Exhaustible Resource Allocation, Intergenerational Equity, and Sustainability

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An OLG model with exhaustible resources and solar energy is developed, and equilibrium time paths are characterized numerically using recursive methods. For the parameter values considered, resource prices increase over time, and extractions, output, and utility decline over time until a steady-state is reached. Decreasing the intertemporal elasticity of substitution or raising consumers' subjective discount rate hastens exhaustion of the resource stock. Market equilibrium can result in much quicker use of the stock than social optimality under a constant discount rate, with consequent higher utility for early generations and lower utility for future generations in contrast to social optimality.

Fundamental issues in the economics of exhaustible resources include market allocation and efficiency of exhaustible resource use over time, and the implications of exhaustibility for economic growth, intergenerational equity, and sustainability. There is a vast literature on these subjects. As a broad generalization, the first issue is typically examined in the context of Hotelling-type, partial equilibrium models with an exogenous interest rate (Hotelling; Devarajan and Fisher). The second issue is traditionally addressed with optimal growth models where annual investment and resource extractions maximize discounted utility as in Stiglitz (1974a) and Dasgupta and Heal. Related work includes competitive equilibria with an exogenous saving rate (Stiglitz 1974b), alternate criterion functions (Solow; Mitra 1980), efficient growth paths (Mitra 1978), and competitive equilibria with infinite-lived agents (van Geldrop, Jilin, and Withagen).

An alternative approach is provided by overlapping generations (OLG) models as originally explored by Samuelson and Diamond and augmented to include natural resources. These extend Hotelling-type models by considering general equilibrium with endogenous interest rates. They differ from the growth literature with exhaustible resources by focusing on market allocations as opposed to optimality, and with finite-lived agents compared with infinite-lived agents. OLG models can result in qualitatively different behavior than models with infinite-lived agents (Kehoe), and they provide a suitable environment for exploring intergenerational equity and sustainability issues in contrast to growth models which implicitly or explicitly consider infinite-lived consumers.

The existing literature on exhaustible resource allocation in the OLG framework is extremely limited and focuses primarily on efficiency. Kemp and Long provide an example of an infinite horizon economy where the competitive allocation is inefficient as a result of the resource being inessential for production. Manresa examines the existence of equilibria and the first and second welfare theorems in a model with capital. The second welfare theorem is also the focus of Howarth (1991a,b) and Howarth and Norgaard. Love derives some qualitative results for an economy where agents have CES utility and production is Cobb-Douglas. Long, Mitra, and Sorger consider the sustainability of consumption when there is capital and an exhaustible resource.

These papers typically assume that the resource is essential to production but do not consider the possibility of a "backstop" technology, i.e., solar power. Here an OLG model that includes both exhaustible resources and a source of continuing energy flows to the economy is developed. Consumers live for two periods. They purchase both exhaustible resource stocks and the technology (land) for capturing solar power in the first lifetime-period out of wage income, and then sell energy

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flows from both stocks in the second lifetime-period to firms, and the stocks themselves to the next generation. Firms convert labor and energy flows supplied by households into a single consumption good which is then purchased by consumers.

Equilibrium conditions are characterized for this model, and a recursive solution procedure is developed. Previous work is primarily theoretical and generally considers open-loop solutions for specific functional forms. The numerical solution procedure developed here is analogous to the closed-loop controller in dynamic programming, and allows for the solution to long horizon problems, including infinite horizon problems, and for general functional forms. Extension to uncertainty is also straightforward with these methods, although only deterministic models are considered here. There are five markets clearing in each period in this model (output, labor and energy flows, exhaustible resource and solar capital stock markets). Using numerical simulations, equilibrium time paths are first characterized for prices, quantities, and utility under CES utility and Cobb-Douglas production. Sensitivity to the underlying fundamentals of the economy (preferences and technology) is then investigated.

