{r^s}})_{s \in \mathbf{N}}$ of the sequence in (1) such that for some $k \in I_n$ with $p_k > 0$, for every $s \in \mathbf{N}$, $l^{r^s}_k$ is not binding for consumer i. Then $x^{i^{r^s}} \in \Delta^i(\tilde{l}^{r^s}, L^{r^s}, p^{r^s})$, where $\tilde{l}^{r^s}$ is the rationing scheme with $\tilde{l}^{r^s}_j = l^{r^s}_j$, $\forall j \in I_n \setminus \{k\}$, and $\tilde{l}^{r^s}_k = -\bar{w}_k$. By considering the sequence $(\tilde{l}^{r^s}, L^{r^s}, \frac{p^{r^s}}{\|p^{r^s}\|}, x^{i^{r^s}})_{s \in \mathbf{N}}$ one obtains a contradiction in the same way as in the previous paragraph. Consequently, $p \cdot l = 0$ and without loss of generality for every $r \in \mathbf{N}$, for every $k \in I_n$ with $p_k > 0$, $l^r_k = x^{i^r}_k - w^i_k$.
Suppose there exists a subsequence $(l^{r^s}, L^{r^s}, \frac{p^{r^s}}{\|p^{r^s}\|}, x^{i^{r^s}})_{s \in \mathbf{N}}$ of the sequence in (1) such that for some $k \in I_n$ with $p_k > 0$, for every $s \in \mathbf{N}$, $l^{r^s}_k < 0$. Since $(l^{r^s}, L^{r^s}, p^{r^s}) \in \mathcal{L}_{r^s}$, it holds for some $k^{r^s} \in I_n$ that $p^{r^s}_{k^{r^s}} \geq p^{r^s}_k$ and $l^{r^s}_{k^{r^s}} = -\bar{w}_{k^{r^s}}$. This contradicts $p \cdot l = 0$. Consequently, $p \cdot l = 0$ and without loss of generality for every $r \in \mathbf{N}$, for every $k \in I_n$ with $p_k > 0$, $0 = l^r_k = x^{i^r}_k - w^i_k$.
Define the set $J = \{k \in I_n \mid p_k > 0\}$, a non-empty proper subset of I_n. Without loss of generality $J = I_n \setminus I_{\hat{n}}$ for some $\hat{n} \in \mathbf{N}$ satisfying $1 \leq \hat{n} < n$. Now consider a consumer with characteristics $\hat{X}^i, \hat{\succeq}^i, \hat{w}^i$, where $\hat{X}^i = \{x^i \in \mathbf{R}^{\hat{n}+1}_+ \mid (x^i, w^i_{\hat{n}+1}, \ldots, w^i_n) \in X^i\}$, $\hat{\succeq}^i = \{(x^i, y^i) \in \hat{X}^i \times \hat{X}^i \mid (x^i, w^i_{\hat{n}+1}, \ldots, w^i_n) \succeq^i (y^i, w^i_{\hat{n}+1}, \ldots, w^i_n)\}$, $\hat{w}^i = (w^i_0, \ldots, w^i_{\hat{n}})$. It is

easily verified that $(\hat{X}^i, \hat{\succeq}^i, \hat{w}^i)$ satisfies Assumptions A1, A2, and A3. Moreover, using that for every $k \in J$, $x_k^{i^r} = w_k^i$, it is clear that the corresponding demand correspondence $\hat{\Delta}^i$ satisfies $\hat{x}^{i^r} \in \hat{\Delta}^i(\hat{l}^r, \hat{L}^r, \hat{p}^r)$, $\forall r \in \mathbf{N}$. Here $\hat{l}^r$ and $\hat{L}^r$ denote the first $\hat{n}$ components of l^r and L^r, respectively, and $\hat{p}^r$ and $\hat{x}^{i^r}$ denote the first $\hat{n}+1$ components of p^r and x^{i^r}, respectively. Since for every $j \in I_n \setminus I_{\hat{n}}$, $l_j = 0$, the definition of $\mathcal{L}_r$ implies that there is no loss of generality in assuming that for every $r \in \mathbf{N}$ there exists $k^r \in I_{\hat{n}}$ such that $\hat{p}_{k^r}^r \geq r$ and $\hat{l}_{k^r}^r = -\bar{w}_{k^r}$. Consider the bounded sequence $(\hat{l}^r, \hat{L}^r, \frac{\hat{p}^r}{\|\hat{p}^r\|}, \hat{x}^{i^r})_{r \in \mathbf{N}}$ in $-\mathbf{R}_+^{\hat{n}} \times \mathbf{R}_+^{\hat{n}} \times \mathbf{R}_+^{\hat{n}+1} \times \hat{X}^i$. Without loss of generality this sequence converges to an element $(\hat{l}, \hat{L}, 0, \hat{p}, \hat{x}^i) \in -\mathbf{R}_+^{\hat{n}} \times \mathbf{R}_+^{\hat{n}} \times \{0\} \times \mathbf{R}_+^{\hat{n}} \times \hat{X}^i$.
Suppose $\hat{p} \cdot \hat{l} > 0$, then again by Drèze (1975) $\hat{\Delta}^i$ is upper semi-continuous at $(\hat{l}, \hat{L}, \hat{p})$ and therefore $\hat{x}^i \in \hat{\Delta}^i(\hat{l}, \hat{L}, \hat{p})$, yielding a contradiction in the same way as before. Repeating the arguments used before, the finiteness of n and the definition of $\mathcal{L}_r$ guarantee that in a finite number of steps the case where $\hat{p} \cdot \hat{l} > 0$ will be reached, contradicting that for every $r \in \mathbf{N}$, $\hat{x}_0^{i^r} \leq \bar{w}_0$.

Q.E.D.

A constrained equilibrium of an economy $\mathcal{E}_{(\underline{p}, \bar{p})}$ is defined as follows (see Drèze (1975)).

Definition 2.2 (Constrained Equilibrium)
A constrained equilibrium of an economy $\mathcal{E}_{(\underline{p}, \bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p}, \bar{p})})$ *is an element* $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ *of the set* $\prod_{i=1}^m X^i \times -\mathbf{R}_+^n \times \mathbf{R}_+^n \times P_{(\underline{p}, \bar{p})}$ *such that*

1. $\forall i \in I_m : x^{*i} \in \delta^i(l^*, L^*, p^*)$;

2. $\sum_{i=1}^m x^{*i} = \bar{w}$;

3. $\forall j \in I_n : x_j^{*h} - w_j^h = L_j^*$ *for some* $h \in I_m$ *implies* $x_j^{*i} - w_j^i > l_j^*$ $\forall i \in I_m$, *and* $x_j^{*h} - w_j^h = l_j^*$ *for some* $h \in I_m$ *implies* $x_j^{*i} - w_j^i < L_j^*$ $\forall i \in I_m$;

4. $\forall j \in I_n : p_j^* < \bar{p}_j$ *implies* $L_j^* > x_j^{*i} - w_j^i$ $\forall i \in I_m$, *and* $p_j^* > \underline{p}_j$ *implies* $l_j^* < x_j^{*i} - w_j^i$ $\forall i \in I_m$.

In Drèze (1975) the existence of a constrained equilibrium is shown for any economy $\mathcal{E}_{(\underline{p}, \bar{p})}$ satisfying Assumptions A1, A2, A3, and A4. Let $(x^{*1}, \ldots, x^{*m}, p^*)$ be a Walrasian equilibrium with $p_0^* = 1$ of an economy $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$. Then it is clear that the element $(x^{*1}, \ldots, x^{*m}, -\bar{w}, \bar{w}, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{((p_1^*, \ldots, p_n^*), (p_1^*, \ldots, p_n^*))}$. Hence the concept of a constrained equilibrium generalizes the concept of a Walrasian equilibrium. The choice of the rationing schemes $l^* = -\bar{w}$ and $L^* = \bar{w}$ guarantees that Conditions 3 and 4 of Definition 2.2 are satisfied. As Lemma 2.3 shows the choice of these rationing schemes is not uniquely determined.

Lemma 2.3

Let an economy $\mathcal{E}_{(\underline{p},\bar{p})}$ satisfying the Assumptions A1, A2, A3, and A4 be given. Let $(x^{*1},\ldots,x^{*m},l^*,L^*,p^*)$ be a constrained equilibrium of $\mathcal{E}_{(\underline{p},\bar{p})}$. If for a rationing scheme $(l,L)\in -\mathbf{R}^n_+\times\mathbf{R}^n_+$, for every $j\in I_n$,

$$\begin{aligned}
&l^*_j = x^{*i}_j - w^i_j \text{ for some consumer } i\in I_m \text{ implies } l_j = l^*_j,\\
&l^*_j < x^{*i}_j - w^i_j \text{ for every consumer } i\in I_m \text{ implies } l_j < x^{*i}_j - w^i_j,\ \forall i\in I_m,\\
&L^*_j = x^{*i}_j - w^i_j \text{ for some consumer } i\in I_m \text{ implies } L_j = L^*_j, \text{ and}\\
&L^*_j > x^{*i}_j - w^i_j \text{ for every consumer } i\in I_m \text{ implies } L_j > x^{*i}_j - w^i_j,\ \forall i\in I_m,
\end{aligned}$$

then $(x^{*1},\ldots,x^{*m},l,L,p^*)$ is a constrained equilibrium of $\mathcal{E}_{(\underline{p},\bar{p})}$.

