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SERVICING THE PUBLIC DEBT: COMMENT

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Servicing the Public Debt: Comment

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ABSTRACT

This comment extends Calvo's (1988) analysis of the relation between expectations and the number of repudiation equilibria in public debt management, in order to analyse the effects of heterogeneity in (nominal) debt holdings. It is shown that the number of equilibria depends on the relative wealth of the agent represented by the government. If this agent is extremely poor no equilibrium exists, while if this agent is relatively wealthy uniqueness is ensured. Under a representative democracy, if the issue of debt precedes the elections, the number of politico-economic equilibria and the implied welfare losses depend on the degree of inequality. However, if this timing is reversed, the Pareto-optimum is attained, because of the existence of a government candidate whose unique equilibrium discretionary policy choice is the first-best repudiation rate.

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In an interesting contribution, Calvo (1988) analyses the role of expectations in the multiplicity of equilibria that might arise in public debt management. In a "good" equilibrium the interest rate on government debt is low because investors expect little or no debt repudiation, and indeed, given the relatively low debt service costs, ex post the government has less of an incentive for repudiation. Conversely, in a "bad" equilibrium, expectations of future repudiation imply higher debt service costs, which lead to stronger incentives for repudiation, thereby fulfilling the expectations of investors.

The issue of multiplicity has been analysed further in related models of Giavazzi and Pagano (1989) and Alesina, Prati and Tabellini (1989). The general finding is that a confidence crisis in public debt management is less likely if the average maturity of debt is longer, roll-over of the outstanding stock of debt is more evenly smoothed out over time and if the government has better access to foreign credit lines in the event of a debt panic. Although Calvo points out that there could be important redistributive effects of debt repudiation, so far no systematic analysis has linked the problem of multiple equilibria to the distribution of debt holdings among investors and the potentially resulting political conflict.

Below, Calvo's analysis is extended to allow for heterogeneity in nominal asset holdings (e.g., private and public debt) among private agents. Agents with different relative wealth prefer a different composition of government revenues for a given level of public expenses. Some agents hold a lot of debt, hence prefer little or no repudiation[1] of debt, while others are poor and prefer repudiation rather than taxation.

Although data on the inequality in nominal asset holdings are rare, which is in particular the case for comparable cross-country data, the figures in, for example, Smith (1988) show that the shares in total bonds and cash holdings by the richest one percent of the U.S. population are typically in the range of 10% to 30% or even higher. Similar figures in Avery, Elliehausen and Kennickel (1988) from the 1983 Survey of Consumer Finances suggest that only less than 50% of nominal wealth is held by the poorer 90% of U.S. households. The few available data for other (industrialised) countries (for example, France) suggest degrees of inequality of comparable magnitude.

The extension of Calvo's model advanced below shows that the number of equilibria depends on the relative wealth of the agent represented by the government. If this agent is very

[1] As in Calvo, "repudiation" is a catchall word here, which refers to anything from open repudiation to an inflation tax on the gross rate of return on nominal debt. The analysis in this comment is mostly concerned with the latter form of repudiation.

poor, no equilibrium exists, because the incentive of the government to redistribute through a high rate of inflation is too strong for agents to be willing to hold debt, while if the agent represented is relatively wealthy, uniqueness of equilibrium is ensured. Therefore, in a representative democracy where the issue of debt precedes the elections, the number of equilibria and their implied welfare losses depend on the degree of inequality among the electorate. Crucially, however, it can be shown that there exists a *unique* type of government candidate whose *unique* discretionary equilibrium policy choice is the Pareto optimal inflation rate. Hence, if elections *precede* the issue of debt, the Pareto optimum is attained. The electorate's perspective is altered in such a way that they do not only take into account the set of alternative policy outcomes, but a wider set of characteristics of the equilibrium on which the economy settles. In particular, the electorate takes into account the effects of their voting behaviour on inflationary expectations as an important determinant of the ex post optimal inflation rate. From a more general point of view, these findings stress the importance of an appropriate timing in the management of (public) debt and the scheduling of elections.

Including international debt in the analysis leaves most of the results unaffected, although, if elections are introduced which follow the issue of debt, the chosen government represents a poorer agent and has a stronger incentive for inflation than before. This makes the existence of an equilibrium in which agents are prepared to hold a given amount of public debt less likely. However, when elections are appropriately scheduled, the economy still attains its Pareto optimum.

The remainder of this comment is organised as follows. Section I extends Calvo's model to allow for heterogeneity in nominal debt holdings. Section II analyses the existence and number of equilibria if the government represents agents with different levels of wealth, while Section III introduces a representative democracy with elections. Section IV includes international debt into the analysis. Section V concludes this comment.

