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Imperfect Knowledge, Inflation Expectations, and Monetary Policy

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Abstract:

This paper investigates the role that imperfect knowledge about the structure of the economy plays in the formation of expectations, macroeconomic dynamics, and the efficient formulation of monetary policy. Economic agents rely on an adaptive learning technology to form expectations and to update continuously their beliefs regarding the dynamic structure of the economy based on incoming data. The process of perpetual learning introduces an additional layer of dynamic interaction between monetary policy and economic outcomes. We find that policies that would be efficient under rational expectations can perform poorly when knowledge is imperfect. In particular, policies that fail to maintain tight control over inflation are prone to episodes in which the public's expectations of inflation become uncoupled from the policy objective and stagflation results, in a pattern similar to that experienced in the United States during the 1970s. Our results highlight the value of effective communication of a central bank's inflation objective and of continued vigilance against inflation in anchoring inflation expectations and fostering macroeconomic stability.

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1 Introduction

Rational expectations provides an elegant and powerful framework that has come to dominate thinking about the dynamic structure of the economy and econometric policy evaluation over the past 30 years. This success has spurred further examination into the strong information assumptions implicit in many of its applications. Thomas Sargent (1993) concludes that "rational expectations models impute much *more* knowledge to the agents within the model ... than is possessed by an econometrician, who faces estimation and inference problems that the agents in the model have somehow solved" (p. 3, emphasis in original).[1] Researchers have proposed refinements to rational expectations that respect the principle that agents use information efficiently in forming expectations, but nonetheless recognize the limits to and costs of information-processing and cognitive constraints that influence the expectations-formation process (Sargent 1999, Evans and Honkapohja 2001, Sims 2003).

In this study, we allow for a form of imperfect knowledge in which economic agents rely on an adaptive learning technology to form expectations. This form of learning represents a relatively modest deviation from rational expectations that nests it as a limiting case. We show that the resulting process of perpetual learning introduces an additional layer of interaction between monetary policy and economic outcomes that has important implications for macroeconomic dynamics and for monetary policy design. As we illustrate, monetary policies that would be efficient under rational expectations can perform poorly when knowledge is imperfect. In particular, with imperfect knowledge, policies that fail to maintain tight control over inflation are prone to episodes in which the public's expectations of inflation become uncoupled from the policy objective. The presence of this imperfection makes stabilization policy more difficult than would appear under rational expectations and

[1] Missing from such models, as Benjamin Friedman (1979) points out, "is a clear outline of the way in which economic agents derive the knowledge which they then use to formulate expectations." To be sure, this does not constitute a criticism of the traditional use of the concept of "rationality" as reflecting the optimal use of information in the formation of expectations, taking into account an agent's objectives and resource constraints. The difficulty is that in Muth's (1961) original formulation, rational expectations are not optimizing in that sense. Thus, the issue is not that the "rational expectations" concept reflects too much rationality but rather that it imposes too little rationality in the expectations formation process. For example, as Sims (2003) has pointed out, optimal information processing subject to a finite cognitive capacity may result in fundamentally different processes for the formation of expectations from those implied by rational expectations. To acknowledge this terminological tension, Simon (1978) suggested that a less misleading term for Muth's concept would be "model consistent" expectations (p. 2).

highlights the value of effectively communicating a central bank's inflation objective and of continued vigilance against inflation in anchoring inflation expectations and fostering macroeconomic stability.

In this paper, we investigate the macroeconomic implications of a process of "perpetual learning." Our work builds on the extensive literature relating rational expectations with learning and the adaptive formation of expectations (Bray 1982, Bray and Savin 1984, Marcet and Sargent 1989, Woodford, 1990, Bullard and Mitra 2002). A key finding in this literature is that under certain conditions an economy with learning converges to the rational expectations equilibrium (Townsend 1978, Bray 1982, 1983, Blume and Easley 1982). However, until agents have accumulated sufficient knowledge about the economy, economic outcomes during the transition depend on the adaptive learning process (Lucas 1986). Moreover, in a changing economic environment, agents are constantly learning and their beliefs converge not to a fixed rational expectations equilibrium, but to an ergodic distribution around it (Sargent 1999, Evans and Honkapohja 2001).[2]

As a laboratory for our experiment, we employ a simple linear model of the U.S. economy with characteristics similar to more elaborate models frequently used to study optimal monetary policy. We assume that economic agents know the correct structure of the economy and form expectations accordingly. But, rather than endowing them with complete knowledge of the parameters of these functions—as would be required by imposing the rational expectations assumption—we posit that economic agents rely on finite memory least squares estimation to update these parameter estimates. This setting conveniently nests rational expectations as the limiting case corresponding to infinite memory least squares estimation and allows varying degrees of imperfection in expectations formation to be characterized by variation in a single model parameter.

We find that even marginal deviations from rational expectations in the direction of

[2]Our work also draws on some other strands of the literature related to learning, estimation, and policy design. One such strand has examined the formation of inflation expectations when the policymaker's objective may be unknown or uncertain, for example during a transition following a shift in policy regime (Taylor 1975, Bomfim et al, 1997, Erceg and Levin, 2003, Kozicki and Tinsley, 2001, Tetlow and von zur Muehlen 2001). Another strand has considered how policymaker uncertainty about the structure of the economy influences policy choices and economic dynamics (Balvers and Cosimano 1994, Wieland 1998, Sargent 1999, and others). Finally, our work relates to explorations of alternative approaches for modeling aggregate inflation expectations, such as Ball (2000), Carroll (2003), and Mankiw and Reis (2002).

imperfect knowledge can have economically important effects on the stochastic behavior of our economy and policy evaluation. An interesting feature of the model is that the interaction of learning and control creates rich nonlinear dynamics that can potentially explain both the shifting parameter structure of linear reduced form characterizations of the economy and the appearance of shifting policy objectives or inflation targets. For example, sequences of policy errors or inflationary shocks, such as experienced during the 1970s, could give rise to stagflationary episodes that do not arise under rational expectations with perfect knowledge.

Indeed, the critical role of the formation of inflation expectations for understanding the successes and failures of monetary policy is a dimension of policy that has often been cited by policymakers over the past two decades but that has received much less attention in formal econometric policy evaluations. An important example is the contrast between the stubborn persistence of inflation expectations during the 1970s when policy placed relatively greater attention on countercyclical concerns and the much improved stability in both inflation and inflation expectations following the renewed emphasis on price stability in 1979. In explaining the rationale for this shift in emphasis in 1979, Federal Reserve Chairman Volcker highlighted the importance of learning in shaping the inflation expectations formation process:[3]

> It is not necessary to recite all the details of the long series of events that have culminated in the serious inflationary environment that we are now experiencing. An entire generation of young adults has grown up since the mid-1960's knowing only inflation, indeed an inflation that has seemed to accelerate inexorably. In the circumstances, it is hardly surprising that many citizens have begun to wonder whether it is realistic to anticipate a return to general price stability, and have begun to change their behavior accordingly. Inflation feeds in part on itself, so part of the job of returning to a more stable and more productive economy must be to break the grip of inflationary expectations.
> (Volcker 1979, p. 888)

This historical episode is a clear example of inflation expectations becoming uncoupled from

[3]Indeed, we would argue that the shift in emphasis towards greater focus on inflation was itself influenced by the recognition of the importance of facilitating the formation of stable inflation expectations—which had been insufficiently appreciated earlier during the 1970s. See Orphanides (2003a) for a more detailed description of the policy discussion at the time and the nature of the improvement in monetary policy since 1979. See also Christiano and Gust (2000) and Sargent (1999) for alternative explanations of the rise in inflation during the 1960s and 1970s.

the intended policy objective and illustrates the point that the design of monetary policy must account for the influence of policy on expectations.

We find that policies designed to be efficient under rational expectations can perform very poorly when knowledge is imperfect. This deterioration in performance is particularly severe when policymakers put a high weight on stabilizing real economic activity relative to price stability. Our analysis yields two conclusions for the conduct of monetary policy when knowledge is imperfect. First, policies that emphasize tight inflation control can facilitate learning and provide better guidance for the formation of inflation expectations. Second, effective communication of an explicit numerical inflation target can help focus inflation expectations and thereby reduce the costs associated with imperfect knowledge. Policies that combine vigilance against inflation with an explicit numerical inflation target mitigate the negative influence of imperfect knowledge on economic stabilization and yield superior macroeconomic performance. Thus, our findings provide analytical support for monetary policy frameworks that emphasize the primacy of price stability as an operational policy objective, for example, the inflation targeting approach discussed by Bernanke and Mishkin (1997) and adopted by several central banks over the past decade or so.

2 The Model Economy

We consider a stylized model that gives rise to a nontrivial inflation-output variability tradeoff and in which a simple one-parameter policy rule represents optimal monetary policy under rational expectations.[4] In this section, we describe the model specification for inflation and output and the central bank's optimization problem; in the next two sections, we take up the formation of expectations by private agents.

Inflation is determined by a modified Lucas supply function that allows for some intrinsic inflation persistence,

$$\pi_{t+1} = \phi\pi^e_{t+1} + (1-\phi)\pi_t + \alpha y_{t+1} + e_{t+1}, \quad e \sim \text{iid}(0, \sigma^2_e), \tag{1}$$

where π denotes the inflation rate, π^e is the private agents' expected inflation rate based on

[4] Since its introduction by Taylor (1979), the practice of analyzing monetary policy rules using such an inflation-output variability tradeoff has been adopted in a large number of academic and policy studies.

time t information, y is the output gap, $\phi \in (0,1)$, $\alpha > 0$, and e is a serially uncorrelated innovation. As discussed by Clark, Goodhart, and Huang (1999) and Lengwiler and Orphanides (2002), this specification incorporates an important role for inflation expectations for determining inflation outcomes while also allowing for some inflation persistence that is necessary for the model to yield a nontrivial inflation-output gap variability tradeoff.[5]

We assume that the policymaker can set policy during period t so as to determine the intended level of the output gap for period $t+1$, x_t, subject to a control error, u_{t+1},

$$y_{t+1} = x_t + u_{t+1} \quad u \sim \text{iid}(0, \sigma_u^2). \tag{2}$$

This is equivalent to assuming that the intended output gap for period $t+1$ is determined by the real rate gap set during period t, $x_t = -\xi(r_t - r^*)$, where r is the short-term real interest rate, and r^* is the equilibrium real rate.[6] As will become clear, with this assumption the model has the property that under perfect knowledge both the optimal policy rule and the optimal inflation forecast rule can be written in terms of a single state variable, the lagged inflation rate. This facilitates our analysis. Inflation expectations are fundamentally anchored by monetary policy, while output expectations are anchored by views of aggregate supply that are presumably less influenced by monetary policy. For this reason, we focus on the interaction between monetary policy and inflation expectations.

The central bank's objective is to design a policy rule that minimizes the loss, denoted by $\mathcal{L}$, equal to the weighted average of the asymptotic variances of the output gap and of deviations of inflation from the target rate,

$$\mathcal{L} = (1-\omega)Var(y) + \omega Var(\pi - \pi^*), \tag{3}$$

where $Var(z)$ denotes the unconditional variance of variable z, and $\omega \in (0,1]$ is the relative weight on inflation stabilization. This completes the description of the structure of the

[5]We have also examined the "New-Keynesian" variant of the Phillips curve studied by Gali and Gertler (1999) and others, which also allows for some intrinsic inflation inertia. As we report in section 6, our main findings are not sensitive to this alternative.

[6]Note, however, that this abstracts from the important complications associated with the real-time measurement of the output gap and and the equilibrium real interest rate for formulating the policy rule. See Orphanides (2003b), Laubach and Williams (2003), and Orphanides and Williams (2002) for analyses of these issues.

model economy, with the exception of the expectations formation process that we examine in detail below.