Proof

Clearly $(x^{*1},\ldots,x^{*m},l,L,p^*)$ is an element of the set $\prod_{i=1}^m X^i\times -\mathbf{R}^n_+\times\mathbf{R}^n_+\times P_{(\underline{p},\bar{p})}$. It is easily seen that Conditions 2, 3, and 4 of a constrained equilibrium are satisfied by $(x^{*1},\ldots,x^{*m},l,L,p^*)$, using the properties of l and L and the fact that $(x^{*1},\ldots,x^{*m},l^*,L^*,p^*)$ is a constrained equilibrium. So it remains to be shown that Condition 1 of a constrained equilibrium is satisfied. Suppose for some $i\in I_m$, $x^{*i}\notin\delta^i(l,L,p^*)$. Clearly $x^{*i}\in\gamma^i(l,L,p^*)$. Hence for some $y^i\in\gamma^i(l,L,p^*)$, $y^i\succ^i x^{*i}$. Since $y^i\notin\gamma^i(l^*,L^*,p^*)$ and $y^i\in\gamma^i(l,L,p^*)$ it has to hold that for some $j\in I_n$, $l_j\le y^i_j-w^i_j<l^*_j$, or for some $j\in I_n$, $L^*_j<y^i_j-w^i_j\le L_j$. Moreover, it holds that

$$[y^i_j-w^i_j<l^*_j\Rightarrow l^*_j<x^{*i}_j-w^i_j] \text{ and } [y^i_j-w^i_j>L^*_j\Rightarrow L^*_j>x^{*i}_j-w^i_j]. \tag{2}$$

Define $\forall\lambda\in(0,1)$, $y^i(\lambda)=\lambda y^i+(1-\lambda)x^{*i}$. By the convexity of $\gamma^i(l,L,p^*)$ it holds that $y^i(\lambda)\in\gamma^i(l,L,p^*)$, $\forall\lambda\in(0,1)$. By the convexity of $\succeq^i$, $y^i(\lambda)\succ^i x^{*i}$, $\forall\lambda\in(0,1)$. However, if λ is close enough to zero then $y^i(\lambda)\in\gamma^i(l^*,L^*,p^*)$, using (2) and the fact that $y^i(\lambda)\in\gamma^i(l,L,p^*)$. This is a contradiction since $x^{*i}\in\delta^i(l^*,L^*,p^*)$. Hence Condition 1 of a constrained equilibrium is satisfied by $(x^{*1},\ldots,x^{*m},l,L,p^*)$.

Q.E.D.

For every commodity both a lower and an upper bound were specified in the model described above. The following theorem makes clear that it is possible to use this model also in case on a market only a minimum price or a maximum price is given, or even in case there are no constraints on the prices at all, corresponding to the Walrasian case. Let $\underline{J}\subset I_n$ denote the possibly empty set of commodities on the market for which a minimum price is present, and let $\bar{J}\subset I_n$ denote the possibly empty set of commodities on the market for which a maximum price prevails. Again, for $j\in\underline{J}$, $\underline{p}_j$ denotes the minimum price on the market of commodity j and for $j \in \bar{J}$, $\bar{p}_j$ denotes the maximum price on the market of commodity j. Now it is possible to make the following assumption, which is weaker than Assumption A4.

A4'. The set of admissible prices is equal to

$$P_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})} = \{p \in \mathbf{R}^{n+1}_{+} \,|\, p_0 = 1,\ \forall j \in \underline{J},\ \underline{p}_j \leq p_j,\ \forall j \in \bar{J},\ p_j \leq \bar{p}_j\},$$

with $\forall j \in \underline{J}$, $\underline{p}_j \in \mathbf{R}_+$, $\forall j \in \bar{J}$, $\bar{p}_j \in \mathbf{R}_+$, and $\forall j \in \underline{J} \cap \bar{J}$, $\underline{p}_j \leq \bar{p}_j$.

A constrained equilibrium of an economy $\mathcal{E}_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})})$ is defined according to Definition 2.2, where if $j \notin \bar{J}$ then $L_j^* > x_j^{*i} - w_j^i$, $\forall i \in I_m$, and if $j \notin \underline{J}$ then $l_j^* < x_j^{*i} - w_j^i$, $\forall i \in I_m$. Hence demand rationing is never allowed to be binding if no maximum price is specified and supply rationing is not binding if no minimum price is present. In case $\underline{J} = \bar{J} = \emptyset$ one obtains the definition of a Walrasian equilibrium, since non-binding rationing schemes are irrelevant. The following theorem shows that there is no loss of generality in making Assumption A4, or equivalently describing price regulations by an element $(\underline{p}, \bar{p}) \in R$, instead of making Assumption A4'.

Theorem 2.4
Let an economy $\mathcal{E}_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})})$ satisfying Assumptions A1, A2, A3, and A4' be given. Then there exists $(\underline{p}', \bar{p}') \in R$ such that $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of $\mathcal{E}_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})}$ if and only if $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p}',\bar{p}')} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p}',\bar{p}')})$.

Proof
Let M be as in Lemma 2.1 for the economy $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ and take $N \in \mathbf{R}$ such that $N \geq M$ and $N > \underline{p}_j$, $\forall j \in \underline{J}$. For $j \in \underline{J}$ let $\underline{p}'_j = \underline{p}_j$, for $j \in I_n \setminus \underline{J}$ let $\underline{p}'_j = 0$, for $j \in \bar{J}$ let $\bar{p}'_j = \bar{p}_j$, and for $j \in I_n \setminus \bar{J}$ let $\bar{p}'_j = N$. This defines the economy $\mathcal{E}_{(\underline{p}',\bar{p}')}$.
Suppose $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})}$ satisfying $\|p^*\|_\infty > N$. Using the same arguments as in Lemma 2.3 and the fact that $\sum_{i=1}^m x^{*i} = \bar{w}$, there is no loss of generality in assuming that $l_j^* = -\bar{w}_j$ if $j \in I_n$ satisfies $l_j^* < x_j^{*i} - w_j^i$, $\forall i \in I_m$, and that $L_j^* = \bar{w}_j$ if $j \in I_n$ satisfies $L_j^* > x_j^{*i} - w_j^i$, $\forall i \in I_m$. Since $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{((\underline{p}_j)_{j\in\underline{J}},(\bar{p}_j)_{j\in\bar{J}})}$ it then holds that $l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j$, $\forall j \in I_n$. For all $h \in I_m$, $x^{*h} \in \delta^h(l^*, L^*, p^*)$, and since $x_0^{*h} - w_0^h \leq \sum_{i=1}^m x_0^{*i} = \bar{w}_0$, $x^{*h} \in \Delta^h(l^*, L^*, p^*)$. If $j \in I_n$ is such that $p_j^* \geq N$ then $N > \underline{p}_j$, $\forall j \in \underline{J}$, implies $l_j^* = -\bar{w}_j$. So $(l^*, L^*, p^*) \in \mathcal{L}_M$ and consequently $x_0^{*i} > \bar{w}_0$ for every $i \in I_m$, contradicting Condition 2 of Definition 2.2. So $p^* \in P_{(\underline{p}',\bar{p}')}$ and therefore $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of $\mathcal{E}_{(\underline{p}',\bar{p}')}$.
Let a constrained equilibrium $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ of $\mathcal{E}_{(\underline{p}',\bar{p}')}$ be given. Using Lemma 2.3 and the fact that $\sum_{i=1}^m x^{*i} = \bar{w}$ there is no loss of generality in assuming that $l_j^* = -\bar{w}_j$ if

$j \in I_n$ satisfies $l_j^* < x_j^{*i} - w_j^i$, $\forall i \in I_m$, and that $L_j^* = \bar{w}_j$ if $j \in I_n$ satisfies $L_j^* > x_j^{*i} - w_j^i$, $\forall i \in I_m$. Since $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p}', \bar{p}')}$ it then holds that $l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j$, $\forall j \in I_n$. Suppose $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is not a constrained equilibrium of $\mathcal{E}_{((\underline{p}_j)_{j \in \underline{J}}, (\bar{p}_j)_{j \in \bar{J}})}$. Then for some $j \in I_n \setminus \underline{J}$, $p_j^* = 0$ and $l_j^* = x_j^{*i} - w_j^i$ for some $i \in I_m$, or for some $j \in I_n \setminus \bar{J}$, $p_j^* = N$ and $L_j^* = x_j^{*i} - w_j^i$ for some $i \in I_m$. Consider the first case. If $p_j^* = 0$ then by the strict monotonicity of the preference relation it holds that $x_j^{*i} - w_j^i = L_j^* > l_j^* = x_j^{*i} - w_j^i$, where for the inequality Condition 3 of Definition 2.2 is used. This is a contradiction. Consider the second case. By Condition 4 of a constrained equilibrium for every $j \in I_n$ for which $p_j^* = N$ it holds that $l_j^* = -\bar{w}_j$, so $(l^*, L^*, p^*) \in \mathcal{L}_M$. Clearly for every $i \in I_m$, $x^{*i} \in \Delta^i(l^*, L^*, p^*)$ and consequently $x_0^{*i} > \bar{w}_0$, contradicting Condition 2 of Definition 2.2.

Q.E.D.

Let be given an economy $\mathcal{E}_{(\underline{p}, \bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p}, \bar{p})})$. In Theorem 2.5 it is shown that it is possible to represent the admissible prices and rationing schemes by the set

$$Q = \{q \in \mathbf{R}^n \mid 0 \leq q_j \leq 1,\ \forall j \in I_n\},$$

of which an element q will be called a pseudo-price vector. For every $j \in I_n$, the j-th component of the functions $\hat{p} : R \times Q \to \{1\} \times \mathbf{R}_+^n$, $\hat{l} : Q \to -\mathbf{R}_+^n$, and $\hat{L} : Q \to \mathbf{R}_+^n$ is defined by

$$\begin{aligned}
&\hat{p}_j(\underline{p}, \bar{p}, q) = \max\{\underline{p}_j, \min\{\bar{p}_j, \underline{p}_j(2 - 3q_j) + \bar{p}_j(3q_j - 1)\}\},\ \forall (\underline{p}, \bar{p}) \in R,\ \forall q \in Q,\\
&\hat{l}_j(q) = -\min\{1, 3q_j\}\,\bar{w}_j,\ \forall q \in Q,\\
&\hat{L}_j(q) = \min\{1, 3 - 3q_j\}\,\bar{w}_j,\ \forall q \in Q.
\end{aligned}$$

Note that $\hat{p}_0(\underline{p}, \bar{p}, q) = 1$, $\forall (\underline{p}, \bar{p}) \in R$, $\forall q \in Q$. In this way a pseudo-price $q \in Q$ determines an admissible price $\hat{p}(\underline{p}, \bar{p}, q)$ and a rationing scheme $(\hat{l}(q), \hat{L}(q))$. Remark that for every $j \in I_n$,

$$\begin{aligned}
&0 \leq q_j < \frac{1}{3} \text{ implies } \hat{p}_j(\underline{p}, \bar{p}, q) = \underline{p}_j,\ \hat{l}_j(q) > -\bar{w}_j, \text{ and } \hat{L}_j(q) = \bar{w}_j,\\
&\frac{1}{3} \leq q_j \leq \frac{2}{3} \text{ implies } \underline{p}_j \leq \hat{p}_j(\underline{p}, \bar{p}, q) \leq \bar{p}_j,\ \hat{l}_j(q) = -\bar{w}_j, \text{ and } \hat{L}_j(q) = \bar{w}_j,\\
&\frac{2}{3} < q_j \leq 1 \text{ implies } \hat{p}_j(\underline{p}, \bar{p}, q) = \bar{p}_j,\ \hat{l}_j(q) = -\bar{w}_j, \text{ and } \hat{L}_j(q) < \bar{w}_j.
\end{aligned}$$

These properties will guarantee that in an equilibrium Conditions 3 and 4 of Definition 2.2 are satisfied if prices and rationing schemes are described using the pseudo-prices. In the following it is shown that describing prices and rationing schemes by pseudo-prices does not exclude any of the constrained equilibrium allocations and prices.