Welfare issues are also considered. It is shown that even when the competitive allocation is Pareto-optimal, it is not necessarily the social optimum as stressed by Howarth and Norgaard in the context of OLG models with exhaustible resources. In particular, we find that, even in contrast to a social optimum with a high discount rate (10%), the market allocation may exhaust the resource much faster, while utility for the early generations starts out higher but rapidly falls to a lower steady-state level.[1]

Model

Utility for the generation born in period t is defined by

$$U(c_{1t}, c_{2,t+1}) \tag{1}$$

where c_{1t} and $c_{2,t+1}$ denote consumption in the first and second (lifetime) periods, respectively. We will generally assume that U is concave and increasing in its arguments. Specific functional forms will be considered in what follows.

In the first lifetime period, consumers earn wage income that is then spent on consumption and investment in the exhaustible resource stock x_{t+1} and the solar energy capital stock s_{t+1}, for carryover into the next period. Thus the first lifetime period budget constraint is given by

$$p_t^q c_{1t} + p_t^x x_{t+1} + p_t^s s_{t+1} = p_t^l l_t \tag{2}$$

where p_t^q, p_t^x, p_t^s, and p_t^l denote output, exhaustible resource stock, solar energy capital stock, and labor prices in period t respectively, and l_t is the (inelastic) labor supply.

In the second period consumers earn income from sale of energy flows to firms and sale of the stocks to the young generation. Income is then used for consumption. Thus, the second period budget constraint for generation t is

$$\begin{aligned} p_{t+1}^q c_{2,t+1} = {} & p_{t+1}^e (r_{t+1} + s_{t+1}) \\ & + p_{t+1}^x (x_{t+1} - r_{t+1}) \\ & + p_{t+1}^s s_{t+1} \end{aligned} \tag{3}$$

where r_t denotes resource flows (extractions) in period t and p_t^e denotes the price of energy flows. Note also that

$$x_t \geq r_t \geq 0 \tag{4}$$

bounds resource extractions in each period, and that there are no extraction costs.

Firms earn profits according to

$$p_t^q q_t - p_t^l l_t - p_t^e e_t \tag{5}$$

where q_t is output, e_t is energy flows, and l_t is labor quantity demanded. The firm's production function is given by

$$q_t = F(l_t, e_t) \tag{6}$$

where F is assumed to be homogenous of degree 1. Constant returns to scale and profit maximization imply zero profits; therefore firm ownership need not be specified.

As noted, there are two resource stocks capable of supplying energy flows to the economy. The first is an exhaustible (depletable) resource such as oil. This stock evolves according to

$$x_{t+1} = x_t - r_t \tag{7}$$

where x_t is the stock at the beginning of period t. The second resource is a nondepletable resource such as solar power. Solar radiation is captured by the solar energy capital stock, which is interpreted here as land. This capital stock (land) is assumed to be fixed in supply, and energy flows to the economy from this source occur at the constant rate of s units per year. Without loss of generality, we therefore assume

$$s_t = s \tag{8}$$

[1] To keep the analysis tractable, this model focuses only on energy resources. A more general treatment would also include nonfuel minerals, renewables, land, and environmental quality.

for all years t, and note that stocks x_t and s_t, extractions r_t, and solar energy flows to the economy s, are measured in commensurate units.

In this model there are five markets that must clear in each year. Market clearing conditions for the output market are

$$q_t = c_{1t} + c_{2t} \tag{9}$$

while labor demanded by the firm must equal l_t. Market clearing for energy flows is

$$r_t + s_t = e_t \tag{10}$$

where the left-hand side is energy supplied by the resource owners (households) and the right-hand side is energy demanded by firms. Investment demand by the young generation for the exhaustible resource (x_{t+1}) must satisfy equation (7), and demand for solar energy capital stocks (land) by the young generation must satisfy equation (8).

