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Abstract

We study the behavior of firms in an imperfectly competitive environment in which firms influence the evolution of the stock of capital equipment. Our model enables us, using analytical characterizations, to show the effect of key ingredients of dynamic competition on firm strategies and industry dynamics in addition to the usual static interaction. These effects are the static market externality (implicit in the static Cournot Equilibrium) as well as the dynamic market externality due to the effect on the market outputs of a capital stock and a dynamic externality that stems from the competition between firms for the capital stock. These strategic elements justify our conclusions, based on the study of four market structures, for the link between industrial organization and industry growth.

Key Words: Cournot competition, oligopolistic non-cooperative dynamic games

JEL classification: D43, D92, L13, O12, Q20

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1. Introduction

The role of capital deepening on economic growth is usually studied through highly aggregated growth models. In these models, typically focusing on the macroeconomy, perfect competition is the prevailing market structure. On the other hand, the effects of capital deepening on the growth of smaller markets, like industries, are very important, and these effects are not generally driven by mechanisms of perfectly competitive capital markets.

Capital is one of the major inputs for the production of output by a firm. Unlike in static environments, when capital evolves over time, the organization of the intermediate good market, the capital market, also plays a major role in industry dynamics. Firms often operate in an imperfectly competitive environment for capital, and thus must take account of the market mechanism behind the dynamics of the capital stock in making their output decision. The determinants of output strategies by firms in such an imperfectly competitive dynamic environment, are hardly ever studied or poorly understood.

Unlike in aggregate models that focus on a single aggregated type of capital, firms in each specific industry use specific equipment. When it is important to study the sectoral growth of certain industries, aggregation is a rather oversimplifying assumption.

The dynamics of capital, especially firm specific capital, is indispensable for studying the dynamic organizations of industry. There are capital structures that are specific to industries, especially in the way that they affect the dynamics. More output means more specific equipment utilization, which leads, in turn, to higher depreciation of this equipment. Pipe lines and transmission grids, shared research efforts and the exploitation of natural resources are examples of capital structure that is so specific to an industry, and capital accumulation is affected by production quantities, as well as by the number of firms in the industry.

We study the effect of the number of firms in an industry on equilibrium market output when production depends on capital that evolves over time, isolating, one by one, the strategic components that constitute the final-good supply behavior of firms. We show the impact of each of these strategic elements on aggregate firm supply and on the evolution of capital.

One of the strategic components of firm behavior is the intratemporal Cournot-Nash quantity competition among firms. The presence of a firm in the market constitutes a *market externality* for the rest of the firms in the market. This strategic element is exactly the same as the one that appears in a static framework.[1]

In contrast to static environments, in the dynamic context, final-good supply strategies of firms involve decisions of optimal intertemporal capital-stock management. Any final good supply decision taken by firms today affects the evolution of capital in the future. Firms incorporate this concern into their final product supply decisions.

Furthermore, the decisions of the firms with respect to how the capital stock evolves over time, are complicated by the fact that other firms also have a direct influence on the evolution of available capital. The presence of competition in the market for the capital good, constitutes a *dynamic externality* for the firms in the capital market.[2]

In a dynamic oligopoly in which other firms appear both in the final good market and the capital market, both the market externality and the dynamic externality influence each firm's decisions. In this paper, we isolate the effect of each externality on the firm's final good supply by setting up a parametric model with a demand function and growth dynamics that yield analytical characterizations for four alternative market structures. In this model there is a natural symmetric equilibrium.

[1] Several recent papers still deal with the issue of existence and uniqueness of Cournot-Nash equilibrium in static frameworks. See, for example, Gaudet and Salant (1991), Novshek (1984a), (1984b) and (1985).
[2] This externality was first analyzed by Levhari and Mirman (1980).

First, we characterize the strategy of a dynamic monopolist. This is the benchmark case. In this case the output market is supplied by a monopolist who is also alone in influencing the evolution of the capital stock. Second, we consider the case of two monopolists whose capital evolution is influenced by the actions of both firms. In other words, there are two markets, each supplied by a monopolist. We compare the results in this market structure to the results of the pure monopolist case. We find that the presence of the dynamic externality leads the two monopolistic firms to supply more in each period, compared to the pure monopolist. But more production in each period means more capital utilization, hence more capital depreciation. Therefore, the dynamic externality reduces capital growth.

Third, when the two firms (both influencing the capital stock) also compete in the same market, their aggregate supply in each period increases even more. This shows that the usual result of the static Cournot model is present in the dynamic context as well. The market externality makes firms engage in quantity Cournot competition. Indeed, in this case there are three externalities all influencing output in the same direction, greater supply. These three externalities are the usual static market externality plus a dynamic market externality and a dynamic externality.

Last (fourth), we compare the benchmark monopolist's strategy with that of two firms that sell in the same market but extract capital from two separate sources. This comparison isolates the impact of the market externality when the dynamic externality is not present. The setup with two different levels of stocks leads to the possibility of a non-symmetric equilibrium, in this paper we continue to study only symmetric equilibrium outcomes. As in the static model, the market externality increases aggregate supply in each period. The growth rate of the capital stocks of the two capital sources in equilibrium depends on the elasticity of demand. In particular, if the demand elasticity is low (high), the market externality leads to higher (lower) capital growth rate compared to the benchmark monopoly.

The presence of the two externalities, the dynamic and the market externality, in dynamic oligopolistic markets was first studied in Mirman (1979). Although Mirman (1979) does not present an analysis of the impact of the two externalities on strategies, the two elements are pointed out in necessary equilibrium conditions. Moreover, Mirman (1979) explores the problems that can arise in dynamic oligopoly models under usual assumptions on the objective of each firm and the dynamic constraints, assumptions that would lead to tractable decision rules in a standard optimal control problem. In particular, in the dynamic oligopoly case supply strategies may not, in general, be continuous functions. However, continuous differentiability of supply strategies of all firms is a convenient property for each firm to determine its equilibrium strategy.

Since not much is understood in the literature about the existence of equilibrium in dynamic games and no general sufficient conditions for strategies to be continuously differentiable functions are known, the fact that we present a parametric model that enables us to have analytical characterizations is not restrictive. On the contrary, studying a class of models is the only way to know that the framework of analysis is well-behaved and an appropriate vehicle for running the thought experiments that isolate the impact of the dynamic and the market externality on equilibrium strategies.

Our analysis does not involve linear demand functions, but isoelastic ones. Mirman (1979) shows how linear demand functions lead to either a corner solution or an interior solution that is exactly the same as the static solution. It is clear from this analysis of Mirman (1979) that the linear demand model is not appropriate for addressing the issues raised in this paper.

Our model can be applied in various other issues of industrial organization, e.g., knowledge accumulation models, industries using specific or vintage capital, natural-resource based industries. Also, the fact that the framework is parametric, enables empirical estimation of the parameters from time-series data, especially because we use iso-elastic demand and production functions. Koulovatianos and Mirman (2003) study the link between market structure and industry growth when firms pursue cost-reducing knowledge accumulation through R&D investment using an alternative model specification, but they point out the same strategic elements behind firm behavior, namely the importance of the dynamic and the market externality.

A dynamic model of imperfect competition has been studied also in Levhari and Mirman (1980). It was made clear that firms in an imperfectly competitive environment change their output in each period when the capital stock is part of the analysis. Dutta and Sundaram (1992) and (1993) are the only papers stating general results about existence of equilibrium in dynamic imperfect-competition setups. On the other hand, Ericson and Pakes (1995) show the importance of Markov-perfect dynamics in an imperfectly competitive environment for empirical work.

Vedenov and Miranda (2001) and Pakes and McGuire (2001) discuss numerical procedures for oligopoly games with capital accumulation. Both studies suggest ways of overcoming the several technical difficulties.

In section 2 we present the benchmark model, the dynamic monopoly. Sections 3 and 4 add the dynamic externality and the market externality in the benchmark model and make comparisons. Section 5 examines the dynamics of the market when the market externality comes in alone.

2. The Dynamic Monopoly

In this section we present the benchmark model, the dynamic monopoly. We show the dynamic structure of the problem and the functional forms that enable us to obtain a closed form solution, which allows us to characterize this solution. Moreover, we compare the optimal behavior of the dynamic monopoly with the static monopoly.

Consider a monopoly operating in infinite horizon, $t = 0, 1, ...$, facing an inverse-demand function $p = D(q)$, in each period. Production of q needs capital and labor as inputs and production technology is given by,

$$q = F(k, l) \ ,$$

with $F_1, F_2 > 0$ and F quasiconcave. The cost of hired labor in each period is given by

$$c = c(l) \ ,$$

where $c' > 0$.

There are two determinants of capital evolution,

(a) An exogenous determinant of intertemporal capital supply (or reproduction), captured by the function f. If the monopolistic firm does not operate at all, capital would evolve according to,

$$k_{t+1} = f(k_t) \ .$$

(b) An endogenous determinant, the amount the firm supplies in each period. Capital stock depreciates over time depending on usage. The more final-good units produced, the more capital-stock units consumed.[3] Capital-stock units

[3] This idea of capital utilization is also studied by Greenwood et. al. (1988) in a general-equilibrium framework. Higher utilization of capital wears equipment out, or it leaves less time for its maintenance.

depreciate according to the function $\psi(q)$, a strictly increasing function of q. Therefore, when the monopolistic firm operates, capital evolves according to the law of motion,

$$k_{t+1} = f(k_t) - \psi(q_t) \ , \tag{1}$$

for $t = 0, 1, ...$

Equation (1) introduces the element of intertemporal choice. While deciding upon the supply in each period, the firm chooses between using the capital stock now or investing it for later use.

We eliminate the variable l in order to facilitate the exposition. Since $F_2 > 0$,

$$l = L(k, q) \, . \tag{2}$$

Since F_1,$F_2 > 0$, $L_1 < 0$ and $L_2 > 0$. So, the cost in each period becomes a function of the quantity produced and the available stock of capital,

$$c = c(L(k, q)) \ , \tag{3}$$

i.e. the cost of production decreases if there is more capital available. The objective of the monopoly is to determine a supply-quantity decision rule as a function of the available capital, $q = Q(k)$, so that it maximizes its life-time profits,

$$\sum_{t=0}^{\infty} \delta^t \left[D(q_t) \, q_t - c(L(k_t, q_t)) \right] \ , \tag{4}$$

given $k_0 > 0$ and with $\delta \equiv \frac{1}{1+r}$, the profit discount factor, determined by an exogenous constant interest rate $r > 0$.

Our goal is to obtain closed form results and study their properties in all monopolistic and duopolistic setups that we examine. In order to achieve this goal, we use specific functional forms that enable us to obtain decision rules of the form $Q(k) = \omega f(k)$, with $\omega \in (0,1)$. In particular,

$$D(q) = q^{-\frac{1}{\eta}} \ , \quad \text{with } \eta > 1, \tag{5}$$

and

$$f(k) = \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}} \ , \tag{6}$$

i.e. the intertemporal production function of capital is a CES function. The depreciation function of capital, ψ is,

$$\psi(q) = q \ ,$$

i.e. the units of capital that are consumed equal the supply of the final good in each period. The final-good production function is,

$$q = F(k,l) = \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{\frac{1}{\eta-1}} l^{1-\frac{1}{\eta}} \ ,$$

so

$$L(k,q) = \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{-\frac{\eta}{(\eta-1)^2}} q^{\frac{\eta}{\eta-1}} \ .$$

Whereas the labor-cost function is

$$c(l) = \nu l^{1-\frac{1}{\eta}} \ , \quad \text{with } \nu < 1.$$

Therefore, the cost function is given by:

$$c(L(k,q)) = C(k)\, q \ ,$$

with

$$C(k) \equiv \nu \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{-\frac{1}{\eta-1}} \ . \tag{7}$$

The problem of the monopolist can be written in a Bellman-equation form,

$$V_M(k) = \max_{q \geq 0} \left\{ D(q)\, q - C(k)\, q + \delta V_M(f(k) - q) \right\} \ , \tag{8}$$

where $D(q)$ is given by (5), $f(k)$ by (6) and $C(k)$ by (7). The functions given by (6) and (7) can be accommodated in our model in two ways. Both of these are consistent with linear decision rules, however, they each imply different dynamics. Specifically:

(i) Set $\phi = 0$ and $\alpha \in \left[1, \frac{1}{\delta}\right]$. In this case, the intertemporal production function of capital is $f(k) = \alpha^{\frac{\eta}{\eta-1}} k = Ak$, a usual ingredient in growth theory, that may lead to perpetual growth of the market if $\alpha \in \left(1, \frac{1}{\delta}\right]$.[4] In case $\alpha = 1$, the model is appropriate for the study of markets trading a non-renewable resource.

(ii) Set $\phi > 0$ and $\alpha \in (0, 1)$. In this case, the production function of capital is a function of current capital and a constant parameter ϕ. Note that this assumption implies that the elasticity of substitution between current capital and the constant factor is the same as the elasticity of demand. There are two reasons for this assumption. First, it provides linear analytic solutions, which is especially important for the duopoly (differential game) case. Even if a solution can be guaranteed in more general cases, studying various implications of the model would be impossible. In other words, the analytical simplicity of this framework allows us to derive the comparative statics (or dynamics) of the model that is the very essences of this study. Second, with $\phi > 0$, the model has a zero-growth-rate steady state.[5]

As mentioned above, using the functional forms (5), (6) and (7), the model gives a decision rule of the form,

$$Q(k) = \omega \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}} . \tag{9}$$

[4] We place the upper bound $\frac{1}{\delta}$ on parameter α in order to guarantee the boundedness of the value function of each firm. The reasoning is clearer in the analysis of each setup.

[5] Moreover, for scholars of empirical applications of our model, the function given by (6) has three parameters, α, η and ϕ, giving enough degrees of freedom for treating data through data-mining approaches.

