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THE D_2-TRIANGULATION FOR SIMPLICIAL HOMOTOPY ALGORITHMS FOR COMPUTING SOLUTIONS OF NONLINEAR EQUATIONS

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The D_2-Triangulation for Simplicial Homotopy Algorithms for Computing Solutions of Nonlinear Equations

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Abstract

We propose a new triangulation of arbitrary refinement of grid sizes of $(0, 1] \times R^n$ for simplicial homotopy algorithms for computing solutions of nonlinear equations. On each level this triangualtion, called the D_2-triangualtion, subdivides R^n according to the D_1-triangulation introduced earlier by the author. It is showed that the D_2-triangulation is superior to the well-known K_2-triangulation and the well-known J_2-triangulation when counting the number of simplices between any two levels.

Keywords: Triangulations, Measures of Efficiency of Triangulations, Simplicial Homotopy Algorithms

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1 Introduction

Simplicial homotopy algorithms for computing solutions of nonlinear equations, which were originally introduced by Eaves and Saigal in [8], are established with triangulations of continuous refinement of grid sizes of $(0,1] \times R^n$. Examples are Eaves and Saigal's K_3-triangulation in [8], Todd's J_3-triangulation in [14], the author's D_3-triangulation in [4], the triangulations of van der Laan and Talman in [10], of Shamir in [12], of Kojima and Yamamoto in [9], of Broadie and Eaves in [2], and of Doup and Talman in [6]. All these triangulations were derived from the well-known K_1-triangulation of R^n or from the well-known J_1-triangulation of R^n except the D_3-triangulation which was derived from the D_1-triangulation of R^n. The latter triangulation was proposed by the author in an earlier paper [3] and it is superior to the K_1-triangulation and the J_1-triangulation according to all measures of efficiency of triangulations. The D_3-triangulation subdivides $(0,1] \times R^n$ with a fixed refinement factor of 2. In this paper, we construct a triangulation of continuous refinement of grid sizes of $(0,1] \times R^n$ by using the D_1-triangulation. The factor of grid refinement of the new triangulation, which we call the D_2-triangulation, can be any positive integer. In addition, in order to compare with the D_2-triangulation we present also the K_2-triangulation and the J_2-triangulation. It is showed that the D_2-triangulation is superior to the K_2-triangulation and the J_2-triangulation when counting the number of simplices.

In Section 2, the D_2-triangulation is constructed. Its algebraic definition is given in Section 3. Its pivot rules are described in Section 4. The comparison of these triangulations is presented in Section 5.

2 The Construction of the D_2-Triangulation

Let n be a positive integer and let $N = \{1, 2, \ldots, n\}$. Let Q denote the set of vectors in R^n whose components are all integers and let $w \in Q$ be given. Then $I_o(w)$ and $I_e(w)$ denote the sets

$$I_o(w) = \{i \in N \mid w_i \text{ is odd}\} \text{ and } I_e(w) = \{j \in N \mid w_j \text{ is even}\}.$$

Furthermore, $A(w)$ denotes the set

$$A(w) = \left\{ x \in R^n \mid \begin{array}{l} w_i - 1 \le x_i \le w_i + 1 \text{ for } i \in I_o(w) \text{ and} \\ x_i = w_i \text{ for } i \in I_e(w) \end{array} \right\}$$

and $B(w)$ denotes the set

$$B(w) = \left\{ x \in R^n \mid \begin{array}{l} x_i = w_i \text{ for } i \in I_o(w) \text{ and} \\ w_i - 1 \le x_i \le w_i + 1 \text{ for } i \in I_e(w) \end{array} \right\}.$$

Let k be a nonnegative integer. Then $D_k(w)$ denotes the set

$$D_k(w) = convexhull \left\{ (\left\{2^{-k}\right\} \times A(w)) \cup (\left\{2^{-(k+1)}\right\} \times B(w)) \right\}.$$

Lemma 2.1(see [4] or [9]). We have

$$D_k(w) = \left\{ d \in [2^{-(k+1)}, 2^{-k}] \times R^n \mid \begin{array}{l} \mid d_i - w_i \mid \le 2^{k+1} d_0 - 1 \text{ for } i \in I_o(w) \\ \mid d_i - w_i \mid \le 2 - 2^{k+1} d_0 \text{ for } i \in I_e(w) \end{array} \right\}.$$

Lemma 2.2(see [4] or [9]). $\cup_{w \in Q} D_k(w) = [2^{-(k+1)}, 2^{-k}] \times R^n$.

Lemma 2.3(see [4] or [9]). For w^1, $w^2 \in Q$, $D_k(w^1) \cap D_k(w^2)$ is either empty or a common face of both $D_k(w^1)$ and $D_k(w^2)$, and when $D_k(w^1) \cap D_k(w^2)$ is not empty,

$$\begin{aligned} D_k(w^1) \cap D_k(w^2) = convexhull \Big\{ & (\left\{2^{-k}\right\} \times (A(w^1) \cap A(w^2))) \\ & \cup (\left\{2^{-(k+1)}\right\} \times (B(w^1) \cap B(w^2))) \Big\}. \end{aligned}$$

For convenience, we first give the definitions of the D_1-triangulation, of the K_1-triangulation, and of the J_1-triangulation. For more details, see [3] and [11].

Let e^i be the i-th unit vector in R^n for $i = 1, 2, \ldots, n$.