Equilibrium consists of a series of prices (p_t^q, p_t^l, p_t^e, p_t^x, p_t^s) and quantities (q_t, c_{1t}, c_{2t}, l_t, e_t, r_t, x_t, s_t) such that consumers maximize utility subject to the budget constraints, producers maximize profits, and markets clear in all periods. The next section derives a system of equations/inequalities characterizing intertemporal equilibrium in this model.

Equilibrium Conditions

This section derives the conditions characterizing equilibrium in the OLG model. To begin, we first note that if (p_t^q, p_t^l, p_t^e, p_t^x, p_t^s) is an equilibrium, then so is $(1/p_t^q)$ (p_t^q, p_t^l, p_t^e, p_t^x, p_t^s). Therefore without loss of generality, we can assume the output price $p_t^q = 1$ and define $w_t = p_t^l\, l_t / p_t^q$, $p_t = p_t^e/p_t^q$, and $z_t = p_t^s/p_t^q$, as the relative prices for wages, energy flows, and solar energy capital stocks respectively. We also let $l_t = 1$.

With these definitions, first-order conditions for utility maximization imply

$$p_t^x = (U_2/U_1)p_{t+1}^x \quad 0 \le r_t < x_t \tag{11a}$$

$$p_t^x \ge (U_2/U_1)p_{t+1}^x \quad r_t = x_t \tag{11b}$$

and

$$z_t = (U_2/U_1)\,(p_{t+1} + z_{t+1}) \tag{12}$$

where U_i is marginal utility for lifetime period i. Condition (11) implies that consumers choose carryover stocks to equate the marginal rate of substitution for consumption in both periods to the resource price ratio provided this is possible. If this is not possible, then carryover stocks are zero. Since solar energy capital stocks are fixed, the first-order condition for consumer demand for these stocks results in the price relation (12).

The first-order conditions for optimal extractions by the old generation also imply $p_t^x \ge p_t$ if $r_t = 0$, $p_t^x = p_t$ if $0 < r_t < x_t$, and $p_t^x \le p_t$ if $r_t = x_t$. In the latter case, carryover stocks of the exhaustible resource are zero. If $p_t^x < p_t$ is the equilibrium in this case, then $p_t^x = p_t$ would also be an equilibrium since raising the price of stocks would keep the young generation demand at zero and it would still be optimal for the old generation to extract all the resource. Thus the price relations

$$p_t^x \ge p_t \quad r_t = 0 \tag{13a}$$

$$p_t^x = p_t \quad r_t > 0 \tag{13b}$$

also follow from utility maximization.

Defining $f(e_t) \equiv F(1,e_t)$ and substituting for e_t, then

$$p_t = f'\,(r_t + s) \tag{14}$$

and

$$w_t = f(r_t + s) - f'(r_t + s)\,(r_t + s) \tag{15}$$

follow from profit-maximization, constant returns to scale, and $l_t = 1$.

A steady-state occurs in this model when $r_t = x_t = 0$ and all prices remain constant over time. Assuming a steady-state exists, then (12) implies

$$z = \frac{U_2}{U_1 - U_2} f'(s) \tag{16}$$

where $U_i = \partial U/\partial c_i$ and where

$$\begin{aligned} c_1 &= f(s) - f'(s)s - zs \\ c_2 &= (f'(s) + z)s \end{aligned} \tag{17}$$

define steady-state consumption levels. Note that equation (16) implies $U_1 > U_2$ and hence $c_1 < c_2$ when utility is separable with identical single-period utility functions and no discounting. Substituting (17) into (16) yields a single equation in one unknown (z).

More generally equations (11)–(13) constitute a system of three equations in three unknowns (r_t, z_t, p_t^x) for each period in a finite-horizon model after substituting for prices from (14) and (15) and consumption from (2) and (3). Thus, equilibrium over T periods could be computed by solving the $3T$ equations in $3T$ variables using an equation solver system. Difficulties with this approach include (a) handling the conditionals in (11b) and (13a) when the inequalities hold strictly, (b) computational effort rising exponentially with the time horizon, and (c) inability to solve infinite horizon problems. The last difficulty is especially limiting given interest in sustainability. The next section develops a recursive algorithm that includes possible inequalities in (11b) and (13a), for which computational effort is linear in the time horizon, and that can solve infinite horizon problems.