I. Heterogeneous Debt Holdings

This section extends Calvo's monetary model of non-indexed public debt management (Section II of his article) to allow for heterogeneity in nominal debt holdings among private sector agents (cf. Beetsma and Van der Ploeg, 1993). Although data on the distribution of nominal asset holdings are rare, in particular comparable cross-country data, recent work of, for example, Smith (1988) and Avery, Elliehausen and Kennickel (1988) on U.S. microdata suggests that the distributions of these asset holdings are extremely skewed. For example, the richest (in terms of *total* net worth) percent of the population has a share in the range of at least 10% to more than 30% (depending on the item and the period) in total nominal asset value holdings by the population. Similar figures are reported for France (Kessler and Wolff, 1991) and Japan (Bauer and Mason, 1992). Other typical characteristics of such data are the strong correlations among the distributions of various wealth items (i.e., the rich members of the population not only hold large shares of nominal wealth, but also of corporate stock, real estate, etcetera), as well as the correlation between the distribution of income and wealth. Roughly speaking, the highest income earners are also the most wealthy in the population. The income distribution, however, is generally much less skewed than the wealth distribution (Bauer and Mason, 1992). These observations suggest that a potentially important further step in the analysis of the role of expectations in (public) debt management would be to allow for heterogeneity in nominal asset holdings, which, in particular, makes it possible to analyse the effects of political pressures for redistribution.

A few qualifications are in order. First, it is assumed that (private and public) debt is in nominal terms, rather than indexed to the price level. On the one hand this can be motivated by the fact that in reality most of the debt, both public and private, is in nominal terms, at least in industrialised countries. On the other hand, the version of Calvo's model with real debt repudiation (Section I of his article) can be extended in a way very similar to the extension below, by allowing for heterogeneity in holdings of indexed debt and a government which chooses a degree of repudiation which depends on the relative wealth of the agent it represents. The second point is that, ideally, one would like to allow for other types of inequality as well. However, as the remainder of the analysis makes clear, this would add a great deal of additional complexity to the model. This is in particular the case when elections are introduced (as in Section III), since the conditions for a stable majority are notoriously restrictive if there is more than one source of conflict (inequality) among voters (although, as noticed above, the available data suggest that the various wealth items are highly concentrated with the same groups in the population). Finally, we should address the observation that usually large amounts of nominal debt are held by institutional investors. To keep the analysis focussed we abstract from the existence of such investors which do not exert direct political influence via the electoral process anyway. Also, in reality, a high rate of inflation might have a significant indirect effect on the wealth of those who hold large amounts of stock in institutional investment companies.

There are two periods, period 0 and period 1, and two types of agents: private sector agents and the government. In period 0, nominal debt is issued both by private agents and the government, while in period 1 the debt is paid off. All debt is assumed to be perfectly substitutable and, hence, has the same nominal interest factor $R_b \equiv 1+i$ (where i is the nominal interest rate). Although in period 1 the value of debt is fixed in nominal terms, its real value depends on the rate of inflation between period 0 and period 1. If we denote by P_t the price level in period t (t=0,1), the real interest factor on debt is $R_b P_0/P_1$. Therefore, we interpret $\theta = 1-P_0/P_1$ as the share of debt that is "repudiated" through inflation. The inflation rate, $\pi \equiv (P_1-P_0)/P_0$, which is assumed to be under direct control of the government[2], is uniquely related to the "repudiation rate", θ. Since $-1 \leq \pi < \infty$, $-\infty < \theta < 1$. Following Calvo, the demand for real money balances is assumed to be constant, hence $M_t/P_t = \kappa \geq 0$, where M_t is the (demand determined) nominal money supply.

Private agents have the possibility to accumulate physical capital, with an exogenous gross real interest factor $R>1$, or to invest in (or issue) nominal bonds. In a perfect foresight equilibrium with positive stocks of capital and outstanding debt (private or public), individuals are indifferent between the two types of assets, which requires

$$(1-\theta)\, R_b = R. \tag{1.1}$$

In period 0, private agents differ in their available resources for investment. In particular, under the assumption that each individual invests the same amount, k, in capital, and that individuals do not differ in other respects, the private sector is modelled as a continuum of agents, who are distributed on **R** according to their individual total (private and public) debt holdings. The aggregate mass of private agents is normalised to unity.