Putting (9) into the objective function of the monopoly, as given by (4), we obtain the value function,

$$V_M(k) = \frac{\alpha\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-\omega)^{1-\frac{1}{\eta}}} k^{1-\frac{1}{\eta}} + \kappa_M \ , \tag{10}$$

where κ_M is a constant.[6] We show that the properties of ω imply that the value function, given by (10), is bounded and the profits of the monopolist are positive. Substituting equation (10) into equation (7), the Bellman equation of the monopolist becomes,

$$V_M(k) = \max_{q\geq 0}\left\{ q^{1-\frac{1}{\eta}} - \nu\left[f(k)\right]^{-\frac{1}{\eta}} q + \frac{\alpha\delta\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-\omega)^{1-\frac{1}{\eta}}}\left[f(k) - q\right]^{1-\frac{1}{\eta}} + \delta\kappa_M \right\} \ , \tag{11}$$

where $f(k)$ is given by (6). The first-order condition implies,

$$\left(1-\frac{1}{\eta}\right)\left[\frac{q}{f(k)}\right]^{-\frac{1}{\eta}} - \nu = \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-\omega)^{1-\frac{1}{\eta}}}\left[1-\frac{q}{f(k)}\right]^{-\frac{1}{\eta}} \ . \tag{12}$$

It is clear from equation (12) that the decision rule $q = \omega f(k)$, as is implied by (9), satisfies the necessary condition, for all $k > 0$. Substituting this decision rule into (12), we find the relationship that gives the condition from which the constant ω is characterized,

$$g_M(\omega) \equiv \frac{\left(1-\frac{1}{\eta}\right)\omega^{-\frac{1}{\eta}} - \nu}{\omega^{-\frac{1}{\eta}} - \nu} = \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\omega}{(1-\omega)^{\frac{1}{\eta}} - \alpha\delta(1-\omega)} \equiv h(\omega) \ . \tag{13}$$

We show that (13) implies that, $\omega \in (0,1)$ and that the value function is bounded, momentary profits and momentary marginal profits are always positive. From the left-hand side of equation (13), $g_M(0) = 1-\frac{1}{\eta}$ and $g_M(1) = 1-\frac{1}{\eta(1-\nu)}$, whereas $g_M'(\omega) = -\frac{\nu}{\eta^2}\frac{\omega^{-\frac{1}{\eta}-1}}{\left[\omega^{-\frac{1}{\eta}} - \nu\right]^2} < 0$.

[6] In particular,

$$\kappa_M = \phi\frac{\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)\left[1-\alpha(1-\delta)(1-\omega)^{1-\frac{1}{\eta}}\right]}{\left[1-\alpha\delta(1-\omega)^{1-\frac{1}{\eta}}\right](1-\delta)} \ .$$

From the right-hand side of equation, (13), $h(0) = 0$, $h(1) = \infty$ and

$$h'(\omega) = \left(1 - \frac{1}{\eta}\right) \alpha\delta \frac{\frac{1-\left(1-\frac{1}{\eta}\right)\omega}{(1-\omega)^{1-\frac{1}{\eta}}} - \alpha\delta}{\left[(1-\omega)^{\frac{1}{\eta}} - \alpha\delta(1-\omega)\right]^2} .$$

Noticing that $1 - \left(1 - \frac{1}{\eta}\right)\omega \geq (1-\omega)^{1-\frac{1}{\eta}}$ for all $\omega \in [0,1]$ with equality if and only if $\omega = 0$, the fact that $\alpha\delta \leq 1$ implies that $h'(\omega) > 0$ for all $\omega \in (0,1]$. These properties of g_M and h are depicted in Figures 1.a and 1.b. We denote the equilibrium constant of the supply decision rule of the monopoly as $\omega_M^{Dynamic}$. It is obvious that since $\omega_M^{Dynamic} \in (0,1)$, $\alpha\delta \leq 1$ and $\nu < 1$, the value function $V_M(k)$ is bounded, for all $k > 0$ and that the momentary profits, $\left(\omega_M^{1-\frac{1}{\eta}} - \nu\omega_M\right)[f(k)]^{1-\frac{1}{\eta}}$, are always positive. Therefore, the life-time profits of the monopoly are also positive. That the momentary marginal profits, $\left[\left(1-\frac{1}{\eta}\right)\omega_M^{-\frac{1}{\eta}} - \nu\right][f(k)]^{-\frac{1}{\eta}}$, are always positive is easy to see by rearranging terms of equation (13).

Since the momentary profit function is inverse-U shaped, positive marginal profits imply that, for a given level of capital, the dynamic monopoly supplies less in each period compared to the static monopoly. It is easy to see that the supply of a static monopoly is of the form,

$$q = \omega_M^{Static} f(k) \ ,$$

where

$$\omega_M^{Static} = \begin{cases} \left(\frac{1-\frac{1}{\eta}}{\nu}\right)^{\eta} & \text{if } \nu > 1 - \frac{1}{\eta} \\ 1 & \text{if } \nu \leq 1 - \frac{1}{\eta} \end{cases} .$$

Setting marginal profit equal to zero implies that $g_M(\omega) = 0$. The two cases of the static strategies are depicted in Figures 1.a and 1.b. It is obvious that in both cases $\omega_M^{Static} > \omega_M^{Dynamic}$.

The dynamic monopoly takes into account the influence that its current supply has on the evolution of capital in the future. Supplying more in the current period reduces the capital stock in the future, so, (i) its cost per unit of output increases and, (ii) momentary profits, $\left(\omega_M^{1-\frac{1}{\eta}} - \nu\omega_M\right)[f(k)]^{1-\frac{1}{\eta}}$, are increasing in the stock of capital, so less capital in the future reduces future profits. This rationale behind the behavior of the dynamic monopolist is transparent in the monopolist's Euler equation. In particular, the first-order condition implied by (8) is,

$$D(q) + D'(q)\,q - C(k) = \delta V_M'\left(\widehat{k}\right) \ .$$

Here $\widehat{k}$ is the capital stock in the subsequent period. Applying the envelope theorem to (8) it is,

$$V_M'(k) = -C'(k)\,q + \delta V_M'\left(\widehat{k}\right) f'(k) \ .$$

Combining the last two equations yields the Euler equation,

$$D(q) + D'(q)\,q - C(k) = \delta\left\{-C'\left(\widehat{k}\right)\widehat{q} + \left[D(\widehat{q}) + D'(\widehat{q})\,\widehat{q} - C\left(\widehat{k}\right)\right] f'\left(\widehat{k}\right)\right\} \ , \tag{14}$$

where $\widehat{q}$ is the output strategy of the firm in the subsequent period. The firm counterbalances its current marginal profit with, (i) the discounted marginal increase in the cost of producing next period's quantity caused by a decrease in next period's capital and, (ii) next period's marginal profit multiplied by the marginal product of next period's capital.[7] Both future considerations on the right-hand side of the Euler equation come with a positive sign, both contributing to a reduction in current supply compared to the static case, for which the right hand side of equation (13) is zero, meaning that in the static case the future plays no role. This is captured by Figures 1.a and 1.b. It should be noted that if the constraint imposed by the capital stock is zero, then capital plays no role in the dynamics of this monopoly. However, in our specification of the model the right hand side of equation (13) is never zero i.e., $\omega_M^{Dynamic} \in (0,1)$ and thus the capital constraint is always binding.

[7] Substituting the policy $q = \omega f(k)$ into equation (14) leads, after some algebra, to the same expression as (13).

3. Two Monopolists Extracting Capital from the Same Source

In this section we look at two identical firms A and B, each selling in its own market as a monopolist, facing the same demand function given by (5), having the same cost function given by (7) and extracting capital from the same source. So, capital evolves according to,

$$k_{t+1} = f(k_t) - q_{A,t} - q_{B,t} \ , \tag{15}$$

with $f(k)$ given by (6). Compared to the monopoly problem of the previous section, the two monopolistic firms have a direct capital-accumulation interaction. We say that the presence of both firms using the same source of capital gives rise to a *dynamic externality*. The goal of this section is to study the impact of the dynamic externality on aggregate final-product supply and capital dynamics.[8]

We denote the value function of the two monopolistic firms with a direct capital-accumulation interaction as $V_{A,m}$ and $V_{B,m}$. Due to the symmetry of the problem we can focus on the problem of firm A without loss of generality. The problem of firm A in a Bellman-equation form is given by,

$$V_{A,m}(k) = \max_{q_A \geq 0} \left\{ D(q_A) q_A - C(k) q_A + \delta V_{A,m}(f(k) - q_A - Q_B(k)) \right\} \ , \tag{16}$$

where $Q_B(k)$ is the supply strategy as a function of the capital stock of firm B. The problem of firm B is given by the same Bellman equation as in (16), with the roles of A and B switched.

Using the functional forms (5), (6) and (7), and taking account of the symmetry of the two firms the model gives a decision rule of the form,

$$Q_A(k) = Q_B(k) = \omega f(k) \ , \tag{17}$$

[8] The term 'dynamic externality' was first introduced by Mirman (1979). Levhari and Mirman (1980) provide another model that offers an explicit analysis of the dynamic externality. The model of this paper enables us to study duopolistic cases in which there is also a market externality.

with $\omega \in (0,1)$. Substituting (17) into the objective of firm A gives the value function,

$$V_{A,m}(k) = \frac{\alpha\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}} k^{1-\frac{1}{\eta}} + \kappa_m \ , \tag{18}$$

where κ_m is a constant.[9] Substituting equation (18) into (16) yields,

$$V_{A,m}(k) = \max_{q_A \geq 0} \left\{ q_A^{1-\frac{1}{\eta}} - \nu [f(k)]^{-\frac{1}{\eta}} q_A + \frac{\alpha\delta\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}} [f(k) - q_A - Q_B(k)]^{1-\frac{1}{\eta}} + \delta\kappa_m \right\} , \tag{19}$$

where $f(k)$ is given by (6). The first-order condition implies,

$$\left(1-\frac{1}{\eta}\right)\left[\frac{q_A}{f(k)}\right]^{-\frac{1}{\eta}} - \nu = \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}}\left[1 - \frac{q_A}{f(k)} - \frac{Q^B(k)}{f(k)}\right]^{-\frac{1}{\eta}} . \tag{20}$$

From equation (20) the decision rules $q_A = q_B = \omega f(k)$ satisfy the necessary conditions for all $k > 0$. Substituting this decision rule into (20), yields the condition that characterizes the constant ω,

$$g_m(\chi) \equiv 2\frac{\left(1-\frac{1}{\eta}\right)2^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}} - \nu}{2^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}} - \nu} = \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\chi}{(1-\chi)^{\frac{1}{\eta}} - \alpha\delta(1-\chi)} \equiv h(\chi) \ , \tag{21}$$

where $\chi \equiv 2\omega$. We focus on χ, the aggregate fraction of $f(k)$ utilized by both firms. Note that the right-hand side of equation (21), $h(\chi)$, is the same function as the right-hand side of equation (13). This similarity helps us compare the monopolistic aggregate supply of this section with the decisions of the monopoly of the previous section.

We show that from (21), $\chi \in (0,1)$ and that the value function is bounded. Moreover, momentary profits and momentary marginal profits are positive. It is easy to see that the

[9] In particular,

$$\kappa_m = \phi\frac{\left(\omega^{1-\frac{1}{\eta}} - \nu\omega\right)\left[1-\alpha(1-\delta)(1-2\omega)^{1-\frac{1}{\eta}}\right]}{\left[1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}\right](1-\delta)} .$$

left-hand side of equation (21), $g_m(\chi)$, has $g_m(0) = 2\left(1-\frac{1}{\eta}\right)$ and $g_m(1) = 2\left[1 - \frac{2^{\frac{1}{\eta}}}{\eta\left(2^{\frac{1}{\eta}}-\nu\right)}\right]$, whereas $g_m'(\chi) = -\frac{\nu}{\eta^2}\frac{2^{\frac{1}{\eta}+1}\chi^{-\frac{1}{\eta}-1}}{\left[2^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu\right]^2} < 0$.

The aggregate supply of two static monopolies extracting from the same source of capital is of the form,

$$q = \chi_m^{Static} f(k) \ ,$$

where

$$\chi_m^{Static} = \begin{cases} \left(\frac{1-\frac{1}{\eta}}{\nu}\right)^{\eta} & \text{if } \nu > 2^{\frac{1}{\eta}}\left(1-\frac{1}{\eta}\right) \\ 1 & \text{if } \nu \leq 2^{\frac{1}{\eta}}\left(1-\frac{1}{\eta}\right) \end{cases} .$$

In the case $\nu > 2^{\frac{1}{\eta}}\left(1-\frac{1}{\eta}\right)$, $g_m\left(\chi_m^{Static}\right) = 0$. All cases are depicted in Figures 2.a and 2.b, in which it is obvious that $\chi_m^{Dynamic} \in (0,1)$, whereas $\chi_m^{Dynamic} < \chi_m^{Static}$. Following the same reasoning as in the pure dynamic monopoly case, we see that the value function of each firm is bounded, the momentary (and the infinite-horizon) profits and the momentary marginal profits are positive, for all $k > 0$.

3.1 Impact of the dynamic externality on aggregate supply and capital dynamics

Since the model of this section contains the dynamic externality, a direct comparison with the benchmark dynamic monopoly of the previous section enables us to evaluate the influence of the dynamic externality on equilibrium strategies.

The first-order condition implied by (16) is,

$$D(q_A) + D'(q_A)\, q_A - C(k) = \delta V_{A,m}'\left(\widehat{k}\right) \ .$$

Recall that $\widehat{k}$ is the capital stock in the next period. Applying the envelope theorem to (16),

$$V_{A,m}'(k) = -C'(k)\, q_A + \delta V_{A,m}'\left(\widehat{k}\right)\left[f'(k) - Q_B'(k)\right] \ .$$

Combining the last two equations yields the Euler equation,

$$D(q_A)+D'(q_A)\,q_A-C(k)=\delta\left\{-C'\left(\widehat{k}\right)\widehat{q}_A+\left[D(\widehat{q}_A)+D'(\widehat{q}_A)\,\widehat{q}_A-C\left(\widehat{k}\right)\right]\left[f'\left(\widehat{k}\right)-Q_B'\left(\widehat{k}\right)\right]\right\}\ . \tag{22}$$

Where $\widehat{q}_A$ is the output strategy of firm A in the next period. Note that the dynamic externality appears in the Euler equation (22) embodied in the term $Q_B'\left(\widehat{k}\right)$. This means that this firm must take account of the effect that the other firm has on any future investments. From (22), the firm counterbalances its current marginal profit with, (i) the discounted marginal increase in the cost of producing next period's quantity caused by a decrease in next period's capital and (ii) next period's marginal profit multiplied by *the difference between the marginal product of next period's capital from the marginal change in next period's supply of the other firm due to a change in next period's capital*, i.e., the dynamic externality.[10] In equilibrium, $f'\left(\widehat{k}\right)-Q_B'\left(\widehat{k}\right)=\left(1-\omega_m^{Dynamic}\right)f'\left(\widehat{k}\right)$,which is positive for all $\widehat{k}>0$.[11] Both future considerations on the right-hand side of the Euler equation come with a positive sign, both contributing to a reduction in current supply compared to the static case. This is captured by Figures 2.a and 2.b.