Theorem 2.5
Let an economy $\mathcal{E}_{(\underline{p},\bar{p})}$ satisfying Assumptions A1, A2, A3, and A4, and a constrained equilibrium $(x^{*1},\ldots,x^{*m},l^*,L^*,p^*)$ of $\mathcal{E}_{(\underline{p},\bar{p})}$ be given. Then there exists a pseudo-price $q^* \in Q$ such that $(x^{*1},\ldots,x^{*m},\hat{l}(q^*),\hat{L}(q^*),\hat{p}(\underline{p},\bar{p},q^*))$ is a constrained equilibrium of $\mathcal{E}_{(\underline{p},\bar{p})}$, with $\hat{p}(\underline{p},\bar{p},q^*) = p^*$.

Proof

Define

$$\begin{aligned} J^1 &= \left\{ j \in I_n \mid \exists i \in I_m : x_j^{*i} - w_j^i = L_j^* \right\}, \\ J^2 &= \left\{ j \in I_n \mid \forall i \in I_m : l_j^* < x_j^{*i} - w_j^i < L_j^* \right\}, \\ J^3 &= \left\{ j \in I_n \mid \exists i \in I_m : l_j^* = x_j^{*i} - w_j^i \right\}. \end{aligned}$$

By Condition 3 of Definition 2.2, $\forall j \in J^1,\ l_j^* < x_j^{*i} - w_j^i,\ \forall i \in I_m$, and $\forall j \in J^3,\ x_j^{*i} - w_j^i < L_j^*,\ \forall i \in I_m$. By Condition 4 of Definition 2.2, $\forall j \in J^1,\ p_j^* = \bar{p}_j$, and $\forall j \in J^3,\ p_j^* = \underline{p}_j$. Obviously $\{J^1, J^2, J^3\}$ is a partition of I_n. By Assumptions A1 and A2, for every $j \in I_n,\ x_j^{*i} - w_j^i \geq -w_j^i > -\bar{w}_j,\ \forall i \in I_m$, and using Assumptions A1 and A2 again, for every $j \in I_n,\ x_j^{*i} - w_j^i < \sum_{i=1}^m x_j^{*i} = \bar{w}_j,\ \forall i \in I_m$. Define $l \in -\mathbf{R}_+^n$ by $l_j = -\bar{w}_j,\ \forall j \in J^1 \cup J^2$, and $l_j = l_j^*,\ \forall j \in J^3$. Define $L \in \mathbf{R}_+^n$ by $L_j = \bar{w}_j,\ \forall j \in J^2 \cup J^3$, and $L_j = L_j^*,\ \forall j \in J^1$. Then by Lemma 2.3 it holds that $(x^{*1},\ldots,x^{*m},l,L,p^*)$ is a constrained equilibrium for the economy $\mathcal{E}_{(\underline{p},\bar{p})}$. Moreover, $l_j \geq -\bar{w}_j$, and $L_j \leq \bar{w}_j,\ \forall j \in I_n$. Let $q^* \in Q$ be defined by

$$\begin{aligned} q_j^* &= 1 - \frac{L_j}{3\bar{w}_j},\ \forall j \in J^1, \\ q_j^* &= \frac{1}{2},\ \forall j \in J^2 \text{ such that } \underline{p}_j = \bar{p}_j, \\ q_j^* &= \frac{p_j^* + \bar{p}_j - 2\underline{p}_j}{3(\bar{p}_j - \underline{p}_j)},\ \forall j \in J^2 \text{ such that } \underline{p}_j < \bar{p}_j, \\ q_j^* &= -\frac{l_j}{3\bar{w}_j},\ \forall j \in J^3. \end{aligned}$$

It is easily checked that $\hat{l}\,(q^*) = l,\ \hat{L}\,(q^*) = L,\ \hat{p}\left(\underline{p},\bar{p},q^*\right) = p^*$, and therefore $(x^{*1},\ldots,x^{*m},\hat{l}(q^*),\hat{L}(q^*),\hat{p}(\underline{p},\bar{p},q^*))$ is a constrained equilibrium of $\mathcal{E}_{(\underline{p},\bar{p})}$.

Q.E.D.

3 Endogenously Determined Price Rigidities

The behaviour of the government will be modelled as being the result of the competition for votes between two political candidates, indexed by $k = 1, 2$. It is not difficult to extend the model in a similar way as in Wittman (1984) and allow for an arbitrary number of political candidates. The electorate consists of the consumers in the economy and chooses between the candidates by a majority vote.

Candidates are assumed to have the possibility to propose price regulations on the markets. For $k \in I_2$, $A^k \subset R$ denotes the set of admissible price regulations among which candidate k can choose. An element a^k in the set A^k of the candidates corresponds with the choice of a lower bound and an upper bound on the set of admissible prices. If candidate $k \in I_2$ chooses a policy position $a^k \in A^k$ then the proposed set P_{a^k} of admissible prices of the economy is given by

$$P_{a^k} = \left\{p \in \mathbf{R}_+^{n+1} \,\middle|\, p_0 = 1,\ \forall j \in I_n,\ a_j^k \leq p_j \leq a_{j+n}^k\right\}$$

and the resulting economy is given by $\mathcal{E}_{a^k} = \left(\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{a^k}\right)$. In general more than one constrained equilibrium allocation and price might result for the economy, given any lower bound and any upper bound on the prices. For instance the results of Debreu (1974) imply that economies exist with an arbitrary number of Walrasian equilibria. It will be assumed that a candidate proposes besides the chosen price regulation also a corresponding equilibrium price, amount of supply rationing (also called unemployment), and amount of demand rationing on each market. By Theorem 2.5 there is no loss of generality in describing prices and rationing schemes by the pseudo-price $q \in Q$. The set C^k will denote the set of admissible actions of candidate $k \in I_2$ and is therefore given by

$$\begin{aligned} C^k \;=\; & \{(a^k, q) \in A^k \times Q \mid \text{there exists a constrained equilibrium} \\ & (x^1, \ldots, x^m, \hat{l}(q), \hat{L}(q), \hat{p}(a^k, q)) \text{ of } \mathcal{E}_{a^k}\}. \end{aligned} \tag{3}$$

When $A^k = R$ then the corresponding set of admissible actions C^k is denoted by C. For $(a, q) \in C$ define the indirect utility function $\bar{v}^i : C \to [0, 1)$ of consumer $i \in I_m$ by

$$\bar{v}^i(a, q) = u^i(x^{*i}),\ \text{for } x^{*i} \in \delta^i(\hat{l}(q), \hat{L}(q), \hat{p}(a, q)).$$

Obviously, this function is well defined.

In order to describe the assumptions with respect to the set of admissible price regulations of the candidates, a new mathematical concept will be introduced first. Define for $\nu \in \mathbf{N}$ and $r \in \mathbf{R}_+$ the ν-dimensional cube $B_r^\nu = \{b \in \mathbf{R}^\nu \mid \|b\|_\infty \leq r\}$ and define the projection function $\beta_r^\nu : \mathbf{R}^\nu \to B_r^\nu$ which assigns to an element $x \in \mathbf{R}^\nu$ the element $b \in B_r^\nu$ minimizing $\|x - b\|_2$, so for every $j \in I_\nu$, $(\beta_r^\nu(x))_j = \min\{\max\{-r, x_j\}, r\}$. Clearly β_r^ν is a continuous function. Now it is possible to give the definition of a property of a subset of a Euclidean space, weaker than compactness but stronger than closedness, and hence called semi-compactness.

Definition 3.1 (Semi-Compactness)
Let a subset S of $\mathbf{R}^\nu$ be given. Then S is semi-compact if for every $r \in \mathbf{R}_+$, $\beta_r^\nu(S)$ is closed in $\mathbf{R}^\nu$.

Since the continuous image of a compact set is compact, every compact subset of $\mathbf{R}^\nu$ is semi-compact. It will be shown that every semi-compact set is closed. Suppose $S \subset \mathbf{R}^\nu$ is semi-compact but not closed. Then there exists a converging sequence of points in S with limit s such that $s \notin S$. Let $r \in \mathbf{R}_+$ be such that $r > \|s\|_\infty$. Then $\beta_r^\nu(S)$ is not closed, which contradicts the semi-compactness of S. Two examples of semi-compact sets not being compact are the sets $\mathbf{R}^\nu$ and $\mathbf{N}^\nu$. An example of a closed set not being semi-compact is the set $\{s \in \mathbf{R}^2 \mid s_2 = \arctan(s_1)\}$. It is not difficult to show that for subsets of $\mathbf{R}$ the concepts of semi-compactness and closedness coincide. When verifying the semi-compactness of a set it is useful to know that if $0 \leq r \leq \bar{r}$ and $\beta_{\bar{r}}^\nu(S)$ is closed then $\beta_r^\nu(S)$ is closed. This follows easily from the property that $\beta_r^\nu(\beta_{\bar{r}}^\nu(S)) = \beta_r^\nu(S)$ for $r \leq \bar{r}$. If $\beta_{\bar{r}}^\nu(S)$ is closed then it has to be compact, and since the continuous image of a compact set is compact and therefore closed, the set $\beta_r^\nu(\beta_{\bar{r}}^\nu(S))$ is closed for any $r \leq \bar{r}$. The following assumption with respect to the set A^k is made.

