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This note studies the implications of the requirement that the budget shares predicted by a demand system lie between zero and one. It is shown that when homotheticity is not assumed, the only known Engel functions that can meet the requirement are fractional, i.e. ratios of functions of income. Regularity conditions on preferences that guarantee budget share positivity are stated in two cases. It is also shown that in these two cases, share positivity is incompatible with flexibility.

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Section 1. Introduction[1]

It is sometimes said that Engel's Law is the most universally accepted proposition in applied econometrics. It is therefore not surprising that many authors have investigated the empirical plausibility of various Engel functions, starting with the classic studies of Working (1943) and Leser (1961). Leser, in particular, found that Engel functions of the form $q_i = a_i + b_i x + c_i x \log x$ provided the best fit to cross-section data on the quantities q_i of commodity i and on income x. However, if this function is rewritten in terms of the budget shares $w_i = p_i q_i / x$, it implies that w_i is unbounded as x tends to infinity, unless $c_i = 0$. Since the budget shares sum to unity, this means that the previous Engel function must predict some negative budget shares for sufficiently high income levels, unless quasi-homotheticity obtains ($c_i = 0$ for all i). The same observation can be made for Engel functions derived from demand systems of the PIGLOG form (such as the AIDS model), where $a_i = 0$ for all i. To the best of our knowledge, this simple fact has never been mentioned in the literature.

In the case of an individual agent, budget share positivity can be guaranteed if the demand functions are derived from the Kuhn-Tucker conditions for utility maximization. It is, however, implausible that the rational representative consumers of Gorman (1953) and Muellbauer (1976) would face corner solutions in their budget allocation problem. Hence, the requirement that a predicted budget share lie between zero and one is a regularity condition of a different nature than the differentiability, monotonicity and concavity of a cost function: whereas the latter conditions must be verified both by an individual agent's preferences and by the preferences of a rational representative agent, requiring the absence of corner solutions only appears reasonable at the aggregate level (for the representative agent) and on empirical grounds (a demand system that can predict negative budget shares is bound to yield non-sensical simulation results with some data sets). Nevertheless, it is intuitively clear that the positivity requirement is likely to conflict with flexibility, as it is the case with the other regularity conditions (see Diewert and Wales (1987)). The purpose of this note is to make this point more precise, and to investigate which classes of demand systems do in fact guarantee the positivity of predicted budget shares.

In order to achieve this, Section 2 of this paper will review five broad classes of demand systems that have already been characterized in the literature. They include most known forms of Engel functions (e.g. homothetic and quasi-homothetic demands, PIGLOG, PIGL, Translog, and the new functional forms in Gorman (1981) and Lewbel (1987a, 1987b)). In most of these cases, exhaustive characterizations (or taxonomies) exist, which identify the precise functional forms that the Engel functions must take if the demand system satisfies regularity conditions such as adding-up, homogeneity and symmetry. This fact will enable us to limit our investigations to two possible candidate classes. The first one is the class of homothetic demands, which violates Engel's Law and can thus be further excluded as empirically implausible. The second one is the EXP class of demand systems characterized by Lewbel (1987b).

Section 3 identifies sufficient monotonicity conditions on preferences which guarantee the positivity of predicted EXP budget shares (the bounds that will be obtained are in fact tighter than 0 and 1). It will also be shown that the EXP budget shares are monotonic in income. Section 4 shows that the Minflex Laurent system of Barnett (1983) also implies budget share positivity when that system is globally regular. Section 5 suggests that a fundamental conflict exists between budget share positivity and flexibility. Section 6 concludes.

Section 2. Some taxonomies of demand systems

In this section, we will use results of Gorman (1961, 1981), Lewbel (1987a, 1987b) and Muellbauer (1975) to identify two classes of demand systems guaranteeing that the budget shares w_i lie between zero and one. Since all the systems satisfy adding-up ($\sum_i w_i = 1$), it is in fact sufficient to ensure that $w_i \geq 0$ for all i.