Individuals consume all of their wealth in period 1. Consumption of an individual with debt holdings $\tilde{b}$ (henceforth, "agent $\tilde{b}$" or an agent of "type $\tilde{b}$") is

$$\tilde{c} = y - z(x) + kR + (1-\theta)\tilde{b}R_b - x - \kappa\theta - \nu(\theta), \tag{1.2}$$

where y is (exogenous) endowment income, kR is the gross real return on investment in physical capital, $(1-\theta)\tilde{b}R_b$ is the net real return on debt holdings, x denotes the tax payment and $\kappa\theta = (M_1 - M_0)/P_1$ the amount of seigniorage paid. Function z(x) represents the "deadweight" cost of taxation and has properties $z(0)=z'(0)=0$ and $z''(x)>0$. Distortionary losses due to inflation are given by $\nu(\theta)$ with analogous properties $\nu(0)=\nu'(0)=0$ and $\nu''(\theta)>0$. Thus, inflation is reflected in a direct welfare cost for agents.

[2] This corresponds to a situation in which the central bank is dependent. Although recently a number of countries have tried to make their central banks less dependent, in practice it turns out that no central bank is completely immune for political pressures, not even the Bundesbank (witness the course of events around the German unification).

Taxes, x, follow from the per capita government budget constraint in period 1,

$$x = (1-\theta)b^{A}R_{b} + g - \kappa\theta, \tag{1.3}$$

where b^{A} is the per capita or average amount of government debt issued in period 0 and g is the exogenous level of government spending. Unless explicitly stated otherwise, we focus on the more realistic case where b^{A} is non-negative. Higher inflation, hence a higher θ, lowers real debt service costs and raises seigniorage revenues, $\kappa\theta$, and, therefore, requires less tax revenues. In the special case where $b^{A}=0$, the net indebtedness of the government is zero and there is no outside debt. All private nominal claims cancel out in the aggregate.

II. Government type and existence of equilibria

Equilibrium consumption of an arbitrary agent $\tilde{b}$ follows upon substitution of the government budget constraint (1.3) and arbitrage condition (1.1) into (1.2):

$$\tilde{c} = y + [k+(\tilde{b}-b^{A})]R - g - z(Rb^{A}+g-\kappa\theta) - \nu(\theta), \tag{2.1}$$

where the term $(\tilde{b}-b^{A})R$ captures the higher (lower) than average consumption possibilities of agent $\tilde{b}$ if $\tilde{b}>(<)b^{A}$.

II.1 Pareto-optimal equilibrium

In equilibrium, consumption only depends on the sum of distortionary losses from taxation and inflation, which are equal for all agents. Therefore, the Pareto-optimal or "first-best" equilibrium corresponds to the minimisation of total distortionary losses $T(\theta)\equiv z(Rb^{A}+g-\kappa\theta)+\nu(\theta)$. Because $T''(\theta)>0$ and excluding the corner solution $\theta=1$, which is ensured if $\nu'(1)>z'(Rb^{A}+g-\kappa)\kappa$, the repudiation share θ^{FB} in a first-best equilibrium follows from

$$\kappa\, z'(Rb^{A}+g-\kappa\theta^{FB}) = \nu'(\theta^{FB}). \tag{2.2}$$

Note that $0\leq\theta^{FB}<(Rb^{A}+g)/\kappa$ and that θ^{FB} increases in the level of government commitments, $Rb^{A}+g$. Since $T''(\theta)>0$, $T'(\theta)<0$ for $\theta<\theta^{FB}$, and $T'(\theta)>0$ for $\theta<\theta^{FB}$. An increase in θ reduces, on the one hand, the real value of money holdings, thereby lowering the required amount of tax revenues, and hence implies lower distortionary losses from taxation, while, on the other hand, it increases the distortionary losses associated with inflation. Therefore, (2.2) says that at $\theta=\theta^{FB}$ the marginal benefit from an increase in θ equals its marginal cost.

II.2 Discretionary equilibria

Suppose that the government in period 1 represents agent $\tilde{b}$ [3] and is not constrained by any commitments. The government objective function, $y-z((1-\theta)b^{A}R_b+g-\kappa\theta)+kR+(1-\theta)(\tilde{b}-b^{A})R_b-g-\nu(\theta)$, which follows upon substitution of (1.3) into (1.2), is strictly concave in θ on $(-\infty,1]$. Therefore, if $\tilde{b}>b_{min}(R_b)\equiv[b^{A}R_b-\nu'(1)+z'(g-\kappa)]/R_b$, the unique optimal choice, $\tilde{\theta}^{SB}<1$ ("SB" for "second-best"), solves:

$$-z'((1-\theta)b^{A}R_b+g-\kappa\theta)\,(b^{A}R_b+\kappa)\;+\;\nu'(\theta)\;+\;(\tilde{b}-b^{A})R_b\;=\;0. \tag{2.3}$$