We compare the total current extraction in the two cases, namely $\chi_m^{Dynamic}\equiv 2\omega_m^{Dynamic}$ with $\chi_M^{Dynamic}\equiv\omega_M^{Dynamic}$.

Let

$$G(\chi,N)\equiv N\frac{\left(1-\frac{1}{\eta}\right)N^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu}{N^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu}\ .$$

Then,

$$G(\chi,1)=g_M(\chi)\quad\text{and}\quad G(\chi,2)=g_m(\chi)\ ,$$

[10]Substituting the policies $q_A=q_B=\omega f(k)$ into equation (22) leads, after some algebra, to the same expression as (21).

[11]Because the strategies are linear in $f(k)$, similar to the strategies in Levhari and Mirman (1980), we know that the strategies of both firms are global maxima of each firm's value function (which is concave for all $k>0$). Mirman (1979) presents examples of difficulties that may arise if the strategy $Q^B(k)$ of the other firm were concave. The concavity of the value function of firm A is not guaranteed.

whereas,

$$\frac{\partial G(\chi,N)}{\partial N}=\frac{\left(1-\frac{1}{\eta}\right)N^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu}{N^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu}+\frac{\nu}{\eta^2}\frac{N^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}}{\left(N^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu\right)^2}>0 \quad \text{for all } N\geq 1.$$

So, for all $\chi\in(0,1)$ such that $\left(1-\frac{1}{\eta}\right)\chi^{-\frac{1}{\eta}}-\nu>0$ (which also implies that $\left(1-\frac{1}{\eta}\right)2^{\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}-\nu>0$), it is,

$$g_m(\chi)>g_M(\chi)\ .$$

The latter inequality,

$$\chi_m^{Dynamic}>\chi_M^{Dynamic}\ ,$$

is depicted in Figure 3, i.e. total extraction is higher when two monopolies extract capital from the same source and: (i) if $\phi=0$ and $\alpha\in\left[1,\frac{1}{\delta}\right]$ the growth rate of the market of the single monopoly is forever higher; and (ii) if $\phi>0$ and $\alpha\in(0,1)$, the single monopoly reaches a higher steady state.

To sum up, the dynamic externality makes firms increase their aggregate supply in each period. Although firms take account of the fact that more extraction reduces capital in the future and, therefore, reduces future profits, both firms extract more capital in aggregate terms in each period, reducing the growth of capital.

4. Duopoly with Firms Extracting Capital from the Same Source

In this section both firms extract capital from the same source, and sell outputs in the same market. Selling in the same market yields a 'market' externality as well as a 'dynamic' externality. We study the impact of the market externality, *while the dynamic externality is present*, by comparing the equilibrium of this section with the equilibrium of the previous sections. It should be noted that, as in the static case, a 'static' market externality is always present in the duopoly solution. In the dynamic case, studied in this section, there is also a 'dynamic' market externality. Both of these externalities have an effect on the equilibrium solution.

We look at two identical firms, A and B, that sell in the same market and extract capital from the same source. The demand function is given by (5). Both firms have the same cost function given by (7). So, capital evolves according to:

$$k_{t+1} = f(k_t) - q_{A,t} - q_{B,t} \ , \tag{23}$$

with $f(k)$ given by (6). We denote the value function of the duopolistic firms with a direct capital-accumulation interaction as $V_{A,d}$ and $V_{B,d}$. Due to the symmetry of the problem we can focus on the problem of firm A without loss of generality. The problem of firm A in a Bellman-equation form is given by,

$$V_{A,d}(k) = \max_{q_A \geq 0} \left\{ D\left(q_A + Q_B(k)\right) q_A - C(k)\, q_A + \delta V_{A,d}\left(f(k) - q_A - Q_B(k)\right) \right\} \ , \tag{24}$$

where $Q_B(k)$ is the supply strategy of firm B. The problem of firm B is given by the Bellman equation as in (24), except that A and B are switched.

Note that, the effect of firm B on the decisions of firm A appears in two places. These are the sources of all the externalities. Compared to the monopoly problem of the previous section, the two duopolistic firms have a direct market interaction. We say that the presence of the other firm in the same market causes a *market externality.*[12] In this section we study the impact of the market externality on aggregate final-product supply and capital dynamics.

Due to the functional forms (5), (6) and (7), and the symmetry of the two firms the model gives a decision rule of the form,

$$Q_A(k) = Q_B(k) = \omega f(k) \ , \tag{25}$$

[12]The term 'market externality' was also used by Mirman (1979).

with $\omega \in (0,1)$. Substituting (25) into the objective of firm A gives the following value function,

$$V_{A,d}(k) = \frac{\alpha\left(2^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}} k^{1-\frac{1}{\eta}} + \kappa_d \ , \tag{26}$$

where κ_d is a constant.[13] Substituting equation (26) into (24) yields,

$$\begin{aligned} V_{A,d}(k) &= \max_{q_A \geq 0} \left\{ [q_A + Q_B(k)]^{1-\frac{1}{\eta}} - \nu\,[f(k)]^{-\frac{1}{\eta}} q_A + \right. \\ &\left. + \frac{\alpha\delta\left(2^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}} [f(k) - q_A - Q_B(k)]^{1-\frac{1}{\eta}} + \delta\kappa_d \right\} \ , \end{aligned} \tag{27}$$

where $f(k)$ is given by (6). The first-order condition implies,

$$\begin{aligned} &\left[\frac{q_A}{f(k)} + \frac{Q_B(k)}{f(k)}\right]^{-\frac{1}{\eta}} - \frac{1}{\eta}\left[\frac{q_A}{f(k)} + \frac{Q_B(k)}{f(k)}\right]^{-\frac{1}{\eta}-1} \frac{q_A}{f(k)} - \nu = \\ &= \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\left(2^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega\right)}{1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}}\left[1 - \frac{q_A}{f(k)} - \frac{Q_B(k)}{f(k)}\right]^{-\frac{1}{\eta}} \ . \end{aligned} \tag{28}$$

From equation (28) the decision rules $q_A = q_B = \omega f(k)$, as it is implied by (25), satisfies the necessary condition, for all $k > 0$. Substituting this decision rule into (28), we arrive at the relationship that gives the condition from which we can characterize the constant ω,

$$g_d(\chi) \equiv 2\frac{\left(1-\frac{1}{2\eta}\right)\chi^{-\frac{1}{\eta}} - \nu}{\chi^{-\frac{1}{\eta}} - \nu} = \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\chi}{(1-\chi)^{\frac{1}{\eta}} - \alpha\delta(1-\chi)} \equiv h(\chi) \ , \tag{29}$$

where $\chi \equiv 2\omega$. We focus on χ, the aggregate fraction of $f(k)$ extracted by both firms. We stress that the right-hand side of equation (29), $h(\chi)$ is the same function as the right-hand side of equation (13). This helps us in comparing the duopolistic aggregate supply of this section with the decisions of the two previous sections.

[13]In particular,

$$\kappa_d = \phi\frac{\left(2^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega\right)\left[1-\alpha(1-\delta)(1-2\omega)^{1-\frac{1}{\eta}}\right]}{\left[1-\alpha\delta(1-2\omega)^{1-\frac{1}{\eta}}\right](1-\delta)} \ .$$

We show that the constant χ that satisfies (29) is such that, $\chi \in (0,1)$. Also, the value function is bounded, momentary profits and momentary marginal profits are always positive. It is easy to see that the left-hand side of equation (29), $g_d(0) = 2\left(1 - \frac{1}{2\eta}\right)$ and $g_d(1) = 2\left[1 - \frac{1}{2\eta(1-\nu)}\right]$, whereas $g_d'(\chi) = -\frac{\nu}{\eta^2}\frac{\chi^{-\frac{1}{\eta}-1}}{\left(\chi^{-\frac{1}{\eta}} - \nu\right)^2} < 0$.

The aggregate supply of a static duopoly extracting capital from the same source is,

$$q = \chi_d^{Static} f(k) \ ,$$

where,

$$\chi_d^{Static} = \begin{cases} \left(\frac{1-\frac{1}{2\eta}}{\nu}\right)^{\eta} & \text{if } \nu > 1 - \frac{1}{2\eta} \\ 1 & \text{if } \nu \leq 1 - \frac{1}{2\eta} \end{cases} .$$

In the case $\nu > 1 - \frac{1}{2\eta}$, $g_d\left(\chi_d^{Static}\right) = 0$. All cases are depicted in Figures 4.a and 4.b, in which $\chi_d^{Dynamic} \in (0,1)$, whereas $\chi_d^{Dynamic} < \chi_d^{Static}$. Following the same reasoning as in the pure dynamic monopoly case, the value function of each firm is bounded, the momentary (and the infinite-horizon) profits and the momentary marginal profits are positive, for all $k > 0$.

4.1 Impact of the dynamic market externality on aggregate supply and capital dynamics

In this section we analyze the effect of the market externality on the equilibrium outputs when the dynamic externality is present. We compare the equilibrium strategies of the model of this section with those of the previous section.

The first-order condition implied by (24) is,

$$D(q_A + Q_B(k)) + D'(q_A + Q_B(k))\, q_A - C(k) = \delta V_{A,d}'\left(\widehat{k}\right) .$$

Applying the envelope theorem to (24) yields,

$$V_{A,d}'(k) = D'(q_A + Q_B(k))\, q_A Q_B'(k) - C'(k)\, q_A + \delta V_{A,d}'\left(\widehat{k}\right)\left[f'(k) - Q_B'(k)\right] .$$

Combining the last two equations yields the Euler equation,

$$D\left(q_A+Q_B\left(k\right)\right)+D'\left(q_A+Q_B\left(k\right)\right)q_A-C\left(k\right)=\delta\left\{D'\left(\widehat{q}_A+Q_B\left(k\right)\right)\widehat{q}_AQ'_B\left(\widehat{k}\right)-\right.$$

$$\left.-C'\left(\widehat{k}\right)\widehat{q}_A+\left[D\left(\widehat{q}_A\right)+D'\left(\widehat{q}_A\right)\widehat{q}_A-C\left(\widehat{k}\right)\right]\left[f'\left(\widehat{k}\right)-Q'_B\left(\widehat{k}\right)\right]\right\}\ . \quad (30)$$

Note that, as in the previous section, the dynamic externality is embodied in the term, $Q'_B\left(\widehat{k}\right)$, appearing at the end of the right hand side of equation (30). However, the market externalities, both 'static' and 'dynamic', are now also apparent. In equation (30), the term, $Q_B\left(k\right)$, is the usual 'static' duopoly market externality while the term, $Q'_B\left(\widehat{k}\right)$, but this time appearing in the first term of the right hand side of (30), is the 'dynamic' market externality. It should be noted that the effect of these externalities can not be isolated. The firm counterbalances its current marginal profit with: (i) the marginal change in next period's price multiplied by next period's supplied quantity of the firm times the marginal change in next period's supply of the other firm due to a change in next period's capital *(dynamic market externality);* (ii) the discounted marginal increase in the cost of producing next period's quantity caused by a decrease in next period's capital; and (iii) next period's marginal profit multiplied by *the difference between the marginal product of next period's capital from the marginal change in next period's supply of the other firm due to a change in next period's capital (dynamic externality).*[14]

Note, finally, that in equilibrium, $f'\left(\widehat{k}\right)-Q'_B\left(\widehat{k}\right)=\left(1-\omega_d^{Dynamic}\right)f'\left(\widehat{k}\right)$, which is positive for all $\widehat{k}>0$, yet the term $D'\left(\widehat{q}_A+Q_B\left(\widehat{k}\right)\right)\widehat{q}_AQ'_B\left(\widehat{k}\right)$ is negative. It is interesting to note that this dynamic equilibrium condition looks strikingly similar to the equilibrium condition of the previous section. However, they are different. The difference is due to the market externalities, which affects the equilibrium strategies of each firm. Since the

[14]Substituting the policies $q_A=q_B=\omega f\left(k\right)$ into equation (30) leads, after some algebra, to the same expression as (29).

strategies of each firm changed, the equilibrium steady state is also changed. In equilibrium, the right-hand side of equation (30) is positive, as it is implied by Figures 4.a and 4.b. As we proved above: $\chi_d^{Dynamic} < \chi_m^{Static}$. The momentary profit function is again inverse-U shaped, so $\chi_d^{Dynamic} < \chi_m^{Static}$, in equilibrium implies that momentary profits are always positive in the dynamic case, hence the right-hand side of equation (30) is also positive, *the market externality is dominated by the dynamic externality in equilibrium.*

We compare the total extraction in the monopolistic versus the duopolistic markets in which both firms extract capital from the same source, namely $\chi_m^{Dynamic} \equiv 2\omega_m^{Dynamic}$ with $\chi_d^{Dynamic} \equiv 2\omega_d^{Dynamic}$. For the comparison, we focus on the interval of χ for which, in both cases, the momentary profits are positive, namely,

$$\left(1 - \frac{1}{\eta}\right) 2^{\frac{1}{\eta}} \chi^{-\frac{1}{\eta}} - \nu > 0 \ , \tag{31}$$

and

$$\left(1 - \frac{1}{2\eta}\right) \chi^{-\frac{1}{\eta}} - \nu > 0 \ . \tag{32}$$

i.e.,

$$\chi < \max \left\{ 2 \left(\frac{1 - \frac{1}{\eta}}{\nu} \right)^{\eta}, \left(\frac{1 - \frac{1}{2\eta}}{\nu} \right)^{\eta} \right\} \ . \tag{33}$$