3 The Perfect Knowledge Benchmark

We begin by considering the "textbook" case of rational expectations with perfect knowledge in which private agents know both the structure of the economy and the central bank's policy. In this case, expectations are rational in that they are consistent with the true data generating process of the economy (the model). In the next section, we use the resulting equilibrium solution as a "perfect knowledge" benchmark against which we compare outcomes under imperfect knowledge, in which case agents do not know the structural parameters of the model but instead must form expectations based on estimated forecasting models.

Under the assumption of perfect knowledge, both the evolution of the economy and optimal monetary policy can be expressed in terms of two variables, the current inflation rate and its target level. These variables determine the formation of expectations and the policy choice, which, together with serially uncorrelated shocks, determine output and inflation in period $t+1$. Specifically, we can write the monetary policy rule in terms of the inflation gap,

$$x_t = -\theta(\pi_t - \pi^*), \tag{4}$$

where $\theta > 0$ measures the responsiveness of the intended output gap to the inflation gap.

Given this monetary policy rule, inflation expectations are:

$$\pi^e_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + \frac{1-\phi-\alpha\theta}{1-\phi}\pi_t. \tag{5}$$

Inflation expectations depend on the current level of inflation, the inflation target, and the parameter θ measuring the central bank's responsiveness to the inflation gap. Substituting this expression for expected inflation into equation (1) yields the rational expectations solution for inflation for a given monetary policy,

$$\pi_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + (1 - \frac{\alpha\theta}{1-\phi})\pi_t + e_{t+1} + \alpha u_{t+1}. \tag{6}$$

One noteworthy feature of this solution is that the first-order autocorrelation of the inflation rate, given by $1-\frac{\alpha\theta}{1-\phi}$, is decreasing in θ and is invariant to the value of π^*. Note that the rational expectations solution can also be written in terms of the "inflation expectations gap"—the difference between inflation expectations for period $t+1$ from the inflation target, $\pi^e_{t+1}-\pi^*_t$,

$$\pi^e_{t+1}-\pi^*_t=\frac{1-\phi-\alpha\theta}{1-\phi}(\pi_t-\pi^*). \tag{7}$$

Equations (4) and (5) close the perfect knowledge benchmark model.

3.1 Optimal Monetary Policy under Perfect Knowledge

For the economy with perfect knowledge, the optimal monetary policy, θ^P, can be obtained in closed form and is given by:[7]

$$\theta^P=\frac{\omega}{2\,(1-\omega)}\left(-\frac{\alpha}{1-\phi}+\sqrt{\left(\frac{\alpha}{1-\phi}\right)^2+\frac{4\,(1-\omega)}{\omega}}\right)\quad \text{for}\quad 0<\omega<1. \tag{8}$$

In the limit, when ω equals unity (that is, when the policymaker is not at all concerned with output stability), the policymaker sets the real interest rate so that inflation is expected to return to its target in the next period. The optimal policy in the case $\omega=1$ is given by: $\theta^P=\frac{1-\phi}{\alpha}$, and the irreducible variance of inflation, owing to unpredictable output and inflation innovations, equals $\sigma_e^2+\alpha^2\sigma_u^2$. More generally, the optimal value of θ depends positively on the ratio $\frac{1-\phi}{\alpha}$, and the parameters α and ϕ enter only in terms of this ratio. In particular, the optimal policy response is larger the greater the degree of intrinsic inertia in inflation, measured by $1-\phi$.

The greater the central bank's weight on inflation stabilization, the greater is the responsiveness to the inflation gap and the smaller the first-order autocorrelation in inflation. Differentiating equation (8) shows that the policy responsiveness to the inflation gap is increasing in ω, the weight the central bank places on inflation stabilization. As a result, the

[7] The optimal policy can be described in terms of the Euler equation that relates the intended output gap to the inflation rate and one the intended output gap expected in the next period:

$$x_t=E_{t-1}\left\{x_{t+1}-\frac{\omega}{1-\omega}\frac{\alpha}{1-\phi}\pi_{t+1}\right\}.$$

Under the assumption of serially uncorrelated shocks, the solution simplifies to the expression given in the text.

autocorrelation of inflation is decreasing in ω, with a limiting value approaching unity when ω approaches zero, and zero when ω equals one. That is, if the central bank cares only about output stabilization, the inflation rate becomes a random walk, while if the central bank cares only about inflation stabilization, the inflation rate displays no serial correlation. And, as noted, this model yields a nontrivial monotonic tradeoff between the variability of inflation and the output gap for all values of $\omega \in (0, 1]$. These results are illustrated in Figure 1. The top panel of the figure shows the variability tradeoff described by optimal policies for values of ω between zero and one. The lower panel plots the optimal values of θ against ω.

4 Imperfect Knowledge

As the perfect knowledge solution shows, private inflation forecasts depend on knowledge of the structural model parameters and of policymaker preferences. In addition, these parameters influence the expectations formation function nonlinearly. We now relax the assumption that private agents have perfect knowledge of all structural parameters and policymaker preferences. Instead, we posit that agents must somehow infer the information necessary for forming expectations by observing historical data, in essence acting like econometricians who know the correct specification of the economy but are uncertain about the parameters of the model.

In particular, we assume that private agents update the coefficients of their model for forecasting inflation using least squares learning with finite memory. We focus on least squares learning because of its desirable convergence properties, straightforward implementation, and close correspondence to what real-world forecasters actually do.[8] Estimation with finite memory reflects agents' concern for changes in the structural parameters of the

[8]This method of adaptive learning is closely related to optimal filtering, where the structural parameters are assumed to follow random walks. Of course, if private agents know the complete structure of the model—including the laws of motion for inflation, output, and the unobserved states and the distributions of the innovations to these processes—then they could compute efficient inflation forecasts that could outperform those based on recursive least squares. However, uncertainty regarding the precise structure of the time variation in the model parameters is likely to reduce the real efficiency gains from a method optimized to a particular model specification relative to a simple method such as least-squares learning. Further, once we begin to ponder how economic agents could realistically model and account for such uncertainty precisely, we quickly recognize the significance of respecting (or the absurdity of ignoring) the cognitive and computational limits of economic agents.

economy. To focus our attention on the role of imperfections in the expectations formation process itself, however, we deliberately abstract from the introduction of the actual uncertainty in the structure of the economy which would justify such concerns in equilibrium. Further, we do not model the policymaker's knowledge or learning, but instead focus on the implications of policy based on simple time-invariant rules of the form given in equation (4) that do not require explicit treatment of the policymaker's learning problem.[9]

We model perpetual learning by assuming that agents use a constant gain in their recursive least squares formula that places greater weight on more recent observations, as in Sargent (1999) and Evans and Honkapohja (2001). This algorithm is equivalent to applying weighted least squares where the weights decline geometrically with the distance in time between the observation being weighted and the most recent observation. This approach is closely related to the use of fixed sample lengths or rolling-window regressions to estimate a forecasting model (Friedman 1979). In terms of the mean "age" of the data used, a rolling-regression window of length l is equivalent to a constant gain κ of $2/l$. The advantage of the constant gain least squares algorithm over rolling regressions is that the evolution of the former system is fully described by a small set of variables, while the latter requires one to keep track of a large number of variables.

4.1 Least Squares Learning with Finite Memory

Under perfect knowledge, the predictable component of next period's inflation rate is a linear function of the inflation target and the current inflation rate, where the coefficients on the two variables are functions of the policy parameter θ and the other structural parameters of the model, as shown in equation (5). In addition, the optimal value of θ is itself a nonlinear function of the central bank's weight on inflation stabilization and the other model structural parameters. Given this simple structure, the least squares regression of inflation on a constant and lagged inflation,

$$\pi_i = c_{0,t} + c_{1,t}\pi_{i-1} + v_i, \tag{9}$$

[9] We also abstract from two other elements that may further complicate policy design: The possibilities that policymakers may rely on a misspecified model or a misspecified information set for computing agent's expectations; see, Levin, Wieland, and Williams (2003) and Orphanides (2003b), respectively, for a discussion of these two issues.

yields consistent estimates of the coefficients describing the law of motion for inflation (Marcet and Sargent 1988 and Evans and Honkapohja 2001). Agents then use these results to form their inflation expectations.[10]

To fix notation, let X_i and c_i be the 2×1 vectors $X_i = (1, \pi_{i-1})'$ and $c_i = (c_{0,i}, c_{1,i})'$. Using data through period t, the least squares regression parameters for equation (9) can be written in recursive form:

$$c_t = c_{t-1} + \kappa_t R_t^{-1} X_t (\pi_t - X_t' c_{t-1}), \tag{10}$$

$$R_t = R_{t-1} + \kappa_t (X_t X_t' - R_{t-1}), \tag{11}$$

where κ_t is the gain. With least squares learning with infinite memory, $\kappa_t = 1/t$, so as t increases, κ_t converges to zero. As a result, as the data accumulate this mechanism converges to the correct expectations functions and the economy converges to the perfect knowledge benchmark solution. As noted above, to formalize perpetual learning—as would be required in the presence of structural change—we replace the decreasing gain in the infinite memory recursion with a small constant gain, $\kappa > 0$.[11]

With imperfect knowledge, expectations are based on the perceived law of motion of the inflation process, governed by the perpetual learning algorithm described above. The model under imperfect knowledge consists of the structural equation for inflation (1), the output gap equation (2), the monetary policy rule (4), and the one-step-ahead forecast for inflation, given by

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t, \tag{12}$$

where $c_{0,t}$ and $c_{1,t}$ are updated according to equations (10) and (11).

We emphasize that in the limit of perfect knowledge (that is, as $\kappa \to 0$), the expectations function above converges to rational expectations and the stochastic coefficients for the

[10] Note that here we assume that agents employ a reduced form of the expectations formation function that is correctly specified under rational expectations with perfect knowledge. However, agents may be uncertain of the correct form and estimate a more general specification, for example, a linear regression with additional lags of inflation which nests (9). In section 6, we also discuss results from such an example.

[11] In terms of forecasting performance, the "optimal" choice of κ depends on the relative variances of the transitory and permanent shocks, as in the relationship between the Kalman gain and the signal-to-noise ratio in the case of the Kalman filter. Here, we do not explicitly attempt to calibrate κ in this way, but instead examine the effects for a range of values of κ.

intercept and slope collapse to:

$$c_0^P = \frac{\alpha\theta\pi^*}{1-\phi},$$

$$c_1^P = \frac{1-\phi-\alpha\theta}{1-\phi}.$$

Thus, this modeling approach accommodates the Lucas critique in the sense that expectations formation is endogenous and adjusts to changes in policy or structure (as reflected here by changes in the parameters θ, π^*, α, and ϕ). In essence, our model is one of "noisy rational expectations." As we show below, although expectations are imperfectly rational, in that agents need to estimate the reduced form equations they employ to form expectations, they are nearly rational, in that the forecasts are close to being efficient.

5 Perpetual Learning in Action

We use model simulations to illustrate how learning affects the dynamics of inflation expectations, inflation, and output in the model economy. First, we examine the behavior of the estimated coefficients of the inflation forecast equation and evaluate the performance of inflation forecasts. We then consider the dynamic response of the economy to shocks similar to those experienced during the 1970s in the United States. Specifically, we compare the outcomes under perfect knowledge and imperfect knowledge with least squares learning that correspond to three alternative monetary policy rules to illustrate the additional layer of dynamic interaction introduced by the imperfections in the formation of inflation expectations.

