



---

No. 2005/23

**Credit Market Competition and Capital Regulation**

Franklin Allen, Elena Carletti,  
and Robert Marquez

---





## Center for Financial Studies

The *Center for Financial Studies* is a nonprofit research organization, supported by an association of more than 120 banks, insurance companies, industrial corporations and public institutions. Established in 1968 and closely affiliated with the University of Frankfurt, it provides a strong link between the financial community and academia.

The CFS Working Paper Series presents the result of scientific research on selected topics in the field of money, banking and finance. The authors were either participants in the Center's Research Fellow Program or members of one of the Center's Research Projects.

If you would like to know more about the *Center for Financial Studies*, please let us know of your interest.

Prof. Dr. Jan Pieter Krahen

Prof. Volker Wieland, Ph.D.



CFS Working Paper No. 2005/23

## **Credit Market Competition and Capital Regulation\***

Franklin Allen<sup>1</sup>, Elena Carletti<sup>2</sup>,  
and Robert Marquez<sup>3</sup>

September 24, 2005

### **Abstract:**

Market discipline for financial institutions can be imposed not only from the liability side, as has often been stressed in the literature on the use of subordinated debt, but also from the asset side. This will be particularly true if good lending opportunities are in short supply, so that banks have to compete for projects. In such a setting, borrowers may demand that banks commit to monitoring by requiring that they use some of their own capital in lending, thus creating an asset market-based incentive for banks to hold capital. Borrowers can also provide banks with incentives to monitor by allowing them to reap some of the benefits from the loans, which accrue only if the loans are in fact paid o.. Since borrowers do not fully internalize the cost of raising capital to the banks, the level of capital demanded by market participants may be above the one chosen by a regulator, even when capital is a relatively costly source of funds. This implies that capital requirements may not be binding, as recent evidence seems to indicate.

**JEL Classification:** G21, G38

**Keywords:** Banking, Costly Capital, Asset Side Market Discipline

\* We would like to thank Martin Hellwig, Moshe Kim and Steven Ongena for useful comments, as well as seminar participants at the Federal Reserve Bank of New York, the Max Planck Institute, the 2005 Federal Reserve Bank of Chicago Bank Structure Conference, and the 2005 CEPR Summer Symposium. The usual disclaimers apply. We are grateful to the Wharton Financial Institutions Center for financial support.

<sup>1</sup> Contact information: Franklin Allen, Wharton, School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104-6367, allenf@wharton.upenn.edu.

<sup>2</sup> Center for Financial Studies

<sup>3</sup> Robert H. Smith School of Business, University of Maryland at College Park

# 1 Introduction

A common justification for capital regulation for banks is the reduction of bank moral hazard. Given the presence of deposit insurance, banks have easy access to deposit funds. If they hold a low level of capital, there is an incentive for them to take on excessive risk. If the risky investment pays off, the banks' shareholders receive the payoff. On the other hand, if it does not, the bulk of the losses are borne either by depositors or by the body providing deposit insurance. Given the widely accepted view that equity capital is more costly for banks than other forms of funds, the common assumption in much of the extant analyses of bank regulation is that capital adequacy standards should be binding as banks attempt to economize on the use of this costly input.

In practice, however, it appears that the amount of capital held by banks has varied substantially over time in a way that is difficult to explain as a function of regulatory changes. For example, Berger et al. (1995) report that in the 1840's and 1850's banks in the U.S. had capital ratios of around 40 to 50 percent. These ratios fell dramatically throughout the twentieth century, reaching a range of 6 to 8 percent in the 1940's where they stayed until the end of the 1980's. More recent evidence in Flannery and Rangan (2004) suggests that bank capital ratios have again increased, with banks in the U.S. now holding capital that is 75% in excess of the regulatory minimum (see also Barth et al., 2005, for international evidence).<sup>1</sup> Given that capital adequacy standards were not in existence during much of the nineteenth century, and have not fluctuated much since their inception, it is hard to find a regulatory rationale to explain movements in banks' capital holdings.

To better understand the role of bank capital and regulation, we present a simple model of bank lending that incorporates two features widely believed to be important for banking markets. First, we incorporate a consideration related to banks' lending behavior into their choice of financing, recognizing that banks' capital structures may have implications for their

---

<sup>1</sup>A recent study by Citigroup Global Markets (2005) finds that "... most European banks have and generate excess capital", with Tier 1 ratios significantly above target. See also Alfon et al. (2004).

ability to attract clients (e.g., borrowers). Second, we assume that banks perform a special role as monitors or as producers of information. With these two features, we show that costly capital is not a sufficient condition to guarantee that banks will minimize how much capital they hold, implying that capital requirements need not be binding if banks operate in a competitive market. Moreover, our model is static in nature, so we obtain this result even abstracting from other, possibly important, dynamic considerations, such as those found in Blum and Hellwig (1995), Bolton and Freixas (2005), or Peura and Keppo (2005).

The starting point of our model is that firms face an agency problem between shareholders and managers, which banks can help resolve by monitoring. Specifically, we assume that the more monitoring a bank does, the greater is the probability that a firm's investment is successful. Bank monitoring therefore has two effects in our model. First, it increases the probability that the firm's loan is repaid, thus increasing the return to the bank. Second, it benefits the firm's owners since it increases the return on their investments. Firms therefore find bank loans more desirable the greater is the underlying agency problem between shareholders and managers of the firm.<sup>2</sup>

Given limited liability for the bank, we argue that borrowers can use two different tools to provide their lending bank with an incentive to monitor. One instrument is embodied in the interest rate on the loan, since a marginal increase in the loan rate gives the bank a greater incentive to monitor in order to receive the higher payoff if the project succeeds. This increased payoff for the bank can also benefit the firm's owners if it exceeds the extra amount they pay the bank for the loan. A borrower can therefore use the interest payment on the loan to pay for bank monitoring in a way that is contingent on the success of the project. The second instrument is the amount of equity capital a bank has. The more capital a bank holds, the greater the loss the bank's owners will face if the loan is not repaid and so the greater is the incentive to monitor. Put differently, capital helps solve the limited

---

<sup>2</sup>There are numerous possible interpretations for bank "monitoring" that are consistent with our analysis. For instance, banks may perform a screening function that allows them to better determine the likelihood of loan repayment for individual borrowers. This screening should benefit borrowers by reducing cross-subsidies and increasing the efficiency of loan pricing.

liability problem banks face due to their extensive reliance on deposit-based financing.<sup>3</sup>

We consider two distinct cases regarding the structure of the credit market in our analysis. In the first case, we assume that the demand for loans by firms with good projects exceeds banks' supply of funds so that borrowers must compete for funds. In the second case, we assume instead that there is a shortage of good projects relative to the funds available so that banks must compete for firms' business and tailor their contracts so as to attract this business.

When there is a shortage of bank funds available, we show that banks optimally choose to hold no capital since equity is more costly than deposits, and limited liability protects them from having to repay depositors when their loans are not repaid. Banks also raise the interest rates on loans to the highest level that is consistent with firms being willing to borrow, and it is this which provides them with an incentive to monitor. We also show that when the cost of equity is not too much greater than the cost of deposits, a regulator interested in maximizing social welfare would impose a requirement that banks hold a positive amount of capital. This "capital requirement" leads to improved monitoring and reduces the cost to the deposit insurance fund, an aspect which is not internalized by the banks. The banks, however, would like to have as low a level of capital as possible so that any capital constraint imposed by a regulator will be binding.

The case where there is an excess supply of bank funds is more complex. In equilibrium, we find that even in the absence of a regulator, banks will hold a positive amount of capital in order to attract borrowers' business. The reason is that capital acts as a commitment device for banks to monitor, which is good for borrowers. Moreover, we also find that the loan rate most attractive to borrowers is also one that is sufficiently high to induce banks to monitor. These findings suggest that market discipline can be imposed not only from the liability side, as has been stressed in the literature on the use of subordinated debt (for a

---

<sup>3</sup>Following the rest of the literature on capital regulation, in the first part of the paper we take it as given that there is deposit insurance. We relax this assumption in the later part of the paper to show that our results are not driven by the existence of deposit insurance.

review, see Flannery and Nikolova, 2004), but also from the asset side of the bank's balance sheet (see Kim et al., 2005, for evidence that borrowers may indeed exert a disciplinary influence on banks' behavior).

In this setting, we show that a regulator will in general want to choose a different level of capital than that obtained in the market equilibrium. Specifically, when the cost of equity capital is relatively low, and is just above the cost of deposits, the regulator will want to impose a capital requirement that is above the level of capital obtained in the market. This occurs for the same reason as above, in that the cost of deposit insurance is not fully internalized by banks or borrowers. By contrast, when the cost of equity capital is high relative to the cost of deposits, the regulator may want to impose a capital requirement that is lower than that in the market. The reason is that the borrowers do not fully internalize the cost of equity capital and demand a high level of capital as a commitment for banks to monitor. In this instance, any capital requirement set by a regulator would not be binding, because competition for borrowers leads banks to hold greater amounts of capital than is socially optimal.

We extend our model to the case where there is no deposit insurance and show that the qualitative results of the base model are unaffected. Specifically, banks may have incentives to hold capital above what would be socially optimal when there is an excess supply of funds and banks have to compete for borrowers. Interestingly, in the absence of deposit insurance, banks may prefer to hold a positive level of capital even in the case where there is an excess demand for credit as a way of reducing their cost of borrowing from depositors.