Noticing that,

$$g_m(0) = 2\left(1 - \frac{1}{\eta}\right) < 2\left(1 - \frac{1}{2\eta}\right) = g^d(0) \ ,$$

we show that $g_m'(\chi) < g_d'(\chi)$ in the interval given by (33). It is easy to verify that

$$g_m'(\chi) < g_d'(\chi) \Leftrightarrow 2^{\frac{1}{\eta}} \frac{2^{\frac{1}{2}\left(1-\frac{1}{\eta}\right)} - 1}{2^{\frac{1}{2}\left(1+\frac{1}{\eta}\right)} - 1} \chi^{-\frac{1}{\eta}} - \nu > 0 \ ,$$

or

$$g_m'(\chi) < g_d'(\chi) \Leftrightarrow \chi < \left[\frac{2^{\frac{1}{\eta}}}{\nu} \frac{2^{\frac{1}{2}\left(1-\frac{1}{\eta}\right)} - 1}{2^{\frac{1}{2}\left(1+\frac{1}{\eta}\right)} - 1} \right]^{\eta} \ .$$

It is easy to show that,

$$\left[\frac{2^{\frac{1}{\eta}}}{\nu}\frac{2^{\frac{1}{2}\left(1-\frac{1}{\eta}\right)}-1}{2^{\frac{1}{2}\left(1+\frac{1}{\eta}\right)}-1}\right]^{\eta}>\max\left\{2\left(\frac{1-\frac{1}{\eta}}{\nu}\right)^{\eta},\left(\frac{1-\frac{1}{2\eta}}{\nu}\right)^{\eta}\right\},$$

so (33) implies that for all values of χ such that momentary marginal profits are strictly positive, $g_m'(\chi) < g_d'(\chi)$. Combining this last inequality with the fact that $g_m'(0) < g_d'(0)$, we can see from Figure 5 that

$$\chi_m^{Dynamic} < \chi_d^{Dynamic}.$$

4.2 Dynamics and steady states

So far we have concluded that,

$$\chi_M^{Dynamic} < \chi_m^{Dynamic} < \chi_d^{Dynamic}.$$

Since the law of motion of capital in equilibrium is,

$$k_{t+1} = (1-\chi) f(k_t).$$

In all three cases capital grows faster for the monopoly and slower for the duopoly when both firms extract from the same source of capital. The growth rate of capital in the case of two monopolies extracting capital from the same source is in-between. In Figure 6 we depict the dynamics and the steady states if $\phi > 0$ and $\alpha \in (0,1)$. For the case of $\phi = 0$ and $\alpha \in \left[1, \frac{1}{\delta}\right]$, there may be perpetual positive growth or gradual shrinkage of capital to zero in the long run.

In brief, *the dynamic externality reduces capital growth and when both the dynamic and the market externality are present, capital growth falls even more.*

It remains to see the case of a duopoly with firms extracting capital from different sources. In this case we are able to isolate the market externality and compare the dynamics with those of a pure monopoly or with the dynamics of a duopoly with both the market and the dynamic externality present.

5. Duopoly with firms extracting capital from different sources

The goal of this section is to isolate the impact of the market externality on the supply behavior of a firm that operates in a dynamic environment. In order to achieve this, we depart from the benchmark monopoly case by adding one more firm in the market, but not in the source of capital that the firm utilizes. In this way we evaluate the impact of the market externality, *while the dynamic externality is not present.*

We examine the behavior of two identical firms, A and B, each utilizing capital from their own, separate capital source. So, we distinguish between two stocks of capital, k_A and k_B, and assume that the initial capital stocks are equal, i.e. it is $k_{A,0} = k_{B,0} > 0$. The demand function is given by (5). Both firms have the same cost function given by (7). So, the capital stocks evolve according to,

$$k_{A,t+1} = f\left(k_{A,t}\right) - q_{A,t}\ , \tag{34}$$

$$k_{B,t+1} = f\left(k_{B,t}\right) - q_{B,t}\ , \tag{35}$$

with $f(k)$ given by (6) with $\phi = 0$ and $\alpha \in \left[1, \frac{1}{\delta}\right]$. The analysis of this section, leading again to strategies of the form $Q(k) = \omega f(k)$, is possible only for these values of parameters α and ϕ, namely the case of linear production of capital.[15] So, comparisons of aggregate supply

[15] To see why it must be that $\phi = 0$ and $\alpha \in \left[1, \frac{1}{\delta}\right]$, we calculate the value function of firm A for the general case of $\phi \geq 0$, using $Q_A(k_A, k_B) = \omega f(k_A)$ and $Q_B(k_A, k_B) = \omega f(k_B)$. The resulting value function is,

$$V_{A,D}(k_A, k_B) = \frac{\alpha\left[\omega^{1-\frac{1}{\eta}}\left(k_A + k_B\right)^{-\frac{1}{\eta}} k_A - \nu\omega k_A^{1-\frac{1}{\eta}}\right]}{1 - \alpha\delta\left(1-\omega\right)^{1-\frac{1}{\eta}}} + \frac{\phi\delta\left[\omega^{1-\frac{1}{\eta}}\left(1 + \frac{f(k_B)}{f(k_A)}\right)^{-\frac{1}{\eta}} k_A - \nu\omega\right]}{\left(1-\delta\right)\left[1 - \alpha\delta\left(1-\omega\right)^{1-\frac{1}{\eta}}\right]}\ .$$

Without $\phi = 0$, this value function does not validate the strategies $Q_A(k_A, k_B) = \omega f(k_A)$ and $Q_B(k_A, k_B) = \omega f(k_B)$.

in this setup with the benchmark monopoly or the other setups is restricted to the case of a linear function $f(k) = \alpha^{\frac{\eta}{\eta-1}} k$.

We denote the value function of the duopolistic firms without a direct capital-accumulation interaction, without a dynamic externality, as $V_{A,D}$ and $V_{B,D}$. These value functions depend on both capital stocks, (k_A, k_B). Due to the symmetry of the problem we can focus on the problem of firm A without loss of generality. The problem of the firm A in a Bellman-equation form is given by,

$$\begin{aligned} V_{A,D}(k_A, k_B) &= \max_{q_A \geq 0} \{ D(q_A + Q_B(k_A, k_B)) q_A - C(k_A) q_A + \\ &\quad + \delta V_{A,D}(f(k_A) - q_A \;,\; f(k_B) - Q_B(k_A, k_B)) \} \;, \end{aligned} \tag{36}$$

where $Q_B(k_A, k_B)$ is the supply strategy of firm B. The problem of firm B is given by switching A and B in the Bellman equation (36).

Compared to the monopoly problem of the first section, the two duopolistic firms have a direct market interaction. The presence of the other firm in the same market causes a 'static' *market externality*, as discussed in the previous section.

Using the functional forms (5), (6) and (7), together with the symmetry of the two firms the model gives decision rules of the form,

$$Q_A(k_A, k_B) = \omega f(k_A) \;, \tag{37}$$

$$Q_A(k_A, k_B) = \omega f(k_B) \;, \tag{38}$$

with $\omega \in (0, 1)$. Substituting (37) and (38) into the objective of firm A gives the following value function,

$$V_{A,D}(k_A, k_B) = \frac{\alpha \left[\omega^{1-\frac{1}{\eta}} (k_A + k_B)^{-\frac{1}{\eta}} k_A - \nu \omega k_A^{1-\frac{1}{\eta}} \right]}{1 - \alpha \delta (1 - \omega)^{1-\frac{1}{\eta}}} \;. \tag{39}$$

Substituting equation (39) into (36) yields,

$$V_{A,D}\left(k_A,k_B\right)=\max_{q_A\geq 0}\left\{\left[q_A+Q_B\left(k_A,k_B\right)\right]^{1-\frac{1}{\eta}}-\nu\left[f\left(k_A\right)\right]^{-\frac{1}{\eta}}q_A+\right.$$

$$\left.+\frac{\alpha\delta\left\{\omega^{1-\frac{1}{\eta}}\left[f\left(k_A\right)-q_A+f\left(k_B\right)-Q_B\left(k_A,k_B\right)\right]^{-\frac{1}{\eta}}\left(f\left(k_A\right)-q_A\right)-\nu\omega\left(f\left(k_A\right)-q_A\right)^{1-\frac{1}{\eta}}\right\}}{1-\alpha\delta\left(1-\omega\right)^{1-\frac{1}{\eta}}}\right\}, \tag{40}$$

where $f(k)$ is given by (6). The first-order condition implies,

$$\left[q_A+Q_B\left(k_A,k_B\right)\right]^{-\frac{1}{\eta}}\left[1-\frac{1}{\eta}\frac{q_A}{q_A+Q_B\left(k_A,k_B\right)}\right]-\nu\left[f\left(k_A\right)\right]^{-\frac{1}{\eta}}=$$

$$=\frac{\alpha\delta\left[\omega^{1-\frac{1}{\eta}}\left(\widehat{k}_A+\widehat{k}_B\right)^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta}\frac{\widehat{k}_A}{\widehat{k}_A+\widehat{k}_B}\right)-\left(1-\frac{1}{\eta}\right)\nu\omega\left(\widehat{k}_A\right)^{-\frac{1}{\eta}}\right]}{1-\alpha\delta\left(1-\omega\right)^{1-\frac{1}{\eta}}}\ . \tag{41}$$

Substituting the decision rules $q_A=\omega f\left(k_A\right)$, $q_B=\omega f\left(k\right)$, together with (34) and (35) into (41) we obtain the condition from which we can characterize the constant ω,

$$g_D\left(\omega\right)\equiv\frac{2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)\omega^{-\frac{1}{\eta}}-\nu}{\frac{2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)}{1-\frac{1}{\eta}}\omega^{-\frac{1}{\eta}}-\nu}=\frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\omega}{\left(1-\omega\right)^{\frac{1}{\eta}}-\alpha\delta\left(1-\omega\right)}\equiv h\left(\omega\right)\ . \tag{42}$$

We show that, $\omega\in(0,1)$ and that the value function is bounded, momentary profits and momentary marginal profits are always positive. It is easy to see that the left-hand side of equation (42), $g_D\left(\omega\right)$, has $g_D\left(0\right)=1-\frac{1}{\eta}$ and $g_D\left(1\right)=\left(1-\frac{1}{\eta}\right)\frac{2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)-\nu}{2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)-\nu\left(1-\frac{1}{\eta}\right)}$, whereas,

$$g_D'\left(\omega\right)=-\frac{\nu}{\eta\left(\eta-1\right)}\frac{2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)\omega^{-\frac{1}{\eta}-1}}{\left[\frac{2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)}{1-\frac{1}{\eta}}\omega^{-\frac{1}{\eta}}-\nu\right]^2}\ .$$

The supply of a static duopoly extracting capital from the same source is of the form,

$$q=\omega_D^{Static}f\left(k\right)\ ,$$

where

$$\omega_D^{Static}=\begin{cases}\frac{1}{2}\left(\frac{1-\frac{1}{2\eta}}{\nu}\right)^{\eta} & \text{if } \nu>2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)\\ 1 & \text{if } \nu\leq 2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)\end{cases}\ .$$

In the case $\nu > 2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)$, $g_D\left(\omega_D^{Static}\right)=0$. All cases are depicted in Figures 7.a and 7.b, from which it is obvious that $\omega_D^{Dynamic}\in(0,1)$, whereas $\omega_D^{Dynamic}<\omega_D^{Static}$. Following the same reasoning as in the dynamic monopoly case, we see that the value function of each firm is bounded, the momentary (and the infinite-horizon) profits and the momentary marginal profits are positive, for all $k>0$.

5.1 Impact of the static market externality on firm supply and capital dynamics

The first-order conditions implied by (36) are,

$$D\left(q_A+Q_B\left(k_A,k_B\right)\right)+D'\left(q_A+Q_B\left(k_A,k_B\right)\right)q_A-C\left(k_A\right)=\delta V'_{A,D}\left(\widehat{k}_A,\widehat{k}_B\right)\ .$$

Applying the envelope theorem on (36) yields,

$$\begin{aligned}\frac{\partial V_{A,D}\left(k_A,k_B\right)}{\partial k_A}&=D'\left(q_A+Q_B\left(k_A,k_B\right)\right)q_A\frac{\partial Q_B\left(k_A,k_B\right)}{\partial k_A}-C'\left(k_A\right)q_A+\\&+\delta\left[\frac{\partial V_{A,D}\left(\widehat{k}_A,\widehat{k}_B\right)}{\partial\widehat{k}_A}f'\left(k_A\right)-\frac{\partial V_{A,D}\left(\widehat{k}_A,\widehat{k}_B\right)}{\partial\widehat{k}_B}\frac{\partial Q_B\left(k_A,k_B\right)}{\partial k_A}\right].\end{aligned}$$

Combining the last two equations yields the necessary condition,

$$\begin{aligned}&D\left(q_A+Q_B\left(k_A,k_B\right)\right)+D'\left(q_A+Q_B\left(k_A,k_B\right)\right)q_A-C\left(k_A\right)=\\&=\delta\Big\{-C'\left(\widehat{k}_A\right)\widehat{q}_A+\left[D\left(\widehat{q}_A+Q_B\left(\widehat{k}_A,\widehat{k}_B\right)\right)+D'\left(\widehat{q}_A+Q_B\left(\widehat{k}_A,\widehat{k}_B\right)\right)\widehat{q}_A-C\left(\widehat{k}_A\right)\right]f'\left(\widehat{k}_A\right)+\\&+\left[D'\left(\widehat{q}_A+Q_B\left(\widehat{k}_A,\widehat{k}_B\right)\right)\widehat{q}_A-\delta\frac{\partial V_{A,D}\left(\widehat{\widehat{k}}_A,\widehat{\widehat{k}}_B\right)}{\partial\widehat{\widehat{k}}_B}\right]\frac{\partial Q_B\left(\widehat{k}_A,\widehat{k}_B\right)}{\partial\widehat{k}_A}\Big\},\end{aligned}\tag{43}$$

where $\widehat{\widehat{k}}$ is the capital stock two periods ahead. The necessary optimal condition of firm B is given by the same equation as (43), except that A and B are switched. Note that the only externality appearing in equation (43) is the market externality, i.e., there is no dynamic externality in this model. The firm counterbalances its current marginal profit with: (i) the discounted marginal increase in the cost of producing next period's quantity caused by a decrease in next period's capital; (ii) next period's marginal profit multiplied by the marginal product of next period's capital period's capital; and (iii) the difference in the marginal change in next period's price multiplied by next period's supplied quantity of the firm, from the discounted marginal change in the value function caused by a change in the capital stock of the other firm two periods ahead, times the marginal change in next period's supply of the other firm due to a change in next period's capital.[16] The last term of the right-hand side of equation (43) vanishes, since, in equilibrium $Q_B(k_A, k_B) = \omega f(k_B)$, so $\frac{\partial Q_B(k_A,k_B)}{\partial k_A} = 0$.