It is possible in many instances to characterize the form of the Engel curves implied by a particular class of demand systems satisfying adding-up, homogeneity and symmetry. This has been done for the following cases:

Class 1: $$q_i = a_i + b_i x$$

Class 2: $$q_i = b_i x + c_i f(x)$$

Class 3: $$q_i = a_i + b_i x + c_i f(x)$$

Class 4: $$q_i = \sum_{r \in R} b_{ir} f_r(x)$$

Class 5: $$q_i = \frac{a_i f(x) + b_i g(x)}{aF(x) + bG(x)}$$

where q_i is the demand for good i; x is nominal income; f, g, F, G, and f_r are differentiable functions of income; and a_i, b_i, c_i, a, b, and b_{ir} are differentiable functions of the prices only.[2]

Class 1 is the Gorman polar form, which obviously does not solve the problem of budget share positivity for all income levels unless $a_i = 0$ for all i (in this case the demands are homothetic). Indeed, it implies $w_i = \alpha_i/x + \beta_i$ with $\alpha_i = a_i p_i$ and $\beta_i = b_i p_i$. Since β_i is homogeneous of degree 0 in the prices, it belongs to a bounded set. Adding-up implies that $\sum \alpha_i = 0$. Hence in the nonhomothetic case, $\alpha_s < 0$ for some s; and as $x \to 0$, w_s must become negative.

Class 2 has been studied by Muellbauer (1975). He shows that if $f(x) \neq 0$, then $f(x)$ must either be equal to $x \log x$ (PIGLOG) or to x^k with $k \neq 1$ (PIGL). Since neither function implies bounded budget shares and since $\sum w_i = 1$, neither class ensures budget share positivity. Homothetic demands are obtained if $f(x) = 0$.

Class 3 has been studied by Lewbel (1987a). He shows that $f(x)$ must be either 0, x^k, $x \log x$, or $\log x$. Again, it is easy to check that none of these functions implies bounded budget shares.

Class 4 has been partially characterized by Gorman (1981). He shows that the rank of $B = [b_{ir}]$ is at most three. When B has rank 3, we must have either that $f_r(x) = x(\log x)^r$ with $0 \in R$, or that $f_r(x) = x^{r+1}$ with $0 \in R$, or that w_i is a linear combination of sines and cosines. Again, neither choice appears to guarantee positive budget shares.

Class 5 has been studied by Lewbel (1987b). He shows that the budget shares can always be written in the following form:

$$w_i = \frac{a_i + b_i f(x)}{1 + b f(x)} \tag{1}$$

where $f(x)$ must be either 0, $\log x$, x^k, or $\tan(k \log x)$ for $k \neq 0$. Homothetic demands are obtained if $f(x) = 0$; adding-up implies $\sum a_i = 1$ and $\sum b_i = b$.

If $b = 0$ in (1), we have $w_i = a_i + b_i f(x)$, which is the Class 2 system and does not imply $0 \leq w_i \leq 1$ as we have seen. If $b \neq 0$, the denominator in (1) will vanish at some positive income level when $f(x) = \log x$ or $f(x) = \tan(k \log x)$. If $b < 0$ and $f(x) = x^k$, then w_i becomes unbounded as x^k tends to $-b^{-1} > 0$. We conclude that when $f(x) \neq 0$, it is necessary for $0 \leq w_i \leq 1$ that $b > 0$ and that $f(x) = x^k$ (the case where $f(x) = x^k$ is called the EXP demand system by Lewbel (1987b)). This proves our claim that among the nonhomothetic systems characterized in this section, only the EXP demands with $b > 0$ can ensure that $0 \leq w_i \leq 1$ at all income levels. In fact, when $bf(x) \geq 0$, w_i in (1) is a convex combination of a_i and b_i/b; it is then necessary and sufficient for $0 \leq w_i \leq 1$ that $b > 0$, $a_i \geq 0$ and $b_i \geq 0$. We will show in the next section that this is guaranteed, for EXP demands, by regularity conditions on preferences.

Section 3. EXP demands and share positivity

The following theorem, which will be used in the proof of Theorem 2 below, can in fact be extended to all the fractional demand systems in Class 5.