The implications of our model are consistent with recent empirical observations, including the capital buildup of banks during the 90's, when the competitiveness of credit markets is thought to have increased significantly (for a discussion of this issue, see Boot and Thakor, 2000). Our model also offers the surprising prediction that, *ceteris paribus*, borrowers should be willing to pay higher interest rates to less-capitalized banks in order to provide them an alternative incentive to monitor. This is consistent with recent work by Hubbard et al.

(2002), who find that borrowing from poorly-capitalized banks is more expensive, but only for informationally-sensitive borrowers (see also Kim et al., 2005). Moreover, our model offers other cross-sectional implications concerning firms' sources of borrowing. An implication of our analysis is that borrowing from a well-capitalized bank that thus commits to monitoring, is of greater value to firms with high agency problems. Firms for which monitoring adds little value should prefer to borrow either from an arm's length source of financing or from a bank with low capital. Billett et al. (1995) finds that lender "identity," in the sense of the lender's credit rating, is an important determinant of the market's reaction to the announcement of a loan. To the extent that capitalization improves a lender's rating and reputation, these results are in line with the predictions of our model.

Recent research on the role of bank capital has studied the interaction between capital and liquidity creation (Diamond and Rajan, 2000) and the role of capital in determining banks' lending capacities and providing incentives to monitor (Holmstrom and Tirole, 1997). Our approach is complementary to these, but instead focuses on how borrower demand for monitoring services can itself lead banks to hold capital. Our paper is also related to studies of the role of capital in reducing risk-taking, recent examples of which are Hellmann et al. (2000) and Repullo (2004).

Section 2 outlines the model. Section 3 considers firms' financing choice and banks' choice of monitoring taking the loan rates and capital amounts as given. The case where there is an excess demand for credit is considered in Section 4, while the case where there is an excess supply of funds is analyzed in Section 5. Section 6 extends the analysis to the case where there is no deposit insurance, and where banks can engage in risk-shifting via their monitoring decisions. Section 7 contains concluding remarks.

## 2 Model

Consider a simple one-period economy, with  $N$  banks and  $M$  firms. There are three parties: firms (borrowers), an arm's length credit market, and banks. We describe each of these below.

**Borrowers:** Each firm can invest in a risky project with gross payoff of  $R$  when successful and 0 when not, and will choose to do so as long as the expected return from the project is greater than what it can earn in its next best alternative,  $r_B \geq 0$ . The manager of the firm can choose how much effort  $E$  to spend in running the project and increase its success probability at a cost of  $\frac{E^2}{2}$ , and how much time to spend on other activities,  $1 - E$ , for which he enjoys a private benefit  $B$ . For simplicity, we also let  $E$  represent the probability of success of the investment project, so that effort increases the expected return of the project and reduces the probability of failure. The firm's shareholders can choose between financing the project with an arm's length loan or with a bank loan.

**Credit market financing:** A competitive arm's length market provides financing at a gross interest rate of  $r_U$ . This loan is unmonitored, giving the firm's manager full discretion in choosing how much effort to exert.

**Banks:** Banks finance themselves with an amount of capital  $k$  at a cost  $r_E$  per unit, and an amount of deposits  $1 - k$  at a cost  $r_D$ , with  $r_E \geq r_D$ . Deposits are fully insured so that the deposit rate  $r_D$  does not depend on the risk of bank portfolios. (We analyze the case where there is no deposit insurance in Section 6.1.) This assumption captures the idea that bank capital is a particularly expensive form of financing, and that depositors don't have the specialized skills necessary to become bankers, therefore having a lower opportunity cost (see Berger et al., 1995, for a discussion of this issue. Hellmann et al., 2000, and Repullo, 2004, make a similar assumption).<sup>4</sup>

---

<sup>4</sup>The assumption that  $r_E \geq r_D$  is fairly standard in the literature, and is generally used to argue why capital requirements should be binding, in that banks wish to minimize the use of the more costly input. Eliminating this assumption only strengthens our results, as banks may then want to use capital as a cheaper source of financing relative to deposits.

Banks can monitor their borrowers and by doing so influence managerial effort. Specifically, each bank chooses a monitoring effort  $q$  and can constrain the manager's actions so that he exerts effort  $E = q$ , but this comes at a cost of  $cq^2$  for the bank. The convex cost function reflects the fact that it is increasingly difficult for a bank to control the actions of a borrower. Our modelling of bank monitoring captures the idea that banks can help to reduce an agency problem between the shareholders and managers of the firm, and thus increase value.<sup>5</sup> We note, however, that there are other equally plausible interpretations of the influence banks have on firms that are consistent with our model. For example, banks may instead have to choose how much effort to dedicate to screening borrowers. Firms would benefit from such screening by enjoying more accurate pricing of the risk associated with their loans and avoiding cross-subsidies to less-efficient borrowers.

**Market structure:** The loan rate  $r_L$  and the amount of capital  $k$  are determined endogenously, and can be set in one of two ways. They can either both be determined by the bank or the amount of capital can be set by a regulator who maximizes social welfare. The market is always competitive, but the solution will depend on the division of surplus between banks and borrowers. We will distinguish between two cases for the allocation of surplus: first, the case where there is a shortage of funds available to lend ( $N < M$ ), and second the case where there is a shortage of firms with good investment projects ( $N > M$ ).

**Timing:** The model can be divided into 4 stages. First, the level of bank capital  $k$  is determined, either by the bank or by a regulator. Second, banks set the loan rate  $r_L$ .<sup>6</sup> Third, borrowers choose the loan that is most attractive to them. Finally, banks choose their monitoring effort  $q$  once the terms of the loan have been set and they have raised capital and deposits.

---

<sup>5</sup>See, e.g., Besanko and Kanatas, 1993, Holmstrom and Tirole, 1997, Carletti, 2004, and Dell'Ariccia and Marquez, 2005, for studies with a similar monitoring technology.

<sup>6</sup>Note that, in the absence of regulation, this timing structure is equivalent to assuming that  $k$  and  $r_L$  are set simultaneously.

### 3 Financing choice and bank monitoring

Before proceeding to the main analysis, it is useful to characterize the borrowers' choice of financing source, as well as the banks' choice of monitoring effort taking as given the amount of capital,  $k$ , and the pricing of the loans,  $r_L$ . With an unmonitored (i.e., arm's length) loan, the manager of each firm chooses effort  $E$  to maximize

$$\max_E \pi_A = E(R - r_U) + (1 - E)B - \frac{E^2}{2}, \quad (1)$$

where  $r_U$  is the interest rate on an unmonitored loan. The solution to this problem yields

$$E^* = \max \{R - r_U - B, 0\}, \quad (2)$$

with the additional condition that  $E \leq 1$ . Note that the manager's effort is decreasing in both the private benefit  $B$  and the loan rate  $r_U$ . The manager exerts the maximal effort  $E^* = R - r_U$  for  $B = 0$ , and reduces it as  $B$  increases. Similarly,  $E \rightarrow 0$  as  $r_U \rightarrow R - B < R$ .

The firm chooses the source of financing that maximizes its value. That is, the firm chooses to obtain a bank loan as long as the return,  $q(R - r_L)$ , is greater than if the loan is unmonitored,  $E(R - r_U)$ . This can be expressed as

$$q(R - r_L) > (R - r_U - B)(R - r_U)$$

or

$$B > R - r_U - q \frac{(R - r_L)}{(R - r_U)}. \quad (3)$$

Condition 3 shows that the choice between an unmonitored loan from the arm's length market and a monitored loan from a bank depends on the level of private benefit for the manager as well as the loan rates. Firms where managers have large private benefits, so that there is a severe agency problem, will prefer to borrow from banks as a way to solve the

manager's agency problem and commit to a higher level of monitoring. Also, a bank loan will be preferred if the arm's length market is not very attractive (if  $r_U$  is high relative to  $r_L$ ).

To find the optimal level of monitoring for the banks, note that each of them chooses a monitoring effort so as to maximize expected profits. Since the bank's revenues is  $r_L - (1 - k)r_D$  if the loan is repaid and zero if the loan defaults, the expected profit can be expressed as

$$\max_q \Pi = q(r_L - (1 - k)r_D) - kr_E - cq^2. \quad (4)$$

The solution to this problem yields

$$q^* = \min \left\{ \frac{r_L - (1 - k)r_D}{2c}, 1 \right\} \quad (5)$$

as the optimal level of monitoring for each bank. Note that, when  $q < 1$ , bank monitoring effort is increasing in the return from lending ( $r_L$ ) as well as in the level of capital ( $k$ ) the bank holds, but is decreasing in the deposit rate ( $r_D$ ) and in  $c$ , a measure of the marginal cost of monitoring.

We note that this framework implies a moral hazard problem in the choice of monitoring when banks raise a positive amount of deposits. Since banks repay depositors only when their portfolios succeed, they do not internalize the full cost of default on depositors. This limited liability biases bank monitoring downwards. Capital forces banks to bear some of the burden associated with non-performing loans, and therefore provides an incentive for banks to monitor. Thus, a possible rationale for regulation is to limit moral hazard and raise the level of monitoring. This is illustrated by noting that, in the absence of limited liability, the equilibrium level of monitoring would be  $\hat{q} = \min \left\{ \frac{r_L}{2c}, 1 \right\} \geq q^*$ , with the inequality strict whenever  $q^* < 1$ . Since our focus is on bank monitoring and regulation, in what follows we restrict attention to the case where firms find it optimal to borrow from a bank.