Comparing growth in the cases of this duopoly setup and the pure monopoly, reveals the impact of the static market externality on industry growth. The right-hand sides of equations (42) and (13) are the same. Moreover, after some algebra, it is,

$$g_M(\omega) < g_D(\omega) \Longleftrightarrow 2^{-\frac{1}{\eta}}\left(1 - \frac{1}{2\eta}\right) > 1 - \frac{1}{\eta}\ .$$

Noticing that $2^{-\frac{1}{\eta}}\left(1 - \frac{1}{2\eta}\right) > 1 - \frac{1}{\eta}$ if and only if $\eta < 2.73$, figures 8.a and 8.b depict that

$$\omega_M < \omega_D \Longleftrightarrow \eta < 2.73\ .$$

For all η's, both in the static and the dynamic case, the total fraction of resources extracted is higher in the duopolistic setup. Yet, the interesting difference in the dynamic setup is that for $\eta < 2.73$ each duopolistic firm utilizes a higher fraction of their capital resources compared to being a monopolist. So, for $\eta < 2.73$, the growth rate of capital is lower in the duopolistic setup compared to the growth rate of a monopoly. The opposite holds if

[16]Substituting the policies $Q^A(k_A, k_B) = \omega f(k_A)$, $Q^B(k_A, k_B) = \omega f(k_B)$ into equation (43) leads, after some algebra, to the same expression as (42).

$\eta > 2.73$. Thus, if $\eta > 2.73$, duopolies can make higher profits in the long run compared to the monopolist, despite the fact that duopolists share the market.

To sum up, the equilibrium industry dynamics change the influence of the 'static' market externality. In particular, *the dynamic externality dominates the market externality.*

REFERENCES

Dutta, Prajit K. and Rangarajan K. Sundaram (1992): "Markovian Equilibrium in a Class of Stochastic Games: Existence Theorems for Discounted and Undiscounted Models," Economic Theory, Vol. 2(2).

Dutta, Prajit K. and Rangarajan K. Sundaram (1993): "How Different Can Strategic Models Be?" Journal of Economic Theory, Vol. 60, pp. 42-61.

Ericson, Richard and Ariel Pakes (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," The Review of Economic Studies, Vol 62(1), pp. 53-82.

Gaudet, Gerhard and Stephen W. Salant (1991): "Uniqueness of Cournot Equilibrium: New Results from Old Methods," The Review of Economic Studies, Volume 58(2), pp. 399-404.

Greenwood, Jeremy, Zvi Hercowitz and Gregory Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," The American Economic Review, Vol. 78(3), pp. 402-417.

Koulovatianos, Christos and Leonard J. Mirman (2003): "R&D Investment, Market Structure, and Industry Growth," Mimeo, University of Virginia.

Levhari, David and Leonard J. Mirman (1980): "The Great Fish War: an Example using a Dynamic Cournot-Nash Solution," The Bell Journal of Economics, Volume 11, Issue 1, pp. 322-334.

Mirman, Leonard J. (1979): "Dynamic Models of Fishing: A Heuristic Approach," Control Theory in Mathematical Economics, Liu and Sutinen, Eds.

Novshek, William (1984a): "Finding All n-firm Cournot Equilibria," International Economic Review, Vol. 25, pp. 62-70.

Novshek, William (1984b): "Perfectly Competitive Markets as the Limits of Cournot Markets," Journal of Economic Theory, Vol. 34, pp. 72-82.

Novshek, William (1985): “On the Existence of Cournot Equilibrium,” The Review of Economic Studies, Volume 52(1), pp. 85-98.

Pakes, Ariel and Paul McGuire (2001): “Stochastic Algorithms, Symmetric Markov-Perfect Equilibria, and the ‘Curse’ of Dimensionality,” Econometrica, Vol. 69(5), pp. 1261-81.

Vedenov, Dmitry V. and Mario J. Miranda (2001): “Numerical Solution of Dynamic Oligopoly Games with Capital Investment,” Economic Theory, Vol. 18 pp.237-261.