Let either

$$D = \{x \in R^n \mid \text{all components of } x \text{ are odd}\}$$

or

$$D = \{x \in R^n \mid \text{all components of } x \text{ are even}\}.$$

Let π denote a permutation of the elements of N and let s denote a sign vector. Let p denote an integer such that $0 \leq p \leq n-1$.

The Definition of the D_1-Triangulation:

Take $y \in D$, and let π, s, and p be taken as above.
If $p = 0$, let $y^0 = y$, and

$$y^j = y + s_{\pi(j)} e^{\pi(j)}, j = 1, 2, \ldots, n.$$

If $p \geq 1$, let $y^0 = y + s$, and

$$y^j = y^{j-1} - s_{\pi(j)} e^{\pi(j)}, j = 1, 2, \ldots, p-1,$$
$$y^j = y + s_{\pi(j)} e^{\pi(j)}, j = p, p+1, \ldots, n.$$

Let D_1 denote the collection of all simplices $D_1(y, \pi, s, p)$ that are the convex hull of y^0, y^1, ..., y^n, as obtained from the above definition. Then D_1 is a triangulation of R^n.

Let

$$K = \{x \in R^n \mid \text{all components of } x \text{ are integers}\}.$$

Let π denote a permutation of the elements of N.

The Definition of the K_1-Triangulation:

Take $y \in K$, and let π be taken as above.
Let $y^0 = y$, and

$$y^j = y^{j-1} + e^{\pi(j)}, j = 1, 2, \ldots, n.$$

Let K_1 denote the collection of all simplices $K_1(y, \pi)$ that are the convex hull of $y^0, y^1, \ldots, y^n$, as obtained from the above definition. Then K_1 is a triangulation of R^n.

Let either

$$J = \{x \in R^n \mid \text{all components of } x \text{ are odd}\}$$

or

$$J = \{x \in R^n \mid \text{all components of } x \text{ are even}\}.$$

Let π denote a permutation of the elements of N and let s denote a sign vector.

The Definition of the J_1-Triangulation:

Take $y \in J$, and let π and s be taken as above.
Let $y^0 = y$, and

$$y^j = y^{j-1} + s_{\pi(j)} e^{\pi(j)}, j = 1, 2, \ldots, n.$$

Let J_1 denote the collection of all simplices $J_1(y, \pi, s)$ that are the convex hull of $y^0, y^1, \ldots, y^n$, as obtained from the above definition. Then J_1 is a triangulation of R^n.

Take G to be one of these triangulations of R^n. Let $\bar{G}$ denote the set of all faces of all simplices in G. Take $\alpha_0 \in (0, 1]$ and $\beta_i \in \{1/j \mid j = 1, 2, \ldots\}$ for $i = 0, 1, \ldots$. Choose α_j such that $\alpha_{j+1} = \alpha_j \beta_j$, for $j = 0, 1, \ldots$.

Let

$$\alpha_k \bar{G} \mid \alpha_k A(w) = \left\{\sigma \subseteq \alpha_k A(w) \mid \sigma \in \alpha_k \bar{G} \text{ and } dim(\sigma) = dim(A(w))\right\}$$

and

$$\alpha_{k+1} \bar{G} \mid \alpha_k B(w) = \left\{\sigma \subseteq \alpha_k B(w) \mid \sigma \in \alpha_{k+1} \bar{G} \text{ and } dim(\sigma) = dim(B(w))\right\}.$$

For the D_1-triangulation, the K_1-triangulation, and the J_1-triangulation, it is obvious that $\alpha_k \bar{G} \mid \alpha_k A(w)$ is a triangulation of $\alpha_k A(w)$ and $\alpha_{k+1} \bar{G} \mid \alpha_k B(w)$ is a triangulation of $\alpha_k B(w)$.

Let a denote the number of elements in the set $I_o(w)$ and b the number of elements in the set $I_e(w)$. Take

$$\sigma_A = convexhull\left\{y_A^0, y_A^1, \ldots, y_A^a\right\} \in \alpha_k \bar{G} \mid \alpha_k A(w)$$

and

$$\sigma_B = convexhull\left\{y_B^0, y_B^1, \ldots, y_B^b\right\} \in \alpha_{k+1} \bar{G} \mid \alpha_k B(w).$$

Let

$$\sigma = convexhull\left\{(\left\{2^{-k}\right\} \times \sigma_A) \cup (\left\{2^{-(k+1)}\right\} \times \sigma_B)\right\}.$$

It can easily be obtained that σ is a simplex and that

$$\begin{aligned}\sigma = convexhull\Big\{&(2^{-k}, y_A^0)^\top, (2^{-k}, y_A^1)^\top, \ldots, (2^{-k}, y_A^a)^\top,\\ &(2^{-(k+1)}, y_B^0)^\top, (2^{-(k+1)}, y_B^1)^\top, \ldots, (2^{-(k+1)}, y_B^b)^\top\Big\}.\end{aligned}$$

Let $T(k, k+1)$ denote the collection of all such simplices σ. Clearly, we have that for $\sigma^1, \sigma^2 \in T(k, k+1)$, $\sigma^1 \cap \sigma^2$ is either empty or a common face of both σ^1 and σ^2. Moreover,

$$\cup_{\sigma \in T(k,k+1)} \sigma = [2^{-(k+1)}, 2^{-k}] \times R^n.$$

Hence $T(k, k+1)$ is a triangulation of $[2^{-(k+1)}, 2^{-k}] \times R^n$.