Recursive Algorithm

A recursive algorithm is used to solve for equilibrium in this model. The algorithm is analogous to the use of dynamic programming methods (Bertsekas) to solve dynamic optimization problems. Similar approaches can be found in Stokey and Lucas (with Prescott) and Coleman. The primary solution concept is that of a series of equilibrium rules g_t

$$\left.\begin{matrix} r_t \\ z_t \\ p_t^x \end{matrix}\right\} = g_t(x_t) \tag{18}$$

where $g:\Re^1 \rightarrow \Re^3$ with component functions g_{it}, $i = 1, 3$, for r_t, z_t, and p_t^x respectively. These rules give equilibrium extractions and prices for solar energy capital stocks and exhaustible resource stocks respectively as a function of the exhaustible resource stock at the beginning of year t, and are estimated by proceeding backward in time from an initial rule. Once the rules have been calculated, then an equilibrium time path for all variables can be traced out by proceeding forward in time using the equilibrium rules and other relations defined above.

To illustrate, consider the last period in a finite horizon model with T periods. In the last period, market clearing requires that $r_T = x_T$. Also $z_T = 0$ and $p_T^x = 0$ since carryover of solar energy capital stocks or exhaustible resource stocks is of no value. This therefore determines the equilibrium rule g_T for the last period.

Now suppose that we know the equilibrium rule g_{t+1} for period $t + 1$. Equilibrium conditions for period t can be derived from (11)–(13) by using the definitions for consumption (2)–(3) and resource stocks (7), resource prices and wages as defined in (14) and (15), and the equilibrium rule (18) for period $t + 1$. These equilibrium conditions are then

$$p_t^x - \frac{U_2}{U_1} g_{3,t+1}(x_t - r_t) \begin{cases} =0 & r_t < x_t \\ \geq 0 & r_t = x_t \end{cases} \tag{19}$$

for carryover stocks,

$$z_t = \frac{U_2}{U_1}(f'[g_{1,t+1}(x_t - r_t) + s] + g_{2,t+1}(x_t - r_t)) \tag{20}$$

for price of the solar energy capital stock, and

$$p_t^x \geq f'(r_t + s) \quad r_t = 0 \tag{21a}$$

$$p_t^x = f'(r_t + s) \quad r_t > 0 \tag{21b}$$

for exhaustible resource prices.

In the above relations,

$$\frac{U_2}{U_1} = \frac{\partial U(c_{1t}, c_{2,t+1})/\partial c_2}{\partial U(c_{1t}, c_{2,t+1})/\partial c_1} \tag{22}$$

$$c_{1t} = f(r_t + s) - f'(r_t + s)(r_t + s) - p_t^x(x_t - r_t) - z_t s \tag{23}$$

$$c_{2,t+1} = f'[g_{1,t+1}(x_t - r_t) + s][g_{1,t+1}(x_t - r_t) + s] + g_{3,t+1}(x_t - r_t)[x_t - r_t - g_{1,t+1}(x_t - r_t)] + g_{2,t+1}(x_t - r_t)s \tag{24}$$

and recalling that g_i are the component functions for r, z, and p^x respectively. Inspection of equations (19)–(24) shows that, after the appropriate substitutions and for a given x_t, this is a three-equation system in the three unknowns r_t, z_t, and p_t^x. Numerical solution for various x_t then determines the equilibrium rule g_t. (Bisection methods were generally used after specifying the various possible cases.)

Proceeding in this manner we can calculate equilibrium rules for all periods in a finite horizon model. To calculate equilibrium time paths, we solve (19)–(24) forward in time starting from the initial resource stock level x_1 and using the stock equation of motion (7) and the appropriate equilibrium rules. Solution for the infinite horizon model is analogous except that the backward recursions proceed until convergence in the equilibrium rule, from which forward simulations determine equilibrium time paths.