B1. For $k \in I_2$ the set A^k is a non-empty semi-compact subset of R.

The assumptions made with respect to A^k, $k \in I_2$, are very weak. Since $\beta_r^{2n}(R) = B_r^{2n} \cap R$, which is an intersection of two closed sets and therefore closed for every $r \geq 0$, the set R itself satisfies Assumption B1. So the case $A^k = R$, $k \in I_2$, is not excluded. This is conceptually the most interesting case, since it corresponds to the situation where in a democratic society price regulations are chosen by political candidates, and where there are no restrictions on the set of admissible price regulations.

However, it is also possible to model that a candidate is not capable of setting arbitrarily chosen lower and upper bounds on the prices, for example because of institutional reasons. This might be the more realistic case, since according to Cox (1980) regulators are not capable of enforcing every possible price regulation. This might be modelled by restricting the set A^k of admissible price regulations to be some non-empty compact subset of R. Another possibility is that each candidate is only capable of considering a finite number of possibilities, in which case the set A^k is a finite set. Assumption B1 also admits many intermediate possibilities for the set A^k, for example cases where candidates are only able to regulate prices on some markets. An example for $n = 2$ is given by the semi-compact set $A^k = \{(\underline{p}_1, \underline{p}_2, \bar{p}_1, \bar{p}_2) \in R \mid \underline{p}_2 = \bar{p}_2 = 1\}$.

According to Kramer (1973) deterministic voting equilibria only exist under extremely restrictive assumptions in case the policy space is more than one-dimensional, which is clearly the case in this paper. This is why attention will also be focused on probabilistic voting models, where candidates do not necessarily have perfect information about the voting decision of consumers. Voting models with some probabilistic aspects were first rigorously analyzed in Hinich and Ordeshook (1969, 1971), and Hinich, Ledyard and Ordeshook (1972). For the sake of simplicity in this paper probabilistic voting models without abstentions will be considered following the approach of among others, Comanor (1976), Coughlin and Nitzan (1981b), and Feldman and Lee (1988). In Wittman (1984) the following two arguments for the probabilistic voting model are given. The first argument is that political candidates do not have perfect information about the preferences and actions of the voters. This is also the point of view taken in Coughlin, Mueller and Murrell (1990) where in the preferences of the voters there is a bias in favour of or against a political candidate not perfectly known to the political candidates. The second argument is that voters do not have perfect information about the positions of the political candidates when casting their vote.

It will be assumed that candidates have the same subjective expectations about the voting behaviour of the consumers. This assumption can easily be relaxed, but is made for notational convenience. For candidate $k \in I_2$ and for every consumer $i \in I_m$ a voting function $\pi^{ki} : \bar{v}^i(C^1) \times \bar{v}^i(C^2) \to [0,1]$ describes the expectations of candidate k about the voting behaviour of consumer i. If the candidates have chosen actions $c^1 \in C^1$ and $c^2 \in C^2$ then $\pi^{ki}(\bar{v}^i(c^1), \bar{v}^i(c^2))$ is the probability candidate $k \in I_2$ assigns to the event that consumer $i \in I_m$ votes for him. This completes the description of the economy $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ with political system $G = (\{A^k, (\{\pi^{ki}\}_{i=1}^m)\}_{k=1}^2)$.

Deterministic voting without abstentions corresponds to the case where $\pi^{1i}(v^{1i}, v^{2i}) = 1$ if $v^{1i} > v^{2i}$, $\pi^{1i}(v^{1i}, v^{2i}) = \frac{1}{2}$ if $v^{1i} = v^{2i}$, $\pi^{1i}(v^{1i}, v^{2i}) = 0$ if $v^{1i} < v^{2i}$, and $\pi^{2i}(v^{1i}, v^{2i}) = 1 - \pi^{1i}(v^{1i}, v^{2i})$, $\forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$. As a matter of realism the assumption that π^{ki} is non-decreasing in v^{ki} and non-increasing in $v^{k'i}$, $k' \neq k$, is often made. For this paper the only assumption needed with respect to the voting functions is the following.

B2. The function $\pi^{ki} : \bar{v}^i(C^1) \times \bar{v}^i(C^2) \to [0,1]$ is continuous, $\forall i \in I_m$, $\forall k \in I_2$.

In case both $\bar{v}^i(C^1)$ and $\bar{v}^i(C^2)$ have no accumulation points Assumption B2 does not exclude any function π^{ki} and therefore does not exclude deterministic voting behaviour. This is for example the case if the sets C^1 and C^2 are finite.

A preference relation $\succeq^i$ on X^i of a consumer $i \in I_m$ satisfying Assumptions A1 and A3 admits many representations by utility functions. Now consider the case where such a representation $\hat{u}^i$ (not necessarily continuous) different from u^i is chosen. Since the voting functions depend on the representation chosen, it is a natural question to ask whether the voting functions $\hat{\pi}^{1i}$ and $\hat{\pi}^{2i}$ associated with $\hat{u}^i$ are continuous if π^{1i} and π^{2i} are continuous, or in other words is Assumption B2 independent of the representation chosen for the preference relation. To show this, let the function $h^i : \hat{u}^i(X^i) \to u^i(X^i)$ be defined by

$h^i(\hat{u}^i(x^i)) = u^i(x^i)$, $\forall x^i \in X^i$. Let $(t^r)_{r\in\mathbb{N}}$ be a sequence in $\hat{u}^i(X^i)$ converging to a point $t \in \hat{u}^i(X^i)$. It is not difficult to show that it is possible to construct a sequence $(x^{i^r})_{r\in\mathbb{N}}$ in X^i satisfying $\hat{u}^i(x^{i^r}) = t^r$ and $(x^{i^r})_{r\in\mathbb{N}} \to x^i$ for some $x^i \in X^i$ with $\hat{u}^i(x^i) = t$. Then $\lim_{r\to\infty} h^i(t^r) = \lim_{r\to\infty} h^i(\hat{u}^i(x^{i^r})) = \lim_{r\to\infty} u^i(x^{i^r}) = u^i(x^i) = h^i(\hat{u}^i(x^i)) = h^i(t)$, and therefore the function h^i is continuous. Let be given $(v^{1i}, v^{2i}) \in \hat{v}^i(C^1) \times \hat{v}^i(C^2)$, where $\hat{v}^i$ is the indirect utility function associated with $\hat{u}^i$. Then $\hat{\pi}^{ki}(v^{1i}, v^{2i}) = \pi^{ki}(h^i(v^{1i}), h^i(v^{2i}))$ and hence $\hat{\pi}^{ki}$ is continuous by the continuity of the functions π^{ki} and h^i.

The political candidates are assumed to maximize either both their expected plurality or both their probability of winning the elections. In the first case the pay-off function $K^1 : C^1 \times C^2 \to \mathbf{R}$ of candidate 1 is defined by

$$K^1\left(c^1, c^2\right) = \sum_{i=1}^{m} \pi^{1i}\left(\bar{v}^i(c^1), \bar{v}^i(c^2)\right) - \sum_{i=1}^{m} \pi^{2i}\left(\bar{v}^i(c^1), \bar{v}^i(c^2)\right), \ \forall c^1 \in C^1,\ \forall c^2 \in C^2. \quad (4)$$

The pay-off function $K^2 : C^1 \times C^2 \to \mathbf{R}$ of candidate 2 is easily seen to be equal to $-K^1$. If candidates maximize their probability of winning the elections then the pay-off function $K^1 : C^1 \times C^2 \to \mathbf{R}$ of candidate 1 is defined by

$$K^1\left(c^1, c^2\right) = \sum_{\{S\subset I_m \| |S| \geq \frac{1}{2}m + \frac{1}{2}\}} \prod_{i\in S} \pi^{1i}\left(\bar{v}^i(c^1), \bar{v}^i(c^2)\right) \prod_{i\in I_m\setminus S} \pi^{2i}\left(\bar{v}^i(c^1), \bar{v}^i(c^2)\right)$$
$$+\frac{1}{2} \sum_{\{S\subset I_m \| |S| = \frac{1}{2}m\}} \prod_{i\in S} \pi^{1i}\left(\bar{v}^i(c^1), \bar{v}^i(c^2)\right) \prod_{i\in I_m\setminus S} \pi^{2i}\left(\bar{v}^i(c^1), \bar{v}^i(c^2)\right) - \frac{1}{2}, \ \forall c^1 \in C^1,\ \forall c^2 \in C^2. \quad (5)$$

Here $|S|$ denotes the number of elements of a set S. In case of a tie, the toss with a fair coin determines the outcome of the elections. Empty sets are included in the summation and the convention is made that $\prod_{i\in\emptyset} \pi^{ki}(\bar{v}^i(c^1), \bar{v}^i(c^2)) = 1$. The pay-off function $K^2 : C^1 \times C^2 \to \mathbf{R}$ of candidate 2 is again easily seen to be equal to $-K^1$. Now price regulations will be determined endogenously as the outcome of a game with sets of admissible actions C^1 and C^2 and pay-off functions K^1 and K^2.

Definition 3.2 (Political Economic Equilibrium)
A political economic equilibrium of the economy $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ with political system $G = (\{A^k, (\{\pi^{ki}\}_{i=1}^m)\}_{k=1}^2)$ is a Nash equilibrium for the mixed extension of the game $\mathcal{G} = \langle C^1, C^2, K^1, K^2 \rangle$, where C^1 and C^2 are as defined in (3), and K^1 and K^2 are either as defined in (4) or as defined in (5).