Theorem 2.4. $\cup_{j=0}^{\infty} T(j, j+1)$ is a triangulation of $(0, 1] \times R^n$.

Proof. From the choice of α_j and β_j for $j = 0, 1, \ldots$, we have this conclusion.

We call the triangulation constructed in **Theorem 2.4** the G_2-triangulation. In this way we obtain the K_2-triangulation, the J_2-triangulation, and the D_2-triangulation of $(0, 1] \times R^n$. In case of the D_2-triangulation, each level 2^{-k} for $k = 0, 1, \ldots$, the set $\left\{2^{-k}\right\} \times R^n$ is triangulated according to the D_1-triangulation. Similarly for the K_2-triangulation and the J_2-triangulation.

3 The Description of the D_2-Triangulation

Let $N_0 = \{0, 1, \ldots, n\}$. Let u^i be the i-th unit vector in R^{n+1} for $i = 0, 1, \ldots, n$.

Take a permutation $\pi = (\pi(0), \pi(1), \ldots, \pi(n))$ of the elements of N_0. Let q denote an integer such that $\pi(q) = 0$. Take a vector y in $(0, 1] \times R^n$ such that for some nonnegative integer k, $y_0 = 2^{-(k+1)}$ and $y_{\pi(i)}/\alpha_{k+1}$ is an integer for $i = 0, \ldots, q-1$ and $y_{\pi(i)}/\alpha_k$ is an integer for $i = q+1, \ldots, n$. Define

$$w_{\pi(i)} = \begin{cases} \lfloor y_{\pi(i)}/\alpha_k \rfloor + 1 & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is odd,} \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor & \text{otherwise,} \end{cases}$$

for $i = 0, 1, \ldots, q-1$, and

$$w_{\pi(i)} = \begin{cases} y_{\pi(i)}/\alpha_k + 1 & \text{if } y_{\pi(i)}/\alpha_k \text{ is even,} \\ y_{\pi(i)}/\alpha_k & \text{otherwise,} \end{cases}$$

for $i = q+1, \ldots, n$.

Definition 3.1.

Take y and π as given above. Then y^{-1}, y^0,..., y^n are defined as follows.

$$\begin{aligned}
&y^{-1} = \textstyle\sum_{j=0}^{q} y_{\pi(j)}u^{\pi(j)} + \alpha_k \sum_{j=q+1}^{n} w_{\pi(j)}u^{\pi(j)},\\
&y^i = y^{i-1} + \alpha_{k+1}u^{\pi(i)}, i = 0, 1, \ldots, q-1,\\
&y^q = \alpha_k \textstyle\sum_{j=0}^{q-1} w_{\pi(j)}u^{\pi(j)} + \sum_{j=q+1}^{n} y_{\pi(j)}u^{\pi(j)} + 2y_0u^0,\\
&y^i = y^{i-1} + \alpha_k u^{\pi(i)}, i = q+1, \ldots, n.
\end{aligned}$$

Let $y^{-1}, y^0, \ldots, y^n$ be obtained in the above manner. Then it is obvious that they are affinely independent. Thus their convex hull is a simplex. Let us denote this simplex by $K_2(y, \pi)$. Then the K_2-triangulation is the set of all such simplices $K_2(y, \pi)$. Following the conclusions in the previous section, we have that this triangulation is a simplicial subdivision of $(0, 1] \times R^n$ such that its factor of refinement can be chosen arbitrarily.

Take a permutation $\pi = (\pi(0), \pi(1), \ldots, \pi(n))$ of the elements of N_0. Let q denote an integer such that $\pi(q) = 0$. Take a vector y in $(0, 1] \times R^n$ such that for some nonnegative integer k, $y_0 = 2^{-(k+1)}$ and either $y_{\pi(i)}/\alpha_k$ is even for $i = q+1, \ldots, n$ and $y_{\pi(i)}/\alpha_{k+1}$ is even for $i = 0, \ldots, q-1$ or $y_{\pi(i)}/\alpha_k$ is odd for $i = q+1, \ldots, n$ and if $1/\beta_k$ is even, $y_{\pi(i)}/\alpha_{k+1}$ is even for $i = 0, \ldots, q-1$ and if $1/\beta_k$ is odd, $y_{\pi(i)}/\alpha_{k+1}$ is odd for $i = 0, \ldots, q-1$. Take s to be a sign vector. If $y_{\pi(j)}/\alpha_k$ is odd for $j = q+1, \ldots, n$, define

$$w_{\pi(i)} = \begin{cases} \lfloor y_{\pi(i)}/\alpha_k \rfloor + 1 & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is odd and} \\ & \text{either } y_{\pi(i)}/\alpha_k \neq \lfloor y_{\pi(i)}/\alpha_k \rfloor \\ & \text{or both } \lfloor y_{\pi(i)}/\alpha_k \rfloor = y_{\pi(i)}/\alpha_k \text{ and } s_{\pi(i)} = 1, \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is even,} \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor - 1 & \text{otherwise,} \end{cases}$$

for $i = 0, 1, \ldots, q-1$ and if $y_{\pi(j)}/\alpha_k$ is even for $j = q+1, \ldots, n$, define

$$w_{\pi(i)} = \begin{cases} \lfloor y_{\pi(i)}/\alpha_k \rfloor + 1 & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is even and} \\ & \text{either } y_{\pi(i)}/\alpha_k \neq \lfloor y_{\pi(i)}/\alpha_k \rfloor \\ & \text{or both } \lfloor y_{\pi(i)}/\alpha_k \rfloor = y_{\pi(i)}/\alpha_k \text{ and } s_{\pi(i)} = 1, \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is odd,} \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor - 1 & \text{otherwise,} \end{cases}$$

for $i = 0, 1, \ldots, q-1$.