The primary advantages of this solution procedure are that, with one state variable, equilibrium can be computed very quickly for a wide class of functional forms, and that it is applicable to infinite horizon problems. Although not exploited here, the method is also applicable to stochastic problems, and where there is distortionary taxation and transfers (Coleman; Stokey and Lucas [with Prescott]).

CES Utility/Cobb-Douglas Production

In this section we consider separable CES utility of the form

$$U(c_{1t}, c_{2,t+1}) = \frac{c_{1t}^{\rho}}{\rho} + \beta \frac{c_{2,t+1}^{\rho}}{\rho} \tag{25}$$

where β is the discount factor, $0 < \beta \leq 1$, and ρ is a parameter reflecting intertemporal substitution with $-\infty < \rho < 1$ and $\rho \neq 0$. The intertemporal elasticity of substitution in this instance is given by $\sigma = 1/(1 - \rho)$ with $0 < \sigma < \infty$. Empirical evidence suggests that σ lies between .03 to .87 (Hall; Epstein and Zin); however, a range of values is considered. Production is assumed to be Cobb-Douglas

$$f(e) = e^{\alpha} \tag{26}$$

where $0 < \alpha < 1$.

Figure 1 illustrates the first twenty years of an infinite horizon equilibrium in this economy for $\rho = -1$ (implying $\sigma = .5$), $\beta = 1$ (i.e., no subjective discounting by consumers), $\alpha = .1$, $s = .1$, and an initial stock of the exhaustible resource $x_1 = 10$. It was always the case in this equilibrium that $p_t^x = p_t$, i.e., the price of the resource stock equaled the price of resource flows. As can be seen, both the price of energy flows and the price of solar energy capital stocks are increasing over time until they reach steady-state values in year 16. Extractions decline over time until the resource stock is exhausted in year 16. This is qualitatively similar to standard Hotelling model results where resource prices rise over time and extractions decline.

As energy flows decline over time, both the wage rate and consumption decline over time. As a result, lifetime utility also declines over time for the successive generations until reaching the steady-state in year 16, after which it is constant forever. Thus the market does result in a sustainable equilibrium in this economy; however, early generations are better off than later generations.

Table 1 reports a sensitivity analysis to the preference and technology parameters. The dynamics in each instance are qualitatively similar to that reported above, so the table focuses on the steady-state. In the steady-state, output and the price of energy are strictly a function of the technology, so these values are invariant to the preference parameters. The preference parameters do influence time to steady-state, the long-run equilibrium price of solar energy capital stocks, and the allocation of consumption within consumer lifetimes. The earlier steady-state derivation showed that more consumption occurs in the second period than in the first period. Reducing the intertemporal elasticity of substitution has the effect of equalizing consumption over time by increasing consumption in the first period and reducing it in the second period. This tends to reduce demand for the solar energy capital stock which is the only means of providing second period income once the resource stock is exhausted. Thus, the price of this stock falls when the intertemporal elasticity of substitution falls. Likewise in the transition to the steady-state, there is more demand for current extractions and less for carryover stocks of the exhaustible resource, so the exhaustible resource is depleted sooner with the lower intertemporal elasticity of substitution.