Theorem 1. *If $f(x) = x^k$ in Equation (1) and if w_i is a differentiable function of x, then w_i is monotonic in x, i.e. the sign of $\partial w_i / \partial x$ does not depend on x.*

Proof: Using (1) and letting $f = x^k$, $f' = kx^{k-1}$, we have:

$$\begin{aligned} \frac{\partial w_i}{\partial x} &= \frac{1}{(1+bf)^2}\left(b_i f'(1+bf) - bf'(a_i + b_i f)\right) \\ &= \frac{1}{(1+bf)^2}(b_i f' - bf' a_i) \\ &= \frac{f'}{(1+bf)^2}(b_i - ba_i). \end{aligned} \tag{2}$$

The sign of this expression obviously does not depend on x. ◇

It is helpful to view EXP as a system derived by means of Roy's identity:

$$w_i = -\frac{\partial \log U / \partial \log p_i}{\partial \log U / \partial \log x} \tag{3}$$

from the following indirect utility function:

$$U(p, x) = \left((k+1)x + A(p)x^{k+1}\right) e^{-B(p)}, \tag{4}$$

where $A(p)$ and $B(p)$ are twice differentiable functions of the prices. Homogeneity of U in (p, x) requires that $A(p)$ be homogeneous of degree $-k$ and that $e^{B(p)}$ be homogeneous of degree 1.

Following Lewbel, we use the notation $A_i = \partial A/\partial \log p_i$ and $B_i = \partial B/\partial \log p_i$. By Euler's theorem, since A_i/A is the price elasticity of A and since:

$$\sum B_i = e^{-B} \sum \frac{\partial e^B}{\partial p_i} p_i = e^{-B} e^B,$$

we must have that $\sum A_i = -kA$ and $\sum B_i = 1$. Using (3) and (4), we may write:

$$\begin{aligned} w_i &= -\frac{x^{k+1}A_i - B_i\left((k+1)x + Ax^{k+1}\right)}{(k+1)x + (k+1)Ax^{k+1}} \\ &= \frac{B_i - \frac{1}{k+1}(A_i - B_i A)x^k}{1 + Ax^k}. \end{aligned} \tag{5}$$

Equation (5) is a special case of (1) with $f(x) = x^k$, $a_i = B_i$, $b_i = (B_i A - A_i)/(k+1)$, and $b = A$.

As is quite obvious from Equation (5), it is sufficient for budget share positivity that $k + 1 > 0$, $A > 0$, $A_i \leq 0$ and $B_i \geq 0$. The following theorem provides tighter bounds, and a justification based on the monotonicity of preferences.

Theorem 2. *If U is increasing in x, A is not increasing in p, B is not decreasing in p, and U is homogeneous of degree zero, then either:*

$$0 \leq B_i \leq w_i \leq \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \leq 1, \qquad \text{or:}$$

$$0 \leq \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \leq w_i \leq B_i \leq 1.$$

Proof: In view of $B_i = p_i \partial B / \partial p_i$ and $A_i = p_i \partial A / \partial p_i$, the monotonicity conditions on A and B imply $B_i \geq 0$ and $A_i \leq 0$. By (4), the monotonicity of U in x implies $k > -1$ and $A > 0$. The fact that $A_i \leq 0$ for all i implies that $k > 0$; indeed, if $k < 0$, then $\sum A_i = -kA$ implies that $A_s > 0$ for some s, contradicting the assumption. We may now proceed to prove the claimed inequalities.

The inequalities $0 \leq B_i \leq 1$ are implied by $B_i \geq 0$ for all i and $\sum B_i = 1$. From Theorem 1, w_i is monotonic in x; since, as we have just seen, $k > 0$, we have $f' > 0$ in Equation (2), and:

$$\text{sign}\left(\frac{\partial w_i}{\partial x}\right) = \text{sign}\,(b_i - ba_i) =$$

$$\text{sign}\left(-\frac{1}{k+1}(A_i - B_i A) - AB_i\right) = \text{sign}\left(\frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) - B_i\right). \quad (6)$$

It is obvious from (5) that $w_i = B_i$ if $x = 0$. Using (6), this implies:

$$w_i \geq B_i \quad \text{if} \quad \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \geq B_i$$

$$w_i \leq B_i \quad \text{if} \quad \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \leq B_i.$$