## 4 Excess demand for credit

We begin with the case where there is a shortage of loanable funds relative to the demand for credit, which implies that banks will be able to obtain their preferred terms. This case reflects a situation where there are fewer banks than investment projects ( $N < M$ ), so that borrowers compete away the return on their projects in order to attract funding.

Banks set  $k$  and  $r_L$  so as to maximize their expected profits, taking into account their subsequent monitoring choice and the fact that borrowers accept the loans only if they have a non-negative surplus. Thus, the profit-maximizing contract solves the following problem:

$$\max_{k, r_L} \Pi = q(r_L - (1 - k)r_D) - kr_E - cq^2 \quad (6)$$

subject to

$$\begin{aligned} q &= \min \left\{ \frac{r_L - (1 - k)r_D}{2c}, 1 \right\}; \\ CS &= q(R - r_L) \geq r_B; \\ 0 &\leq k \leq 1. \end{aligned}$$

The first constraint represents the monitoring effort that banks choose in order to maximize expected profits after lending to borrowers, which was obtained above. The second constraint is the participation constraint of borrowers, labelled as consumer surplus ( $CS$ ), and states that borrowers will be willing to accept loans only if they can earn an expected return no less than  $r_B$ . The last constraint is simply a physical constraint on the level of capital, in that banks can choose between raising only deposits, a mixture of deposits and capital, or being entirely equity financed.

The solution to this maximization problem yields the following result.

**Proposition 1** *When there is an excess demand for credit, banks maximize profits by holding no capital ( $k = 0$ ) and offering a loan rate equal to the maximum possible return on the*

project, minus an adjustment to account for the borrower's reservation value ( $r_L = R - \frac{r_B}{q}$ ). Banks exert monitoring effort  $q = \min \left\{ \frac{R-r_D}{2c}, 1 \right\}$  and earn positive expected profits ( $\Pi > 0$ ).

**Proof:** See the appendix.  $\square$

The intuition behind Proposition 1 is simple. When there is an excess supply of profitable lending opportunities, banks will retain all the surplus from investment projects as borrowers compete away their own returns in order to attract funds. Since equity is more costly to banks, they choose to finance themselves entirely with deposits. Banks benefit from a high loan rate in two ways. First, a high loan rate provides them with a large return, all things equal. Second, a high loan rate also give banks greater incentives to monitor. Loan rates and capital are indeed two alternative ways to provide banks with monitoring incentives, but they differ in their impact. Raising capital entails a direct cost only for banks, whereas increasing loan rates has a negative impact only for borrowers. Banks therefore offer to lend at the highest rate that borrowers' are willing to accept.

Given banks' desire to minimize their holdings of capital, there may be scope for capital regulation in this context. Due to limited liability, banks do not internalize the full cost of default, and simply choose their level of capital and loan prices so as to maximize their expected profits. By contrast, a regulator interested in maximizing social welfare, which includes the cost borne by the deposit insurance fund, would solve the following problem:

$$\begin{aligned} \max_k SW &= \Pi + CS - (1 - q)(1 - k)r_D \\ &= qR - (1 - k)r_D - kr_E - cq^2 \end{aligned} \tag{7}$$

subject to

$$\begin{aligned}
q &= \min \left\{ \frac{r_L - (1 - k)r_D}{2c}, 1 \right\}; \\
r_L &= \arg \max_r \Pi(r); \\
CS &= q(R - r_L) \geq r_B; \\
0 &\leq k \leq 1.
\end{aligned}$$

The optimization problem is similar to before, with the important difference that the regulator chooses only the level of capital, and that it does so in order to maximize social welfare. The loan rate is still set as part of the market solution, as given in Proposition 1.

**Proposition 2** *When there is an excess demand for credit, capital regulation that maximizes social welfare requires that banks hold capital equal to  $k = 1 - \frac{2c}{r_D^2}(r_E - r_D)$ , which is positive as long as  $r_D > \max \left\{ R - 2c, \sqrt{c(c + 2r_E)} - c \right\}$ .*

**Proof:** See the appendix.  $\square$

Proposition 2 implies that welfare-maximizing capital regulation requires a positive level of capital due to its positive incentive effect on bank monitoring. This occurs when the required return for depositors  $r_D$  is sufficiently high that banks would not monitor fully when they have no capital (i.e., when  $r_D > R - 2c$ ), and also high enough that the positive incentive effect on social welfare of raising capital outweighs the cost  $r_E$  (i.e., when  $r_D > \sqrt{c(c + 2r_E)} - c$ ).

Comparing Propositions 1 and 2 leads to the following immediate result.

**Proposition 3** *When there is an excess demand for credit, capital regulation requires banks to hold a higher amount of capital than the market if  $r_D > \max \left\{ R - 2c, \sqrt{c(c + 2r_E)} - c \right\}$ .*

This result establishes that a regulator will often require a higher amount of capital than the amount that maximizes banks' profits, and never a lower amount. Regulation can thus

be beneficial as it increases social welfare relative to what would be obtained under the market solution. In these instances, there is a rationale for capital regulation as a way of providing banks with incentives to monitor.

## 5 Excess supply of funds

We now turn to the case where there is a shortage of good lending opportunities for banks relative to the funds the banking system has available to lend. In this case, banks will have to set contract terms competitively in order to attract borrowers, who will generally be able to appropriate most, if not all, of the surplus associated with their projects. In contrast to the previous section, this case reflects the situation where there are fewer investment projects than banks ( $M < N$ ).

The capital holdings and interest rate that maximize borrower surplus solve the following problem:

$$\max_{k, r_L} CS = q(R - r_L) \tag{8}$$

subject to

$$\begin{aligned} q &= \min \left\{ \frac{r_L - (1 - k)r_D}{2c}, 1 \right\}; \\ \Pi &= q(r_L - (1 - k)r_D) - kr_E - cq^2 \geq 0; \\ 0 &\leq k \leq 1; \\ 1 &\leq r_L \leq R; \end{aligned}$$

where, as before,  $\Pi$  represents bank profits,  $q$  is the monitoring effort that each bank chooses as a function of  $r_L$  and  $k$ , and  $CS$  represents consumer surplus. Note that, in contrast to the previous section, we now impose a participation constraint for banks, in that they must earn non-negative profits, and a constraint that the loan rate not be higher than the maximum return from the project. We will assume throughout that there is enough surplus generated

from lending that the borrowers' participation constraint is always satisfied when consumer surplus is being maximized. We can now state the following result, which focuses on the case of an interior solution for bank monitoring. The more general case is relegated to the appendix.

**Proposition 4** *Assume that  $R < 4c$ . When there is an excess supply of funds, consumer surplus is maximized by setting a loan rate of  $r_L = \frac{R+(1-k^{CS})r_D}{2}$  and having banks hold capital equal to  $k^{CS} = \min \left\{ \frac{8cr_E - Rr_D + r_D^2 - 4\sqrt{r_E c(4cr_E - Rr_D + r_D^2)}}{r_D^2}, 1 \right\}$ . Equilibrium monitoring is  $q = \frac{R-(1-k^{CS})r_D}{4c} < 1$ . For  $c > \frac{R^2}{16r_E}$ , banks earn zero expected profits, while for  $c < \frac{R^2}{16r_E}$ ,  $k^{CS} = 1$ , and banks earn positive expected profits.*

**Proof:** See the appendix, which contains a full characterization of the equilibrium.  $\square$

The results in Proposition 4 highlight the incentive mechanisms for bank monitoring provided by a competitive credit market. There are two ways of providing banks with incentives to monitor: by requiring that they hold a minimum amount of capital  $k^{CS}$ , and by setting the rate  $r_L$  on the loan so as to compensate them for their monitoring when the project is successful and the loan is repaid. Both of these variables increase bank monitoring, but differ in terms of their costs and their effects on consumer surplus and bank profits. Borrowers would like banks to hold large amounts of capital so as to commit to exert a high level of monitoring, as borrowers' returns increase with  $q$  but they do not fully internalize the costs of capital and of monitoring. By contrast, since capital is a costly input (i.e.,  $r_E \geq r_D$ ), banks would prefer to minimize its use and to instead receive incentives through a higher loan rate,  $r_L$ . However, while increasing  $r_L$  is good for incentive purposes, its direct effect is to reduce the surplus to the borrowers. Raising  $r_L$  will therefore eventually reduce borrower surplus, and this occurs when the positive incentive effect of a higher loan rate on bank monitoring is dominated by the negative direct effect on consumer surplus,  $R - r_L$ . Thus, when borrowers obtain the surplus, banks have to raise a positive amount of capital to attract borrowers.

The exact amounts of monitoring and capital in equilibrium depend on the return of investment projects  $R$ , the cost of capital  $r_E$ , and the cost of monitoring  $c$ . When projects are not very profitable ( $R < 4c$ ) but capital is not too costly ( $r_E < c$ ), banks raise the highest level of capital, but do not monitor fully since the cost of doing so would be too high. If capital is relatively costly, however, so that  $r_E > c$ , market incentives lead banks to choose a lower level of capital ( $k^{CS} < 1$ ), less monitoring ( $q < 1$ ), or both. The participation constraint of banks prevents them from raising the highest level of capital, thus leading to a lower level of monitoring. In the appendix, we also present the case where projects are highly profitable ( $R \geq 4c$ ), and show that, when capital is not too costly, banks exert the maximum effort,  $q = 1$ , and raise the highest level of capital,  $k^{CS} = 1$ . Borrowers want banks to monitor fully as projects are very profitable, and can induce banks to do so by raising only capital, as long as this is not too costly and banks' profits are positive. When capital is costly, however, banks will again choose a lower level of capital and/or less monitoring.