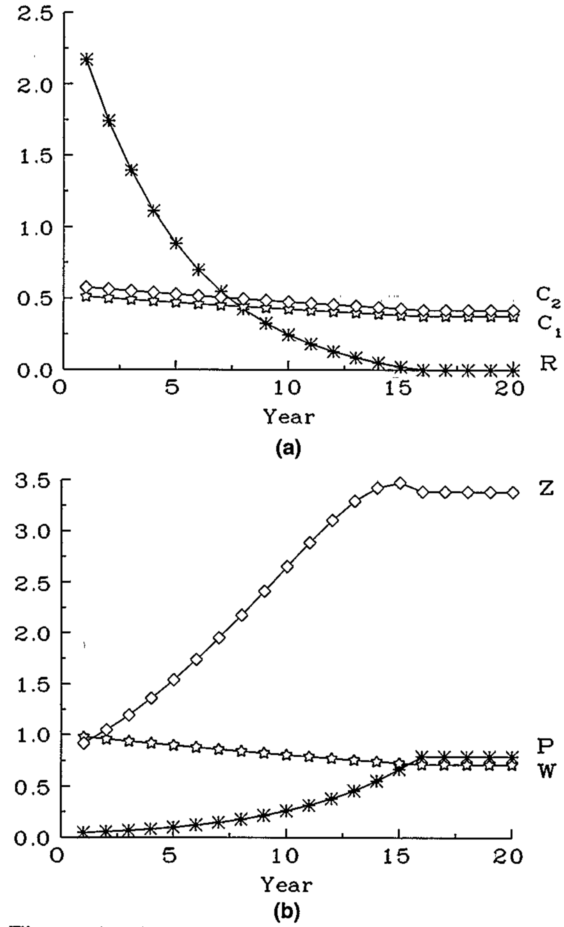

Figure 1. Equilibrium dynamics in the OLG model with $\rho = -1$, $\beta = 1$, $\alpha = .1$, $s = .1$, and $x_1 = 10$. (a) Extractions (r_t) and consumption (c_{1t}, c_{2t}). (b) Prices: energy (p_t), wages (w_t), and solar stocks (z_t).

The effect of raising the consumer's discount

Table 1. Sensitivity Analysis for Steady-State Equilibrium Values in the OLG Economy

Parameter	t_{ss}	p_{ss}^e	z_{ss}	q_{ss}	$c_{1,ss}$	$c_{2,ss}$	U_{ss}
Base	16	.79	3.39	.79	.38	.42	−5.05
$\rho = .5$ ($\sigma = 2$)	18	.79	3.90	.79	.32	.47	2.51
$\rho = -3$ ($\sigma = .25$)	16	.79	3.28	.79	.39	.41	−10.69
$\beta = .6$	15	.79	2.91	.79	.42	.37	−3.98
$\alpha = .05$	27	.45	4.12	.89	.43	.46	−4.49
$\alpha = .2$	10	1.26	2.24	.63	.28	.35	−5.95
$s = .5$	10	.19	.80	.93	.44	.49	−4.30

Base parameter values: $\rho = -1$ ($\sigma = .5$), $\beta = 1$, $\alpha = .1$, $s = .1$, $x_1 = 10$.
t_{ss} = time to steady-state ($x_t = 0$) in years.
p_{ss}^e, z_{ss} = price of energy flows and solar stock respectively.
q_{ss}, $c_{1,ss}$, $c_{2,ss}$ = output and consumption in the first and second lifetime periods respectively.
U_{ss} = lifetime utility.

rate is similar to that of reduced intertemporal elasticity of substitution. Raising the discount rate tends to de-emphasize second period consumption in favor of first period consumption, implying less demand for investment in the first lifetime period. This implies a reduced solar energy capital stock price in the steady-state, and increased extractions in the transition period with hastened exhaustion of the resource stock. The latter effect is consistent with qualitative predictions from a partial equilibrium Hotelling-type argument.

Since the labor supply is fixed in this model, long-run energy prices are determined by the technology. An increase in the production parameter α implies that energy is now more productive in the economy and hence the long-run equilibrium price is bid up by firms. In the particular instances considered here, an increase in α also reduces long-run utility, reflecting that energy is scarce relative to labor. However, this would not necessarily occur in all cases. For example, when $s = 1.5$, then an increase in α from .2 to .4 increases steady-state utility.