Using (5) and L'Hôpital's rule, we see that:

$$\lim_{x \to \infty} w_i = \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right).$$

Using (6) again, we then have:

$$w_i \leq \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \quad \text{if} \quad \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \geq B_i$$

$$w_i \geq \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \quad \text{if} \quad \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \leq B_i.$$

Finally, the inequalities:

$$0 \leq \frac{1}{k+1}\left(B_i - \frac{A_i}{A}\right) \leq 1$$

are guaranteed by $B_i \geq 0$ for all i, $A_i \leq 0$ for all i, $\sum A_i = -kA$ and $\sum B_i = 1$. ◊

Section 4. Share positivity in the Minflex Laurent system

We will show in this section that the Minflex Laurent (ML) system of Barnett (1983) also satisfies budget share positivity under regularity conditions on preferences. The ML system may be written as:

$$w_i = \frac{\alpha_i + \beta_i x^{1/2} + \gamma_i x^2}{\alpha + \beta x^{1/2} + \gamma x^2} \tag{7}$$

with $\alpha_i = a_{ii}p_i + \sum_{j\neq i} a_{ij}^2 p_i^{1/2} p_j^{1/2}$, $\beta_i = a_i p_i^{1/2}$, $\gamma_i = \sum_{j\neq i} b_{ij}^2 p_i^{-1/2} p_j^{-1/2}$, and $\alpha = \sum \alpha_i$, $\beta = \sum \beta_i$, $\gamma = \sum \gamma_i$.

By inspection of (7), it is seen that $\lim_{x\to 0} w_i = \alpha_i/\alpha$ and that $\lim_{x\to\infty} w_i = \gamma_i/\gamma > 0$. It is interesting to compare this with the observation in Barnett (1985) that the regular region of the ML system grows with real income. However, budget share positivity is not guaranteed at all income levels unless the coefficients a_{ii} and a_i are constrained. In particular, the requirements that $a_i > 0$ and $a_{ii} > 0$ are *sufficient* to ensure that $0 \leq w_i \leq 1$

for all i (Barnett (1983) shows that these requirements are also sufficient for global regularity). Unfortunately these constraints destroy the flexibility of the ML system, as emphasized by Diewert and Wales (1987).

Section 5. Share positivity and flexibility

It is clear from the preceding section that a conflict exists between share positivity and flexibility in the ML system. Indeed, the minimality property of the ML system implies that *any* constraints on a_i and a_{ii} will destroy flexibility.

An analogous result can be obtained for the EXP system. We have shown in Theorem 2 that the monotonicity of $A(p)$ and $B(p)$ is *sufficient* for budget share positivity. In this case, of course, $A(p)$ and $B(p)$ cannot be flexible. What has not been shown is that budget share positivity in the EXP system actually *contradicts* the flexibility of A and B. This is the topic of the following theorem.

Theorem 3. *If Equation (5) implies* $0 \leq w_i \leq 1$ *for all* $0 \leq x < +\infty$, *then neither* A *nor* B *can be flexible.*

Proof: Since $\lim_{x\to 0} w_i = B_i$, we must have $B_i = p_i \partial B/\partial p_i \geq 0$ for w_i to be positive at all income levels. This means that the derivatives of B cannot be equated to those of a decreasing function, contradicting flexibility.

Since $B_i \geq 0$ and $\sum B_i = 1$, B_i is contained in a bounded set. Since $\lim_{x\to\infty} w_i = (B_i - A_i/A)/(k+1)$, $w_i \geq 0$ for all x implies $A_i/A \leq B_i$ if $k+1 > 0$. So the elasticities of A are bounded above, contradicting the flexibility of A. The argument when $k+1 < 0$ is similar. ◊

Section 6. Concluding remarks

This note has examined the implications for budget share positivity of most known (nondifferential) demand systems. It has been shown that among the nonhomothetic systems under consideration, only the EXP demands of Lewbel (1987b) and the ML demands of Barnett (1983) are compatible with the requirement that $0 \leq w_i \leq 1$ for all income levels. For these two systems, the requirement appears to be incompatible with flexibility. However, monotonicity conditions on preferences do guarantee budget share positivity.