In calibrating the model for the simulations, each period corresponds to about half a year. We consider values of κ of 0.025, 0.05, and 0.075, which roughly correspond to using 40, 20, or 13 years of data, respectively, in the context of rolling regressions. We consider two values for ϕ, the parameter that measures the influence of inflation expectations on inflation. As a baseline case, we set ϕ to 0.75, which implies a significant role for intrinsic inflation inertia, consistent with the contracting models of Buiter and Jewitt (1981), and Fuhrer and Moore (1995), and Brayton, et al. (1997).[12] In the alternative specification, we

[12]Other researchers suggest an even smaller role for expectations relative to intrinsic inertia; see Fuhrer (1997), Roberts (2001), and Rudd and Whelan (2001).

allow for a greater role for expectations and correspondingly give less weight to inflation inertia by setting $\phi = 0.9$, consistent with the findings of Gali and Gertler (1999) and others. To ease comparisons between the two values of ϕ, we set α so that the optimal policy under perfect knowledge is identical in the two cases. Specifically, for $\phi = 0.75$, we set $\alpha = 0.25$, and for $\phi = 0.9$, we set $\alpha = 0.1$. In all cases, we assume $\sigma_e = \sigma_u = 1$.

The three alternative policies we consider correspond to the values of θ, $\{0.1, 0.6, 1.0\}$, which represent the optimal policies under perfect knowledge for policymakers whose preferences reflect a relative weight on inflation, ω, of 0.01, 0.5, and 1, respectively. Hence, $\theta = 0.1$ corresponds to an "inflation dove" policymaker who is primarily concerned about output stabilization, $\theta = 0.6$ corresponds to a policymaker with "balanced preferences" who weighs inflation and output stabilization equally, and $\theta = 1$ corresponds to an "inflation hawk" policymaker who cares exclusively about inflation.

5.1 The Performance of Least-Squares Inflation Forecasts

Even absent shocks to the structure of the economy, the process of least squares learning generates time variation in the formation of inflation expectations and thereby in the processes of inflation and output. The magnitude of this time variation is increasing in κ—which is equivalent to using shorter samples (and thus less information from the historical data) in rolling regressions. Table 1 reports summary statistics of the estimates of agents' inflation forecasting model based on stochastic simulations of the model economy for the two calibrations we consider. As seen in the table, the unconditional standard deviations of the estimates increase with κ. This dependence of the variation in the estimates on the rate of learning is portrayed in Figure 2, which shows the steady-state distributions of the estimates of c_0 and c_1 for the case of $\phi 0.75$. For comparison, the vertical lines in each panel indicate the values of c_0 and c_1 in the corresponding perfect knowledge benchmark.

The median values of the coefficient estimates are nearly identical to the values implied by the perfect knowledge benchmark; however, the mean estimates of c_1 are biased downward slightly. Although not shown in the table, the mean and median values of c_0 are nearly zero, consistent with the assumed inflation target of zero. There is contemporaneous correlation between estimates of c_0 and c_1 is nearly zero. Each of these estimates, however,

Table 1: Least Squares Learning

	RE	$\phi = 0.75, \alpha = 0.25$			$\phi = 0.90, \alpha = 0.10$		
κ:	0	0.025	0.050	0.075	0.025	0.050	0.075
$\theta = 0.1$							
Mean c_1	0.90	0.86	0.83	0.81	0.88	0.89	0.93
Median c_1	0.90	0.89	0.88	0.88	0.95	0.97	0.98
SD c_0	–	0.37	0.67	1.01	0.79	2.06	4.92
SD c_1	–	0.12	0.17	0.21	0.18	0.23	0.20
$\theta = 0.6$							
Mean c_1	0.40	0.37	0.34	0.32	0.37	0.35	0.33
Median c_1	0.40	0.38	0.37	0.36	0.40	0.41	0.42
SD c_0	–	0.25	0.38	0.50	0.40	0.66	0.91
SD c_1	–	0.20	0.28	0.33	0.31	0.42	0.50
$\theta = 1.0$							
Mean c_1	0.00	-0.02	-0.04	-0.05	-0.03	-0.03	-0.04
Median c_1	0.00	-0.02	-0.04	-0.05	-0.03	-0.04	-0.06
SD c_0	–	0.24	0.35	0.44	0.37	0.58	0.74
SD c_1	–	0.21	0.29	0.35	0.33	0.44	0.51

is highly serially correlated, with first-order autocorrelations just below unity. This serial correlation falls only slightly as κ increases.

Note that a more aggressive policy response to inflation reduces the variation in the estimated intercept, c_0, but increases the magnitude of fluctuations in the coefficient on the lagged inflation rate, c_1. In the case of $\theta = 1$, the distribution of estimates of c_1 is nearly symmetrical around zero. For $\theta = 0.1$ and 0.6, the distribution of estimates of c_1 is skewed to the left, reflecting the accumulation of mass around unity, but the absence of much mass above 1.1.

Finite-memory least squares forecasts perform very well in this model economy. As shown in Table 2, the mean-squared error of agents' one-step-ahead inflation forecasts is only slightly above the theoretical minimum given in the first line of the table (labeled "Perfect knowledge").[13] Only when both inflation displays very little intrinsic inertia and the policymaker places very little weight on inflation stabilization does the performance of finite-

[13]This is consistent with earlier findings regarding least squares estimation. Anderson and Taylor (1976), for example, emphasize that least squares forecasts can be accurate even when consistent estimates of individual parameter estimates are much harder to obtain.

Table 2: Forecasting Performance: Mean-squared Error

		$\phi = 0.75, \alpha = 0.25$			$\phi = 0.90, \alpha = 0.10$		
Forecast method	κ :	0.025	0.050	0.075	0.025	0.050	0.075
Perfect knowledge		1.03	1.03	1.03	1.01	1.01	1.01
$\theta = 0.1$							
LS (finite memory)		1.04	1.05	1.06	1.03	1.19	1.57
LS (infinite memory)		1.05	1.06	1.09	1.08	1.72	3.49
Long-lag Phillips curve		1.05	1.06	1.07	1.06	1.08	1.11
$\theta = 0.6$							
LS (finite memory)		1.04	1.04	1.05	1.01	1.01	1.02
LS (infinite memory)		1.06	1.09	1.12	1.10	1.20	1.31
Long-lag Phillips curve		1.05	1.07	1.08	1.07	1.11	1.17
$\theta = 1.0$							
LS (finite memory)		1.04	1.04	1.05	1.01	1.01	1.02
LS (infinite memory)		1.06	1.10	1.14	1.12	1.28	1.51
Long-lag Phillips curve		1.05	1.07	1.08	1.07	1.12	1.18

memory least squares forecasts break down. Not surprisingly, given that we assume that the structure of the economy is fixed, agents' forecasting performance deteriorates somewhat as κ increases. Nonetheless, finite-memory least squares estimates perform better than those with infinite memory (based on the full sample), and the difference in performance is more pronounced the greater the role of inflation expectations in determining inflation. In an economy where inflation is determined by the forecasts of other agents who use finite-memory least squares, it is better to follow suit rather than to use estimates that would have better forecast properties under perfect knowledge (Evans and Ramey 2001).

With imperfect knowledge, the private agents ability to forecast inflation depends on the monetary policy in place, with forecast errors on average smaller when policy responds more aggressively to inflation. This effect is more pronounced the greater the role of inflation expectations in determining inflation. The marginal benefit from tighter inflation control on the ability of private agents to forecast accurately is greatest when the policymaker places relatively little weight on inflation stabilization. In this case, inflation is highly serially correlated, and the estimates of c_1 are frequently in the vicinity of unity. Evidently, the ability to forecast inflation deteriorates when inflation is nearly a random walk. As seen by comparing the cases of θ of 0.6 and 1.0, the marginal benefit of tight inflation control disappears once the first-order autocorrelation of inflation is well below one.

Finally, even though only one lag of inflation appears in the equations for inflation and inflation expectations, it is possible to improve on infinite-memory least squares forecasts by including additional lags of inflation in the estimated forecasting equation. This result is similar to that found in empirical studies of inflation, where relatively long lags of inflation help predict inflation (Staiger, Stock, and Watson 1997, Stock and Watson 1999, Brayton, Roberts, and Williams 1999). Evidently, in an economy where agents use adaptive learning, multi-period lags of inflation are a reasonable proxy for inflation expectations. This result may also help explain the finding that survey-based inflation expectations do not appear to be "rational" using standard tests (Roberts 1997, 1998). With adaptive learning, inflation forecast errors are correlated with data in the agents' information set; the standard test for forecast efficiency applies only to stable economic environments in which agents' estimates of the forecast model have converged to the true values.

5.2 Least Squares Learning and Inflation Persistence

The time variation in inflation expectations resulting from perpetual learning induces greater serial correlation in inflation. As shown in Table 3, the first-order unconditional autocorrelation of inflation increases with κ. The first column shows the autocorrelations for inflation under perfect knowledge ($\kappa = 0$); note that these figures are identical across the two specifications of ϕ and α. In the case of the "inflation dove" policymaker ($\theta = 0.1$), the existence of learning raises the first-order autocorrelation from 0.9 to very nearly unity. For the policymaker with moderate preferences ($\theta = 0.6$), increasing κ from 0 to 0.075 causes the autocorrelation of inflation to rise from 0.40 to 0.60 when $\phi = 0.75$, or to 0.88 when $\phi = 0.9$.

Thus, in a model with a relatively small amount of intrinsic inflation persistence, the autocorrelation of inflation can be very high, even with a monetary policy that places significant weight on inflation stabilization. Even for the "inflation hawk" policymaker whose policy under perfect knowledge results in no serial persistence in inflation, the perpetual learning generates a significant amount of positive serial correlation in inflation. As we

Table 3: Inflation Persistence: First-order Autocorrelation

			$\phi = 0.75, \alpha = 0.25$			$\phi = 0.90, \alpha = 0.10$		
θ	κ:	0	0.025	0.050	0.075	0.025	0.050	0.075
0.1		0.90	0.97	0.98	0.99	1.00	1.00	1.00
0.6		0.40	0.47	0.54	0.60	0.61	0.78	0.88
1.0		0.00	0.02	0.06	0.08	0.07	0.18	0.25

discuss below, the rise in inflation persistence associated with perpetual learning in turn affects the optimal design of monetary policy.

5.3 The Economy Following Inflationary Shocks

Next, we consider the dynamic response of the model to a sequence of unanticipated shocks, similar in spirit to those that arose in the 1970s. The responses of inflation expectations and inflation do not depend on the "source" of the shocks, that is, on whether we assume the shocks are due to policy errors or to other disturbances.[14] The configuration of shocks we have in mind would not be expected to occur frequently, of course. It is, however, instructive in that it illustrates how in these infrequent episodes the evolution of inflation expectations with learning could dramatically deviate from the perfect knowledge benchmark under some policies. Inflation expectations in these episodes can become uncoupled from the policymakers' objectives, resulting in a period of stagflation that cannot occur under the perfect knowledge benchmark.

Note that under least squares learning, the model responses depend nonlinearly on the initial values of the states c and R. In the following, we report the average response from 1000 simulations, each of which starts from initial conditions drawn from the relevant steady-state distribution. The shock is 2 percentage points in period one and it declines in magnitude from periods two through eight. In period nine and beyond there is no shock. For these experiments we assume the baseline values for ϕ and α, and set $\kappa = 0.05$.

With perfect knowledge, the series of inflationary shocks causes a temporary rise in

[14]The policy error we have in mind is the systematic misperception of the economy's non-inflationary potential supply following an unobserved shift in potential output growth or an increase in the natural rate of unemployment, as apparently experienced in the 1970s. (See, for example, Orphanides and Williams, 2002, and Orphanides, 2003c.) Because such changes can only be perceived with the passage of time, they yield errors that are recognized to be serially correlated only in retrospect. In our model, the effect of such errors on inflation dynamics is isomorphic to that of an exogenous serially correlated inflation shock.

inflation and a decline in the output gap, as shown by the dashed lines in Figure 3. The speed at which inflation is brought back to target depends on the monetary policy response, with the more aggressive policy yielding a relatively sharp but short decline in output and a rapid return of inflation to target. With the inflation hawk or moderate policymaker, the peak increase in inflation is no more than 2-1/2 percentage points and inflation returns to its target within 10 periods. With the inflation dove policymaker, the modest policy response avoids the sharp decline in output, but inflation is allowed to rise to a level about 4-1/2 percentage points above target, and the return to target is more gradual, with inflation still remaining one percentage point above target after 20 periods.