Definition 3.2.

Take y, π and s as given above. Then $y^{-1}, y^0, \ldots, y^n$ are defined as follows.

$$\begin{aligned}
&y^{-1} = y,\\
&y^i = y^{i-1} + \alpha_{k+1} s_{\pi(i)} u^{\pi(i)}, i = 0, 1, \ldots, q-1,\\
&y^q = \alpha_k \textstyle\sum_{j=0}^{q-1} w_{\pi(j)} u^{\pi(j)} + \sum_{j=q+1}^{n} (y_{\pi(j)} - \alpha_k s_{\pi(j)}) u^{\pi(j)} + 2y_0 u^0,\\
&y^i = y^{i-1} + \alpha_k s_{\pi(i)} u^{\pi(i)}, i = q+1, \ldots, n.
\end{aligned}$$

Let $y^{-1}, y^0, \ldots, y^n$ be obtained in the above manner. Then it is obvious that they are affinely independent. Thus their convex hull is a simplex. Let us denote this simplex by $J_2(y, \pi, s)$. Then the J_2-triangulation is the set of all such simplices $J_2(y, \pi, s)$. Then following the conclusions in the previous section, we have that this triangulation is a simplicial subdivision of $(0, 1] \times R^n$ such that its factor of refinement can be chosen arbitrarily.

Take a permutation $\pi = (\pi(0), \pi(1), \ldots, \pi(n))$ of the elements of N_0. Let q denote an integer such that $\pi(q) = 0$. Take a vector y in $(0, 1] \times R^n$ such that for some nonnegative integer k, $y_0 = 2^{-(k+1)}$ and either $y_{\pi(i)}/\alpha_k$ is even for $i = q+1, \ldots, n$ and $y_{\pi(i)}/\alpha_{k+1}$ is even for $i = 0, \ldots, q-1$ or $y_{\pi(i)}/\alpha_k$ is odd for $i = q+1, \ldots, n$ and if $1/\beta_k$ is even, $y_{\pi(i)}/\alpha_{k+1}$ is even for $i = 0, \ldots, q-1$ and if $1/\beta_k$ is odd, $y_{\pi(i)}/\alpha_{k+1}$ is odd for $i = 0, \ldots, q-1$. Take s to be a sign vector. If $y_{\pi(j)}/\alpha_k$ is odd for $j = q+1, \ldots, n$, define

$$w_{\pi(i)} = \begin{cases} \lfloor y_{\pi(i)}/\alpha_k \rfloor + 1 & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is odd and} \\ & \text{either } y_{\pi(i)}/\alpha_k \neq \lfloor y_{\pi(i)}/\alpha_k \rfloor \\ & \text{ or both } \lfloor y_{\pi(i)}/\alpha_k \rfloor = y_{\pi(i)}/\alpha_k \text{ and } s_{\pi(i)} = 1, \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is even,} \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor - 1 & \text{otherwise,} \end{cases}$$

for $i = 0, 1, \ldots, q-1$ and if $y_{\pi(j)}/\alpha_k$ is even for $j = q+1, \ldots, n$, define

$$w_{\pi(i)} = \begin{cases} \lfloor y_{\pi(i)}/\alpha_k \rfloor + 1 & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is even and} \\ & \text{either } y_{\pi(i)}/\alpha_k \neq \lfloor y_{\pi(i)}/\alpha_k \rfloor \\ & \text{ or both } \lfloor y_{\pi(i)}/\alpha_k \rfloor = y_{\pi(i)}/\alpha_k \text{ and } s_{\pi(i)} = 1, \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor & \text{if } \lfloor y_{\pi(i)}/\alpha_k \rfloor \text{ is odd,} \\ \lfloor y_{\pi(i)}/\alpha_k \rfloor - 1 & \text{otherwise,} \end{cases}$$

for $i = 0, 1, \ldots, q-1$.

Let p_1 and p_2 denote two integers such that $-1 \le p_1 \le q-2$ and $0 \le p_2 \le n-q-1$.

Definition 3.3. Take y, π, s, p_1 and p_2 as given above. Then y^{-1}, y^0, ..., y^n are defined as follows.

When $p_1 = -1$, let $y^{-1} = y$,

$$y^i = y + \alpha_{k+1} s_{\pi(i)} u^{\pi(i)}, i = 0, 1, \ldots, q-1,$$

and when $p_1 \ge 0$, let

$$\begin{aligned} y^{-1} &= y + \alpha_{k+1} \textstyle\sum_{j=0}^{q-1} s_{\pi(j)} u^{\pi(j)}, \\ y^i &= y^{i-1} - \alpha_{k+1} s_{\pi(i)} u^{\pi(i)}, i = 0, 1, \ldots, p_1 - 1, \end{aligned}$$

and if $p_1 < q-2$, let

$$y^i = y + \alpha_{k+1} s_{\pi(i)} u^{\pi(i)}, i = p_1, \ldots, q-1,$$

and if $p_1 = q-2$, let

$$\begin{aligned} y^{q-2} &= y^{q-3} - \alpha_{k+1} s_{\pi(q-2)} u^{\pi(q-2)}, \\ y^{q-1} &= y^{q-3} - \alpha_{k+1} s_{\pi(q-1)} u^{\pi(q-1)}. \end{aligned}$$