For $k \in I_2$, let $D(C^k)$ be the set of Borel probability measures on C^k. Then, formally, a Nash equilibrium for the mixed extension of the game $\mathcal{G}$ defined above is a pair of probability measures $(\mu^{*1}, \mu^{*2}) \in D(C^1) \times D(C^2)$ such that

$$\int_{(c^1,c^2)\in C^1\times C^2} K^1(c^1, c^2) d(\mu^{*1}(c^1) \times \mu^{*2}(c^2)) \ = \ \max_{\mu^1 \in D(C^1)} \int_{(c^1,c^2)\in C^1\times C^2} K^1(c^1, c^2) d(\mu^1(c^1) \times \mu^{*2}(c^2)),$$

and

$$\int_{(c^1,c^2)\in C^1\times C^2} K^2(c^1,c^2)d(\mu^{*1}(c^1)\times\mu^{*2}(c^2)) = \max_{\mu^2\in D(C^2)}\int_{(c^1,c^2)\in C^1\times C^2} K^2(c^1,c^2)d(\mu^{*1}(c^1)\times\mu^2(c^2)),$$

see for example Dasgupta and Maskin (1986).

Usually, theorists in voting theory are reluctant to use an equilibrium in mixed strategies as a solution to a game as described in Definition 3.2. The main objection against mixed strategies is that the game in Definition 3.2 does not take into account the dynamic features of campaigns in real world elections. During the campaign political candidates have the possibility to sequentially adjust their proposals. In case an equilibrium in pure strategies exists a political candidate can use his pure strategy during the entire campaign. In case political candidates adopt a mixed strategy then the proposals made by them at a specific point in time will in general not be best responses to each other. Therefore on the next point in time a candidate might want to adjust his strategy. However, as has been pointed out in McKelvey and Ordeshook (1976) this criticism is not completely justified and it is interesting to consider mixed strategy equilibria too, their main reasons being the following. First if one wants to analyze dynamic issues, then this should be explicitly incorporated in the game defined. Considering mixed strategy equilibria for the static game of Definition 3.2 is an essential first step. Second, it is possible to give several reasonable dynamic models where indeed equilibria corresponding to the mixed strategy equilibria of the static game are obtained. It seems reasonable that even in a dynamic context candidates make proposals in the support set of mixed strategy equilibria of the static game.

4 Existence Results

The existence of a political economic equilibrium will be shown in this section. The first step is to show that the set of admissible actions of each candidate is non-empty. The following theorem is about the existence of a constrained equilibrium for a given lower and upper bound on the prices.

Theorem 4.1
Let an economy $\mathcal{E}_{(\underline{p},\bar{p})}$ satisfying Assumptions A1, A2, A3, and A4 be given. Then there exists a constrained equilibrium $(x^{*1},\ldots,x^{*m},l^*,L^*,p^*)$ of the economy $\mathcal{E}_{(\underline{p},\bar{p})}$.

Proof
The conditions given in Drèze (1975) are satisfied, where the existence of a constrained equilibrium is shown.

Q.E.D.

In the following theorem it is shown that there exists a number $M \in \mathbf{R}_+$ such that if in a constrained equilibrium the price of a commodity is higher than M, then every consumer keeps his initial endowments of this commodity. The number M can be chosen independently of the price regulations imposed. This is quite remarkable since the following intuition behind this result is wrong. If the price of a commodity is very high, then every consumer wants to supply this commodity and the only way to obtain an equilibrium is therefore to ration the excess supplies of this commodity completely. This intuition is not correct, since there might be prices of other commodities being even higher. The right argument goes along the following lines. If the price of a commodity is very high and a consumer demands this commodity then he also has to demand an amount of the numeraire commodity which exceeds the total endowments of the numeraire commodity, giving a contradiction since this can not happen in equilibrium. It has to be remarked that it is indeed possible that an equilibrium price is greater than M since the minimum price of a commodity could be greater than M. In case the assumptions of Theorem 4.2 are not satisfied it is possible that a number M with the desired properties does not exist. This follows immediately from the example in Benassy (1975, Section 6). The constrained equilibria derived in this example satisfy the equilibrium concepts of both Benassy and Drèze, while for every price regulation in R on every market some trade occurs in the corresponding constrained equilibrium.

Theorem 4.2

Let an economy $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ satisfying Assumptions A1, A2, and A3 be given. Let M be as in Lemma 2.1. Then, for every $(\underline{p}, \bar{p}) \in R$, if $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p},\bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})})$ and $p_k^* \geq M$ for some $k \in I_n$ then $x_k^{*i} = w_k^i$ and $l_k^* = 0$ for every $i \in I_m$.

Proof

Suppose $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p},\bar{p})}$ with for some $k \in I_n$, $p_k^* \geq M$ and $l_k^* < 0$. Moreover, let k be such that for every $j \in I_n$, $p_j^* > p_k^*$ implies $l_j^* = 0$. Suppose that l_k^* is binding for every consumer $i \in I_m$. Then it holds that $\sum_{i=1}^m (x_k^{*i} - w_k^i) = \sum_{i=1}^m l_k^* < 0$, a contradiction. Consequently some consumer $h \in I_m$ is not rationed on his supply on market k, and so $x^{*h} \in \Delta^h(\tilde{l}^*, L^*, p^*)$, with $\tilde{l}_k^* = -\bar{w}_k$ and $\tilde{l}_j^* = l_j^*$, $\forall j \in I_n \setminus \{k\}$. By Lemma 2.3 there is no loss of generality in assuming that for every $j \in I_n$, $l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j$. So $(\tilde{l}^*, L^*, p^*) \in \mathcal{L}_M$ and therefore, by Lemma 2.1, $x_0^{*h} > \bar{w}_0$, a contradiction. Hence $p_k^* \geq M$ for some $k \in I_n$ implies $l_k^* = 0$. Therefore, for every $i \in I_m$, $x_k^{*i} \geq w_k^i$, and since $\sum_{i=1}^m x_k^{*i} = \sum_{i=1}^m w_k^i$ it holds that $x_k^{*i} = w_k^i$.

Q.E.D.

Lemma 4.3 shows that there is a compact subset of R such that for every element of R outside this compact set, the set of corresponding constrained equilibrium allocations is the same as the set of constrained equilibrium allocations corresponding with some element of R in this compact set. This is not surprising since by Theorem 4.2 we know that if the constrained equilibrium price on some market is greater than or equal to M then no trade takes place on this market.

Lemma 4.3

Let an economy $\mathcal{E}_{(\underline{p},\bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})})$ satisfying Assumptions A1, A2, A3, and A4 be given. Let M be as in Lemma 2.1 for the economy $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$. For every $j \in I_n$, let $\tilde{\underline{p}}_j = \min\{M, \underline{p}_j\}$ and $\tilde{\bar{p}}_j = \min\{M, \bar{p}_j\}$. Then $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p},\bar{p})}$ for some $p^* \in P_{(\underline{p},\bar{p})}$ if and only if $(x^{*1}, \ldots, x^{*m}, l^*, L^*, \tilde{p}^*)$ is a constrained equilibrium of the economy $\mathcal{E}_{(\tilde{\underline{p}},\tilde{\bar{p}})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\tilde{\underline{p}},\tilde{\bar{p}})})$, for some $\tilde{p}^* \in P_{(\tilde{\underline{p}},\tilde{\bar{p}})}$.

Proof

Using Lemma 2.3 there is no loss of generality in assuming that for every $j \in I_n$, $l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j$.

Let $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ be a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p},\bar{p})}$ for some $p^* \in P_{(\underline{p},\bar{p})}$. Let $\tilde{p}^* \in P_{(\tilde{\underline{p}},\tilde{\bar{p}})}$ be defined by $\tilde{p}_j^* = \min\{M, p_j^*\}$, $\forall j \in I_n$, $\tilde{p}_0^* = 1$. Clearly Conditions 2 and 3 of Definition 2.2 are satisfied for $(x^{*1}, \ldots, x^{*m}, l^*, L^*, \tilde{p}^*)$. If $\tilde{p}_j^* < \tilde{\bar{p}}_j$ then $p_j^* < \bar{p}_j$ and if $\tilde{p}_j^* > \tilde{\underline{p}}_j$ then $p_j^* > \underline{p}_j$, and therefore Condition 4 of Definition 2.2 is satisfied. Define $J = \{j \in I_n \mid \tilde{p}_j^* = M\}$. For every $j \in J$ it holds that $p_j^* \geq M$ and hence by Theorem 4.2 for every $i \in I_m$, $0 = l_j^* = x_j^{*i} - w_j^i$, so $x^{*i} \in \Gamma^i(l^*, L^*, \tilde{p}^*)$. It remains to be shown that for every $i \in I_m$, $x^{*i} \in \delta^i(l^*, L^*, \tilde{p}^*)$. Suppose for some $h \in I_m$ there exists $y^h \in \delta^h(l^*, L^*, \tilde{p}^*)$ such that $y^h \succ^h x^{*h}$. Then, using the convexity of preferences, there exists $\hat{y}^h \in \Delta^h(l^*, L^*, \tilde{p}^*)$ such that $\hat{y}^h \succ^h x^{*h}$. Clearly $\hat{y}^h \notin \Gamma^h(l^*, L^*, p^*)$, so $p^* \cdot \hat{y}^h > \tilde{p}^* \cdot \hat{y}^h$, and using that $p_j^* = \tilde{p}_j^*$, $\forall j \in I_n^0 \setminus J$, $p_j^* \geq \tilde{p}_j^*$, $\forall j \in J$, and $l_j^* = 0$, $\forall j \in J$, it has to hold that $\hat{y}_k^h > w_k^h$ for some $k \in J$. Define $p(\lambda) = \lambda \tilde{p}^* + (1 - \lambda) p^*$. Define $\lambda^* = \sup\{\lambda \in [0,1] \mid x^h \in \Delta^h(l^*, L^*, p(\lambda))$ implies $x_j^h = w_j^h$, $\forall j \in J\}$. So there exists a sequence $(\lambda^r)_{r \in \mathbb{N}}$ such that $\lambda^r \to \lambda^*$, $0 \leq \lambda^r \leq \lambda^*$, and $x^{*h} \in \Delta^h(l^*, L^*, p(\lambda^r))$. Hence $x^{*h} \in \Delta^h(l^*, L^*, p(\lambda^*))$ using the upper semi-continuity of Δ^h (since $p_0(\lambda^*) = 1$, see Drèze (1975)) at the point $(l^*, L^*, p(\lambda^*))$. As a consequence of the existence of $\hat{y}^h$, $\lambda^* < 1$. Now let $(\lambda^r)_{r \in \mathbb{N}}$ be a sequence such that $\lambda^r \to \lambda^*$ and $\lambda^r > \lambda^*$. Then there exists $\hat{y}^{h^r} \in \Delta^h(l^*, L^*, p(\lambda^r))$ such that for some $k^r \in J$, $\hat{y}_{k^r}^{h^r} > w_{k^r}^h$. Hence $\hat{y}^{h^r} \in \Delta^h(\tilde{l}^{*^r}, L^*, p(\lambda^r))$ where $\tilde{l}_{k^r}^{*^r} = -\bar{w}_{k^r}$ and $\tilde{l}_j^{*^r} = l_j^*$, $\forall j \in I_n \setminus \{k^r\}$. Without loss of generality $(\hat{y}^{h^r}, \tilde{l}^{*^r})_{r \in \mathbb{N}} \to (\bar{y}^h, \bar{l}^*)$, where for some $k \in J$, $\bar{l}_k^* = -\bar{w}_k$ and $\bar{l}_j^* = l_j^*$, $\forall j \in I_n \setminus \{k\}$. Moreover, $\bar{y}^h \in \Delta^h(\bar{l}^*, L^*, p(\lambda^*))$ using the upper semi-continuity of Δ^h at the point $(\bar{l}^*, L^*, p(\lambda^*))$. Since $\hat{y}^{h^r} \in \Delta^h(l^*, L^*, p(\lambda^r))$ it follows that $\bar{y}^h \in \Delta^h(l^*, L^*, p(\lambda^*))$. So $x^{*h} \sim^h \bar{y}^h$ and therefore $x^{*h} \in \Delta^h(\bar{l}^*, L^*, p(\lambda^*))$.