Imperfect knowledge with learning amplifies and prolongs the response of inflation and output to the shocks, especially when the central bank places significant weight on output stabilization. The solid lines in the figure show the responses of inflation and output under imperfect knowledge for the three policy rules. The inflation hawk's aggressive response to inflation effectively keeps inflation from drifting away from target and the responses of inflation and output differ only modestly from those under perfect knowledge. In the case of balanced preferences, the magnitude of the peak responses of inflation and the output gap is a bit larger than under perfect knowledge, but the persistence of these gaps is markedly higher. The outcomes under the inflation dove, however, are dramatically different. The inflation dove attempts to finesse a gradual reduction in inflation without incurring a large decline in output, but the timid response to rising inflation causes the perceived process for inflation to become uncoupled from the policymaker's objectives. Stagflation results, with the inflation rate stuck over 8 percentage points above target, while output remains well below potential.

The striking differences in the responses to the shocks under imperfect knowledge are a product of the interaction between learning, the policy rule, and inflation expectations. The solid lines in Figure 4 show the responses of the public's estimates of the intercept and the slope parameter of the inflation forecasting equation under imperfect knowledge. Under the inflation hawk policymaker, inflation expectations are well anchored to the policy objective. The serially correlated inflationary shocks cause some increase in both estimates, but the implied increase in the inflation target peaks at only 0.3 percentage point (not shown in the figure). Even for the moderate policymaker who accommodates some of the inflationary shock for a time, the perceived inflation target rises by just one-half percentage point. In contrast, under the inflation dove policymaker, the estimated persistence of inflation, already very high owing to the policymaker's desire to minimize output fluctuations while responding to inflation shocks, rises steadily, approaching unity. With inflation temporarily perceived to be a near-random walk with positive drift, agents expect inflation to continue to rise. The policymaker's attempts to constrain inflation are too weak to counteract this adverse expectations process, and the public's perception of the inflation target rises by 5 percentage points. Despite the best of intents, the gradual disinflation prescription that would be optimal with perfect knowledge yields stagflation—the simultaneous occurrence of persistently high inflation and low output.

Interestingly, the inflation dove simulation appears to capture some key characteristics of the United States economy at the end of the 1970s, and it accords well with Chairman Volcker's assessment of the economic situation at the time:

> Moreover, inflationary expectations are now deeply embedded in public attitudes, as reflected in the practices and policies of individuals and economic institutions. After years of false starts in the effort against inflation, there is widespread skepticism about the prospects for success. Overcoming this legacy of doubt is a critical challenge that must be met in shaping–and in carrying out–all our policies.
>
> Changing both expectations and actual price performance will be difficult. But it is essential if our economic future is to be secure.
> (Volcker 1981, p. 293)

In contrast to this dismal experience, the model simulations suggest that the rise in inflation—and the corresponding costs of disinflation—would have been much smaller if policy had responded more aggressively to the inflationary developments of the 1970s. Although this was apparently not recognized at the time, Chairman Volcker's analysis suggests that the stagflationary experience of the 1970s played a role in the subsequent recognition of the value of continued vigilance against inflation in anchoring inflation expectations.

6 Imperfect Knowledge and Monetary Policy

6.1 Naive Application of the Rational Expectations Policy

We now turn to the design of efficient monetary policy under imperfect knowledge. We start by considering the experiment in which the policymaker sets policy under the assumption that private agents have perfect knowledge when, in fact, they have only imperfect knowledge and base their expectations on the perpetual learning mechanism described above. That is, policy follows (4) with the response parameter, θ, computed using (8).

Figure 5 compares the variability pseudo-frontier corresponding to this equilibrium to the frontier from the perfect knowledge benchmark. The top panel shows the outcomes in terms of inflation and output gap variability with the baseline parameterization, $\phi = 0.75$. The bottom panel shows the results of the same experiment with the more forward-looking specification for inflation, $\phi = 0.9$. In each case, we show the imperfect knowledge equilibria corresponding to three different values of κ.

With imperfect knowledge, the perpetual learning mechanism introduces random errors in expectations formation, that is, deviations of expectations from the values that would correspond to the same realization of inflation and the same policy rule. These errors are costly for stabilization and are responsible for the deterioration in performance shown in Figure 5.

This deterioration in performance is especially pronounced for the policymaker who places relatively low weight on inflation stabilization. As seen in the simulations of the inflationary shocks reported above, for such policies the time variation in the estimated autocorrelation of inflation in the vicinity of unity associated with learning can be especially costly. Furthermore, the deterioration in performance relative to the case of the perfect knowledge benchmark is larger the greater is the role of expectations in determining inflation. With the higher value for ϕ, if a policymaker's preference for inflation stabilization is too low, the resulting outcomes under imperfect knowledge are strictly dominated by the outcomes corresponding to the naive policy equilibrium for higher values of ω.

6.2 Efficient Simple Rule

Next we examine imperfect knowledge equilibria when the policymaker is aware of the imperfection in expectations formation and adjusts policy accordingly. To allow for a straightforward comparison with the perfect knowledge benchmark, we concentrate on the efficient choice of the responsiveness of policy to inflation, θ^S, in the simple linear rule:

$$x_t = -\theta^S(\pi_t - \pi^*),$$

which has the same form as the optimal rule under the perfect knowledge benchmark.[15]

The efficient policy response with imperfect knowledge is to be more vigilant against inflation deviations from the policymaker's target relative to the optimal response under perfect knowledge. Figure 6 shows the efficient choices for θ under imperfect knowledge for the two model parameterizations; the optimal policy under perfect knowledge—which is the same for the two parameterizations considered—is shown again for comparison. As before, we present results for three different values of κ, our baseline $\kappa = 0.05$ and also a smaller and a larger value. The increase in the efficient value of θ is especially pronounced when the policymaker places relatively little weight on inflation stabilization, that is, when inflation would exhibit high serial correlation under perfect knowledge. Under imperfect knowledge, it is efficient for a policymaker to bias the response to inflation upward relative to that implied by perfect knowledge. This effect is especially pronounced with the more forward-looking inflation process. Consider, for instance, the baseline case $\kappa = 0.05$. In the parameterization with $\phi = 0.9$, it is never efficient to set θ below 0.6, the value that one would choose under balanced preferences ($\omega = 0.5$) under perfect knowledge.

Accounting for imperfect knowledge can significantly improve stabilization performance relative to outcomes obtained when the policymaker naively adopts policies that are efficient under perfect knowledge. Figure 7 compares the loss to the policymaker with perfect and imperfect knowledge for different preferences ω. The top panel shows the outcomes for the

[15] In Orphanides and Williams (2003), we explore policies that respond directly to private expectations of inflation, in addition to actual inflation. These rules are not fully optimal; with imperfect knowledge, the fully optimal policy would be a nonlinear function of all the states of the system, including the elements of c and R. However, implementation of such policies would assume the policymaker's full knowledge of the structure of the economy an assumption we find untenable in practice.

baseline parameterization, $\phi = 0.75, \alpha = 0.25$; the bottom panel reports the outcomes for the alternative parameterization of inflation, $\phi = 0.9, \alpha = 0.1$. In both panels, the results we show for imperfect knowledge correspond to our benchmark case, $\kappa = 0.05$. The payoff to reoptimizing θ is largest for policymakers who place a large weight on output stabilization, with the gain huge in the case of $\phi = 0.9$. In contrast, the benefits from reoptimization are trivial for policymakers who are primarily concerned with inflation stabilization regardless of ϕ.

The key finding that the public's imperfect knowledge raises the efficient policy response to inflation is not unique to the model considered here and carries over to models with alternative specifications. In particular, we find the same result when the equation for inflation is replaced with the "New Keynesian" variant studied by Gali and Gertler (1999) (see also Gaspar and Smets 2002). Moreover, we find that qualitatively similar results obtain if agents include additional lags of inflation in their forecasting models.

6.3 Dissecting the Benefits of Vigilance

In order to gain insight into the interaction of imperfections in the formation of expectations and efficient policy, we consider a simple example where the parameters of the inflation forecast model vary according to an exogenous stochastic process.

From equation (5), recall that expectations formation is driven by the stochastic coefficient expectations function:

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t. \tag{13}$$

For the present purposes, let $c_{0,t}$ and $c_{1,t}$ vary relative to their perfect knowledge benchmark values; i.e., $c_{0,t} = c^P_0 + v_{0,t}$ and $c_{1,t} = c^P_1 + v_{1,t}$, where $v_{0,t}$ and $v_{1,t}$ are independent zero mean normal distributions with variances σ^2_0 and σ^2_1.

Substituting expectations into the Phillips curve and rearranging terms results in the following reduced form characterization of the dynamics of inflation in terms of the control variable x:

$$\pi_{t+1} = (1 + \phi v_{1,t})\pi_t + \frac{\alpha}{1-\phi} x_t + \alpha u_{t+1} + e_{t+1} + \phi v_{0,t}. \tag{14}$$

In this case, the optimal policy with stochastic coefficients has the same linear structure as the optimal policy with fixed coefficients and perfect knowledge, and the optimal policy response is monotonically increasing in the variance σ_1^2.[16]

Although informative, the simple case examined above ignores the important effect of the serial correlation in v_0 and v_1 that obtains under imperfect knowledge. The efficient choice of θ cannot be written in closed form in the case of serially correlated processes for v_0 and v_1, but a set of stochastic simulations is informative. Consider the efficient choice of θ for our benchmark economy with balanced preferences, $\omega = 0.5$. Under perfect knowledge, the optimal choice of θ is approximately 0.6. Instead, simulations assuming an exogenous autoregressive process for either c_0 or c_1 with a variance and autocorrelation matching our economy with imperfect knowledge suggest an efficient choice of θ approximately equal to 0.7—regardless of whether the variation is due to c_0 or to c_1. For comparison, with the endogenous variation in the parameters in the economy with learning, the efficient choice of θ is 0.75.

As noted earlier, for a fixed policy choice of policy responsiveness in the policy rule, θ, the uncertainty in the process of expectations formation with imperfect knowledge raises the persistence of the inflation process relative to the perfect knowledge case. This can be seen by comparing the solid and dashed lines in the two panels of Figure 8, which plot the persistence of inflation when policy follows the RE-optimal rule and agents have perfect and imperfect knowledge (with $\kappa = 0.05$), respectively. This increase in inflation persistence complicates stabilization efforts as it raises, on average, the output costs associated with restoring price stability when inflation deviates from its target.

The key benefit of adopting greater vigilance against inflation deviations from the policymaker's target in the presence of imperfect knowledge comes from reducing this excess

[16]See Turnovsky (1977) and Craine (1979) for early applications of the well-known optimal control results for this case. For our model, specifically, the optimal response can be written as:

$$\theta = \frac{\alpha(1-\phi)s}{(1-\phi)(1-\omega)+\alpha^2 s},$$

where s is the positive root of the quadratic equation:

$$0 = \omega(1-\omega)(1-\phi)^2 + (\omega\alpha^2 + (1-\omega)(1-\phi)^2\phi^2\sigma_1^2)s + (\phi^2\sigma_1^2 - 1)\alpha^2 s^2.$$

While the optimal policy response to inflation deviations from target, θ, is independent of σ_0^2, the variance of the $v_{0,t}$ differentiation reveals that it is increasing in σ_1^2, the variance of $v_{1,t}$. As $\sigma_1^2 \to 0$, of course, this solution collapses to the optimal policy with perfect knowledge.

serial persistence of inflation. More aggressive policies reduce the persistence of inflation, thus facilitating its control. The resulting efficient choice of reduction in inflation persistence is reflected by the dash-dot lines in Figure 8.