When $p_2 = 0$, let

$$\begin{aligned} y^q &= \alpha_k \textstyle\sum_{j=0}^{q-1} w_{\pi(j)} u^{\pi(j)} + \sum_{j=q+1}^{n} (y_{\pi(j)} - \alpha_k s_{\pi(j)}) u^{\pi(j)} + 2y_0 u^0, \\ y^i &= y^q + \alpha_k s_{\pi(i)} u^{\pi(i)}, i = q+1, \ldots, n, \end{aligned}$$

and when $p_2 \ge 1$, let

$$\begin{aligned} y^q &= \alpha_k \textstyle\sum_{j=0}^{q-1} w_{\pi(j)} u^{\pi(j)} + \sum_{j=q+1}^{n} y_{\pi(j)} u^{\pi(j)} + 2y_0 u^0, \\ y^i &= y^{i-1} - \alpha_k s_{\pi(i)} u^{\pi(i)}, i = q+1, \ldots, q+p_2-1, \end{aligned}$$

and if $p_2 < n-q-1$, let

$$\begin{aligned} y^* &= \alpha_k \textstyle\sum_{j=0}^{q-1} w_{\pi(j)} u^{\pi(j)} + \sum_{j=q+1}^{n} (y_{\pi(j)} - \alpha_k s_{\pi(j)}) u^{\pi(j)} + 2y_0 u^0, \\ y^i &= y^* + \alpha_k s_{\pi(i)} u^{\pi(i)}, i = q+p_2, \ldots, n, \end{aligned}$$

and if $p_2 = n-q-1$, let

$$\begin{aligned} y^{n-1} &= y^{n-2} - \alpha_k s_{\pi(n-1)} u^{\pi(n-1)}, \\ y^n &= y^{n-2} - \alpha_k s_{\pi(n)} u^{\pi(n)}. \end{aligned}$$

Let $y^{-1}, y^0, \ldots, y^n$ be obtained in the above manner. Then it is obvious that they are affinely independent. Thus their convex hull is a simplex. Let us denote this simplex by $D_2(y, \pi, s, p_1, p_2)$. Then the D_2-triangulation is the set of all such simplices $D_2(y, \pi, s, p_1, p_2)$. Then following the conclusions in the previous section, we have that this triangulation is a simplicial subdivision of $(0, 1] \times R^n$ such that its factor of refinement can be chosen arbitrarily.

4 The Pivot Rules of the D_2-Triangualtion

Let $f : R^n \to R^n$ be a continuous function. Suppose that we want to compute a zero point of f, i.e., a vector $x^* \in R^n$ such that $f(x^*) = 0$. Let v be an arbitrary point in R^n. Then the function $g : (0, 1] \times R^n \to R^n$ is defined by $g(t, x) = f(x)$ if $0 < t < 1$ and $g(t, x) = x - v$ if $t = 1$. Let $(0, 1] \times R^n$ be triangulated according to one of the G_2-triangulations defined before. Next, let H be the piecewise linear approximation of g with respect to one of the G_2-triangulations. More precisely, let $x = \sum_{i=-1}^{n} \lambda_i y^i$ be a vector in some simplex of one of the G_2-triangulations with vertices y^{-1}, y^0, $\ldots$, y^n, where $\lambda_i \geq 0$ for all i and $\sum_{i=-1}^{n} \lambda_i = 1$. Then $H(x)$ is defined by

$$H(x) = \sum_{i=-1}^{n} \lambda_i g(y^i).$$

Clearly $H(1, v) = 0$ and $H(1, w) \neq 0$ for $w \neq v$. Now the simplicial homotopy algorithm follows the piecewise linear path, P, of zero points of H originating at $(1, v)$. The path P is linear on each simplex σ of one of the G_2-triangulations it passes. Such a linear piece can be generated by making a linear programming (l.p.) pivoting step in the system of linear equations

$$\sum_{i=-1}^{n} \lambda_i (g(y^i), 1)^{\mathsf{T}} = (0, 1)^{\mathsf{T}}.$$

When implementing a (l.p.) pivoting step, some λ_i becomes zero, then the vertex y^i of σ must be replaced by a new vertex of a simplex, say $\bar{\sigma}$, of one of the G_2-triangulations, adjacent to σ and sharing with it the facet opposite to y^i.