Hence, some function ϕ exists, such that,

$$\tilde{\theta}^{SB}\;=\;\phi(b^{A},R_b,g,b^{A}-\tilde{b}), \tag{2.4}$$

where, as follows from the implicit function theorem, $\partial\phi/\partial g>0$ and $\partial\phi/\partial(b^{A}-\tilde{b})>0$ (the signs of the other partials depend on the sign of $b^{A}-\tilde{b}$). Given average debt holdings, *the poorer the agent represented by the government, the higher the rate of inflation*. This reflects the incentive to use inflation as a device for redistribution once the value of debt has been fixed in nominal terms. Obviously, if $\tilde{b}=b^{A}$ this incentive vanishes and the problem of the government reduces to the one analysed by Calvo.

A pair $\{\theta,R_b\}$, $\theta<1$, is an equilibrium combination if and only if it jointly fulfills (1.1) and (2.3). Substituting $R_b=R/(1-\theta)$ into (2.3) and rearranging yields:

$$\tilde{b}R\;=\;z'(Rb^{A}+g-\kappa\theta)[Rb^{A}+\kappa(1-\theta)]\;-\;(1-\theta)\nu'(\theta)\;+\;Rb^{A},\;\theta<1. \tag{2.5}$$

The following lemma summarises the relation between the number of equilibria and the relative wealth of the agent represented by the government:

Lemma 1: Let b^{A} be given.

(a) A level of debt holdings $b_L\in\mathbf{R}$ exists, such that, if the government represents an agent poorer than b_L, no equilibrium exists (with per capita amount of public debt equal to b^{A}).

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[3] I.e., this government chooses θ to maximise consumption of agent $\tilde{b}$. Such government is said to be of "type $\tilde{b}$" or is simply referred to as "government $\tilde{b}$".

(b) A level of debt holdings $b^U \in \mathbf{R}$ exists, such that, if the government represents an agent richer than b^U, a unique equilibrium exists.

(c) The government type (b^{FB}, say) compatible with $\theta=\theta^{FB}$ as an equilibrium discretionary policy choice is *unique*. Vice versa, if the government type is b^{FB}, its equilibrium discretionary policy choice is *unique* and given by $\theta=\theta^{FB}$.

Proof: (a) Differentiating the right hand side (LHS) of (2.5) with respect to θ, yields that the RHS of (2.5) is decreasing for $\theta \leq 0$ and increasing for $\theta \geq \max[1, 1+Rb^A/\kappa, (Rb^A+g)/\kappa]$. By continuity, it follows that the RHS of (2.5) is bounded from below for $\theta<1$. (b) The RHS of (2.5) is bounded from above on the interval (0,1). Moreover, it is strictly decreasing for $\theta \leq 0$ and its limit is ∞, as $\theta \rightarrow -\infty$. (c) An equilibrium where $\theta^{SB}=\theta^{FB}$ requires that (2.5) be fulfilled for $\theta=\theta^{FB}$. Rearranging and using (2.2), it follows immediately that $\tilde{b}$ is uniquely given by

$$b^{FB} \equiv b^A + z'(Rb^A+g-\kappa\theta^{FB})\, b^A. \tag{2.6}$$

To prove the reverse part of (c), substitute (2.6) for $\tilde{b}$ into (2.5), rearrange and use $-z'(Rb^A+g-\kappa\theta)\kappa+\nu'(\theta)=T'(\theta)$, so that,

$$(1-\theta)\, T'(\theta) = [z'(Rb^A+g-\kappa\theta) - z'(Rb^A+g-\kappa\theta^{FB})]\, Rb^A. \tag{2.7}$$

It follows from the properties of $T(\theta)$ and z(.) that (2.7) is fulfilled if $\theta=\theta^{FB}$, that the LHS of (2.7) is negative while the RHS is positive if $\theta<\theta^{FB}$ and vice versa if $\theta^{FB}<\theta<1$. Hence, for government b^{FB} the discretionary equilibrium policy choice is uniquely given by θ^{FB}. □