7 Learning with a Known Inflation Target

Throughout the preceding discussion and analysis, we have implicitly assumed that agents do not rely on explicit knowledge regarding the policymaker's objectives in forming expectations. Arguably, this assumption best describes situations where a central bank does not successfully communicate to the public an explicit numerical inflation target and, perhaps, a clear weighting of its price and economic stability objectives. Since the adoption and clear communication of an explicit numerical inflation target is one of the key characteristics of inflation targeting regimes, it is of interest to explore the implications of this dimension of inflation targeting in our model. To do so, we consider the case where the policymaker explicitly communicates the ultimate inflation target to the public; that is, we assume that the public exactly knows the value of π^* and explicitly incorporates this information in forming inflation expectations. Of course, even in an explicit inflation targeting regime, the public may remain somewhat uncertain regarding the policymaker's inflation target, π^*, so that this assumption of a perfectly known inflation target may not be obtainable in practice and may be seen as an illustrative limiting case.

The assumption of a known numerical inflation target simplifies the public's inflation forecasting problem. From equations (7) and (8), the reduced form equation for inflation under rational expectations is given by:

$$\pi_{t+1} - \pi^* = (1 - \frac{\alpha\theta}{1-\phi})(\pi_t - \pi^*) + e_{t+1} + \alpha u_{t+1}. \tag{15}$$

With a known inflation target, the inflation forecasting model consistent with rational expectations is simply:

$$\pi_i - \pi^* = c_{1,t}(\pi_{i-1} - \pi^*) + v_i. \tag{16}$$

Note that this forecasting equation only the slope parameter, c_1 is estimated; thus, in terms of the forecasting equation, the assumption of a known inflation target corresponds to a zero restriction on c_0 (when the forecasting regression written in terms of deviations of inflation from its target). As in the case of an unknown inflation target, constant gain versions of equations (10) and (11) can be used to model the evolution of the formation of inflation expectations in this case. The one-step-ahead forecast of inflation is given by:

$$\pi^e_{t+1} = \pi^* + c_{1,t}(\pi_t - \pi^*), \tag{17}$$

and again, in the limit of perfect knowledge (that is, as $\kappa \to 0$), the expectations function above converges to rational expectations with the slope coefficient $c_1^P = \frac{1-\phi-\alpha\theta}{1-\phi}$. This formulation captures a key rationale for adopting an explicit inflation targeting regime: to reduce the public's uncertainty and possible confusion about the central bank's precise inflation objective and thereby to anchor the public's inflation expectations to the central bank's objective.[17]

Eliminating uncertainty about the inflation target improves macroeconomic performance, in terms of both inflation and output stability. The dotted lines in the upper panel of Figure 9 trace the RE-policy pseudo-frontiers in the case of a known inflation target. For comparison, the dashed lines show the pseudo-frontiers assuming that the inflation target is not known by the public (this repeats the curves shown in Figure 5 for our benchmark case, $\kappa = 0.05$). Recall that the pseudo-frontier is obtained by evaluating the performance of the economy under imperfect knowledge for the set of policies for $\omega \in (0, 1]$ given by equation (8) that would be optimal under perfect knowledge. As seen in the figure, economic outcomes are clearly more favorable when the inflation target is assumed to be perfectly known than otherwise. Still, the resulting pseudo-frontiers lie well to the northeast of those that would obtain under perfect knowledge. Evidently, imperfect knowledge of the dynamic process for inflation alone has large costs in terms of performance, especially when expectations are very important for determining inflation outcomes, represented by the case of $\phi = 0.9$.

[17] The adoption of inflation targeting may affect the private formation of expectations in other ways than by tying down the ultimate inflation objective. For instance, Svensson (2002) argues that inflation-targeting central banks should also make explicit their preference weighting, ω, which in principle could further reduce the public's uncertainty about policy objectives. However, given the remaining uncertainty about model parameters (α and ϕ in our model), the uncertainty about the value of c_1 is not eliminated in this case. The extent to which this uncertainty may be reduced is left to further research.

The basic finding that, relative to the perfect knowledge benchmark, policy should be more vigilant against inflation under imperfect knowledge also obtains in the case of a known inflation target. The lower panels of Figure 9 show the optimal values of θ for the three cases we consider: perfect knowledge, imperfect knowledge with known π^*, and imperfect knowledge with unknown π^*. When π^* is known, the optimal choice of θ is slightly lower than when π^* is unknown. Even with a known inflation target, however, it remains optimal to be more vigilant against inflation relative to the perfect knowledge case. An exception is the extreme case of $\omega = 1$ when the optimal value of θ is exactly unity, the same value that obtains under perfect knowledge.[18]

A striking result, seen most clearly in the case of $\phi = 0.9$, is that the optimal value of θ is relatively insensitive over a large range of values for the stabilization preference weight, ω, whether the inflation is known or unknown. By contrast, under perfect knowledge, the optimal value of θ is quite sensitive to ω. An implication of this finding is that with imperfect knowledge, there is relatively little "cost" associated with policies designed as if inflation were the central bank's primary objective, even when policymakers place substantial value in reducing output variability in fact. By contrast, as shown above, the costs of optimizing policies that incorrectly place a large weight on output stability under the assumption of perfect knowledge can be quite large. This asymmetry suggests that the practice of concentrating attention primarily on price stability in the formulation of monetary policy may be seen as a robust strategy for achieving *both* a high degree of price stability *and* a high degree of economic stability.

8 Conclusion

We examine the effects of a relatively modest deviation from rational expectations resulting from perpetual learning on the part of economic agents with imperfect knowledge. The presence of imperfections in the formation of expectations makes the monetary policy problem considerably more difficult than would appear under rational expectations. Using a simple

[18] In this limiting case, estimates of c_1 are symmetrically distributed around zero. Hence, in terms of a simple rule of the form given by equation 4, there is no gain from over-responding, relative to the case of perfect knowledge, to actual inflation.

linear model, we show that although inflation expectations are nearly efficient, imperfect knowledge raises the persistence of inflation and distorts the policymaker's tradeoff between inflation and output stabilization. As a result, policies that appear efficient under rational expectations can result in economic outcomes significantly worse than would be expected by analysis based on the assumption of perfect knowledge. The costs of failing to account for the presence of imperfect knowledge are particularly pronounced for policymakers who place a relatively greater value on stabilizing output: A strategy emphasizing tight inflation control can yield superior economic performance, in terms of both inflation and output stability, than can policies that appear efficient under rational expectations. More generally, policies emphasizing tight inflation control reduce the persistence of inflation and the incidence of large deviations of expectations from the policy objective, thereby mitigating the influence of imperfect knowledge on the economy. In addition, tighter control of inflation makes the economy less prone to costly stagflationary episodes.

The adoption and effective communication of an explicit numerical inflation target also mitigate the influence of imperfect knowledge on the economy. Communication of an inflation target may greatly improve attainable macroeconomic outcomes and afford greater economic stability relative to the outcomes that are attainable when the public perceives the policymaker's ultimate inflation objective less clearly. These results highlight the potential value of communicating central bank's inflation objective and of continued vigilance against inflation in anchoring inflation expectations and fostering macroeconomic stability.

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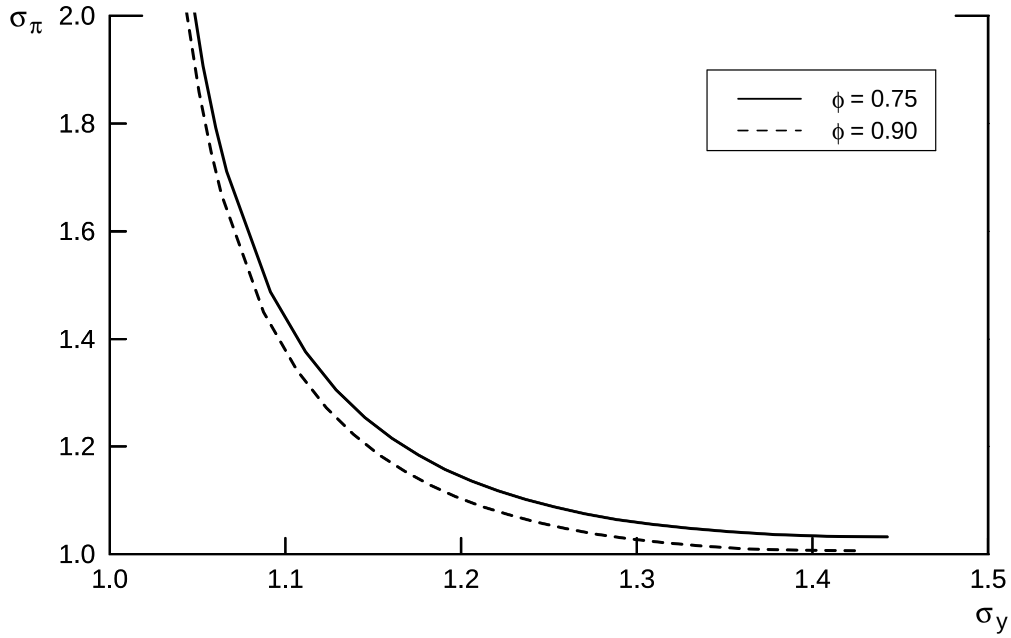
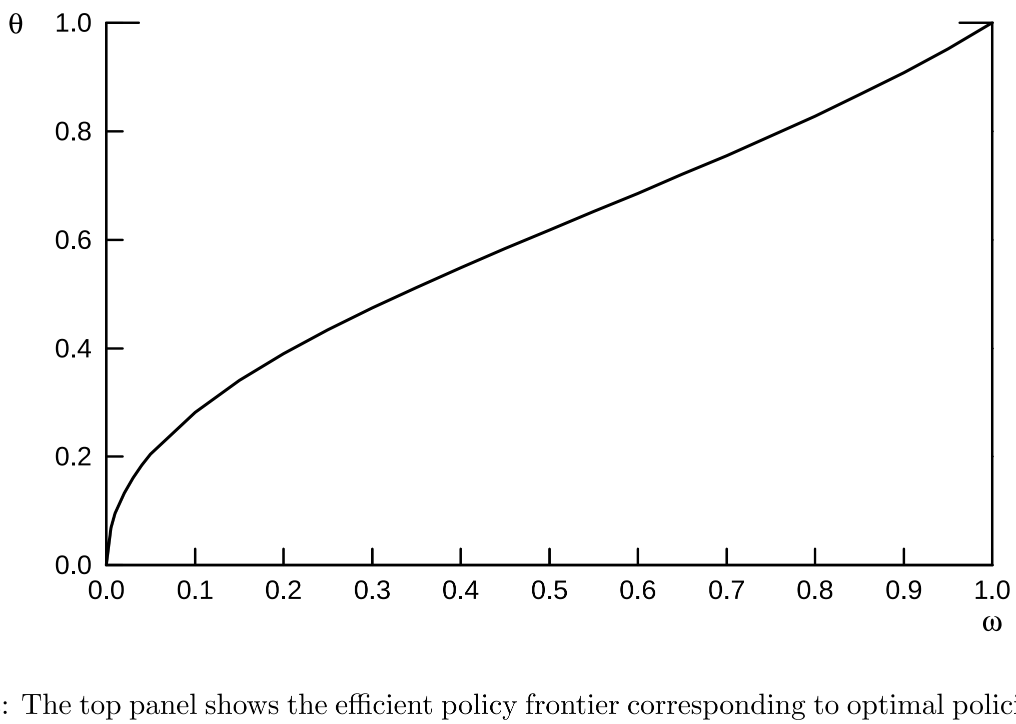
Figure 1

Efficient Policy Frontier with Perfect Knowledge



Optimal Policy Response to Inflation



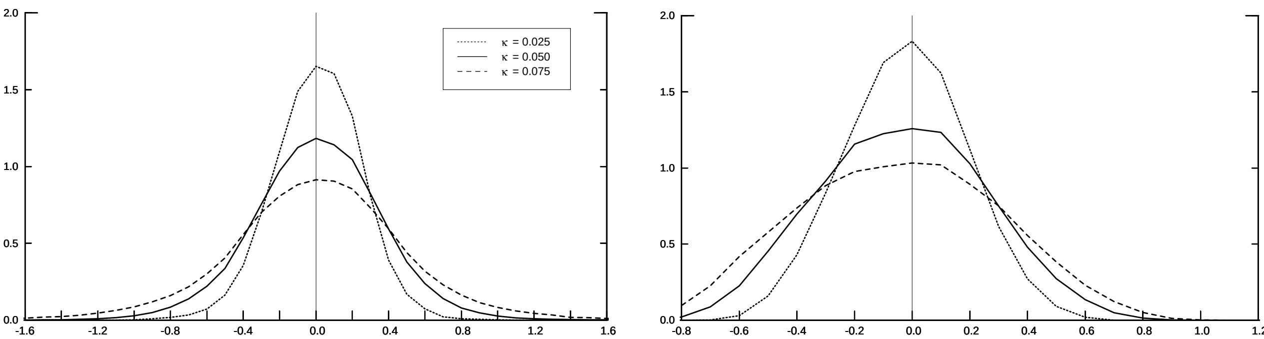
Notes: The top panel shows the efficient policy frontier corresponding to optimal policies for different values of the relative preference for inflation stabilization ω, for the two specified parameterizations of α and ϕ. The bottom panel shows the optimal response to inflation corresponding to the alternative weights ω which are identical for the two parameterizations.