Interestingly, borrowers may be willing to give up some of the return on the loans to the banks in order to provide them with incentives to monitor. They accomplish this by allowing the loan rates to reflect the returns of the projects, and to be increasing in such returns as long as there are incentive effects from doing so (as long as  $q < 1$ ):  $\frac{\partial r_L}{\partial R} > 0$ . In other words, the loan's price need not be set only to compensate banks for the credit risk associated with granting the loan, but also to induce them to exert effort in monitoring the projects and thus improve the expected returns of the loans. Furthermore, since capital and loan rates are alternative instruments for providing banks with an incentive to monitor, we note that the equilibrium value of  $r_L$  is decreasing in the level of capital  $k$ . This implies that these are substitute instruments from the point of view of borrowers, who only trade off their relative costs from the perspective of reducing consumer surplus. The findings in Hubbard et al. (2002) and Kim et al. (2005) lend support to this result, in that they find that interest rates are higher on loans from less-capitalized banks.

The complement to Proposition 2 from the previous section is to analyze the optimal

choice of capital from a social welfare perspective when we assume that there is an excess supply of funds, and rates are set as part of a market solution to maximize the return to borrowers. In other words, a regulator would solve the following problem.

$$\begin{aligned}\max_k SW &= \Pi + CS - (1 - q)(1 - k)r_D \\ &= qR - (1 - k)r_D - kr_E - cq^2\end{aligned}\tag{9}$$

subject to

$$\begin{aligned}q &= \min \left\{ \frac{r_L - (1 - k)r_D}{2c}, 1 \right\}; \\ r_L &= \arg \max_r CS = q(R - r) \\ 0 &\leq k \leq 1.\end{aligned}$$

Again, we focus here on the case of an interior solution, and leave the other cases, which are qualitatively similar, to the appendix.

**Proposition 5** *Assume that  $R < 2c\frac{2r_E - r_D}{r_D}$ . When there is an excess supply of funds, capital regulation that maximizes social welfare requires banks to hold capital equal to  $k^{reg} = \min \left\{ \frac{Rr_D + r_D^2 - 8c(r_E - r_D)}{r_D^2}, 1 \right\}$ , which is less than 1 for  $R < 8c\frac{(r_E - r_D)}{r_D}$  and equal to 1 otherwise. Equilibrium monitoring is  $q = \frac{R - (1 - k^{reg})r_D}{4c} < 1$ .*

**Proof:** See the appendix, which contains a full characterization of the equilibrium.  $\square$

While the interest rate on the loan is determined in a competitive market setting and not subject to regulatory interference, a regulator may want to impose a capital requirement for banks in order to ensure they have sufficient incentives to monitor. In contrast to Proposition 2, now the regulator is more likely to require that banks hold a positive amount of capital, and this amount is greater than in the case where bank funds are in short supply. The reason is that the market sets a lower loan rate when borrowers obtain the surplus than when

banks obtain it and, therefore, the regulator has to use more capital to provide banks with incentives to monitor. Optimal regulation, however, does not necessarily call for “narrow banking” in the sense of having fully capitalized intermediaries, but rather allows for a mix between capital and deposit-based financing. This will generally be true when the cost of capital relative to deposits,  $r_E - r_D$ , is high, or when the aggregate return from encouraging greater monitoring,  $R$ , is relatively low.

We now turn to one of the main results in the paper, which is whether a pure market-based system is likely to provide sufficient incentives for bank discipline and monitoring, and whether capital regulation can be an effective tool for providing such incentives. For that, we have the following:

**Proposition 6** *For all  $R, r_E$ , and  $c$ , there exists a value  $\tilde{r}_D(R, r_E, c) > 0$  such that  $k^{reg} < k^{CS}$  for  $r_D < \tilde{r}_D$ , and  $k^{reg} \geq k^{CS}$  for  $r_D \geq \tilde{r}_D$ .*

**Proof:** See the appendix.  $\square$

While the exact expressions for  $\tilde{r}_D$  are provided in the proof, the interpretation of this result can be stated quite generally for all parameter values as follows. When capital is much more expensive than deposits ( $r_D \ll r_E$ ), it is socially optimal to economize on capital and instead rely more heavily on deposits for financing the bank. The market solution, by contrast, will demand that banks hold an excessive level of capital when there is a shortage of good lending opportunities and credit markets are competitive. At the other extreme, when bank capital is not significantly more costly than deposits, regulators desire that banks hold more capital than the market requires. The reason is that the regulator internalizes both the cost of raising capital as well as the cost to the deposit insurance fund. Thus, when the difference  $r_E - r_D$  is high, the regulator chooses to impose a lower capital requirement because capital is socially more costly than repaying depositors in case of bank default. By contrast, when the difference  $r_E - r_D$  is low, the regulator prefers to require a high level of capital and reduce the costs of bank default.

The result of Proposition 6 also suggests that, when capital is relatively expensive, capital regulation is not likely to be binding as market incentives will induce banks to hold greater amounts of capital than is socially optimal. In other words, a minimum capital requirement imposed by a regulator, as well as changes in this requirement, would have no effect on banks' aggregate holdings of capital.<sup>7</sup> In fact, social welfare maximization would call for a ceiling being placed on the level of capital, or a tax on its use so as to discourage banks from holding excessive capital.

The contrast between the finding in Proposition 6 and that in Proposition 3 is clear. When there is a shortage of bank funds and banks are able to appropriate most of the surplus from lending, capital regulation plays a clear role in increasing bank monitoring and reducing the probability of failure. However, when there is an excess supply of funds and loan rates are relatively low, the market may demand bank monitoring by requiring that banks hold a higher amount of capital. The effectiveness of capital regulation, therefore, clearly depends on the structure of the market for bank credit. When there is an excess demand for credit, establishing a capital adequacy requirement can be a useful way of imposing bank discipline, reducing the burden to the insurance fund and raising social welfare. By contrast, when there is an excess supply of funds, the incentives provided in the market as banks compete to attract borrowers may lead banks to hold excessive amounts of capital, so that capital adequacy requirements become ineffective and unnecessary.

## 6 Extensions

In this section we look at two important extensions. First, we consider the case where there is no deposit insurance, so that banks must internalize the cost imposed on depositors of their inability to repay deposits when their projects fail. Second, we consider a simple extension to allow for banks to shift risk in their choice of investments.

---

<sup>7</sup>This is consistent with the findings of Ashcraft (2001), who finds little evidence that tougher capital requirements were responsible for the increase in capital ratios throughout the 1980's.

## 6.1 The Case Without Deposit Insurance

Up to now we have considered only the case where deposits are fully insured, so that the interest rate paid on deposits is determined entirely by depositors' opportunity cost, given by  $r_D$ . A concern, however, is that banks' incentives to economize on the use of capital may be driven by the fixed-cost nature of deposits, which are not sensitive to risk when they are fully insured. In this section we analyze the case where deposits are not insured, so that the promised repayment must compensate depositors for the risk they face when placing their money in banks that may not repay. This introduces a liability-side disciplining force on bank behavior and capital holdings since banks will have to bear the cost of their risk-taking through a higher deposit rate.

Consider the following slight change to the model. The timing is modified as follows. First, banks choose how much to raise in deposits  $(1 - k)$  and capital  $(k)$ ; the promised repayment on deposits (i.e., the deposit rate)  $c_D$  is then also set. Second, the loan rate  $r_L$  is determined. Third, borrowers choose the loan that is most attractive to them. Fourth, banks choose their monitoring effort  $q$  once the terms of the loan have been set. Note that the only change is the introduction of the setting of the deposit rate  $c_D$  in stage 1.

Deposits are uninsured, so that the expected value of their promised payment  $c_D$  must be equal to depositors' opportunity cost  $r_D$ . Given  $k$ , depositors conjecture a level of monitoring for the bank,  $q^c$ , and set the deposit rate to meet their reservation return, which is given by  $r_D$ . This implies that  $q^c c_D = r_D$ , or that  $c_D = \frac{r_D}{q^c}$ .