According to Lemma 1(a), no equilibrium exists if the government represents a sufficiently poor agent and plans to issue a per capita amount of debt b^A. If the government represents such a poor agent, once this amount of public debt has been issued, the incentives for redistribution are so strong that the government chooses to completely inflate away the real value of any outstanding nominal debt (both public and private). No investor who anticipates this would be willing to hold any debt. However, note that if $b^A>(\kappa-g)/R$, the RHS of (2.5) is increasing in b^A for all $\theta<1$. Therefore, given such a government, it is not excluded that there may exist equilibria in which a smaller amount of public debt is issued. Lemma 1(b) says that if the agent represented is rich enough, a unique equilibrium exists. With respect Lemma 1(c), note that $b^{FB}>b^A$ (if $b^A>0$), i.e. the government compatible with θ^{FB} as its equilibrium discretionary policy choice is more "conservative" than the average voter.[4] Although a government of type $b^{\wedge}>0$ has no incentive to redistribute, it still has an incentive to inflate away part of the real value of government debt, once debt service costs are fixed in nominal terms. Given that in equilibrium these attempts to redistribute are futile, the government type which is prepared to implement the first-best inflation rate should be more conservative than the average agent $b^{\wedge}$.

For a more detailed assessment of the number of equilibria, we discuss two special cases:

No demand for money ($\kappa=0$):

In a first-best situation, the only way distortionary losses from taxation can be reduced is through increased seigniorage revenues. If $\kappa=0$, this channel vanishes, however, and the inflation rate which minimises distortionary losses corresponds to $\theta^{FB}=0$. Discretionary equilibria correspond to solutions of:

$$(b^{\wedge}-\tilde{b}) + z'(Rb^{\wedge}+g)\, Rb^{\wedge} = (1-\theta)\nu'(\theta),\ \theta<1. \tag{2.8}$$

The RHS of (2.8) is increasing for $\theta\leq 0$, decreasing for $\theta\geq 1$, equals zero if $\theta=0$ or $\theta=1$, and is strictly greater than zero for $0<\theta<1$. Hence, the RHS of (2.8) reaches a global maximum on the interval (0,1). It follows immediately that:

Lemma 2: Let $b^{\wedge}$ be given, $\kappa=0$ and the government represent agent $\tilde{b}$. Define b_L as the minimum value of $\tilde{b}$ for which (2.8) has a solution $\theta<1$.

(a) If $\tilde{b}<b_L$, no equilibrium exists (with an amount of public debt $b^{\wedge}$).

(b) If $\tilde{b}=b_L$, at least one equilibrium exists.

(c) If $b_L<\tilde{b}<b^{FB}$, more than one equilibrium exists.

(d) If $\tilde{b}\geq b^{FB}$, a *unique* equilibrium exists.

Compared with Lemma 1, Lemma 2 presents a more detailed assessment of the role of the government type in the number of equilibria. In particular, if the government is not too "radical", so that it represents a "middle class" agent ($b_L<\tilde{b}<b^{FB}$), a multiplicity of equilibria exists. Moreover, note that, however poor the agent represented, there always exists an amount of public debt (maybe negative, in which case the government is a net creditor) compatible with the

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[4] A related result was pointed out by Rogoff (1985) in the context of the appointment of a central banker with stronger anti-inflationary preferences than the representative member of society. Persson and Tabellini (1990) discuss similar findings in various contexts.

existence of an equilibrium.

No public debt ($b^A=0$):

Even though the stock of outstanding government debt is zero, nominal debt holdings need not be equal among private sector agents, due to claims of private agents against each other. If $b^A=0$, (2.6) reduces to $b^{FB}=b^A=0$. The average government type ($b^{FB}=b^A=0$) has no other incentive for inflation than to balance revenues from taxation and inflation in such a way that total distortionary losses are minimised. Discretionary equilibria correspond to solutions of

$$\tilde{b}R = (1-\theta)[z'(g-\kappa\theta)\kappa-\nu'(\theta)] = -(1-\theta)T'(\theta),\ \theta<1. \tag{2.9}$$

If $\nu'(1)>z'(g-\kappa)\kappa$ (a necessary and sufficient condition for the existence of a first-best equilibrium policy $\theta^{FB}<1$), the RHS of (2.9) is strictly decreasing for $\theta\leq\theta^{FB}$ and strictly increasing at $\theta=1$. Hence, the RHS of (2.9) reaches its global minimum at some $\theta\in(\theta^{FB},1)$, which corresponds to some government of type b_L. Moreover, the RHS of (2.9) equals zero for $\theta=\theta^{FB}$ and $\theta=1$, and is negative for $\theta\in(\theta^{FB},1)$. Analogous to Lemma 2, it follows that

Lemma 3: Let $b^A=0$, $\nu'(1)>z'(g-\kappa)\kappa$ and the government represent agent $\tilde{b}$. Define b_L (<0) as the minimum value of $\tilde{b}$ for which (2.9) has a solution $\theta<1$.