Figure 2

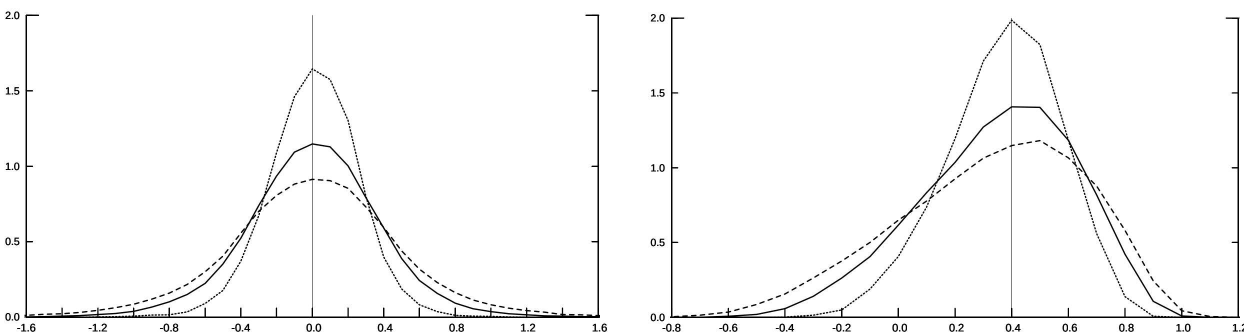
Estimated Expectations Function Parameters
($\phi = 0.75, \alpha = 0.25$)

Intercept | Slope

Inflation Hawk: $\theta = 1$



Balanced Preferences: $\theta = 0.6$



Inflation Dove: $\theta = 0.1$

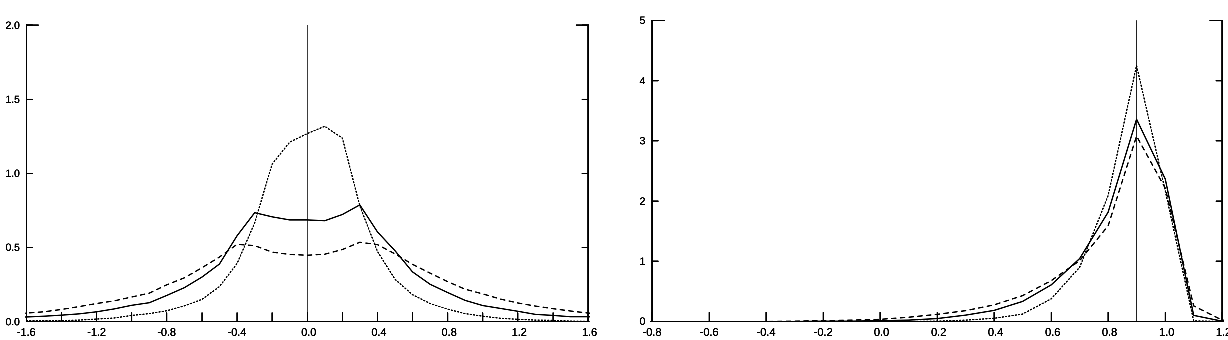
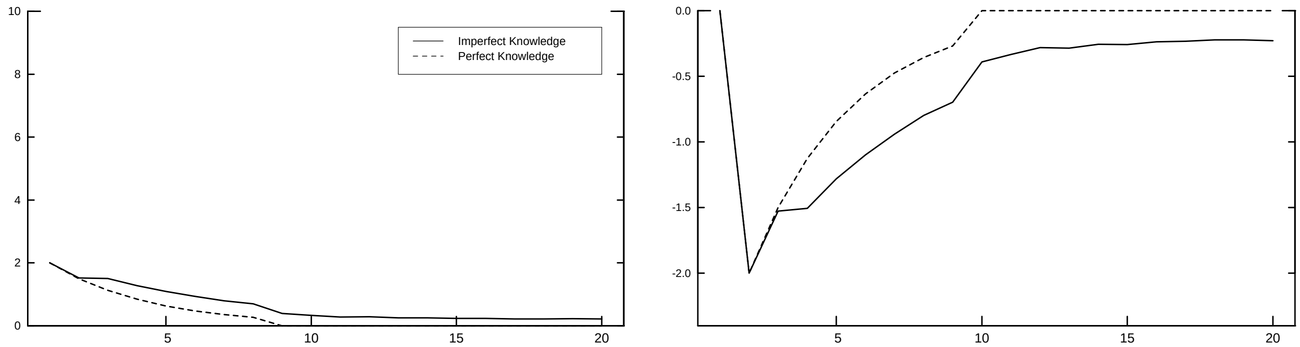


Figure 3

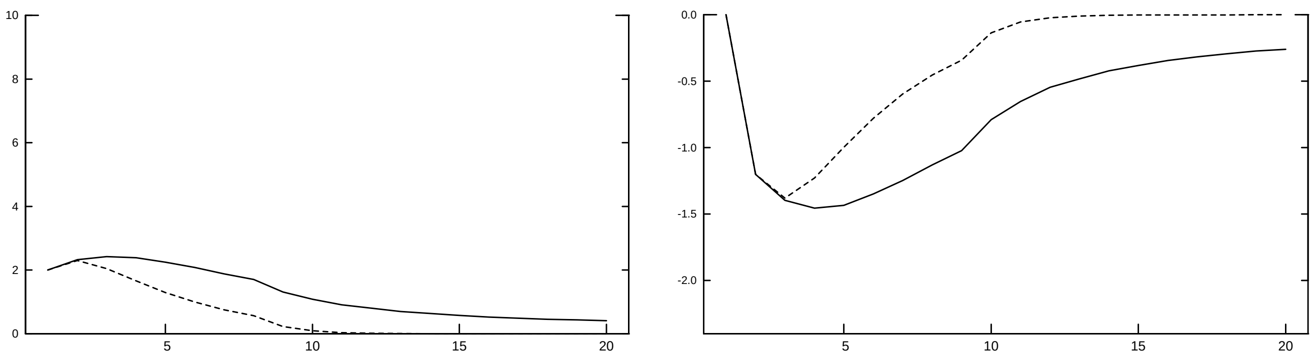
Evolution of Economy Following Inflation Shocks
$(\phi = 0.75, \alpha = 0.25)$

Inflation Output

Inflation Hawk: $\theta = 1$



Balanced Preferences: $\theta = 0.6$



Inflation Dove: $\theta = 0.1$

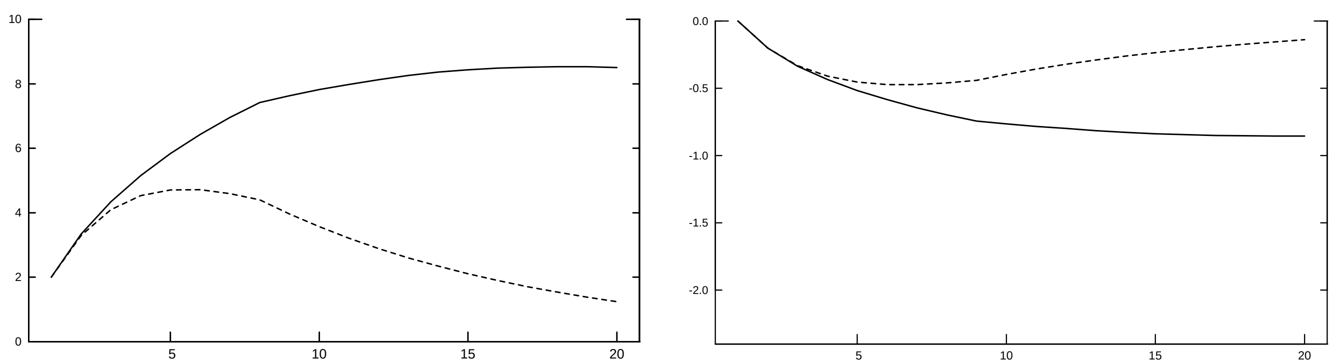
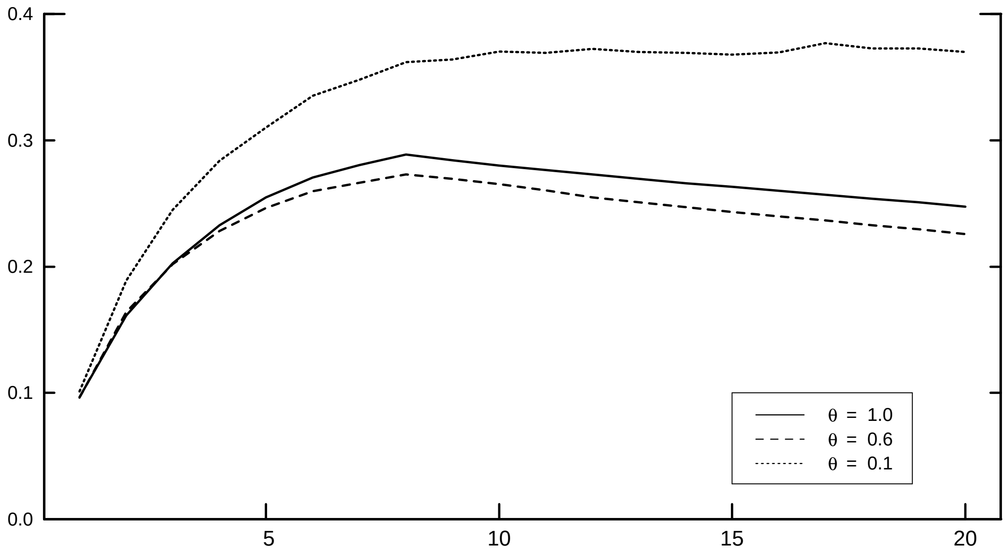