We now solve the model by backward induction. For a given  $c_D$ , banks choose monitoring to maximize

$$\max_q \Pi = q(r_L - (1 - k)c_D) - kr_E - cq^2. \quad (10)$$

For an interior solution, this problem yields  $q^* = \frac{r_L - (1 - k)c_D}{2c}$ . In equilibrium, depositors' conjecture about monitoring must be correct, so that  $q^c = q^*$ . We can therefore substitute  $c_D = \frac{r_D}{q^*}$  into the solution above for  $q$  and solve for the equilibrium value of monitoring. There are

two solutions,  $q_1 = \frac{1}{4c} \left( r_L + \sqrt{r_L^2 - 8cr_D(1-k)} \right)$  and  $q_2 = \frac{1}{4c} \left( r_L - \sqrt{r_L^2 - 8cr_D(1-k)} \right)$ , with  $q_1 > q_2$ . However, it is straightforward to show that both banks and borrowers are better off with the higher level of monitoring. To see this, note that, in equilibrium, bank profits are given by

$$\Pi(q) = q \left( r_L - (1-k) \frac{r_D}{q} \right) - kr_E - cq^2 = qr_L - (1-k)r_D - kr_E - cq^2, \quad (11)$$

which is strictly increasing in  $q$  for  $q \leq \frac{r_L}{2c}$ . Since  $q_2 < q_1 < \frac{r_L}{2c}$ , banks prefer the equilibrium with the higher level of monitoring. From the firm's perspective, its equilibrium return is either equal to  $r_B$  when borrowers compete for funds or it is just  $CS(q) = q(R - r_L)$  when banks compete for borrowers. In the former case, the borrowers are indifferent to the choice of  $q$ , whereas in the latter case, substituting for the equilibrium interest rate  $r_L = \frac{R+(1-k)c_D}{2}$ , we have

$$CS(q) = q \left( R - \frac{R + (1-k) \frac{r_D}{q}}{2} \right) = \frac{1}{2} (qR - (1-k)r_D), \quad (12)$$

which again is strictly increasing in  $q$ . Since depositors are indifferent between the two levels of monitoring, the higher level of monitoring,  $q_1$ , yields a Pareto-superior equilibrium. We focus on this equilibrium in what follows.

Having solved the last stage, stages 2 and 3 follow along the lines of the previous sections. The rate on the loan,  $r_L$ , is given either by the maximum rate that is consistent with borrowers' participation constraints when there is an excess supply of projects, or by the rate that maximizes the return to borrowers,  $\frac{R+(1-k)c_D}{2}$ , when there is an excess supply of funds. Solving the first stage, where banks or the regulator choose the level of capital, we obtain the following result.

**Proposition 7** *When there is an excess supply of funds ( $N > M$ ) and no deposit insurance, there exists a value  $\hat{r}_D(R, r_E, c) > 0$  such that  $k^{reg} < k^{CS}$  for  $r_D < \hat{r}_D$ .*

**Proof:** See the appendix.  $\square$

The proposition establishes that one of our main results, that market-driven incentives can lead banks to hold more capital than is socially optimal, continues to hold even for the case where deposits are not insured. Moreover, it holds under similar conditions as before: when the opportunity cost of deposits is low relative to the cost of bank capital. The intuition is similar to that in the previous section, in that the market solution does not fully internalize all the costs and benefits associated with holding capital. Specifically, when there is an excess supply of funds banks need to adjust their offers to attract borrowers, and end up competing their return away. However, the trade-off between the benefit of increased monitoring and the cost of capital is not the same for borrowers as it is for the society. From a social welfare perspective, requiring that banks hold large amounts of capital has a large negative impact on bank profits when capital is relatively costly, and this reduction in profits may more than offset any gains to borrowers from increased monitoring. Borrowers, of course, do not fully internalize this effect, and may demand a higher level of capital than what is socially optimal.

It is also worth pointing out that, unlike the case with deposit insurance, banks may now wish to hold some capital even when funding is in short supply and borrowers must compete for funding. The following proposition formalizes this result.

**Proposition 8** *When there is an excess demand for credit ( $N < M$ ), banks will hold a positive amount of capital (i.e.,  $k > 0$ ) for  $r_D$  sufficiently large.*

**Proof:** See the appendix.  $\square$

The intuition for this result is as follows. When the deposit rate is constant, banks have no incentive to hold any capital since capital is a costly form of financing. In the absence of deposit insurance, however, capital signals a commitment to monitor on the part of the bank. Depositors recognize this greater incentive to monitor by banks and reduce the interest payment they demand on their deposits. Banks may therefore have an incentive to hold capital as a way of reducing their cost of funding, and this incentive is greatest when the

cost of deposits is high relative to the cost of equity (i.e., when  $r_D$  is close to  $r_E$ ). One way of interpreting this finding is that capital can be valuable even when the payment to depositors accurately reflects the risk they bear, so that banks internalize the cost of their risk-taking. In other words, there is scope for bank capital to improve welfare even when there is a clear channel for discipline from the liability side of the bank's balance sheet. Finally, we note that since borrowers compete away their surplus, the regulator's maximization problem is equivalent to the market's problem, so there is no additional role for regulation.

## 6.2 Banks and Asset Substitution

In the analysis above, we have assumed throughout that bank monitoring is strictly value-increasing, in that greater monitoring increases the expected return of the projects at the same time that it reduces the probability of failure. Here, we briefly analyze the case where banks may suffer from an asset substitution problem due to limited liability: since they are highly leveraged, they may have an incentive to take on risky projects, shifting some of the risk onto creditors. Specifically, we assume that the maximum return on the projects,  $R$ , is a decreasing function of  $q$ ,  $R' < 0$ , so that for low monitoring, the return on the projects is high but risky. By contrast, when a bank monitors a lot and  $q$  is high, the return on the project is lower, but the project is safer. For simplicity, we focus only on the case where there is a shortage of funds.

Given  $c_D$ , expected profits for the bank can be expressed as

$$\max_q \Pi = q\left(R - \frac{r_B}{q} - (1 - k)c_D\right) - kr_E - cq^2. \quad (13)$$

The FOC for an interior solution is

$$R - (1 - k)c_D - 2cq + q\frac{\partial R}{\partial q} = 0, \quad (14)$$

which defines the equilibrium value of monitoring,  $q^*$ . Note that, if  $\frac{\partial R}{\partial q}$  is sufficiently negative,

it could be that the no monitoring is ever optimal. We assume therefore that  $\frac{\partial R}{\partial q}$ , while negative, is of sufficiently small magnitude that at least some monitoring is always optimal.

We can now use the FOC to find  $\frac{\partial q}{\partial k}$  by use of the implicit function theorem (IFT). Define  $G \equiv R - (1 - k)c_D - 2cq + q\frac{\partial R}{\partial q}$ , which is identically equal to 0. By the IFT,  $\frac{\partial q}{\partial k} = -\frac{\frac{\partial G}{\partial k}}{\frac{\partial G}{\partial q}}$ . However, since the problem is concave in  $q$ , we have that  $\frac{\partial G}{\partial q} < 0$ , which means that the sign of  $\frac{\partial q}{\partial k}$  is the same as the sign of  $\frac{\partial G}{\partial k} = c_D > 0$ . Therefore,  $\frac{\partial q}{\partial k} > 0$ , implying that capital provides incentives to monitor. Therefore, we conclude that our results continue to hold even in the case where limited liability for banks creates a risk-shifting problem.

## 7 Concluding remarks

A standard view of capital regulation is that it offsets the risk-taking incentives provided by deposit insurance. A common approach in the study of bank regulation has been to assume that any capital requirements will be binding, since equity capital is generally believed to more costly than other forms of finance. However, in many cases such as the U.S. in the 1990's they appear not to be binding. In this paper we have developed an alternative view of capital that is consistent with the observation that capital constraints may or may not be binding. In particular, when there is an excess supply of funds relative to the number of attractive projects available so that banks compete for projects, the level of capital determined by the market can be higher than the level required by a regulator that maximizes social welfare.

Our main results continue to hold even in the absence of deposit insurance. First, we show that the market equilibrium can still involve a positive level of capital, whether there is an excess supply or a shortage of bank funds. In addition, the optimal amount of capital from a social welfare point of view can be above or below the equilibrium level in the market.

Our model has a number of implications which are in line with recent empirical observations. First, it suggests that capital requirements may not bind when, as in the last decade (see, e.g., Boot and Thakor, 2000), the competitiveness of credit markets increases. Second,

in line with the results in Kim et al. (2005), the model shows that also borrowers can impose discipline on banks by demanding they hold capital as a commitment device to exert monitoring. Finally, an interesting empirical implication of our analysis is that bank monitoring, and thus capital holdings, are of more value to firms with high agency problems. Firms for which monitoring has little value should prefer to borrow either from an arm's length source of financing, or from a bank with low capital, and thus lower costs. Billett et al. (1995) finds that lender "identity", in the sense of the lender's credit rating, is an important determinant of the market's reaction to the announcement of a loan. To the extent that capitalization improves a lender's rating and reputation, these results are consistent with the predictions of our model.

## A Proofs

**Proof of Proposition 1:** Substituting  $q$  in bank profits, we obtain

$$\Pi = \frac{(r_L - (1 - k)r_D)^2}{4c} - kr_E.$$

Differentiating profits with respect to  $k$  gives

$$\begin{aligned} \frac{d\Pi}{dk} &= \frac{(r_L - (1 - k)r_D)r_D}{2c} - r_E \\ &= qr_D - r_E \leq 0, \text{ as } q \leq 1 \text{ and } r_E \geq r_D. \end{aligned}$$

This implies that  $k = 0$ . Furthermore, for  $r_L \geq r_D$ ,

$$\frac{d\Pi}{dr_L} = \frac{(r_L - (1 - k)r_D)}{2c} \geq 0 \quad \forall r_L \leq R,$$

implying that bank profits are always increasing in the interest rate  $r_L$ . However, since the bank must satisfy the borrower's participation constraint, the maximum interest rate that can be charged satisfies  $q^c(R - r_L) \geq r_B$ , where  $q^c$  is the level of monitoring that borrowers conjecture will take place. Since in equilibrium borrowers anticipate correctly how much monitoring takes place, we have that  $r_L = R - \frac{r_B}{q}$ .  $\square$

**Proof of Proposition 2:** Substituting  $r_L = R - \frac{r_B}{q}$  and  $k = 0$  in the expression for  $q$  gives

$$q = \min \left\{ \frac{R - r_D}{2c}, 1 \right\}.$$

Thus,  $q = 1$  if  $R - r_D \geq 2c$ , and  $q < 1$  if  $R - r_D < 2c$ .