(a) If $\tilde{b}<b_L$, no equilibrium exists.

(b) If $\tilde{b}=b_L$, at least one equilibrium exists.

(c) If $b_L<\tilde{b}<b^{FB}$, a multiplicity of equilibria exists.

(d) If $\tilde{b}\geq b^{FB}(=0)$, a *unique* equilibrium exists.

As already pointed out by Calvo, if $b^A=0$ and the government represents $b^A=0=b^{FB}$, the equilibrium will be unique, because θ^{SB} does not depend on R_b in (2.3), and $\theta^{SB}=\theta^{FB}$. If $\tilde{b}\neq b^A$, however, dependence of θ^{SB} on R_b is reintroduced via the third term on the LHS of (2.3), which causes a potential for multiple equilibria. As in the case of zero money holdings ($\kappa=0$), if the government represents an "middle class" agent (i.e., $b_L<\tilde{b}<b^{FB}$), a multiplicity of equilibria exists.

III. Representative Democracy, Timing and Inequality

Consider a political system with elections, where for each possible type of agent $b\in\mathbf{R}$ a government candidate of the same type exists. It is assumed that no candidate is able to precommit to announcements of his future policies. The elected government type is determined by majority rule.

Politico-economic equilibria can be found by solving backwards. The final decision, the choice of θ, given R_b and the new government's type, has been discussed in Section II. As will become clear, it is of interest to consider the following alternative timings:

III.1 Debt issue precedes elections

Consumption of an agent $\tilde{b}$, if a government of type b' is in power, is

$$\begin{aligned} \tilde{c}' &= y - z(g-\kappa) + kR - g - \nu(1), \text{ if } b' \leq b_{min}(R_b), \\ \tilde{c}' &= y - z((1-\theta')b^A R_b + g - \kappa\theta') + kR + (1-\theta')(\tilde{b}-b^A)R_b - g - \nu(\theta'),\ b' > b_{min}(R_b), \end{aligned} \tag{3.1}$$

where θ' is the value of θ chosen by government b'. It is easy to establish that: $\partial\tilde{c}'/\partial b'=0$, if $b'<b_{min}(R_b)$; using (2.4), $\partial\tilde{c}'/\partial b'>0$ if $b_{min}(R_b)<b'<\tilde{b}$, $\partial\tilde{c}'/\partial b'=0$ if $b'=\tilde{b}$, and $\partial\tilde{c}'/\partial b'<0$ if $b'>\max[b_{min}(R_b),\tilde{b}]$; $\tilde{c}'$ is continuous at $b'=b_{min}(R_b)$. Hence, if $b^M>[b^A R_b-\nu'(1)+z'(g-\kappa)]/R_b$, the only type of candidate which cannot be defeated by an alternative in a majority voting contest is b^M, where b^M is the median level of debt holdings. Therefore, irrespective of the nominal interest factor, the candidate of type b^M is elected[5]. His policy choice $\theta=\theta^M$ follows from (2.3) by replacing $\tilde{b}$ with b^M.[6] Policy outcomes in politico-economic equilibria correspond to solutions $\theta=\theta^{M,E}$ of

$$b^M R = z'(Rb^A+g-\kappa\theta)(Rb^A+\kappa(1-\theta)) - (1-\theta)\nu'(\theta) + Rb^A,\ \theta<1. \tag{3.2}$$

The most interesting results with respect to politico-economic equilibria and their existence are summarised by:

Lemma 4: Assume that the issue of debt precedes the elections.

(a) If median debt holdings are less than or equal to average debt holdings ($b^M \leq b^A > 0$), any

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[5] Although the type of the elected candidate does not depend on R_b, his choice of θ, hence also welfare, do depend on R_b.

[6] If $b^M \leq [b^A R_b-\nu'(1)+z'(g-\kappa)]/R_b$, all agents of type $b \leq b^M$ are indifferent among any of the candidates of type $b \leq [b^A R_b-\nu'(1)+z'(g-\kappa)]/R_b$. Therefore, the elected government is some arbitrary candidate from this set and this candidate chooses $\theta=1$. Equilibria with $\theta=1$ do not exist, however. Hence, in situations where a politico-economic equilibrium exists, the government always represents the median debt holder(s).

politico-economic equilibrium repudiation rate $\theta^{M,E}$ exceeds θ^{FB}.

(b) Given b^A, there is a upper bound on b^A-b^M above which no politico-economic equilibrium exists.