Table 1: The Pivot Rules of the K_2-Triangulation

i	q		$\bar{y}$	$\bar{\pi}$	$\bar{q}$	$\bar{k}$
-1	0		y	$(\pi(1), \ldots, \pi(n), \pi(0))$	n	$k-1$
	$q \geq 1$	$y^0_{\pi(0)} = \alpha_k(w_{\pi(0)} + 1)$	$y - (y_{\pi(0)} - \alpha_k w_{\pi(0)}) u^{\pi(0)}$	$(\pi(1), \ldots, \pi(n), \pi(0))$	$q-1$	k
		$y^0_{\pi(0)} \neq \alpha_k(w_{\pi(0)} + 1)$	$y + \alpha_{k+1} u^{\pi(0)}$	$(\pi(1), \ldots, \pi(q-1), \pi(0),$ $\pi(q), \ldots, \pi(n))$	q	k
$0 \leq i$ $< q-1$			y	$(\pi(0), \ldots, \pi(i+1),$ $\pi(i), \ldots, \pi(n))$	q	k
$q-1$	$q \geq 1$	$y_{\pi(q-1)} = \alpha_k(w_{\pi(q-1)} - 1)$	y	$(\pi(0), \ldots, \pi(q),$ $\pi(q-1), \ldots, \pi(n))$	$q-1$	k
		$y_{\pi(q-1)} \neq \alpha_k(w_{\pi(q-1)} - 1)$	$y - \alpha_{k+1} u^{\pi(q-1)}$	$(\pi(q-1), \pi(0), \ldots, \pi(q-2),$ $\pi(q), \ldots, \pi(n))$	q	k
q	$q < n$	$y^{q+1}_{\pi(q+1)} = \alpha_k(w_{\pi(q+1)} + 1)$	y	$(\pi(0), \ldots, \pi(q+1),$ $\pi(q), \ldots, \pi(n))$	$q+1$	k
		$y^{q+1}_{\pi(q+1)} \neq \alpha_k(w_{\pi(q+1)} + 1)$	$y + \alpha_k u^{\pi(q+1)}$	$(\pi(0), \ldots, \pi(q),$ $\pi(q+2), \ldots, \pi(n), \pi(q+1))$	q	k
$q < i$ $< n$			y	$(\pi(0), \ldots, \pi(i+1),$ $\pi(i), \ldots, \pi(n))$	q	k
n	$q < n$	$y_{\pi(n)} = \alpha_k(w_{\pi(n)} - 1)$	y	$(\pi(n), \pi(0), \ldots, \pi(n-1))$	$q+1$	k
		$y_{\pi(n)} \neq \alpha_k(w_{\pi(n)} - 1)$	$y - \alpha_k u^{\pi(n)}$	$(\pi(0), \ldots, \pi(q), \pi(n),$ $\pi(q+1), \ldots, \pi(n-1))$	q	k
	n		y	$(\pi(n), \pi(0), \ldots, \pi(n-1))$	0	$k+1$

Table 2: The Pivot Rules of the J_2-Triangulation

i	q		$\bar{y}$	$\bar{s}$	$\bar{\pi}$	$\bar{q}$	$\bar{k}$
-1	0		$y - \alpha_k s$	s	$(\pi(1), \ldots, \pi(n), \pi(0))$	n	$k-1$
	$q \geq 1$		$y + 2\alpha_{k+1} s_{\pi(0)} u^{\pi(0)}$	$s - 2s_{\pi(0)} u^{\pi(0)}$	π	q	k
$0 \leq i < q - 1$			y	s	$(\pi(0), \ldots, \pi(i+1), \pi(i), \ldots, \pi(n))$	q	k
$q-1$	$q \geq 1$	$y_{\pi(q-1)} = \alpha_k(w_{\pi(q-1)} - s_{\pi(q-1)})$	y	$s - 2s_{\pi(q-1)} u^{\pi(q-1)}$	$(\pi(0), \ldots, \pi(q-2), \pi(q), \ldots, \pi(n), \pi(q-1))$	$q-1$	k
		$y_{\pi(q-1)} \neq \alpha_k(w_{\pi(q-1)} - s_{\pi(q-1)})$	y	$s - 2s_{\pi(q-1)} u^{\pi(q-1)}$	π	q	k
q	$q < n$		y	$s - 2s_{\pi(q+1)} u^{\pi(q+1)}$	π	q	k
$q < i < n$			y	s	$(\pi(0), \ldots, \pi(i+1), \pi(i), \ldots, \pi(n))$	q	k
n	$q < n$		y	$s - 2s_{\pi(n)} u^{\pi(n)}$	$(\pi(0), \ldots, \pi(q-1), \pi(n), \pi(q), \ldots, \pi(n-1))$	$q+1$	k
	n		$y + \alpha_{k+1} s$	s	$(\pi(n), \pi(0), \ldots, \pi(n-1))$	0	$k+1$