Substituting  $r_L = R - \frac{r_B}{q}$  and keeping  $k > 0$ , social welfare becomes

$$SW = \frac{(R - (1 - k)r_D)^2}{4c} - kr_E - \left[ 1 - \frac{(R - (1 - k)r_D)}{2c} \right] (1 - k)r_D.$$

Differentiating SW with respect to  $k$ , we have

$$\begin{aligned}\frac{dSW}{dk} &= \frac{(R - (1 - k)r_D)r_D}{2c} - r_E - \left[ -\frac{(1 - k)r_D^2}{2c} - r_D + \frac{(R - (1 - k)r_D)r_D}{2c} \right] = 0 \\ &= \frac{(1 - k)r_D^2}{2c} + r_D - r_E = 0.\end{aligned}$$

Calculating this expression at the two extreme levels of capital gives

$$\left. \frac{dSW}{dk} \right|_{k=1} = r_D - r_E \leq 0,$$

and

$$\left. \frac{dSW}{dk} \right|_{k=0} = \frac{r_D^2}{2c} + r_D - r_E \geq 0,$$

implying that the welfare-maximizing level of capital is  $k^* \in (0, 1)$  if  $r_D > \sqrt{c(c + 2r_E)} - c$ , and is given by

$$k^* = 1 - \frac{2c}{r_D^2}(r_E - r_D) < 1,$$

thus establishing the proposition.  $\square$

**Proposition 4B** *When there is an excess supply of funds, maximizing borrower surplus yields the following equilibrium:*

1) For  $R \geq 4c$ , monitoring is  $q = 1$ . The loan rate is  $r_L = (1 - k^{CS})r_D + 2c$ , and banks are required to hold capital  $k^{CS}$  equal to  $k^{CS} = \min\left\{\frac{c}{r_E}, 1\right\}$ . For  $k^{CS} = 1$  (i.e., if  $c > r_E$ ), banks earn profits  $\Pi = c - r_E > 0$ , otherwise  $\Pi = 0$ .

2) For  $R < 4c$ , monitoring is  $q = \frac{R - (1 - k^{CS})r_D}{4c} < 1$ . The loan rate is  $r_L = \frac{R + (1 - k^{CS})r_D}{2}$ , and banks hold capital equal to  $k^{CS} = \min\left\{\frac{8cr_E - Rr_D + r_D^2 - 4\sqrt{r_Ec(4cr_E - Rr_D + r_D^2)}}{r_D^2}, 1\right\}$ , which is less than 1 for  $c > \frac{R^2}{16r_E}$  and equal to one otherwise. For  $k^{CS} = 1$ ,  $\Pi = \frac{R^2}{16c} - r_E > 0$ , and  $\Pi = 0$  for  $k^{CS} < 1$ .

**Proof:** Start by noting that, since  $q = \min\left\{\frac{r_L - (1 - k)r_D}{2c}, 1\right\}$ , if  $r_L < (1 - k)r_D + 2c$  then  $q = \frac{r_L - (1 - k)r_D}{2c} < 1$ . Since  $CS = q(R - r_L)$ , we have that  $\frac{\partial CS}{\partial k} = \frac{\partial q}{\partial k}(R - r_L) = \frac{r_D}{2c}(R - r_L) > 0$

for  $q < 1$ . Therefore, more capital increases consumer surplus.

We proceed in two stages, starting by maximizing  $CS$  with respect to the loan's price,  $r_L$ , for a fixed  $k$ , which yields

$$\frac{\partial CS}{\partial r_L} = \frac{\partial q}{\partial r_L}(R - r_L) - q = \frac{R - 2r_L + (1 - k)r_D}{2c} = 0.$$

Solving the FOC yields  $r_L = \frac{R + (1 - k)r_D}{2}$ .

We can now maximize  $CS$  with respect to the choice of capital,  $k$ . However, we know from above that the combination of  $r_L = \frac{R + (1 - k)r_D}{2}$  and the highest possible  $k$  will be optimal for borrowers. We therefore introduce the participation constraint for the bank, that  $\Pi = q(r_L - (1 - k)r_D) - kr_E - cq^2 \geq 0$ . Substituting for  $q = \frac{r_L - (1 - k)r_D}{2c}$  as well as for  $r_L$ , we obtain

$$\Pi = \frac{(R - (1 - k)r_D)^2}{16c} - kr_E \geq 0, \quad k \leq 1.$$

We can solve this for the value of  $k$  that satisfies the constraint with equality ( $\Pi = 0$ ). Since  $\Pi$  is strictly convex in  $k$ ,  $0 \leq k \leq 1$ , and consumer surplus is increasing in  $k$ , the relevant solution must be either the smaller root or a corner solution at  $k = 1$ . The solution is then

$$k^{CS} = \min \left\{ \frac{8cr_E - Rr_D + r_D^2 - 4\sqrt{r_EC(4cr_E - Rr_D + r_D^2)}}{r_D^2}, 1 \right\}.$$

Note that if  $k^{CS} = 1$ , then  $r_L = \frac{R}{2}$ .

We now check when in fact  $q < 1$ . From the definition of the optimal level of monitoring  $q = \min \left\{ \frac{r_L - (1 - k)r_D}{2c}, 1 \right\}$ , we see that, for  $r_L \geq (1 - k)r_D + 2c$ ,  $q = 1$ . Substituting in the optimal value for  $r_L$  gives the following condition:

$$\frac{R + (1 - k)r_D}{2} \geq (1 - k)r_D + 2c.$$

The right hand side is maximized at  $k = 0$ . Thus, a sufficient condition for  $q = 1$  is that  $R - r_D - 4c \geq 0$ . In this case, there is no benefit in terms of greater monitoring to having

a higher interest rate on the loan, and so borrowers should just require the lowest possible interest rate consistent with  $q = 1$ , which is satisfied by  $r_L = (1 - k)r_D + 2c$ . If we again substitute this value of  $r_L$  into the expression for bank profits we obtain

$$\Pi = (r_L - (1 - k^{CS})r_D) - k^{CS}r_E - c = c - k^{CS}r_E,$$

which, after setting equal to zero, yields  $k^{CS} = \frac{c}{r_E}$  as long as  $c < r_E$ . In this case, we have  $r_L = (1 - \frac{c}{r_E})r_D + 2c = (\frac{r_E - c}{r_E})r_D + 2c$ .

Otherwise, for  $c > r_E$ ,  $k^{CS} = 1$ , which implies that  $r_L = 2c$ . Moreover, substituting this value of  $r_L$  into  $q = \min\left\{\frac{r_L - (1 - k^{CS})r_D}{2c}, 1\right\}$  and observing that  $k^{CS} = 1$ , we obtain that  $\Pi > 0$  and  $q = \min\left\{\frac{R}{4c}, 1\right\} = 1$  for  $R \geq 4c$ , and is less than 1 otherwise. All together, this implies that  $q \geq 1$  for  $R \geq 4c$ , and  $q < 1$  for  $R < 4c$ .  $\square$

**Proposition 5B** *When there is an excess supply of funds, capital regulation that maximizes social welfare requires:*

1) For  $R > 2c\frac{2r_E - r_D}{r_D}$ , monitoring is  $q = \min\left\{\frac{R}{4c}, 1\right\}$ , and capital equals  $k^{reg} = \min\left\{\frac{4c + r_D - R}{r_D}, 1\right\}$ , which is less than 1 for  $R > 4c$  and equal to 1 otherwise.

2) For  $R < 2c\frac{2r_E - r_D}{r_D}$ , monitoring is  $q = \frac{R - (1 - k^{reg})r_D}{4c} < 1$  and capital equals  $k^{reg} = \min\left\{\frac{Rr_D + r_D^2 - 8c(r_E - r_D)}{r_D^2}, 1\right\}$ , which is less than 1 for  $R < 8c\frac{(r_E - r_D)}{r_D}$  and equal to 1 otherwise.

**Proof:** Start by maximizing social welfare with respect to  $k$ , assuming that the loan rate is set to maximize CS, i.e., that  $r_L = \frac{R + (1 - k)r_D}{2}$ . Social welfare is given by

$$\max_k SW = \Pi + CS - (1 - q)(1 - k)r_D = qR - (1 - k)r_D - kr_E - cq^2.$$

We can now take the first order condition to get

$$\frac{\partial SW}{\partial k} = \frac{\partial q}{\partial k} (R - 2cq) - r_E + r_D.$$

We know that, for  $q < 1$ , the optimal level of monitoring is  $q = \frac{r_L - (1-k)r_D}{2c}$ . Substituting in the value of  $r_L$  above we get  $q = \frac{R - (1-k)r_D}{4c}$ . We therefore have that

$$\frac{\partial SW}{\partial k} = \frac{r_D}{4c} \left( \frac{R + (1-k)r_D}{2} \right) + r_D - r_E = 0.$$

The solution we obtain is

$$k^{reg} = \min \left\{ \frac{Rr_D + r_D^2 - 8c(r_E - r_D)}{r_D^2}, 1 \right\}.$$

From this expression, we obtain that  $k^{reg} < 1$  for  $c > \frac{Rr_D}{8(r_E - r_D)} \Leftrightarrow R < 8c \frac{(r_E - r_D)}{r_D}$ . Otherwise, for  $R > 8c \frac{(r_E - r_D)}{r_D}$ , we have that  $k^{reg} = 1$ .