(c) Given the degree of inequality (as measured by $b^A - b^M > 0$), there is an interval $[(b^A)^*, \infty)$, $(b^A)^* \geq 0$, for the level of public debt such that no politico-economic equilibrium exists.

Proof: (a) Follows by combining (3.2) with the properties of total distortionary costs $T(\theta)$ and the properties of $z(Rb^A + g - \kappa\theta)$. (b) follows immediately from Lemma 1(a). (c) Rewrite (3.2) as

$$R(b^A - b^M) + z'(Rb^A + g - \kappa\theta)[Rb^A + \kappa(1-\theta)] = (1-\theta)\nu'(\theta), \; \theta < 1. \tag{3.3}$$

Fix $(b^A - b^M)$. If $b^A > \max[(\kappa - g)/R, 0]$, one has that the partial derivative of the LHS of (3.3) with respect to b^A is greater than and bounded away from zero for all $\theta < 1$, that the partial derivative of the LHS of (3.3) with respect to θ is less than zero for all $\theta < 1$, and that $z'(Rb^A + g)Rb^A \geq 0$. Combining this with the property that the RHS of (3.3) increases in θ for $\theta < 0$, gives the desired result. □

Lemma 4(a) implies that a representative democracy in which the issue of debt precedes the elections leads to welfare losses in equilibrium. Furthermore, depending on the degree of inequality, there is a possibility for zero, a unique or multiple politico-economic equilibria. Lemma 4(b) says that if inequality, as measured by $b^A - b^M > 0$, is too high, it is optimal for government b^M to inflate the real value of debt away completely. In that case no equilibrium exists in which the private sector is prepared to hold an average amount of public debt equal to b^A. Finally, according to Lemma 4(c), there is always a limit to the amount of public debt that can be issued. If the amount of debt the government plans to issue is too large, then nobody would be willing to buy any debt for fear of being "expropriated" completely.

III.2 Elections precede debt issue

An alternative timing changes the perspective of agents at the elections and, hence, influences their voting behaviour. The implication is that:

Proposition 1: If elections precede the issue of debt, the elected candidate is b^{FB}, and the first-best equilibrium is attained.

Proof: Recall from Lemma 1(c) that b^{FB} is the unique government candidate whose unique discretionary equilibrium policy choice is $\theta=\theta^{FB}$. Moreover, from the *ex ante* viewpoint at election date, each agent's preferred equilibrium is that in which total distortionary losses are minimised. It follows that, for each individual agent, b^{FB} is the preferred government type. □

If the debt issue comes first, then, at the moment elections take place, the nominal interest factor is given and, in particular, does not depend on the policy choices of the government to be elected. Therefore, poor agents prefer a government which engages in a high inflation policy, so that the tax rate can be lowered and the rich are forced to contribute a larger share of the revenues required by the government. However, if the issue of debt takes place *after* the elections, then, at the moment of the elections, voters take into account that *in equilibrium* redistribution will not be possible. The reason is that the nominal rate of return demanded by investors is determined only when debt is issued and adjusts to the anticipated ex post inflationary incentives which depend on the new government's type. If elections take place before debt is issued, at election date inflationary expectations still have to be incorporated in the nominal interest rate. Hence, the electorate takes into account the effects of their voting behaviour on inflationary expectations. The perspective of the electorate is altered such that each voter prefers a government type compatible with a discretionary equilibrium in which total distortionary losses are minimised.

IV. Extension: International Debt

There are two major ways of issuing debt to foreigners: issuing debt denominated in domestic currency or debt denominated in some foreign currency. The former type of debt is more important for the rich and industrialised democratic countries and its role is studied below in a further extension of the monetary model. Issuing debt denominated in dollars is an important way of attracting funds for Latin American countries. Although the real value of this type of debt is protected against a high domestic inflation rate, it might still become victim of repudiation (recall the example of Mexico, 1982, which announced to no longer honour its foreign debt). Therefore, the introduction of debt denominated in foreign currency may be studied within the context of an analogous extension of the real debt repudiation model of Calvo (1988, Section I).

The analysis with international debt included largely parallels the analysis in previous sections and is therefore kept short and confined to ceteris paribus changes of the results. In other words, given everything else, the question is what happens if the amount of debt issued to foreigners is increased.