Since $(l^*, L^*, p(\lambda^*)) \in \mathcal{L}_M$, $x_0^{*h} > \bar{w}_0$, a contradiction since $\sum_{i=1}^m x_0^{*i} = \bar{w}_0$. So indeed $x^{*i} \in \delta^i(l^*, L^*, \tilde{p}^*)$ for every $i \in I_m$.
Let $(x^{*1}, \ldots, x^{*m}, l^*, L^*, \tilde{p}^*)$ be a constrained equilibrium of the economy $\mathcal{E}_{(\underline{\tilde{p}},\tilde{p})}$ for some $\tilde{p}^* \in P_{(\underline{\tilde{p}},\tilde{p})}$. Define the sets J^1, J^2, and J^3 by

$$\begin{aligned} J^1 &= \{j \in I_n \mid \tilde{p}_j^* = M \text{ and } \exists i \in I_m,\ x_j^{*i} - w_j^i = L_j^*\}, \\ J^2 &= \{j \in I_n \mid \tilde{p}_j^* = M \text{ and } \forall i \in I_m,\ l_j^* < x_j^{*i} - w_j^i < L_j^*\}, \\ J^3 &= \{j \in I_n \mid \tilde{p}_j^* = M \text{ and } \exists i \in I_m,\ l_j^* = x_j^{*i} - w_j^i\}. \end{aligned}$$

Let $p^* \in P_{(\underline{p},\bar{p})}$ be defined by $p_j^* = \bar{p}_j$, $\forall j \in J^1 \cup J^2$, $p_j^* = \underline{p}_j$, $\forall j \in J^3$, and $p_j^* = \tilde{p}_j^*$, $\forall j \in I_n^0 \setminus (J^1 \cup J^2 \cup J^3)$. It is easily verified that $(x^{*1}, \ldots, x^{*m}, l^*, L^*, p^*)$ satisfies Conditions 2, 3, and 4 of Definition 2.2. For $j \in I_n$, if $\tilde{p}_j^* < M$ then $p_j^* = \tilde{p}_j^*$, and if $\tilde{p}_j^* = M$ then $p_j^* \geq \tilde{p}_j^*$ and by Theorem 4.2, $0 = l_j^* = x_j^{*i} - w_j^i$, $\forall i \in I_m$. Consequently, for every $i \in I_m$, $x^{*i} \in \gamma^i(l^*, L^*, p^*) \subset \gamma^i(l^*, L^*, \tilde{p}^*)$, and hence $x^{*i} \in \delta^i(l^*, L^*, p^*)$. So Condition 1 of Definition 2.2 is satisfied.

Q.E.D.

For a given economy E let M be as in Lemma 2.1 and for $k \in I_2$ define $A_M^k = \beta_M^{2n}(A^k)$, and define C_M^k as in (3) where A^k is replaced by A_M^k. The following lemma states that under the assumptions made the set of actions C_M^k is non-empty and compact for $k \in I_2$.

Lemma 4.4

Let be given $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ and $(\{A^k\}_{k=1}^2)$ satisfying the Assumptions A1, A2, A3, and B1. Let M be as in Lemma 2.1. Then the set C_M^k, $k \in I_2$, is non-empty and compact.

Proof

For $k \in I_2$ it follows immediately from Theorem 4.1, Theorem 2.5, and the set A_M^k being non-empty according to Assumption B1, that the set C_M^k is non-empty. Clearly the set C_M^k is bounded. The set C_M^k is closed if every convergent sequence in C_M^k has a limit in C_M^k. Let $(\underline{p}^r, \bar{p}^r, q^r)_{r\in\mathbb{N}}$ be a convergent sequence in C_M^k with limit $(\underline{p}, \bar{p}, q)$. By definition of C_M^k there exists a sequence $(x^r, \underline{p}^r, \bar{p}^r, q^r)_{r\in\mathbb{N}}$, such that $(x^r, \hat{l}(q^r), \hat{L}(q^r), \hat{p}(\underline{p}^r, \bar{p}^r, q^r))$ is a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p}^r,\bar{p}^r)} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p}^r,\bar{p}^r)})$. This sequence is obviously bounded and therefore has a convergent subsequence $(x^{r^s}, \underline{p}^{r^s}, \bar{p}^{r^s}, q^{r^s})_{s\in\mathbb{N}}$ with limit, say, $(x, \underline{p}, \bar{p}, q)$. Since the sets $X^1, \ldots, X^m, Q$, and A_M^k are closed (by Assumption B1), it holds that $\forall i \in I_m$, $x^i \in X^i$, $q \in Q$, and $(\underline{p}, \bar{p}) \in A_M^k$. It will be shown that $(x, \hat{l}(q), \hat{L}(q), \hat{p}(\underline{p}, \bar{p}, q))$ satisfies the four conditions of a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p},\bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})})$. Clearly, for $j \in I_n$,

$$\hat{p}_j(\underline{p}^{r^s}, \bar{p}^{r^s}, q^{r^s}) = \max\{\underline{p}_j^{r^s}, \min\{\bar{p}_j^{r^s}, \underline{p}_j^{r^s}(2 - 3q_j^{r^s}) + \bar{p}_j^{r^s}(3q_j^{r^s} - 1)\}\}$$

$$\to \max\{\underline{p}_j, \min\{\bar{p}_j, \underline{p}_j(2-3q_j)+\bar{p}_j(3q_j-1)\}\} = \hat{p}_j(\underline{p},\bar{p},q), \quad (6)$$

$$\hat{l}_j(q^{r^s}) = -\min\{1, 3q_j^{r^s}\}\bar{w}_j \to -\min\{1, 3q_j\}\bar{w}_j = \hat{l}_j(q), \quad (7)$$

$$\hat{L}_j(q^{r^s}) = \min\{1, 3-3q_j^{r^s}\}\bar{w}_j \to \min\{1, 3-3q_j\}\bar{w}_j = \hat{L}_j(q). \quad (8)$$

Using the results in Drèze (1975) it follows that for every $i \in I_m$, δ^i is an upper semi-continuous correspondence from $-\mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \{1\} \times \mathbf{R}^n_+$ into X^i. Using (6), (7), and (8), this yields $x^i \in \delta^i(\hat{l}(q), \hat{L}(q), \hat{p}(\underline{p},\bar{p},q))$. Consequently, Condition 1 is satisfied. Condition 2 holds since $\sum_{i=1}^m x^i = \sum_{i=1}^m \lim_{s\to\infty} x^{ir^s} = \lim_{s\to\infty} \sum_{i=1}^m x^{ir^s} = \bar{w}$. Obviously $0 \leq x^i_j \leq \bar{w}_j$, $\forall i \in I_m$, $\forall j \in I_n$. This property will be used in order to prove that Conditions 3 and 4 are satisfied. Suppose there exists a consumer $i \in I_m$ and a commodity $j \in I_n$ such that $x^i_j - w^i_j = \hat{L}_j(q)$. Then $q_j > \frac{2}{3}$ and so $\hat{l}_j(q) = -\bar{w}_j$. Hence for all consumers $h \in I_m$ it holds that $x^h_j - w^h_j \geq -w^h_j > -\bar{w}_j = \hat{l}_j(q)$. Suppose now there exists a consumer $i \in I_m$ and a commodity $j \in I_n$ such that $x^i_j - w^i_j = \hat{l}_j(q)$. Then $q_j < \frac{1}{3}$ and so $\hat{L}_j(q) = \bar{w}_j$. Hence for all consumers $h \in I_m$ it holds that $x^h_j - w^h_j \leq \bar{w}_j - w^h_j < \bar{w}_j = \hat{L}_j(q)$, which shows that Condition 3 is satisfied.

Suppose there exists a commodity $j \in I_n$ such that $\hat{p}_j(\underline{p},\bar{p},q) < \bar{p}_j$. Then $q_j < \frac{2}{3}$ and consequently $\hat{L}_j(q) = \bar{w}_j > \bar{w}_j - w^i_j \geq x^i_j - w^i_j$, $\forall i \in I_m$. Finally, suppose there exists a commodity $j \in I_n$ such that $\hat{p}_j(\underline{p},\bar{p},q) > \underline{p}_j$. Then $q_j > \frac{1}{3}$ and consequently $\hat{l}_j(q) = -\bar{w}_j < -w^i_j \leq x^i_j - w^i_j$, $\forall i \in I_m$, which proves that Condition 4 is satisfied. So, $(x, \hat{l}(q), \hat{L}(q), \hat{p}(\underline{p},\bar{p},q))$ satisfies all conditions of Definition 2.2 for a constrained equilibrium of the economy $\mathcal{E}_{(\underline{p},\bar{p})}$. Since $(\underline{p},\bar{p}) \in A^k_M$ it holds that $(\underline{p},\bar{p},q)$ is an element of C^k_M.