Table 1 also shows the effects of increasing the solar energy capital stock. In this instance the exhaustible resource is extracted faster. This is also consistent with a partial equilibrium Hotelling model with a backstop technology. Here lowering the backstop price would also lower the resource price in each period, hence increasing the extraction rate and hastening exhaustion of the resource. Returning to the OLG model, since both energy and the solar energy capital stock are now more abundant, their price goes down, and output, consumption, and utility increase as would be expected.

Equilibrium and Welfare

Even when the OLG equilibrium is Pareto-optimal, it may not be a welfare optimum when intergenerational equity effects are accounted for. This point is stressed by Howarth (1991a,b) and Howarth and Norgaard using first-order conditions and numerical examples for two generations and three time periods. The issue is further investigated here by comparing social optimality to market equilibrium with an infinite horizon.

The objective function for the social optimum problem is

$$\sum_{t=1}^{\infty} \sigma^t U(c_{1t}, c_{2,t+1}) \tag{27}$$

where σ is the social discount factor and $U(c_{1t}, c_{2,t+1})$is lifetime utility for generation t. This is just the present value of utility over the horizon, with σ^t representing the relative weight attached to each generation's utility by the social planner. Lifetime utility is separable

$$U(c_{1t}, c_{2,t+1}) = u(c_{1t}) + \beta u(c_{2,t+1}) \tag{28}$$

where β is again consumer's subjective discount factor and $u(c)$ is periodic utility. Substituting (28) into (27) and rearranging yields

$$\sum_{t=1}^{\infty} \sigma^t [u(c_{1t}) + \frac{\beta}{\sigma} u(c_{2,t})] \tag{29}$$

as the objective function.

The optimization problem is to choose r_t, c_{1t}, and c_{2t} to maximize (29) subject to

$$c_{1t} + c_{2t} = f(r_t + s) \tag{30}$$

and

$$x_{t+1} = x_t - r_t \tag{31}$$

with $0 \leq r_t \leq x_t$ and nonnegative c_{it}. This problem is solved here using dynamic programming: backward recursions on the value function yield the infinite horizon decision rule, which is then simulated forward in time to obtain optimal time paths.

I assume here CES utility with $\rho = -1$, $\beta = 1$, Cobb-Douglas production with $\alpha = .7$, $s = .1$, and $x_1 = 10$. Figure 2 compares results for market equilibrium in the OLG model with the social optimum under social discount rates of 1% and 10%. It can be seen that this range of discount rates implies a significant difference in time to exhaustion for the resource in the social optimum. For example, 1% implies exhaustion after 100 years, while 10% implies exhaustion by 35 years. However, market equilibrium implies exhaustion within 4 years, compared with exhaustion in year 35 for a discount rate of 10%. Thus even the largest discount rate considered here results in exhaustion of the resource much delayed from that implied by market equilibrium in the OLG model.