Figure 4

Estimated Intercept Following Inflation Shocks
$(\phi = 0.75, \alpha = 0.25)$



Estimated Slope Following Inflation Shocks

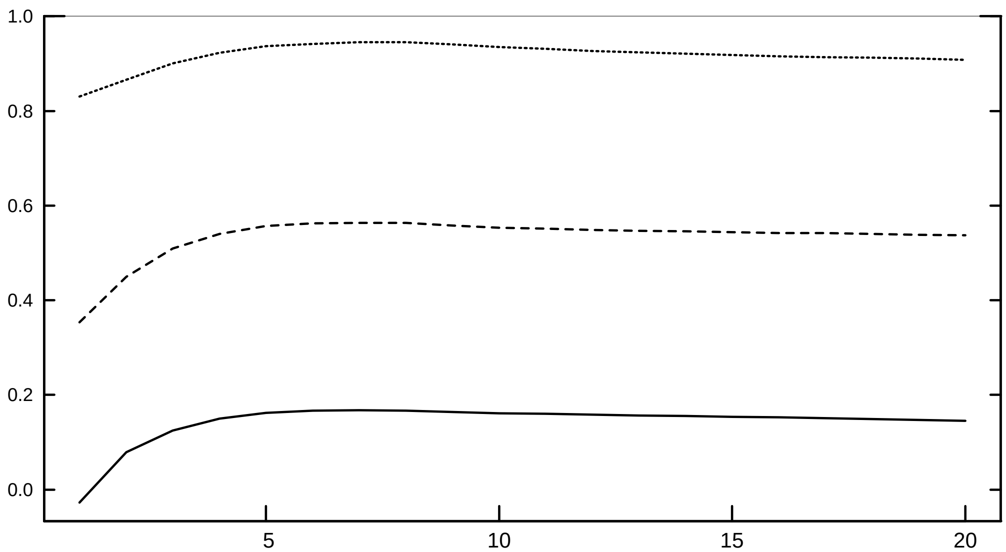
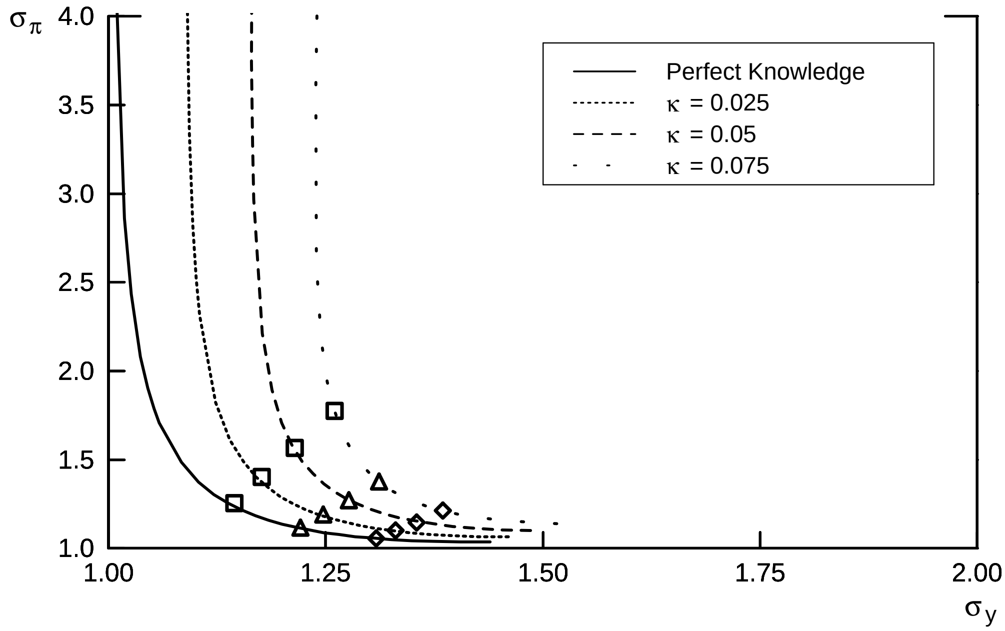
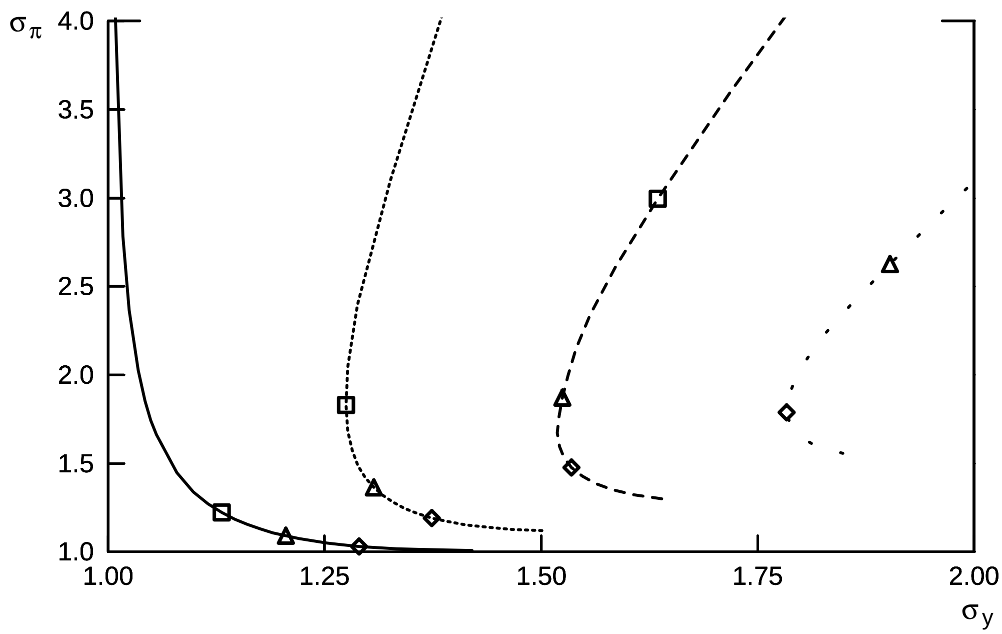


Figure 5

Outcomes with RE-policy ($\phi = 0.75, \alpha = 0.25$)



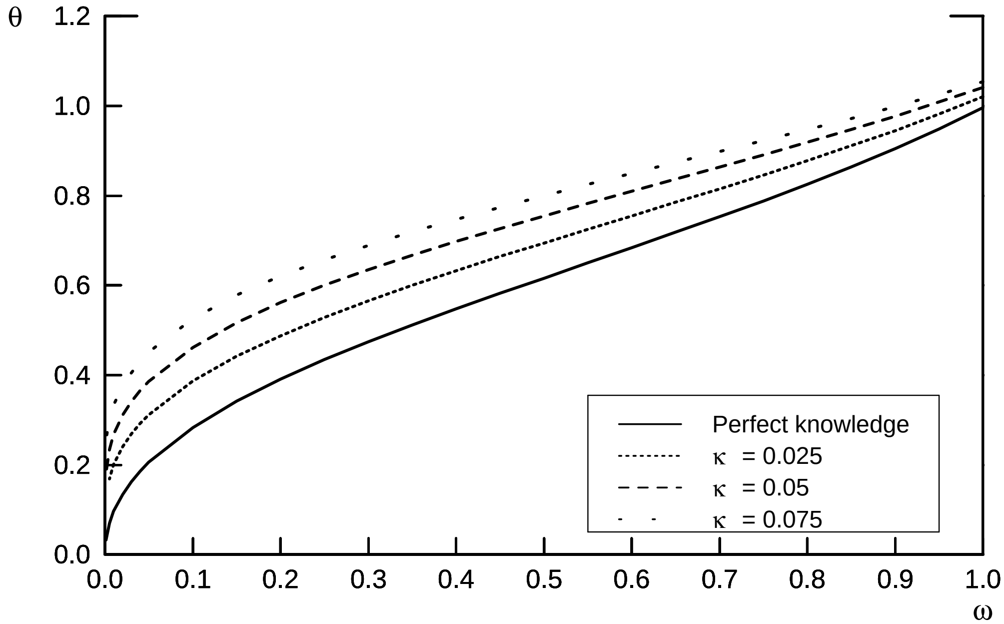
Outcomes with RE-policy ($\phi = 0.9, \alpha = 0.1$)



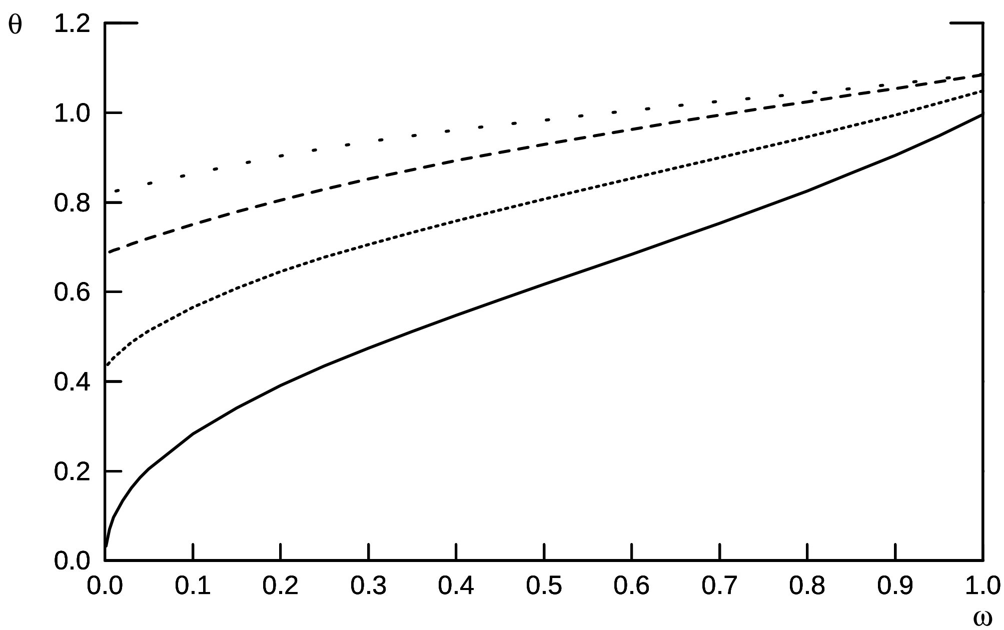
Notes: Each panel shows the efficient frontier with perfect knowledge and corresponding outcomes when the RE-optimal policies are adopted while, in fact, knowledge is imperfect. The square, triangle, and diamond correspond to preference weights $\omega = \{0.25, 0.5, 0.75\}$, respectively.

Figure 6

Efficient Policy Response to Inflation ($\phi = 0.75, \alpha = 0.25$)



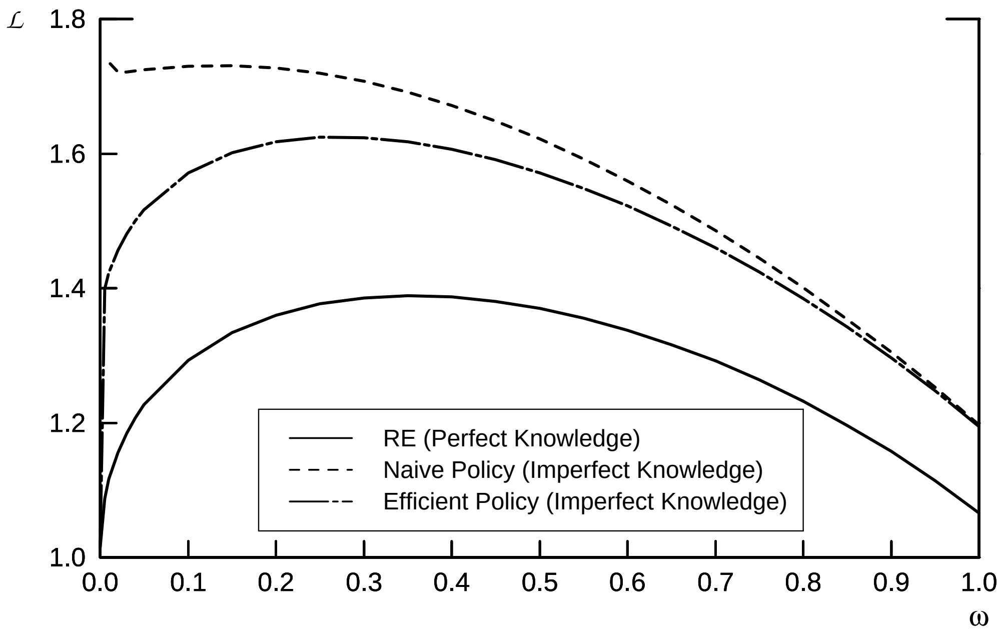
Efficient Policy Response to Inflation ($\phi = 0.9, \alpha = 0.1$)