Table 3(1): The Piovt Rules of the D_2-Triangulation

i	q		p_1	p_2	$\bar{y}$	$\bar{s}$	$\bar{\pi}$	$\bar{q}$	$\bar{p}_1$	$\bar{p}_2$	$\bar{k}$
-1	0				$y - \alpha_k s$	s	$(\pi(1), \ldots, \pi(n), \pi(0))$	n	$p_2 - 1$	0	$k - 1$
	1				$y + 2\alpha_{k+1} s_{\pi(0)} u^{\pi(0)}$	$s - 2s_{\pi(0)} u^{\pi(0)}$	π	q	p_1	p_2	k
	$q \geq 2$		-1		y	s	π	q	$p_1 + 1$	p_2	k
			0		y	s	π	q	$p_1 - 1$	p_2	k
		$y_{\pi(0)} = \alpha_k(w_{\pi(0)} - s_{\pi(0)})$	$p_1 \geq 1$	0	y	$s - 2s_{\pi(0)} u^{\pi(0)}$	$(\pi(1), \ldots, \pi(n), \pi(0))$	$q - 1$	$p_1 - 1$	p_2	k
				$p_2 \geq 1$	y	$s - 2s_{\pi(0)} u^{\pi(0)}$	$(\pi(1), \ldots, \pi(q), \pi(0), \pi(q+1), \ldots, \pi(n))$	$q - 1$	$p_1 - 1$	$p_2 + 1$	k
		$y_{\pi(0)} \neq \alpha_k(w_{\pi(0)} - s_{\pi(0)})$			y	$s - 2s_{\pi(0)} u^{\pi(0)}$	π	q	p_1	p_2	k
$0 \leq i \leq q - 1$		$y_{\pi(i)} = \alpha_k(w_{\pi(i)} - s_{\pi(i)})$	-1	0	y	$s - 2s_{\pi(i)} u^{\pi(i)}$	$(\pi(0), \ldots, \pi(i-1), \pi(i+1), \ldots, \pi(n), \pi(i))$	$q - 1$	p_1	p_2	k
				$p_2 \geq 1$	y	$s - 2s_{\pi(i)} u^{\pi(i)}$	$(\pi(0), \ldots, \pi(i-1), \pi(i+1), \ldots, \pi(q), \pi(i), \pi(q+1), \ldots, \pi(n))$	$q - 1$	p_1	$p_2 + 1$	k
		$y_{\pi(i)} \neq \alpha_k(w_{\pi(i)} - s_{\pi(i)})$			y	$s - 2s_{\pi(i)} u^{\pi(i)}$	π	q	p_1	p_2	k
			$i < p_1 - 1$		y	s	$(\pi(0), \ldots, \pi(i+1), \pi(i), \ldots, \pi(n))$	q	p_1	p_2	k
			$i = p_1 - 1$		y	s	π	q	$p_1 - 1$	p_2	k
			$i > p_1 - 1$ $0 \leq p_1 < q - 2$		y	s	$(\pi(0), \ldots, \pi(p_1 - 1), \pi(i), \pi(p_1), \ldots, \pi(i-1), \pi(i+1), \ldots, \pi(n))$	q	$p_1 + 1$	p_2	k
			$i \geq p_1$ $0 \leq p_1 = q - 2$		$y + 2\alpha_{k+1} s_{\pi(i)} u^{\pi(i)}$	$s - 2s_{\pi(i)} u^{\pi(i)}$	π	q	p_1	p_2	k

Table 3(2): The Piovt Rules of the D_2-Triangulation

i	q		p_1	p_2	$\bar{y}$	$\bar{s}$	$\bar{\pi}$	$\bar{q}$	$\bar{p}_1$	$\bar{p}_2$	$\bar{k}$
q	$q \leq n$ -2			0	y	s	π	q	p_1	p_2+1	k
				1	y	s	π	q	p_1	p_2-1	k
			-1	$p_2 \geq 2$	y	$s-2s_{\pi(q+1)}u^{\pi(q+1)}$	$(\pi(0),\ldots,\pi(q+1),$ $\pi(q),\ldots,\pi(n))$	$q+1$	p_1	p_2-1	k
			$p_1 \geq 0$		y	$s-2s_{\pi(q+1)}u^{\pi(q+1)}$	$(\pi(q+1),\pi(0),\ldots,\pi(q),$ $\pi(q+2),\ldots,\pi(n))$	$q+1$	p_1+1	p_2-1	k
	$n-1$				y	$s-2s_{\pi(q+1)}u^{\pi(q+1)}$	π	q	p_1	p_2	k
$q<i$ $\leq n$			-1	0	y	$s-2s_{\pi(i)}u^{\pi(i)}$	$(\pi(0),\ldots,\pi(q-1),\pi(i),$ $\pi(q),\ldots,\pi(i-1),$ $\pi(i+1),\ldots,\pi(n))$	$q+1$	p_1	p_2	k
			$p_1 \geq 0$		y	$s-2s_{\pi(i)}u^{\pi(i)}$	$(\pi(i),\pi(0),\ldots,\pi(i-1),$ $\pi(i+1),\ldots,\pi(n))$	$q+1$	p_1+1	p_2	k
				$i<q$ $+p_2-1$	y	s	$(\pi(0),\ldots,\pi(i+1),$ $\pi(i),\ldots,\pi(n))$	q	p_1	p_2	k
				$i=q$ $+p_2-1$	y	s	π	q	p_1	p_2-1	k
				$i>q$ $+p_2-1$ $1 \leq p_2 <$ $n-q-1$	y	s	$(\pi(0),\ldots,\pi(q+p_2-1),$ $\pi(i),\pi(q+p_2),\ldots,\pi(i-1),$ $\pi(i+1),\ldots,\pi(n))$	q	p_1	p_2+1	k
				$i>p_2+q$ $p_2=n-q$ -1	y	$s-2s_{\pi(i)}u^{\pi(i)}$	π	q	p_1	p_2	k
n	n				$y+\alpha_{k+1}s$	s	$(\pi(n),\pi(0),\ldots,\pi(n-1))$	0	-1	p_1+1	$k+1$

Let a simplex of the K_2-triangulation, $\sigma = K_2(y, \pi)$, be given with vertices $y^{-1}, y^0, \ldots, y^n$. We wish to obtain the simplex of the K_2-triangulation, $\bar{\sigma} = K_2(\bar{y}, \bar{\pi})$, such that all vertices of σ are also vertices of $\bar{\sigma}$ except the vertex y^i. Table 1 shows how $\bar{y}$ and $\bar{\pi}$ depend on y, π and i.