Q.E.D.

For $k \in I_2$ define the function $K^k_M : C^1_M \times C^2_M \to \mathbf{R}$ as in (4) or (5). For every $i \in I_m$ it follows from Lemma 4.3 that $\bar{v}^i(C^k) = \bar{v}^i(C^k_M)$, so the function $\pi^{ki} : \bar{v}^i(C^1) \times \bar{v}^i(C^2) \to [0,1]$ is well defined on $\bar{v}^i(C^1_M) \times \bar{v}^i(C^2_M)$ and therefore the function K^k_M is well defined on $C^1_M \times C^2_M$. In the next lemma it is shown that the function K^k_M is continuous.

Lemma 4.5

Let be given $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ and $G = (\{A^k, (\{\pi^{ki}\}_{i=1}^m)\}_{k=1}^2)$ satisfying the Assumptions A1, A2, A3, B1, and B2. Let M be as in Lemma 2.1. Then for $k \in I_2$ the function $K^k_M : C^1_M \times C^2_M \to \mathbf{R}$ as defined in (4) or (5) is continuous.

Proof

It is first shown that the function $\bar{v}^i$ is continuous on C, $\forall i \in I_m$. Let be given a sequence $(\underline{p}^r, \bar{p}^r, q^r)_{r\in\mathbf{N}}$ in the set C converging to an element $(\underline{p},\bar{p},q)$. Let $x^{ir} \in \delta^i(\hat{l}(q^r), \hat{L}(q^r), \hat{p}(\underline{p}^r, \bar{p}^r, q^r))$ then, similar to the proof of Lemma 4.4, without loss of generality the sequence $(x^{ir})_{r\in\mathbf{N}}$ converges to an element x^i and $x^i \in \delta^i(\hat{l}(q), \hat{L}(q), \hat{p}(\underline{p},\bar{p},q))$. Hence, using the continuity of u^i, $\bar{v}^i(\underline{p}^r, \bar{p}^r, q^r) = u^i(x^{i^r}) \to u^i(x^i) = \bar{v}^i(\underline{p}, \bar{p}, q)$. By Lemma 4.3 for every $i \in I_m$ and $k \in I_2$ it holds that $\bar{v}^i(C^k) = \bar{v}^i(C^k_M)$ and therefore by Assumption B2 the function π^{ki} is continuous on $v^i(C^1_M) \times v^i(C^2_M)$. Now the continuity of K^k_M follows from the continuity of $\bar{v}^i$ and π^{ki}, $\forall i \in I_m$.

Q.E.D.

In the next theorem the existence of a political economic equilibrium is shown.

Theorem 4.6
Let be given $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ and $G = (\{A^k, (\{\pi^{ki}\}_{i=1}^m)\}_{k=1}^2)$ satisfying the Assumptions A1, A2, A3, B1, and B2. Then there exists a political economic equilibrium for the economy E with political system G.

Proof
In order to show the existence of a political economic equilibrium, the existence of a Nash equilibrium of the mixed extension of the game $\mathcal{G} = \langle C^1, C^2, K^1, K^2 \rangle$ has to be shown. Let M be as in Lemma 2.1 and consider the game $\mathcal{G}_M = \langle C^1_M, C^2_M, K^1_M, K^2_M \rangle$. According to Lemma 4.4 the sets C^1_M and C^2_M are non-empty and compact. By Lemma 4.5 K^1_M and K^2_M are continuous functions on $C^1_M \times C^2_M$. Hence using a theorem of Glicksberg (1952) the mixed extension of $\mathcal{G}_M$ has at least one Nash equilibrium outcome. Obviously, using Lemma 4.3, this Nash equilibrium outcome yields a Nash equilibrium outcome for the game $\mathcal{G}$.

Q.E.D.

It is interesting to have sufficient conditions for the existence of a Nash equilibrium in pure strategies. Usually it is sufficient to assume certain concavity and convexity conditions with respect to the pay-off functions and strategy sets in order to prove the existence of a pure strategy Nash equilibrium, see for example Feldman and Lee (1988). However, since the set C^k is not necessarily convex these conditions might not be satisfied. In Theorem 4.7 other sufficient conditions for the existence of a pure strategy political economic equilibrium are given. It is clear that these conditions are very strong, since the following separability assumption is needed.

B3. For every $i \in I_m$ there exists a function $f^i : \bar{v}^i(C^1) \to \mathbf{R}$ and a function $g^i : \bar{v}^i(C^2) \to \mathbf{R}$ such that $\pi^{1i}(v^{1i}, v^{2i}) = f^i(v^{1i}) - g^i(v^{2i}) + \frac{1}{2}$, $\forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$.

Clearly, for all $i \in I_m$, $\pi^{2i}(v^{1i}, v^{2i}) = 1 - \pi^{1i}(v^{1i}, v^{2i}) = \frac{1}{2} + g^i(v^{2i}) - f^i(v^{1i})$, $\forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$ and hence π^{2i} also satisfies the separability assumption. It is not difficult to show that the continuity of π^{1i} implies that f^i and g^i must be continuous for every $i \in I_m$.

Since the voting functions depend on the utility representation chosen for the preference relation of a consumer, it is important to show that Assumption B3 holds independent of this representation. Let $\hat{u}^i$ be a representation of $\succeq^i$ of consumer $i \in I_m$ different from u^i and define the function $h^i : \hat{u}^i(X^i) \to u^i(X^i)$ in the same way as has been done below Assumption B2. Denote the indirect utility functions and the voting functions corresponding with $\hat{u}^i$ by $\hat{v}^i$, $\hat{\pi}^{1i}$, and $\hat{\pi}^{2i}$, respectively, for every $i \in I_m$. Let be given $(v^{1i}, v^{2i}) \in \hat{v}^i(C^1) \times \hat{v}^i(C^2)$. Then $\hat{\pi}^{1i}(v^{1i}, v^{2i}) = \pi^{1i}(h^i(v^{1i}), h^i(v^{2i})) = f^i(h^i(v^{1i})) - g^i(h^i(v^{2i})) + \frac{1}{2}$, so $\hat{\pi}^{1i}$ satisfies Assumption B3.

Theorem 4.7
Let be given $E = (\{X^i, \succeq^i, w^i\}_{i=1}^m)$ and $G = (\{A^k, (\{\pi^{ki}\}_{i=1}^m)\}_{k=1}^2)$ satisfying the Assumptions A1, A2, A3, B1, B2, and B3. Then there exists a pure strategy political economic equilibrium of the economy E with political system G in case for $k \in I_2$ the function K^k is defined as in (4).

Proof
Let $c^{*1} \in C^1$ be such that $\sum_{i=1}^m f^i(\bar{v}^i(c^{*1})) = \max_{c^1 \in C^1} \sum_{i=1}^m f^i(\bar{v}^i(c^1))$, and let $c^{*2} \in C^2$ be such that $\sum_{i=1}^m g^i(\bar{v}^i(c^{*2})) = \max_{c^2 \in C^2} \sum_{i=1}^m g^i(\bar{v}^i(c^2))$. Then c^{*1} and c^{*2} are well defined using Lemma 4.3, Assumption B1, and the continuity for every $i \in I_m$ of the functions f^i, g^i, and $\bar{v}^i$. Moreover,

$$\begin{aligned}
K^1(c^{*1}, c^{*2}) &= 2\sum_{i=1}^m f^i(\bar{v}^i(c^{*1})) - 2\sum_{i=1}^m g^i(\bar{v}^i(c^{*2})) \\
&\geq 2\sum_{i=1}^m f^i(\bar{v}^i(c^1)) - 2\sum_{i=1}^m g^i(\bar{v}^i(c^{*2})), \ \forall c^1 \in C^1, \\
K^2(c^{*1}, c^{*2}) &= 2\sum_{i=1}^m g^i(\bar{v}^i(c^{*2})) - 2\sum_{i=1}^m f^i(\bar{v}^i(c^{*1})) \\
&\geq 2\sum_{i=1}^m g^i(\bar{v}^i(c^2)) - 2\sum_{i=1}^m f^i(\bar{v}^i(c^{*1})), \ \forall c^2 \in C^2.
\end{aligned}$$

Hence (c^{*1}, c^{*2}) is a pure strategy political economic equilibrium of the economy E with political system G.

Q.E.D.

Although Assumption B3 is very restrictive, it is of some interest since the voting model given in Coughlin, Mueller and Murrell (1990) satisfies this assumption. Let $(c^1, c^2) \in C^1 \times C^2$ be given. In the model of Coughlin, Mueller and Murrell (1990), a consumer $i \in I_m$ votes for candidate 1 if $\bar{v}^i(c^1) - \bar{v}^i(c^2) > b^i$, does not vote if $\bar{v}^i(c^1) - \bar{v}^i(c^2) = b^i$, and votes for candidate 2 if $\bar{v}^i(c^1) - \bar{v}^i(c^2) < b^i$, where the information of the candidates is that b^i is a random variable being uniformly distributed in some given interval $[-r^i, r^i] \subset \mathbf{R}$.

It is assumed that $|v^i(c^1) - v^i(c^2)| \leq r^i$, $\forall (c^1, c^2) \in C^1 \times C^2$. This implies for every $i \in I_m$,

$$\begin{aligned}
\pi^{1i}(c^1, c^2) &= \frac{1}{2} + \frac{1}{2r^i}(v^i(c^1) - v^i(c^2)),\ \forall (c^1, c^2) \in C^1 \times C^2,\\
\pi^{2i}(c^1, c^2) &= \frac{1}{2} + \frac{1}{2r^i}(\bar{v}^i(c^2) - \bar{v}^i(c^1)),\ \forall (c^1, c^2) \in C^1 \times C^2,
\end{aligned}$$

so in this case the choice $f^i(v^{1i}) = \frac{1}{2r^i}v^{1i}$, $\forall v^{1i} \in \bar{v}^i(C^1)$, and $g^i(v^{2i}) = \frac{1}{2r^i}v^{2i}$, $\forall v^{2i} \in \bar{v}^i(C^2)$, satisfies Assumption B3.