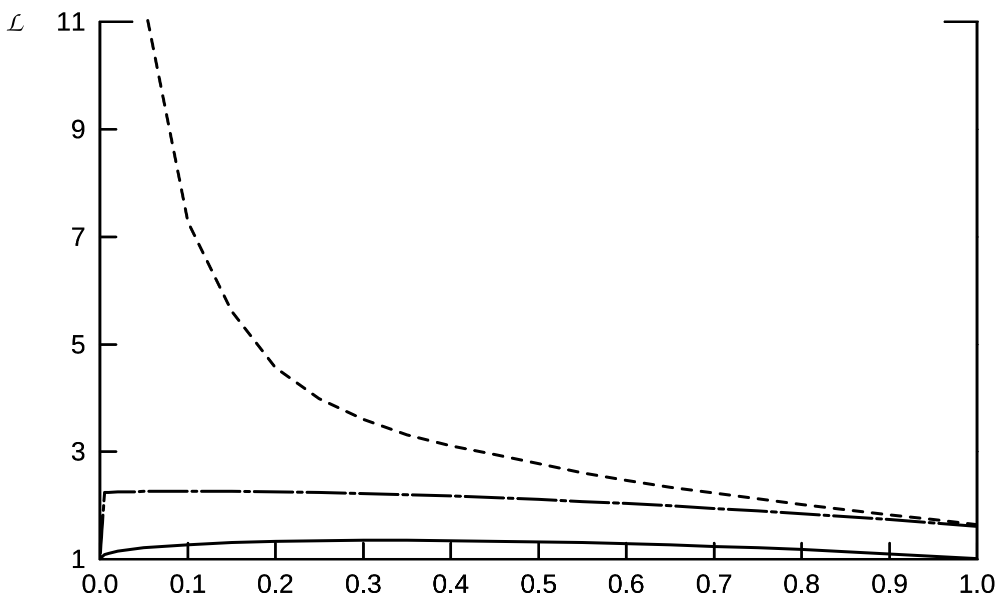
Notes: The solid line in each panel shows the optimal value of θ under perfect knowledge for alternative values of the relative preference for inflation stabilization ω. Remaining lines show the efficient one-parameter policy under imperfect knowledge.

Figure 7

Policymaker Loss ($\phi = 0.75, \alpha = 0.25$)



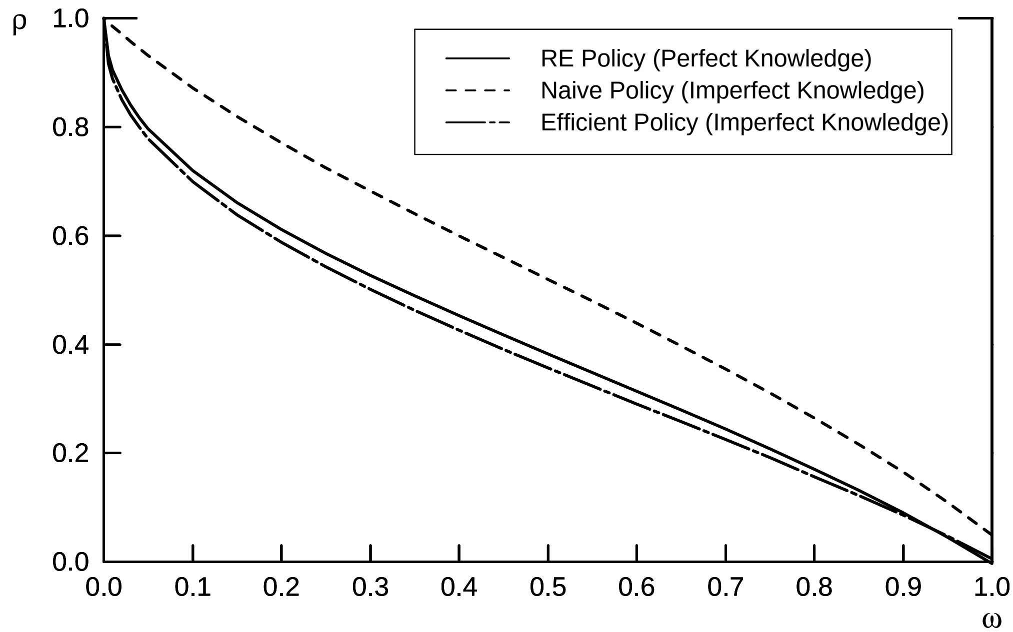
Policymaker Loss ($\phi = 0.9, \alpha = 0.1$)



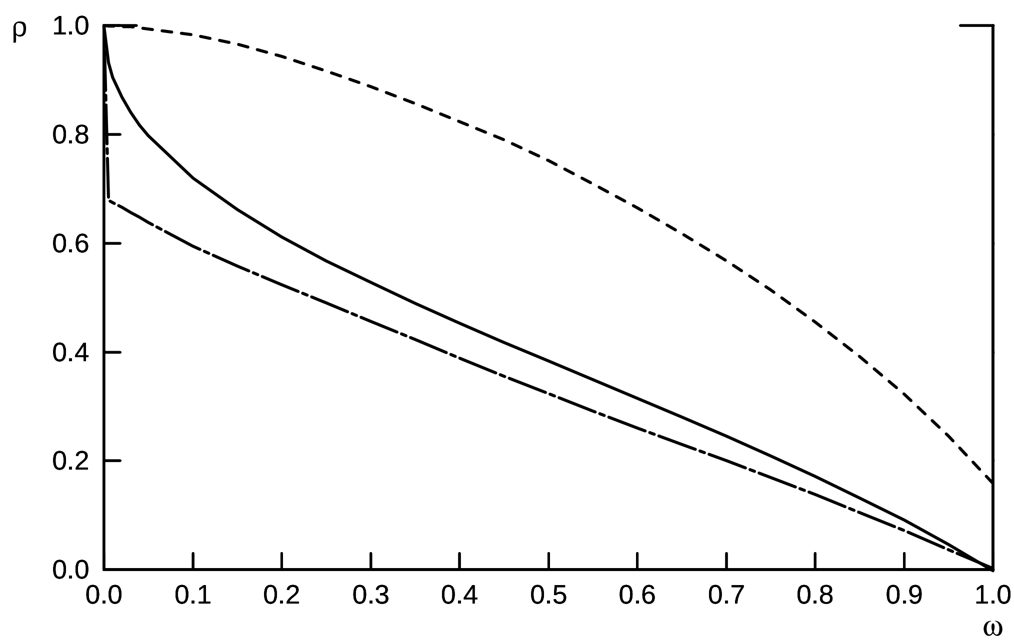
Notes: The two panels show the loss corresponding to alternative values of the relative preference for inflation stabilization ω for different assumptions regarding knowledge and different model parameterizations. The solid line shows the case of perfect knowledge. The dashed line shows the outcomes assuming the policymaker chooses θ assuming perfect knowledge when knowledge is in fact imperfect. The dashed-dotted line shows the outcomes for the efficient one-parameter policy under imperfect knowledge.

Figure 8

Inflation Persistence ($\phi = 0.75, \alpha = 0.25$)



Inflation Persistence ($\phi = 0.9, \alpha = 0.1$)



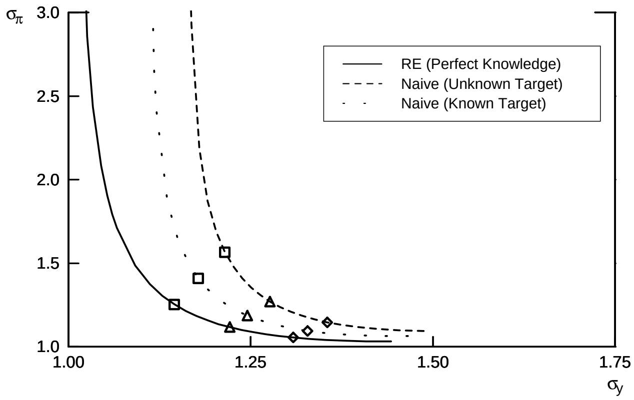
Notes: The figure shows the population first-order autocorrelation of inflation corresponding to policies based on alternative inflation stabilization weights ω. For each value of ω, the solid line shows the inflation persistence in the benchmark case of rational expectations with perfect knowledge. The dashed line shows the corresponding persistence when policy follows the RE-optimal solution but knowledge is imperfect. The dash-dot line shows the persistence associated with the efficient one-parameter rule with imperfect knowledge.

Figure 9

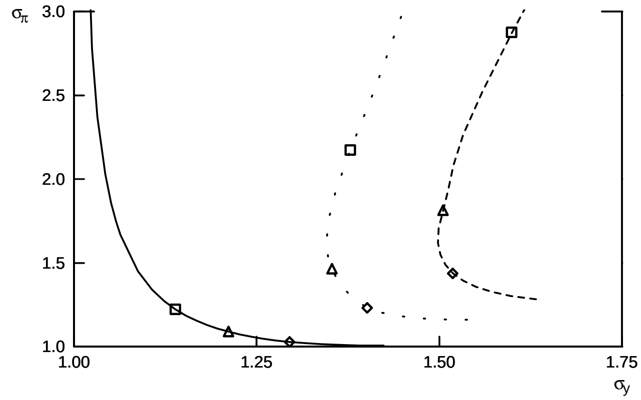
Comparing Policies with a Known and Unknown Inflation Target

Outcomes with RE-policy

$(\phi = 0.75, \alpha = 0.25)$

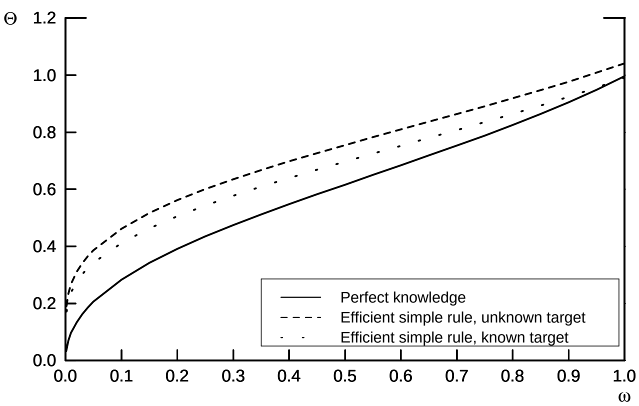


$(\phi = 0.9, \alpha = 0.1)$

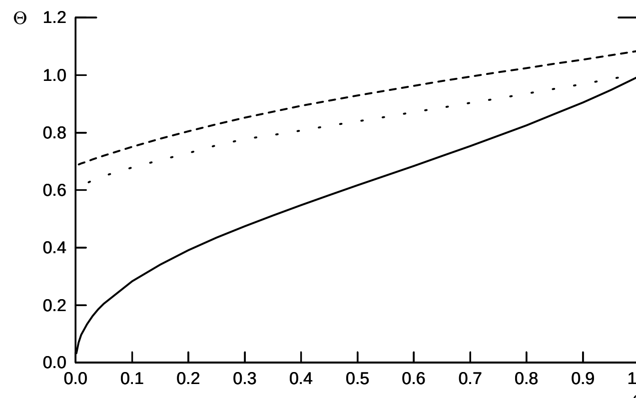


Efficient Response to Inflation

$(\phi = 0.75, \alpha = 0.25)$



$(\phi = 0.9, \alpha = 0.1)$



Notes: The dotted lines indicate economic outcomes (top panel) and efficient policy responses (bottom panel) with perpetual learning when the policymaker's inflation target is assumed to be perfectly known. The solid and dashed lines correspond, respectively, to the perfect knowledge benchmark and the case of perpetual learning with an unknown inflation target. See also the notes to figures 5 and 6.

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