The previous solution assumed that  $q < 1$ . To get the bounds on when  $q = 1$ , substitute the solution for  $k^{reg}$ , assuming  $k^{reg} < 1$ , into

$$q = \frac{R - (1 - k^{reg})r_D}{4c} = \frac{Rr_D - 4c(r_E - r_D)}{2cr_D}.$$

From here, we see that for  $c > \frac{Rr_D}{4r_E - 2r_D} \Leftrightarrow R < 2c \frac{2r_E - r_D}{r_D}$ ,  $q < 1$ . Otherwise, for  $R > 2c \frac{2r_E - r_D}{r_D}$ ,  $q = 1$  and  $k$  should be set such that  $q(k) = 1 \Leftrightarrow k^{reg} = \frac{4c + r_D - R}{r_D}$ . Note, however, that for  $R < 4c$  this solution would imply that  $k^{reg} > 1$ , which is not feasible. Therefore, for  $R < 4c$ , we obtain that  $k^{reg} = 1$ , which implies that  $q = \frac{R}{4c} < 1$ .

One final point that needs to be verified is that, for  $R < 2c \frac{2r_E - r_D}{r_D}$ , then  $q = \frac{R - (1-k)r_D}{4c} < 1$ , but that for  $R > 8c \frac{(r_E - r_D)}{r_D}$ , we have that  $k^{reg} = 1$ , which would imply that  $q = \frac{R}{4c}$ . Note, however, that for both of these conditions to be true at the same time requires that  $8c \frac{(r_E - r_D)}{r_D} < 2c \frac{2r_E - r_D}{r_D}$ . This will be satisfied if and only if  $4(r_E - r_D) < 2r_E - r_D \Leftrightarrow r_E < \frac{3}{2}r_D$ . We can now use this in the necessary condition for  $q < 1$ , which is  $R < 2c \frac{2r_E - r_D}{r_D}$ . Given the restriction on  $r_E$  and  $r_D$ , the right hand side must be less than  $2c \frac{2(\frac{3}{2}r_D) - r_D}{r_D} = 4c$ . Therefore, the joint assumption that  $R < 2c \frac{2r_E - r_D}{r_D}$  and  $R > 8c \frac{(r_E - r_D)}{r_D}$  implies that  $R < 4c$ , and consequently that  $q = \frac{R}{4c} < 1$ , as desired.  $\square$

**Proof of Proposition 6:** We begin with the case of parameter values such that  $q, k < 1$ , and show that there exists a value  $\tilde{r}_D > 0$  such that  $k^{reg} < k^{CS}$  if and only if  $r_D < \tilde{r}_D$ . Consider the solution that maximizes consumer surplus,  $k^{CS}$ , and assume that  $R < 4c$  and  $c > \frac{R^2}{16r_E}$ , which implies that  $q, k^{CS} < 1$ . From the condition defining  $k^{CS}$ ,

$$\Pi = \frac{(R - (1 - k^{CS})r_D)^2}{16c} - k^{CS}r_E = 0,$$

one can clearly see that, as  $r_D \rightarrow 0$ ,  $k^{CS} \rightarrow \frac{R^2}{16cr_E} < 1$  for  $c > \frac{R^2}{16r_E}$ .

By contrast,  $k^{reg}$  is defined by

$$\frac{\partial SW}{\partial k} = \frac{r_D}{4c} \left( \frac{R + (1 - k^{reg})r_D}{2} \right) + r_D - r_E = 0.$$

For  $r_D \rightarrow 0$ ,  $k^{reg} \rightarrow 0$  as well, since it is optimal to just have deposit-based finance. These two results together imply that there is some threshold  $\underline{r}_D$  such that, for  $r_D < \underline{r}_D$ ,  $k^{reg} < k^{CS}$ .

At the other extreme, we consider the solutions as  $r_D \rightarrow r_E$ . For  $c > \frac{R^2}{16r_E}$ ,  $k^{CS} = \frac{8c - R + r_E - 4\sqrt{(4c - R + r_E)c}}{r_E} < 1$ . By comparison,  $k^{reg} \rightarrow 1$  as  $r_D \rightarrow r_E$  for all parameter values. Therefore, we can also conclude that there must exist some threshold  $\bar{r}_D$  such that, for  $r_D > \bar{r}_D$ ,  $k^{reg} > k^{CS}$ .

Comparing the two values of  $k$ ,  $k^{reg} < k^{CS}$  if and only if

$$k^{reg} = \frac{Rr_D + r_D^2 - 8c(r_E - r_D)}{r_D^2} < \frac{8cr_E - Rr_D + r_D^2 - 4\sqrt{r_Ec(4cr_E - Rr_D + r_D^2)}}{r_D^2} = k^{CS}$$

Rearranging, we obtain the condition for  $k^{reg} - k^{CS} < 0$  as:

$$\frac{2}{r_D^2} \left( Rr_D + 4cr_D - 8cr_E + 2\sqrt{cr_E(4cr_E - Rr_D + r_D^2)} \right) < 0$$

Since we know that for low values of  $r_D$  this condition will be satisfied, but not for higher values, we can establish that there is a unique threshold where the inequality flips (i.e., that

$\underline{r}_D = \bar{r}_D$ ) if the difference  $k^{reg} - k^{CS}$  is either concave or convex in  $r_D$ . For this, we only need the second derivative of the term inside the parenthesis, which yields

$$\begin{aligned} & \frac{\partial^2}{\partial r_D^2} \left( Rr_D + 4cr_D - 8cr_E + 2\sqrt{cr_E(4cr_E - Rr_D + r_D^2)} \right) \\ &= \frac{\frac{1}{2}(16cr_E - R^2)(\sqrt{cr_E})}{(4cr_E - Rr_D + r_D^2)(\sqrt{4cr_E - Rr_D + r_D^2})} > 0, \end{aligned}$$

since by assumption  $c > \frac{R^2}{16r_E}$ .

The finding that the function  $k^{reg} - k^{CS}$  is convex implies that  $k^{reg} - k^{CS}$  can at most cross zero twice, the first time from above and the second from below. However, two crossings are inconsistent with the finding in the proposition above that for low values of  $r_D$ ,  $k^{reg} - k^{CS} < 0$ , while for high values of  $r_D$ ,  $k^{reg} - k^{CS} > 0$ . Therefore,  $k^{reg} - k^{CS} = 0$  at one unique point, which implies that  $\underline{r}_D = \bar{r}_D = \tilde{r}_D$ , and we have just one threshold, as desired.

We next proceed to the case where  $q = 1$  in both cases, which is true for sufficiently large  $R$ , but that  $k < 1$ . Start with the case of consumer surplus maximization, where, for  $R > 4c$ ,  $q = 1$  and  $k^{CS} = \frac{c}{r_E}$ . For the case with regulation, we have that for  $R > \max\{2c\frac{2r_E - r_D}{r_D}, 4c\}$ ,  $q = 1$  and  $k^{reg} = \frac{4c + r_D - R}{r_D}$ . Therefore,  $k^{reg} < k^{CS} \Leftrightarrow$

$$\frac{4c + r_D - R}{r_D} < \frac{c}{r_E}$$

This last inequality can be solved for  $r_D$  to yield the condition

$$r_D < r_E \left( \frac{R - 4c}{r_E - c} \right),$$

which establishes that  $k^{reg} - k^{CS} < 0$  if and only if  $r_D < \tilde{r}_D = r_E \left( \frac{R - 4c}{r_E - c} \right)$ , as desired.

The last case is a possible ‘‘mixed’’ case, in which monitoring may be at a maximum for one solution but not the other. It is straightforward to show that the only case of relevance is where, with a slight abuse of notation,  $q^{CS} = 1$  but  $q^{reg} < 1$ . This occurs for

$\frac{Rr_D}{2(2r_E - r_D)} < c < \frac{R}{4}$ , and in this range  $k^{CS} = \frac{c}{r_E}$  and  $k^{reg} = \frac{Rr_D + r_D^2 - 8c(r_E - r_D)}{r_D^2}$ . The difference  $k^{reg} - k^{CS}$  simplifies to

$$(r_E - c)r_D^2 + (R + 8c)r_E r_D - 8cr_E^2 = 0$$

The relevant solution is

$$\tilde{r}_D = \frac{r_E(R+8c) - \sqrt{32c^2 + R^2 + 16c(R+2r_E)}}{2(c - r_E)},$$

which implies  $k^{reg} > k^{CS}$  only if  $r_D > \tilde{r}_D$ .  $\square$

**Proof of Proposition 7:** Assume that there is an excess supply of funds so that, as before, consumer surplus is maximized by setting

$$r_L = \frac{R + (1 - k)c_D}{2}.$$

We can substitute this into the equation for  $q$ , recalling that  $c_D = \frac{r_D}{q}$ , and solve for  $q$  to obtain  $q_1 = \frac{1}{8c} \left( R + \sqrt{R^2 - 16cr_D(1 - k)} \right)$  and  $q_2 = \frac{1}{8c} \left( R - \sqrt{R^2 - 16cr_D(1 - k)} \right)$ . We focus again on the Pareto dominant equilibrium with a higher level of monitoring ( $q_1$ ).