5 An Example

In this section an example of the model of the previous sections will be examined. The example makes clear that by the imposition of price regulations it is possible to obtain politically more desired allocations than the Walrasian equilibrium allocation. First the set of constrained equilibrium allocations in an economy will be considered given some price regulation. Consider an economy with two consumers, two commodities, and Cobb-Douglas utility functions. Commodity 0 is a numeraire commodity with price equal to one. The consumers have the following characteristics, $X^1 = X^2 = \mathbf{R}^2_+$, $u^1(x_0, x_1) = (x_0)^{\frac{1}{5}}(x_1)^{\frac{4}{5}}$, $\forall x \in X^1$, $u^2(x_0, x_1) = (x_0)^{\frac{4}{5}}(x_1)^{\frac{1}{5}}$, $\forall x \in X^2$, $w^1 = w^2 = (4, 1)$. Let be given some $(\underline{p}, \bar{p}) \in R$ such that $0 < \underline{p} = \bar{p} = p$. It is again useful to work with pseudo-prices. The rationing scheme will be a function of the pseudo-price q, where q is an element of the set $Q = [0, 1]$. The functions $\hat{l}$ and $\hat{L}$ are given by

$$\begin{aligned}
\hat{l}(q) &= -\min\{1, 2q\}\,\bar{w}_1 = -2\min\{1, 2q\},\ \forall q \in Q,\\
\hat{L}(q) &= \min\{1, 2 - 2q\}\frac{\bar{w}_0}{p} = \frac{8}{p}\min\{1, 2 - 2q\},\ \forall q \in Q.
\end{aligned}$$

Notice that the functions $\hat{l}$ and $\hat{L}$ are slightly different from the ones used before. In case of a fixed price the functions given above are easier to work with. After some computations it follows that

$$\delta^1(\hat{l}(q), \hat{L}(q), (1, p)) = \begin{cases}
(\frac{p+4}{5}, \frac{4p+16}{5p}), & 0 < p,\ \max\{0, \frac{p-16}{20p}\} \leq q \leq \min\{1, \frac{p+64}{80}\},\\
(16q - 12, \frac{16(1-q)}{p} + 1), & 0 < p \leq 16,\ \frac{p+64}{80} \leq q \leq 1,\\
(4pq + 4, 1 - 4q), & 16 \leq p,\ 0 \leq q \leq \frac{p-16}{20p},
\end{cases}$$

and

$$\delta^2(\hat{l}(q), \hat{L}(q), (1, p)) = \begin{cases}
(\frac{4p+16}{5}, \frac{p+4}{5p}), & 0 < p,\ \max\{0, \frac{p-1}{5p}\} \leq q \leq \min\{1, \frac{p+19}{20}\},\\
(16q - 12, \frac{16(1-q)}{p} + 1), & 0 < p \leq 1,\ \frac{p+19}{20} \leq q \leq 1,\\
(4pq + 4, 1 - 4q), & 1 \leq p,\ 0 \leq q \leq \frac{p-1}{5p}.
\end{cases}$$

By solving $\sum_{i=1}^{2} \delta^i(\hat{l}(q), \hat{L}(q), (1,p)) = \bar{w}$ all pseudo-prices corresponding with a constrained equilibrium for the economy $\mathcal{E}_{(p,p)}$ are obtained. Let the set of admissible price regulations for both political candidates be some non-empty subset $\bar{A}$ of R such that $0 < a_1 = a_2$, $\forall a \in \bar{A}$. It follows that the set $\bar{C}$ of admissible actions corresponding with the set of admissible price regulations A is given by

$$\begin{aligned} \bar{C} &= \{(p,p,q) \in \bar{A} \times Q \mid 0 < p \leq 1 \text{ and } q = 1, \text{ or } 1 < p < 4 \text{ and } q = \frac{21-p}{20}, \\ &\quad \text{or } p = 4 \text{ and } \frac{3}{20} \leq q \leq \frac{17}{20}, \text{ or } 4 < p < 16 \text{ and } q = \frac{16-p}{20p}, \text{ or } p \geq 16 \text{ and } q = 0\}. \quad (9) \end{aligned}$$

Although there is an interval of equilibrium pseudo-prices in the case where $p = 4$, they all correspond to the same constrained equilibrium allocation, which is the unique Walrasian equilibrium allocation. Therefore, considering the remarks made at Theorem 2.4, the possibility of not specifying price regulations is not excluded. Moreover, there is no loss of generality in considering only the admissible action corresponding with $q = \frac{1}{2}$ in this case. It can be verified that in case $0 < p \leq 1$ or $p \geq 16$ every consumer keeps his initial endowments in a constrained equilibrium corresponding with these values of p. If the price regulation is this extreme, no trade takes place. Using the functions u^i and δ^i it is easy to derive the indirect utility function of consumer $i \in I_2$, $\bar{v}^i : \bar{C} \to \mathbf{R}$. The voting functions of both consumers are assumed to be the same as in Coughlin and Nitzan (1981a), i.e., $\forall i \in I_2$, $\forall k \in I_2$,

$$\pi^{ki}\left(v^{1i}, v^{2i}\right) = \frac{e^{v^{ki}}}{e^{v^{1i}} + e^{v^{2i}}}, \; \forall v^{1i}, v^{2i} \in \bar{v}^i(\bar{C}).$$

Suppose candidates attempt to maximize their expected plurality. It is easy to show that for every $(c^1, c^2) \in \bar{C} \times \bar{C}$ it holds that

$$\sum_{i=1}^{2} \pi^{1i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) - \sum_{i=1}^{2} \pi^{2i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) \geq 0 \Leftrightarrow e^{\bar{v}^1(c^1)} e^{\bar{v}^2(c^1)} \geq e^{\bar{v}^1(c^2)} e^{\bar{v}^2(c^2)}.$$

Now suppose candidates attempt to maximize their probability of winning the elections. Then it is easy to show that

$$\begin{aligned} &\pi^{11}(\bar{v}^1(c^1), \bar{v}^1(c^2))\pi^{12}(\bar{v}^2(c^1), \bar{v}^2(c^2)) + \frac{1}{2}\pi^{11}(\bar{v}^1(c^1), \bar{v}^1(c^2))\pi^{22}(\bar{v}^2(c^1), \bar{v}^2(c^2)) \\ &+ \frac{1}{2}\pi^{12}(\bar{v}^2(c^1), \bar{v}^2(c^2))\pi^{21}(\bar{v}^1(c^1), \bar{v}^1(c^2)) - \frac{1}{2} \geq 0 \Leftrightarrow e^{\bar{v}^1(c^1)} e^{\bar{v}^2(c^1)} \geq e^{\bar{v}^1(c^2)} e^{\bar{v}^2(c^2)}. \end{aligned}$$

Using the symmetry of the game, it is then easily seen that both in the case where political candidates maximize expected plurality and in the case where political candidates maximize their probability of winning the elections, in a political economic equilibrium both candidates choose an action $c^* \in \bar{C}$ that maximizes $\sum_{i=1}^{2} \bar{v}^i(c)$ over $c \in \bar{C}$. Consider the case where $\bar{A} = \{(3,3), (4,4), (5,5)\}$. Hence the set $\bar{C}$ is given by $\{(3,3,\frac{9}{10}), (4,4,\frac{1}{2}), (5,5,\frac{11}{100})\}$.

	$p=3$	$p=4$	$p=5$
q	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{11}{100}$
$\delta^1(\hat{l}(q), \hat{L}(q), (1,p))$	$(2\frac{2}{5}, 1\frac{8}{15})$	$(1\frac{3}{5}, 1\frac{3}{5})$	$(1\frac{4}{5}, 1\frac{11}{25})$
$\delta^2(\hat{l}(q), \hat{L}(q), (1,p))$	$(5\frac{3}{5}, \frac{7}{15})$	$(6\frac{2}{5}, \frac{2}{5})$	$(6\frac{1}{5}, \frac{14}{25})$
$\hat{l}(q)$	-2	-2	$-\frac{11}{25}$
$\hat{L}(q)$	$\frac{8}{15}$	2	$\frac{8}{5}$
$\bar{v}^1(p,p,q)$	1.677	1.6	1.506
$\bar{v}^2(p,p,q)$	3.407	3.676	3.833
$\sum_{i=1}^2 \bar{v}^i(p,p,q)$	5.084	5.276	5.339

Table I: Constrained equilibria.

The constrained equilibria are summarized in Table I. From Table I it follows immediately that in the political economic equilibrium both candidates choose for a price regulation where $p = 5$. Compared with the Walrasian equilibrium allocation, this is advantageous for consumer 2 and disadvantageous for consumer 1. It should be remarked that the action where $\underline{p} = 5$ and no maximum price is specified is equivalent in this case. Consumer 2 is rationed on his excess supply of commodity 1.

Now consider the case where $A^1 = A^2 = R$. Using the definition of a constrained equilibrium and the remarks made below (9) it is not difficult to show that the set of admissible actions $\bar{C}$ corresponding with the set of admissible price regulations $\{(\underline{p}, \bar{p}) \in R \mid 1 \leq \underline{p} = \bar{p} \leq 16\}$ gives each candidate the same strategic possibilities as the set of admissible actions corresponding with the set R. In a political economic equilibrium both candidates therefore choose an action c^* such that $c^* = \arg\max_{c \in \bar{C}} \sum_{i=1}^2 \bar{v}^i(c)$. Clearly $c^* \neq (4, 4, \frac{1}{2})$. So in the case where every price regulation is allowed, a price regulation is chosen where the Walrasian equilibrium price system and the Walrasian equilibrium allocation is not allowed. It can be shown that $c^* = (5.035, 5.035, 0.1089)$. The corresponding constrained equilibrium will therefore be characterized by rationing on the supply of commodity 1.

Grandmont (1982) explains the occurrence of temporary price rigidities and quantity rationing by making the observation that in the short run quantities move faster than prices. The example considered in this section demonstrates that price rigidities may exist in the long run too.

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