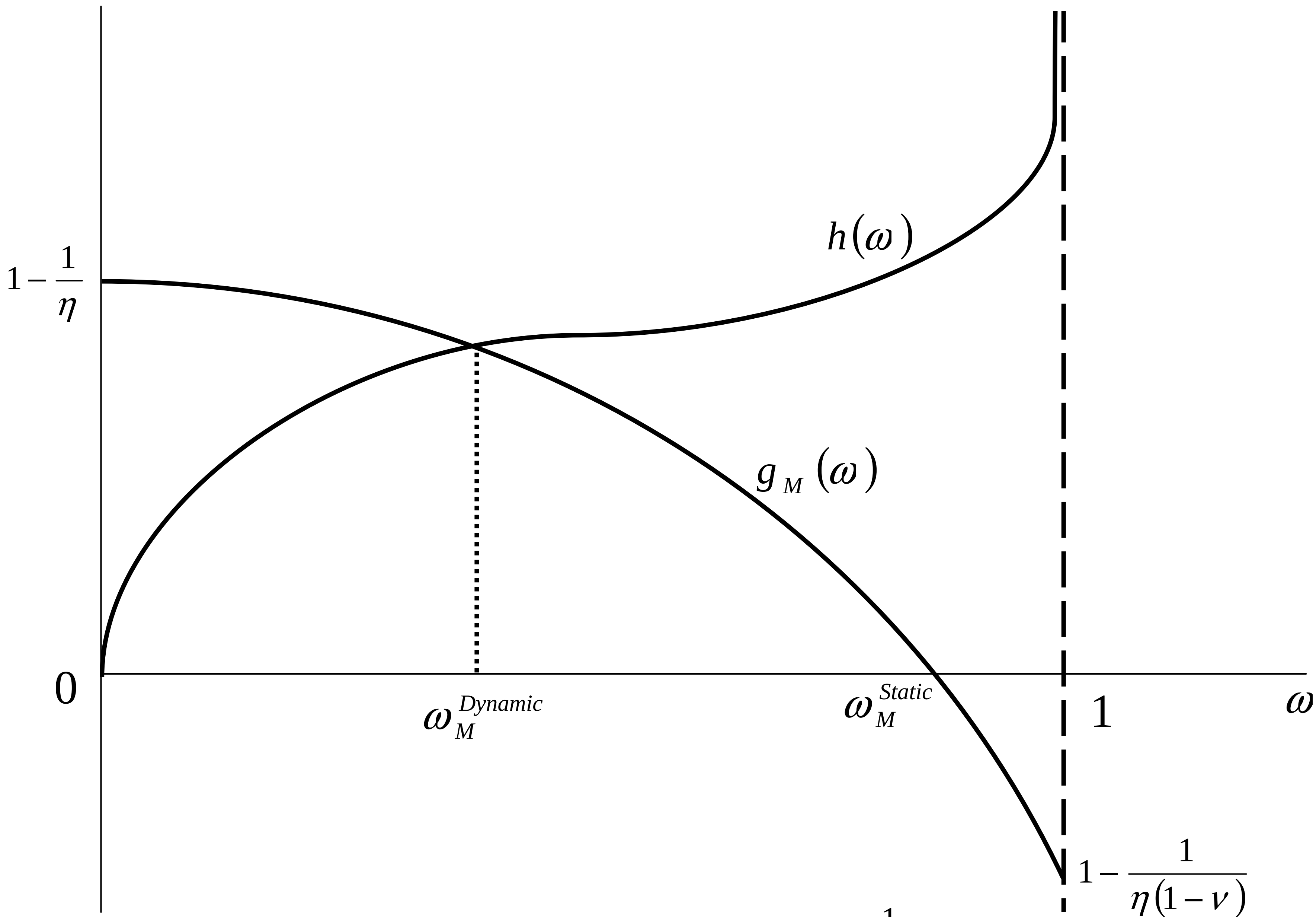


Figure 1.a Case where $\nu > 1-\frac{1}{\eta}$

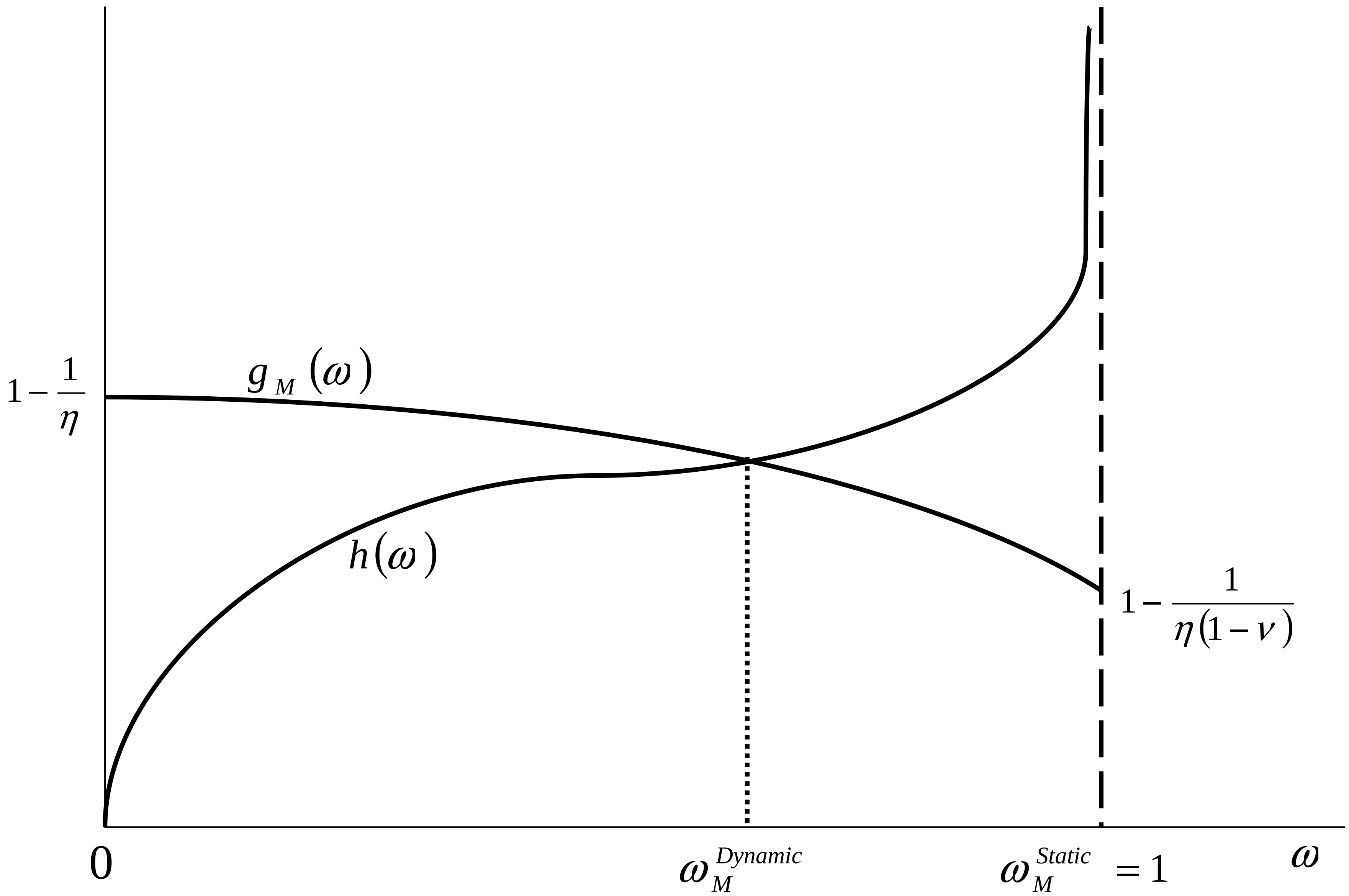


Figure 1.b Case where $\nu \leq 1-\frac{1}{\eta}$

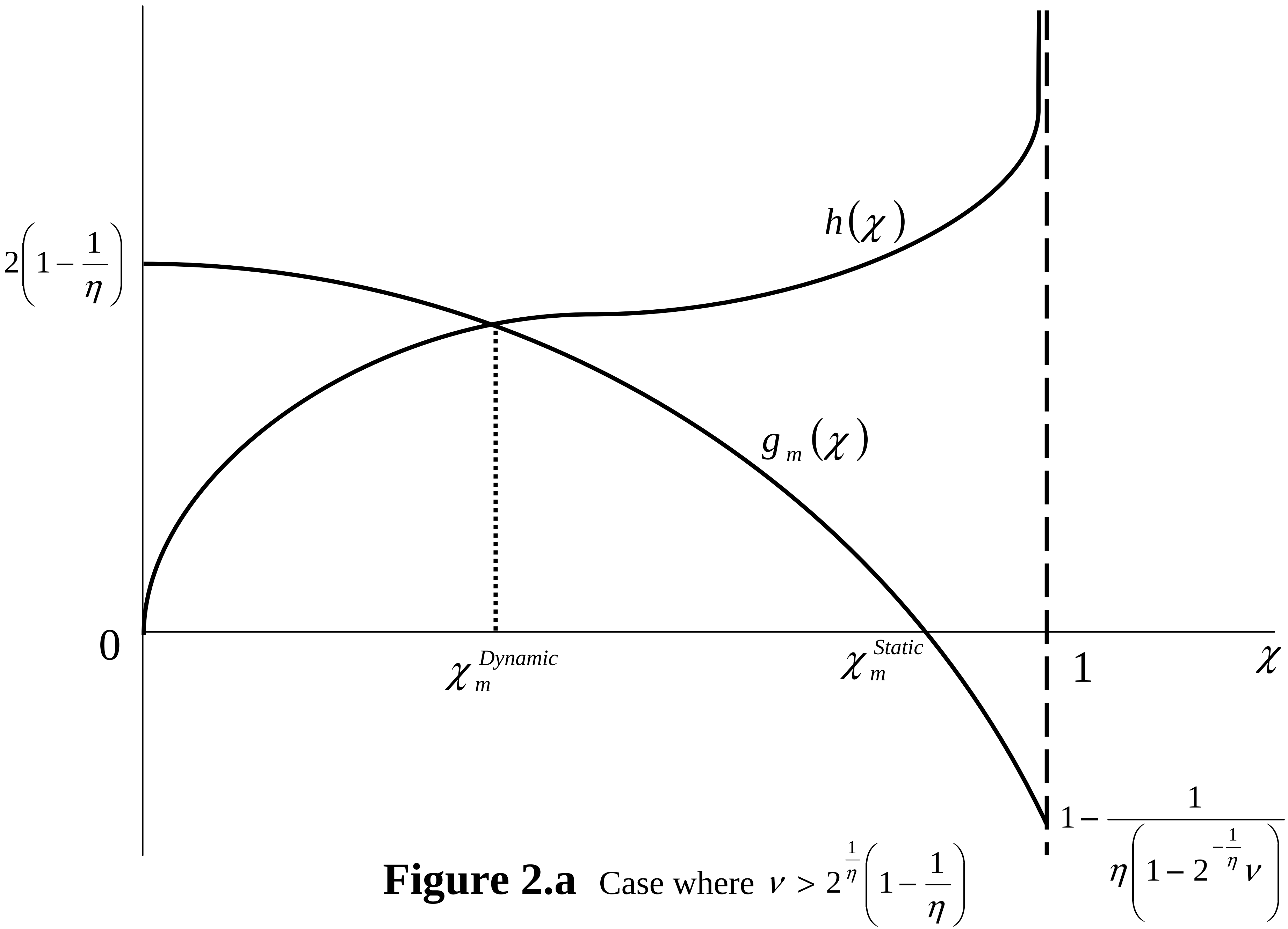


Figure 2.a Case where $\nu > 2^{\frac{1}{\eta}}\left(1-\frac{1}{\eta}\right)$

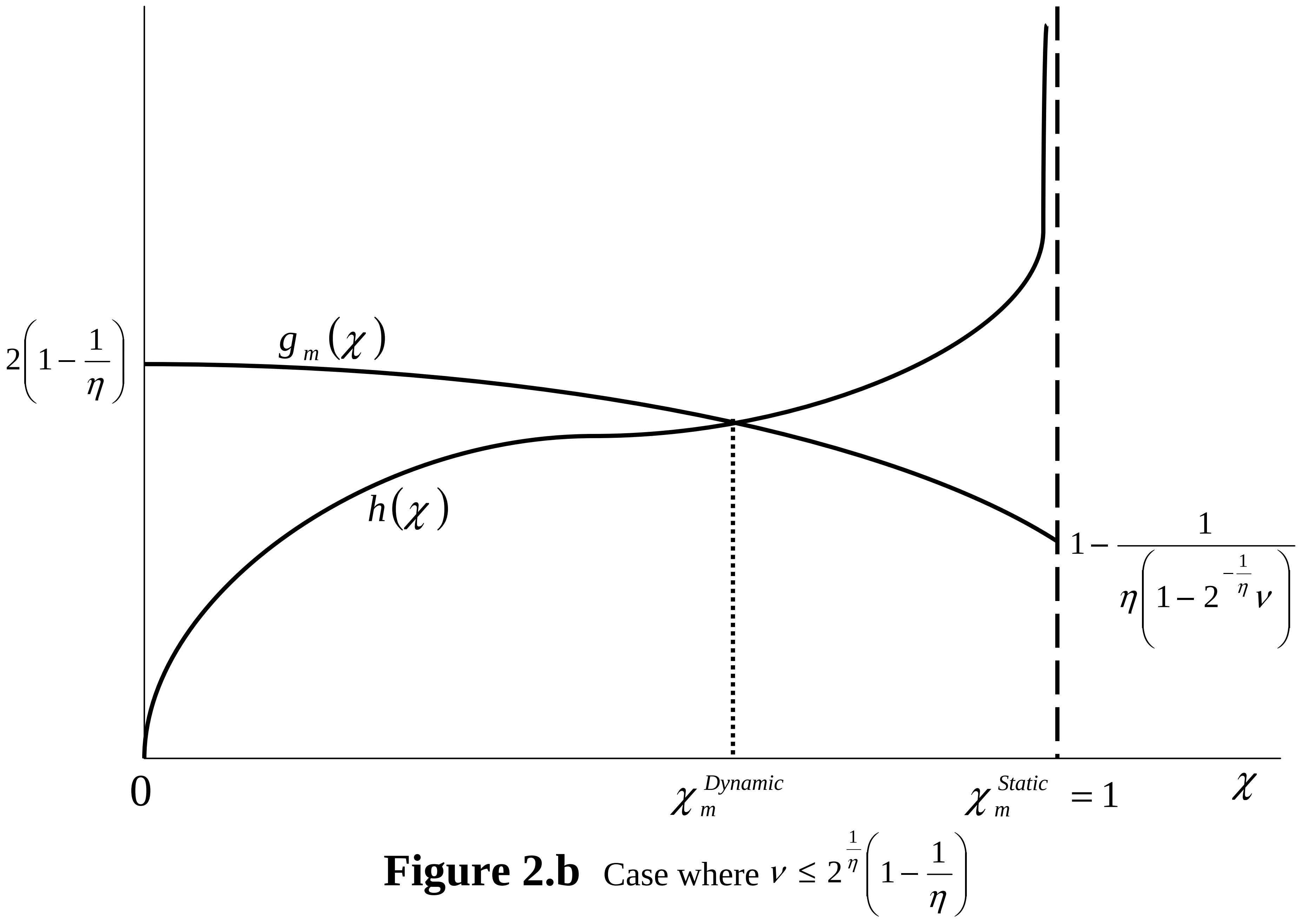
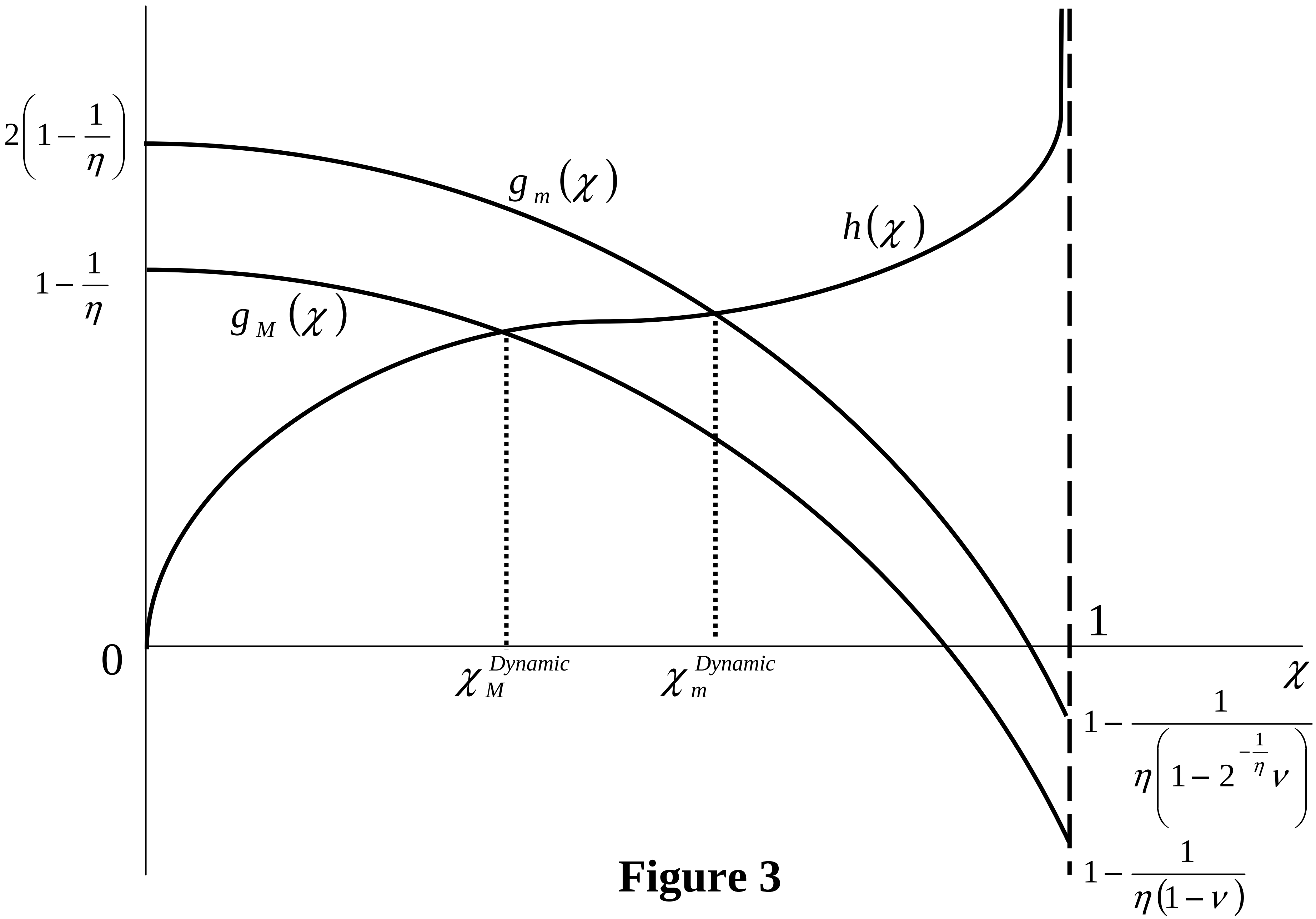


Figure 2.b Case where $\nu \le 2^{\frac{1}{\eta}}\left(1-\frac{1}{\eta}\right)$



$2\left(1-\frac{1}{\eta}\right)$
$1-\frac{1}{\eta}$
$g_m(\chi)$
$g_M(\chi)$
$h(\chi)$
0
1
χ
$\chi_M^{Dynamic}$
$\chi_m^{Dynamic}$
$1-\frac{1}{\eta\left(1-2^{-\frac{1}{\eta}}\nu\right)}$
$1-\frac{1}{\eta(1-\nu)}$