We proceed by maximizing  $CS$  with respect to the choice of capital,  $k$ . However, we know from above that the combination of  $r_L = \frac{R+(1-k)c_D}{2}$  and the highest possible  $k$  will be optimal for borrowers. We therefore introduce the participation constraint for the bank, that  $\Pi = q(r_L - (1 - k)c_D) - kr_E - cq^2 \geq 0$ . Substituting for  $r_L$  gives  $q = \frac{1}{8c} \left( R + \sqrt{R^2 - 16cr_D(1 - k)} \right)$ , and we obtain

$$\Pi = cq^2 - kr_E = c \left( \frac{1}{8c} \left( R + \sqrt{R^2 - 16cr_D(1 - k)} \right) \right)^2 - kr_E = 0$$

Focusing on parameter values for which there is an interior solution for  $k$ , the solution is

$$k^{CS} = \frac{1}{2c(4r_E - r_D)^2} \left( R^2 r_E + 2cr_D^2 - 8cr_D r_E - R\sqrt{r_E(R^2 r_E + 4cr_D^2 - 16r_E c r_D)} \right).$$

To maximize social welfare in the absence of deposit insurance, we need to solve

$$\max_k SW = \Pi + CS = qR - (1 - k)r_D - kr_E - cq^2.$$

The FOC yields

$$\frac{\partial SW}{\partial k} = \frac{\partial q}{\partial k} (R - 2cq) + r_D - r_E = 0.$$

Using  $\frac{\partial q}{\partial k} = \frac{r_D}{\sqrt{16ckr_D - 16cr_D + R^2}}$ , we can substitute into the FOC and solve for  $k$ , which yields

$$k^{reg} = \frac{1}{2} \frac{18cr_D^3 - 48cr_D^2r_E + 32cr_Dr_E^2 + 3R^2r_Dr_E - 2R^2r_E^2}{cr_D(4r_E - 3r_D)^2}.$$

Comparing these two solutions, we note that as  $r_D \rightarrow 0$ , we have that, as before,  $k^{CS} \rightarrow \frac{R^2}{16cr_E}$ . By contrast, for the case of social welfare maximization it is easy to see from the FOC that as  $r_D \rightarrow 0$ ,  $k^{reg} \rightarrow 0$  as well. Therefore, for  $r_D$  sufficiently small, we have that  $k^{reg} < k^{CS}$ , as desired.  $\square$

**Proof of Proposition 8:** Note that bank profit maximization with respect to  $k$  yields the following FOC:

$$\begin{aligned} \frac{\partial \Pi}{\partial k} &= \frac{\partial q}{\partial k} (r_L - (1 - k)c_D) + qc_D - q(1 - k) \frac{\partial c_D}{\partial k} - r_E - 2cq \frac{\partial q}{\partial k} \\ &= qc_D - r_E - q(1 - k) \frac{\partial c_D}{\partial k}. \end{aligned}$$

Since  $qc_D = r_D$ ,  $\frac{\partial c_D}{\partial q} = -\frac{r_D}{q^2}$ , and, given  $q = \frac{1}{4c} \left( R + \sqrt{R^2 - 8cr_D(1 - k)} \right)$  when  $r_L = R - \frac{r_E}{q}$ , that  $\frac{\partial q}{\partial k} = \frac{r_D}{\sqrt{R^2 - 8cr_D(1 - k)}} > 0$ , the FOC becomes :

$$r_D - r_E + (1 - k) \frac{r_D}{q} \frac{r_D}{\sqrt{R^2 - 8cr_D(1 - k)}} = 0.$$

Evaluating at  $k = 0$ ,  $\frac{\partial \Pi}{\partial k}$  is clearly positive for  $r_D \rightarrow r_E$ . Substituting in for  $q$ , we can find

the limiting value of  $\frac{\partial \Pi}{\partial k}$  explicitly as  $r_D \rightarrow r_E$ , which becomes

$$\lim_{r_D \rightarrow r_E} \left( r_D - r_E + \frac{r_D}{q} \frac{r_D}{\sqrt{R^2 - 8cr_D}} \right) = \frac{1}{4} r_E \frac{3R - \sqrt{R^2 - 8cr_E}}{\sqrt{R^2 - 8cr_E}} > \frac{1}{4} r_E \frac{2R}{\sqrt{R^2 - 8cr_E}} > 0$$

Therefore,  $k > 0$  even for the case where there is an excess demand for credit.  $\square$

## References

- [1] Alfon I., I. Argimon and P. Bascunana-Ambros, 2004, “What determines how much capital is held by UK banks and building societies?”, FSA Occasional Paper Series 22, London.
- [2] Ashcraft A.B., 2001, “Do Tougher Bank Capital Requirements Matter? New Evidence from the Eighties”, mimeo, Federal Reserve Bank of New York.
- [3] Barth, J., G. Caprio, and R. Levine, 2005, *Rethinking Bank Regulation: Till Angels Govern*, Cambridge University Press, forthcoming.
- [4] Berger, A.N., R.J. Herring and G.P. Szego, 1995, “The Role of Capital in Financial Institutions”, *Journal of Banking and Finance*, 19, 393-430.
- [5] Besanko, D. and G. Kanatas, 1993, “Credit Market Equilibrium with Bank Monitoring and Moral Hazard”, *Review of Financial Studies*, 6, 213-232.
- [6] Billett, M., M. Flannery and J. Garfinkel, 1995, The Effect of Lender Identity on a Borrowing Firm’s Equity Return, *Journal of Finance*, 699-718.
- [7] Blum, J. and M. Hellwig, 1995, “The Macroeconomic Implications of Capital Adequacy Requirements for Banks,” *European Economic Review*, 39, 739-749.
- [8] Bolton, P. and X. Freixas, 2005, “Corporate Finance and the Monetary Transmission Mechanism,” forthcoming, *Review of Financial Studies*.
- [9] Boot A.W.A. and A.V. Thakor, 2000, “Can Relationship Banking Survive Competition?,” *Journal of Finance*, 55, 679-713.
- [10] Carletti, E., 2004, “The Structure of Relationship Lending, Endogenous Monitoring and Loan Rates”, *Journal of Financial Intermediation*, 13, 58-86.
- [11] Citigroup Global Markets Industry Report, 2005, “Equity Research: Europe Banks”.

- [12] Dell’Ariccia, G. and R. Marquez, 2005, “Competition Among Regulators and Credit Market Integration”, forthcoming, *Journal of Financial Economics*.
- [13] Diamond, D. and R. Rajan, 2000, “A Theory of Bank Capital”, *Journal of Finance*, 55, 2431-2465.
- [14] Flannery M. and S. Nikolova, “Market Discipline of U.S. Financial Firms: Recent Evidence and Research Issues,” in *Market Discipline across Countries and Industries*, edited by C. Borio, W. Hunter, G. Kaufman, and K. Tsatsaronis, MIT Press, Cambridge, MA.
- [15] Flannery, M.J. and K.P. Rangan, 2004, “Market Forces at Work in the Banking Industry: Evidence from the Capital Buildup from the 1990s”, mimeo, University of Florida.
- [16] Hellmann, T., Murdock, K., and J. Stiglitz, 2000, “Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?”, *American Economic Review*, 90, 147-165.
- [17] Holmstrom, B. and J. Tirole, 1997, “Financial Intermediation, Loanable Funds, and the Real Sector”, *Quarterly Journal of Economics*, 112, 663-691.
- [18] Kim, M., E. Kristiansen, and B. Vale, 2005, “Endogenous product differentiation in credit markets: What do borrowers pay for?” *Journal of Banking and Finance*, 29, 681-699.
- [19] Peura, S. and J. Keppo, (2005), “Optimal Bank Capital with Costly Recapitalization,” forthcoming, *Journal of Business*.
- [20] Repullo, R., 2004, “Capital Requirements, Market Power, and Risk-Taking in Banking”, *Journal of Financial Intermediation*, 13, 156-182.

## CFS Working Paper Series:

No.	Author(s)	Title
2005/14	<b>Berc Rustem Volker Wieland Stan Zakovic</b>	Stochastic Optimization and Worst-Case Analysis in Monetary Policy Design
2005/15	<b>Dirk Krueger Fabrizio Perri</b>	Does Income Inequality Lead to Consumption Inequality? Evidence and Theory
2005/16	<b>Klaus Adam Roberto Billi</b>	Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates
2005/17	<b>Roberto Billi</b>	The Optimal Inflation Buffer with a Zero Bound on Nominal Interest Rates
2005/18	<b>Christopher D. Carroll</b>	The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems
2005/19	<b>Yunus Aksoy Athanasios Orphanides David Small Volker Wieland David Wilcox</b>	A Quantitative Exploration of the Opportunistic Approach to Disinflation
2005/20	<b>Yannis Biliass Dimitris Georgarakos Michael Haliassos</b>	Equity Culture and the Distribution of Wealth
2005/21	<b>Silke Brandts Christian Laux</b>	ART Versus Reinsurance: The Disciplining Effect of Information Insensitivity
2005/22	<b>Sean D. Campbell Francis X. Diebold</b>	Stock Returns and Expected Business Conditions: Half a Century of Direct Evidence
2005/23	<b>Franklin Allen Elena Carletti Robert Marquez</b>	Credit Market Competition and Capital Regulation

Copies of working papers can be downloaded at <http://www.ifk-cfs.de>