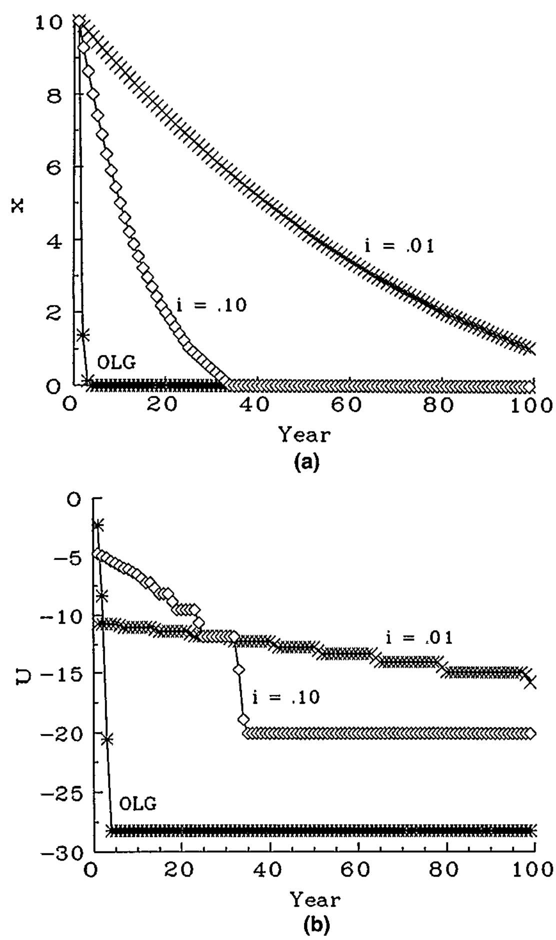

Figure 2. Comparison of social optimum under alternate discount rates to market equilibrium for the OLG model with $\rho = -1$, $\beta = 1$, $\alpha = .7$, $s = .1$, and $x_1 = 10$. (a) Exhaustible resource stocks (x_t). (b) Utility ($U[c_{1t}, c_{2,t+1}]$).

Figure 2(b) compares intergenerational utility under equilibrium and social optimality for the different discount rates. A clear pattern emerges from the figure. In each instance utility starts out high and then declines over time to a steady-state value. Comparing across regimes, market equilibrium starts out at the highest utility level but then drops off rapidly resulting in the lowest steady-state level of utility. In the social optimum, decreasing the social discount rate tends to even out consumption and utility over time. More specifically, a decrease in the social discount rate increases the utility of later generations but at the expense of reduced utility for the early generations. These results suggest that even when resource allocations are Pareto-optimal, market equilibrium and social discount rates can have dramatic effects on intergenerational utility.

Conclusions

This paper develops an OLG model that includes both exhaustible resources and a source of continuing energy flows to the economy. Consumers purchase output and exhaustible and solar energy capital stocks out of wage income, and sell energy flows to firms and stocks to the next generation. Firms convert labor and energy into output. In each period there are five markets clearing (output, labor, energy, and exhaustible and solar energy capital stocks). A recursive solution procedure is developed for computing equilibria in finite and infinite horizon models.

Equilibrium time paths for prices, quantities, and utilities are characterized for CES utility functions with Cobb-Douglas production. The numerical results exhibit increasing resource prices over time until exhaustion of the resource, after which they are constant. This is qualitatively similar to results from partial equilibrium Hotelling-type models with a backstop technology. Price of the solar energy capital stock also increases over time as the resource is exhausted, while utility of each generation is declining over time until reaching a steady-state value. Thus this model exhibits a kind of "sustainability," although early generations with positive levels of the exhaustible resource are better off than later generations for the range of values considered.

Increasing the intertemporal elasticity of substitution or decreasing consumers' subjective discount rate makes carryover of the resource more valuable and extends the life of the exhaustible resource. Making the resource more valuable in production or increasing the solar energy capital stock reduces the life of the exhaustible resource. These results therefore provide insight into how the fundamentals of an economy (preferences and technology) affect prices and allocation for exhaustible resources.

Welfare implications are also investigated. The social optimum problem is solved for various social discount rates over an infinite horizon, with the higher discount rates implying less weight on the future and reduced time to exhaustion for the resource. However, even with the highest interest rate considered (10%), time to exhaustion in the social optimum is orders of magnitude longer than market equilibrium in the OLG model, and intergenerational utility is higher in the steady-state. This is additional evidence that even if equilibrium is Pareto-optimal, there may be very strong intergenerational equity effects associated with market allocations of exhaustible resources.

In this paper we are considering an economy without capital in the traditional sense; thus there is no growth in productive capacity. Since capital provides a way for output to expand over time, it can be anticipated that there will be at least some conditions such that the investment process is sufficiently productive to outweigh the inevitable decline in energy flows, and output, consumption, and utility may increase over time before reaching a steady-state. However, it can also be anticipated that some of the results observed here will still hold, namely, that the market equilibrium will result in skewed distributions of consumption and utility to the present in contrast to social optima with traditional discount rates. A useful extension of the present model and solution algorithm is therefore the incorporation of physical and human capital stocks.

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