Next, let $\sigma = J_2(y, \pi, s)$, be a simplex of the J_2-triangulation with vertices $y^{-1}, y^0, \ldots, y^n$. Suppose that we want to obtain the simplex of the J_2-triangulation, $\bar{\sigma} = J_2(\bar{y}, \bar{\pi}, \bar{s})$, such that all vertices of σ are also vertices of $\bar{\sigma}$ except the vertex y^i. Table 2 shows how $\bar{y}, \bar{\pi}$ and $\bar{s}$ depend on y, π, s and i.

Finally, let a simplex of the D_2-triangualtion, $\sigma = D_2(y, \pi, s, p_1, p_2)$, be given with vertices y^{-1}, y^0, $\ldots$, y^n. If we want to obtain a simplex of the D_2-triangulation, $\bar{\sigma} = D_2(\bar{y}, \bar{\pi}, \bar{s}, \bar{p_1}, \bar{p_2})$, such that all vertices of σ are also vertices of $\bar{\sigma}$ except the vertex y^i, then Table 3 shows how $\bar{y}, \bar{\pi}, \bar{s}, \bar{p_1}$ and $\bar{p_2}$ depned on y, π, s, p_1, p_2 and i.

In these tables,

$$y_0 = 2^{-(k+1)} \text{ and } y = (y_1, y_2, \ldots, y_n)^\top$$

and

$$\bar{y}_0 = 2^{-(\bar{k}+1)} \text{ and } \bar{y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_n)^\top.$$

5 The Comparison of Triangulations for Simplicial Homotopy Algorithms

Let H^n denote the set $H^n = \{x \in R^n \mid 0 \leq x_i \leq 2 \text{ for } i = 1, 2, \ldots, n\}$. Let $\alpha_0 = 1$ and α denote $1/\beta_0$.

Theorem 5.1. The number of simplices of the K_2-triangulation and one of the J_2-triangulation in the set $[2^{-1}, 1] \times H^n$ are both equal to $p_n(\alpha)$, where

$$p_n(\alpha) = \begin{cases} (1 - \alpha^{n+1})2^n n!/(1 - \alpha) & \text{if } \beta_0 \neq 1 \\ (n+1)2^n n! & \text{otherwise.} \end{cases}$$

The number of simplices of the D_2-triangulation in the set $[2^{-1}, 1] \times H^n$ is equal to $q_n(\alpha)$, where

$$q_n(\alpha) = 2^n \sum_{m=0}^{n} (\alpha^m C_n^m d_m d_{n-m})$$

where

$$d_j = j + j(j-1) + \cdots + j(j-1)\cdots 4 \cdot 3 + 2$$

for $j \geq 2$, $d_0 = d_1 = 1$, and $C_n^m = n!/m!(n-m)!$.

Proof. Let $\bar{Q}$ denote the set

$$\bar{Q} = \{w \in R^n \mid w_i \in \{0,1,2\} \text{ for } i = 1,2,\ldots,n\}.$$

Take $w \in \bar{Q}$. Let $\bar{A}(w)$ denote the set

$$\bar{A}(w) = \left\{ x \in R^n \mid \begin{array}{l} w_i - 1 \leq x_i \leq w_i + 1 \text{ for } i \in I_o(w) \text{ and} \\ x_i = w_i \text{ for } i \in I_e(w) \end{array} \right\}$$

and let $\bar{B}(w)$ denote the set

$$\bar{B}(w) = \left\{ x \in R^n \mid \begin{array}{l} x_i = w_i \text{ for } i \in I_o(w) \text{ and} \\ w_i \leq x_i \leq w_i + 1 \text{ for } i \in I_e(w) \text{ and } w_i = 0 \\ w_i - 1 \leq x_i \leq w_i \text{ for } i \in I_e(w) \text{ and } w_i = 2 \end{array} \right\}.$$

Let $\bar{D}(w)$ denote the set

$$\bar{D}(w) = convexhull \left\{ (\{1\} \times \bar{A}(w)) \cup (\left\{2^{-1}\right\} \times \bar{B}(w)) \right\}.$$

Then it is obvious that

$$[2^{-1}, 1] \times H^n = \cup_{w \in \bar{Q}} \bar{D}(w).$$

Let m denote the number of elements in $I_e(w)$. Then there are $2^m C_n^m$ elements in $\bar{Q}$ such that m components of each of them are even. Thus the numbers of simplices of the K_2-triangulation and of the J_2-triangulation in the set $\cup_{w \in \bar{Q}, |I_e(w)|=m} \bar{D}(w)$ are both equal to

$$2^m 2^{n-m} \alpha^m C_n^m m!(n-m)!.$$

The number of simplices of the D_2-triangulation in the same set is equal to

$$2^m 2^{n-m} \alpha^m C_n^m d_m d_{n-m}.$$

Since

$$\cup_{m=0}^n (\cup_{w \in \bar{Q}, |I_e(w)|=m} \bar{D}(w)) = [2^{-1}, 1] \times H^n,$$

the theorem follows immediately.

Theorem 5.2. When $n \geq 3$, $q_n(\alpha) < p_n(\alpha)$. As n goes to infinity, $q_n(\alpha)/p_n(\alpha)$ converges to some number μ such that $\mu \leq e - 2$.

Proof. The conclusion is obvious, the proof is omitted.

From **Theorem 5.2**, we have that the number of simplices of the D_2-triangulation is the smallest of ones of these triangulations for simplicial homotopy algorithms. The author conjectures that the average directional density of the D_2-triangulation is the smallest of ones of these triangulations. For details on the average directional density of a triangulation, we refer to Todd [14].

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