Figure 3

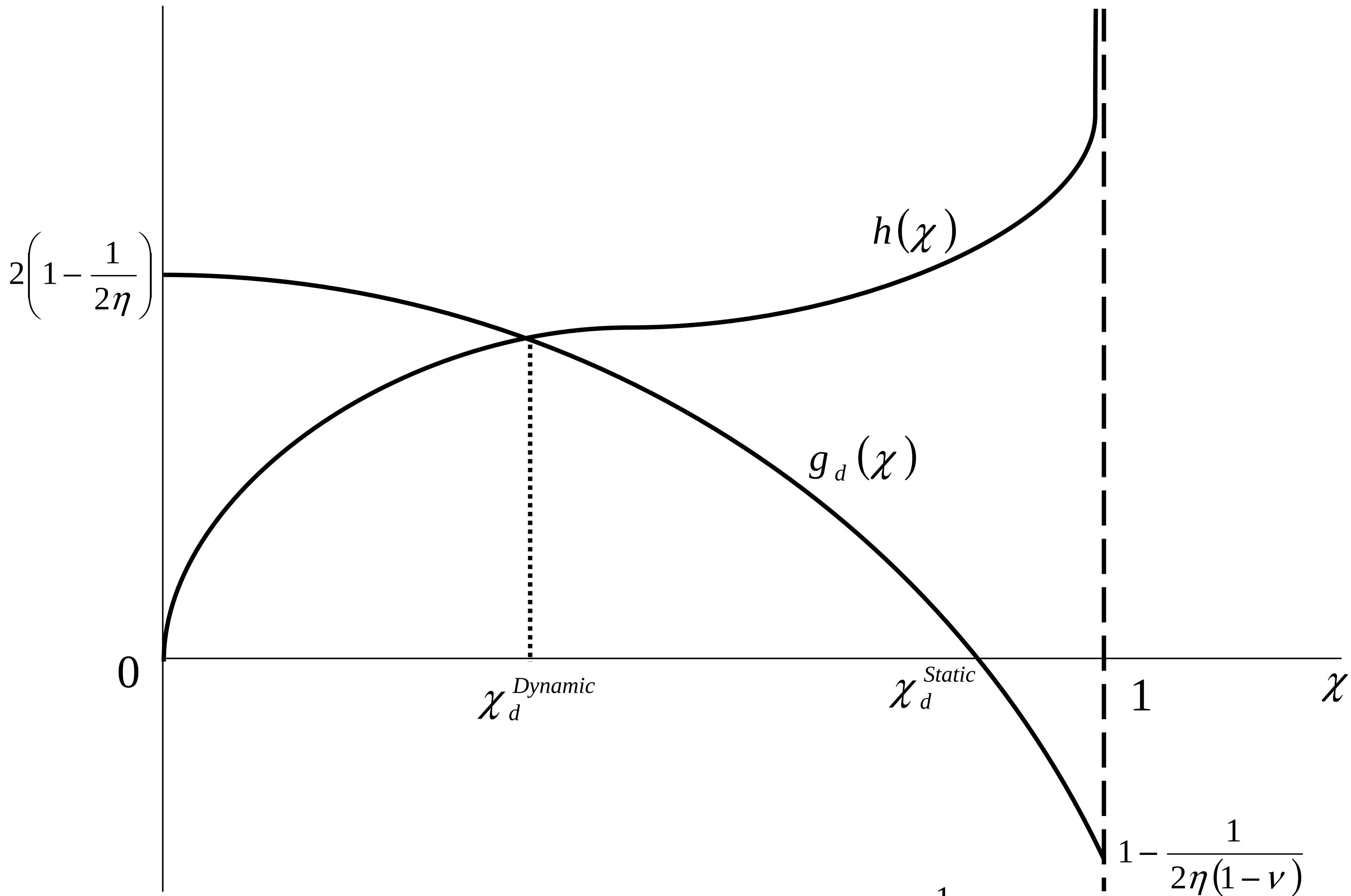


Figure 4.a Case where $\nu > 1-\frac{1}{2\eta}$

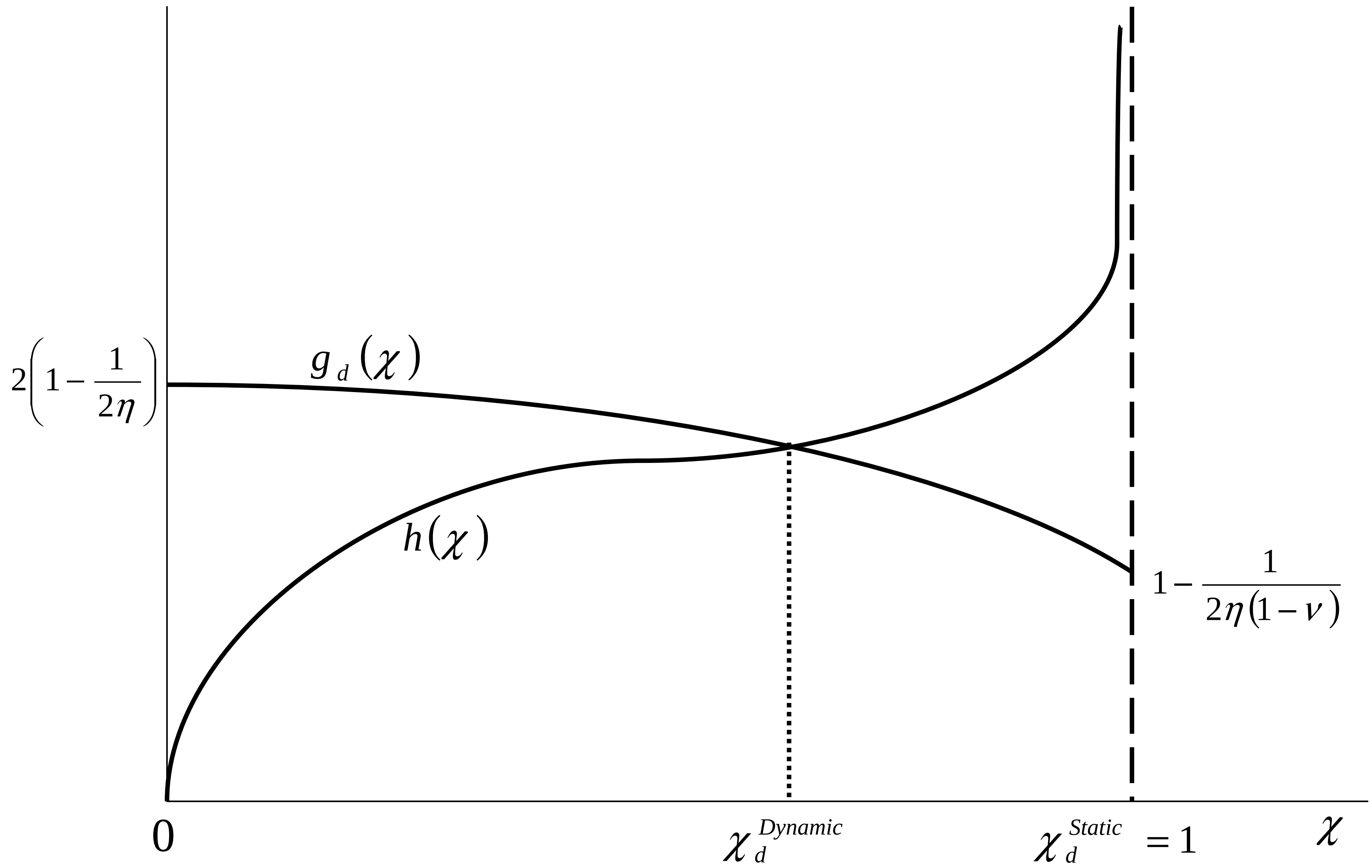
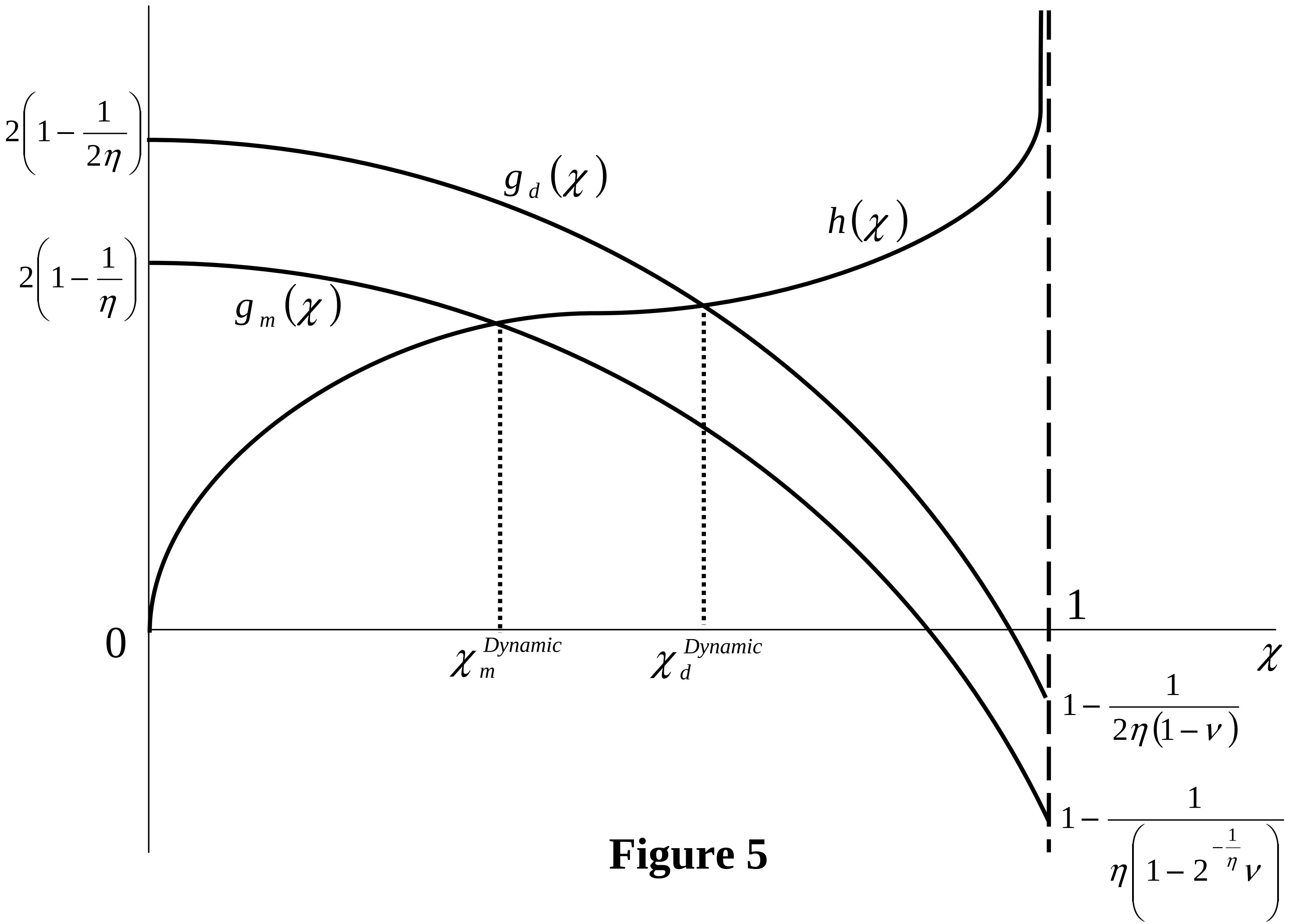


Figure 4.b Case where $\nu \leq 1 - \frac{1}{2\eta}$



$2\left(1-\frac{1}{2\eta}\right)$
$2\left(1-\frac{1}{\eta}\right)$
$g_d(\chi)$
$h(\chi)$
$g_m(\chi)$
0
$\chi_m^{Dynamic}$
$\chi_d^{Dynamic}$
1
χ
$1-\frac{1}{2\eta(1-\nu)}$
$1-\frac{1}{\eta\left(1-2^{-\frac{1}{\eta}}\nu\right)}$

Figure 5

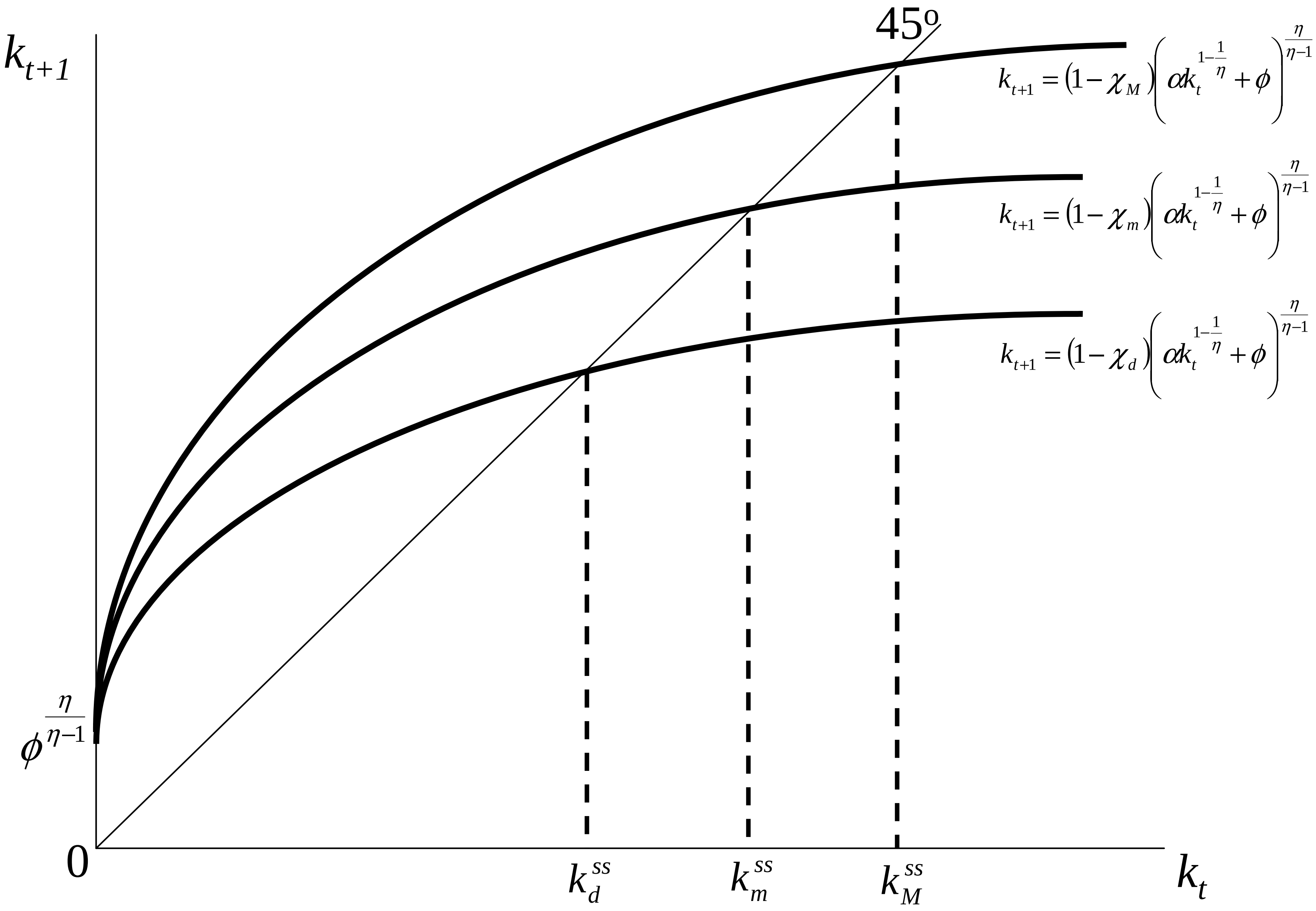

k_{t+1}
45^{o}
$k_{t+1} = (1-\chi_M)\left(\alpha k_t^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}}$
$k_{t+1} = (1-\chi_m)\left(\alpha k_t^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}}$
$k_{t+1} = (1-\chi_d)\left(\alpha k_t^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}}$
$\phi^{\frac{\eta}{\eta-1}}$
0
k_d^{ss}
k_m^{ss}
k_M^{ss}
k_t