We wish to emphasize that this conflict between positivity and flexibility is not an argument against the specification of demand systems implying positive predicted budget shares. Indeed, a concept of flexibility which neglects the fact that the *observations* on w_i lie between zero and one obviously misses an important aspect of the data generating process. Our implication is that the (local) concept of flexibility should be revised in the light of the (global) requirements of regularity.

In this respect, an interesting topic for further research is the specification of positive budget share equations that retain sufficient flexibility for most purposes. In particular, Diewert and Wales (1987) have shown that globally regular cost functions exist which satisfy a requirement of "quasi-flexibility". EXP demands might have an advantage over ML demands in this respect, since A and B need only satisfy general requirements of differentiability, monotonicity, and homogeneity: they can thus be chosen with some leeway. The simplest choice for A and B meeting all requirements (apart from flexibility) appears to be:

$$\log A(p) = \alpha_0 - \sum \alpha_i^2 \log p_i$$

$$B(p) = \beta_0 + \sum \beta_i^2 \log p_i$$

with $\sum \alpha_i^2 = k$ and $\sum \beta_i^2 = 1$. Even this simple choice for A and B, however, leads to a highly nonlinear demand system. Moreover, contrary to EXP demands (where luxury goods remain luxuries at all income levels, and similarly for necessities), the regular ML system does not imply monotonic budget shares: it can thus approximate a wider class of income responses. The cost of this is a larger number of estimated parameters.

Finally, we wish to emphasize that we have not exhausted the list of theoretically plausible Engel functions that can be specified. In particular, one could increase the order of the Laurent series expansion in the Barnett model (at the cost of a considerable increase in the number of parameters, and consequent estimation difficulties); or investigate possible taxonomies of more general fractional demand systems than the ones considered here (a fairly ambitious endeavour); or investigate the Engel functions implied by the Fourier demand system of Gallant (1981). (This does not appear straightforward, since the Fourier system is not separable in income, so that the Engel functions are not explicit). These topics, however, are beyond the scope of this paper.

FOOTNOTES

1. I wish to thank Professor A.P. Barten for suggesting the topic of this paper, and for helpful comments. Any errors are my own.
2. It should be noted that a_i, b_i, c_i, a, and b can be characterized further; the reader is referred to the literature for details.

REFERENCES

W.A. BARNETT (1983), "New Indices of Money Supply and the Flexible Laurent Demand System", *Journal of Business and Economic Statistics* 1, 7–23

W.A. BARNETT (1985), "The Minflex Laurent Translog Flexible Functional Form", *Journal of Econometrics* 30, 33–44

W.E. DIEWERT and T.J. WALES (1987), "Flexible Functional Forms and Global Curvature Conditions", *Econometrica* 55, 43–68

A.R. GALLANT (1981), "On the Bias in Flexible Functional Forms and an Essentially Unbiased Form: the Fourier Flexible Form", *Journal of Econometrics* 15, 211–245

W.M. GORMAN (1953), "Community Preference Fields", *Econometrica* 21, 63–80

W.M. GORMAN (1961), "On a Class of Preference Fields", *Metroeconomica* 13, 53–56

W.M. GORMAN (1981), "Some Engel Curves", in: *Essays in the Theory and Measurement of Consumer Behaviour in Honour of Sir Richard Stone*, ed. by Angus Deaton, Cambridge University Press, Cambridge, 7–29

C.E.V. LESER (1963), "Forms of Engel Functions", *Econometrica* 31, 694–703

A.S. LEWBEL (1987a), "Characterizing Some Gorman Engel Curves", *Econometrica* 55, 1451–1459

A.S. LEWBEL (1987b), "Fractional Demand Systems", *Journal of Econometrics* 36, 311–337

J. MUELLBAUER (1975), "Aggregation, Income Distribution, and Consumer Demand", *Review of Economic Studies* 62, 525–543

H. WORKING (1943), "Statistical Laws of Family Expenditure", *Journal of the American Statistical Association* 38, 43–56

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