As before, let b^A be the per capita[7] level of public debt. Now, we assume that $b^A = b^{DA} + b^F$, where b^{DA} and $b^F \geq 0$ denote the per capita domestic, respectively foreign, holdings of public debt issued by the domestic authorities. Keeping b^A and the available resources for private sector agents' investments fixed, an agent who originally invested an amount $\tilde{b}$ in nominal claims, now invests an amount $\tilde{b}^D = \tilde{b} - b^F$ in nominal claims and an amount $(k+b^F)$ in physical capital. As tax payments are still given by (1.3), his period 1 consumption becomes,

$$\tilde{c} = y - z[(1-\theta)b^A R_b + g - \kappa\theta] + (k+b^F)R + (1-\theta)[\tilde{b} - (b^F + b^A)]R_b - g - \nu(\theta). \tag{4.1}$$

Combining (4.1) with (1.1) shows that the first-best repudiation rate θ^{FB} remains unchanged, which is not surprising given that the government budget constraint (1.3) is unaffected by the introduction of international debt. It is easy to see that all results summarised in Lemma's 1-3 (Section II) continue to hold. In particular, the government type which implements θ^{FB} as its unique discretionary equilibrium policy is still uniquely given by $b^{FB} = b^A + z'(Rb^A + g - \kappa\theta^{FB})b^A$. However, a ceteris paribus increase in b^F implies that b^{FB} becomes "more conservative" relative to an agent with average debt holdings b^{DA}. The intuition is that the larger the amount of debt held by foreigners, the stronger the temptation of a government which represents the average domestic debt holder, b^{DA}, to use inflation to reduce the real value of the debt. The negative effect of the higher inflation rate on the real value of the average agent's debt holdings is more than offset by the reduction in his tax payments. Hence, to compensate for the stronger incentive for inflation, b^{FB} should be relatively more conservative.

Consider the introduction of elections (as in Section III) which take place after all debt has been issued. Going through the same steps as in Section III.1, one finds again that the chosen government represents the median debt holder. Now, however, the median amount of debt holdings equals $b^{DM} \equiv b^M - b^F$ and, hence, the agent represented in any politico-economic equilibrium is poorer (in terms of nominal asset holdings) than when no debt is issued to foreigners. Equilibrium policy outcomes correspond to solutions $\theta = \theta^{M,E}$ of

$$b^{DM}R = (b^M - b^F)R = z'(Rb^A + g - \kappa\theta)(Rb^A + \kappa(1-\theta)) - (1-\theta)\nu'(\theta) + Rb^A, \; \theta < 1. \tag{4.2}$$

The results of Lemma 4 continue to hold. Hence, if median debt holdings, b^{DM}, are less than or

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[7] The term "per capita" denotes an amount per individual in the domestic private sector. Remember that the number of domestic private agents was normalised to unity.

equal to the per capita level of public debt, b^A, any equilibrium repudiation rate $\theta^{M,E}$ exceeds the the first-best repudiation rate. Moreover, provided that (4.2) has at least a single solution $\theta < 1$, the lowest equilibrium repudiation rate increases with a ceteris paribus increase in the amount of debt issued to foreigners[8]. Therefore, if there were some way to coordinate expectations towards the equilibrium with the lowest repudiation rate, a ceteris paribus decrease in the amount of debt sold to foreigners would lead to a Pareto improvement. Finally, for a given per capita level of public debt, b^A, the limiting degree of inequality, $b^{DA}-b^{DM}=b^A-b^M$, compatible with the existence of an equilibrium falls with an increase in b^F.

The final step of this extension is to consider the alternative timing where elections precede the issue of debt. As noted above, b^{FB} still is the unique government type which has θ^{FB} as its unique discretionary equilibrium policy choice. Hence, the results of Proposition 1 apply, so that, *irrespective of the amount of debt that is sold to foreigners, the economy attains the first-best if elections are appropriately scheduled vis-à-vis the issue of debt.*

V. Conclusion

The monetary version of Calvo's (1988) debt repudiation model has been extended to a context with heterogeneity in debt holdings among the private sector. An unequal distribution of debt holdings may provide an additional incentive for a government which represents the poor to redistribute by inflating away (part of) the real debt value. Therefore, the degree of inequality plays an important role in the possible number of equilibria. Moreover, if elections are introduced, their timing relative vis-à-vis the issue of debt is important. If elections precede the issue of debt, voters are induced to take account of a wider set of aspects (in particular, the level of inflationary expectations and its effects on nominal interest rates) of any potentially resulting equilibrium. This might lead to Pareto improvements and reduce the need for debt indexation as a way to attain the first best (see Calvo). The latter could have other advantages, since indexation against inflation might convey the wrong signals about the government's anti-inflationary preferences or distort the incentives of the government in other ways not modeled here (see Fischer and Summers, 1989).

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[8] Note that the RHS of (4.2) is decreasing at the minimum level of θ for which (4.2) is fulfilled.

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