Figure 6

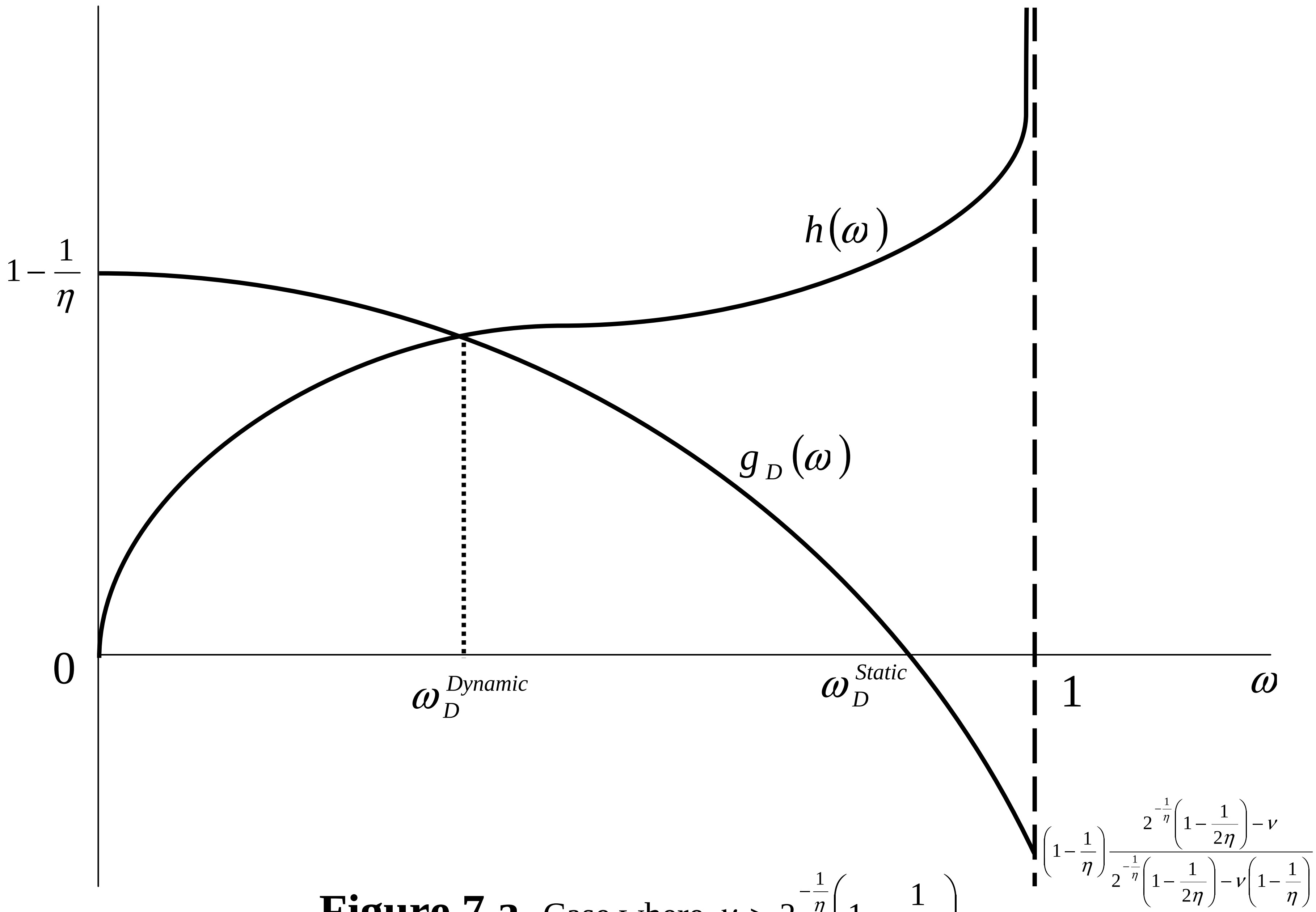


Figure 7.a Case where $\nu > 2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)$

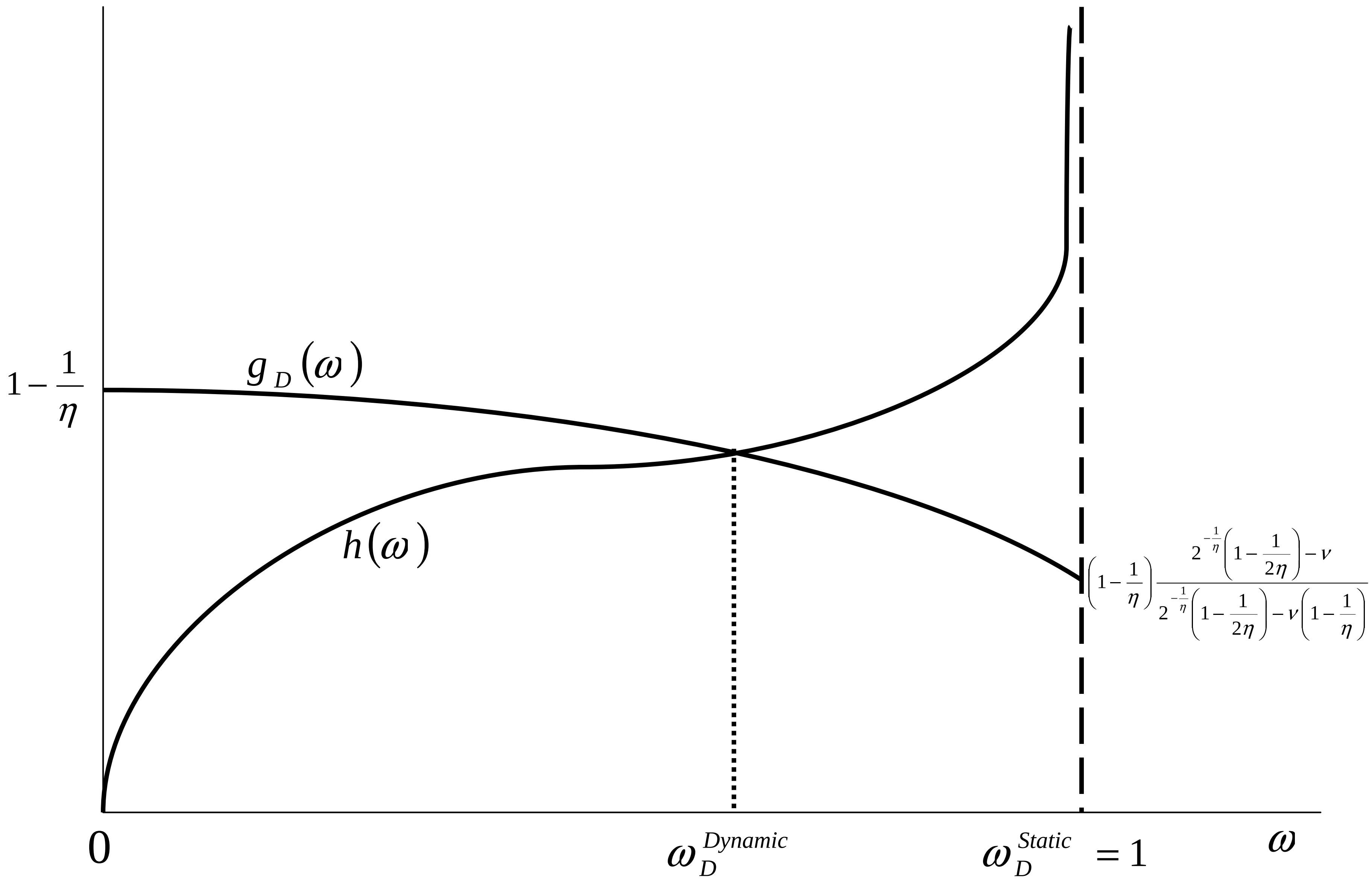


Figure 7.b Case where $\nu > 2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)$

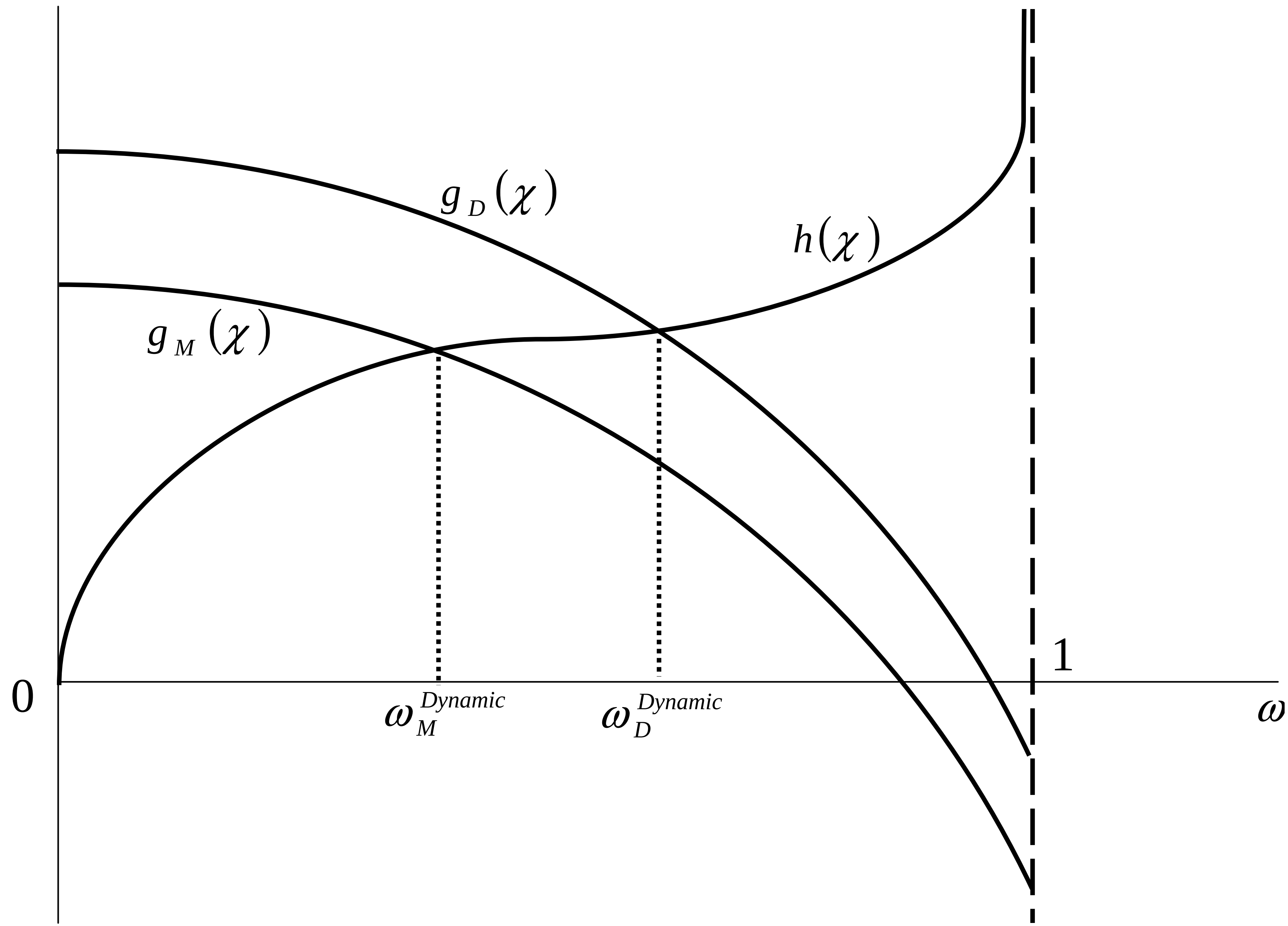


Figure 8.a Case where $\eta < 2.73$

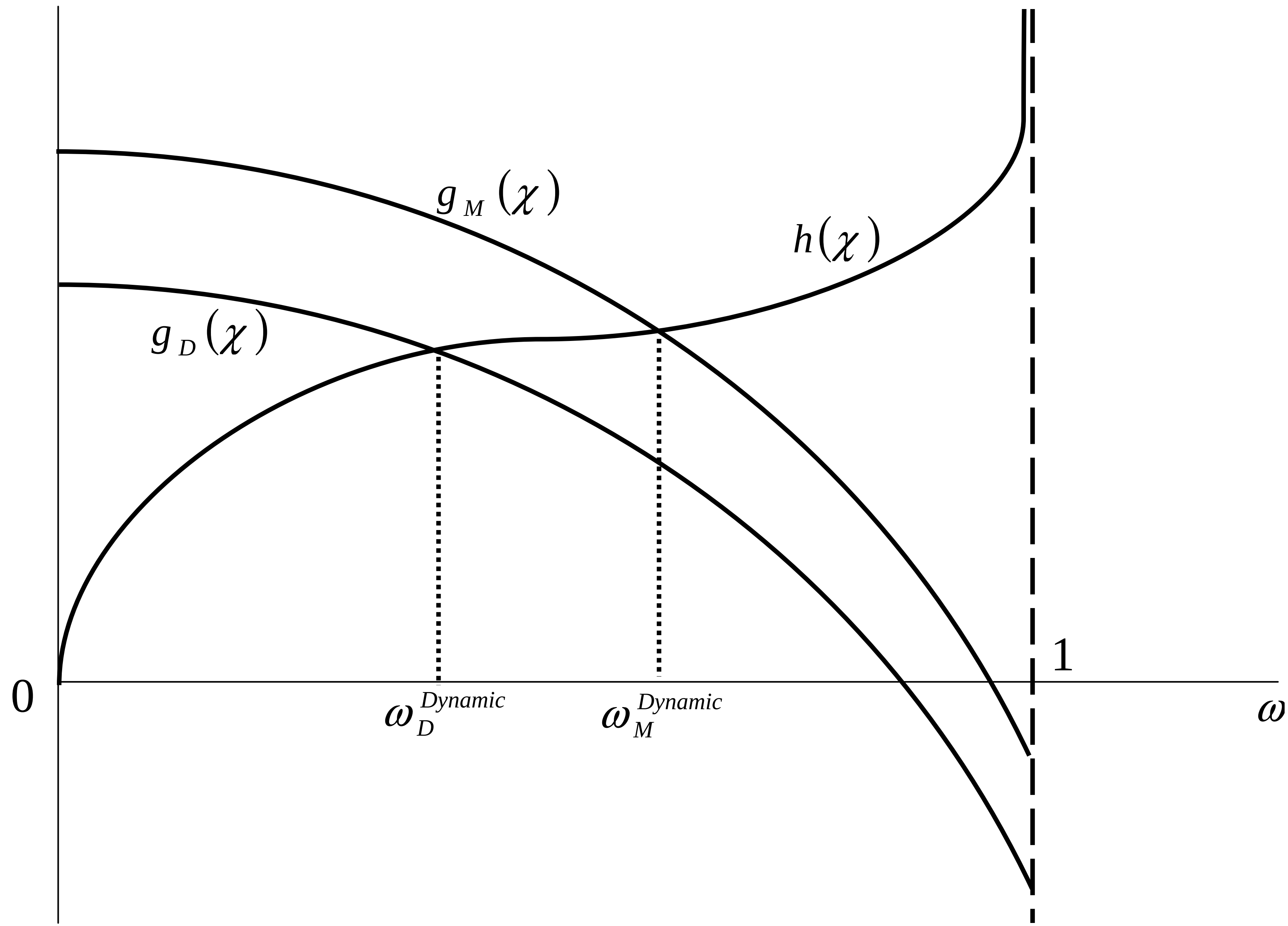


Figure 8.b Case where $\eta > 2.73$