Department of Agricultural and Resource Economics
University of California Davis

Sensitivity of the GME Estimates to Support Bounds

by

Quirino Paris and Michael R. Caputo

The authors are Professors, Department of Agricultural and Resource Economics, University of California, and members of the Giannini Foundation of Agricultural Economics.

August, 2001

Working Paper No. 01-008

California Agricultural Experiment Station
Giannini Foundation for Agricultural Economics

Sensitivity of the GME Estimates to Support Bounds

The generalized maximum entropy (GME) estimator was introduced by Golan *et al.* as a way to overcome two empirical problems that hamper traditional econometrics: multi-collinearity and ill-posed models. Despite its recent origin, several papers based on the GME approach have appeared already in this journal (Paris and Howitt; Miller and Plantinga; Lence and Miller; Zhang and Fan). The distinguishing feature of the GME estimator consists in its requirement of a specific assumption and non-sample information about parameters and error terms. In particular, its implementation relies on subjective information about the range of variation of parameters and error terms that must be provided by the researcher.

For reference ease, we state the GME estimator of a classical linear statistical model following the notation of Golan *et al*. (chapter 6): The linear model to estimate is given by

$$\text{(1)} \qquad \mathbf{y} = \mathbf{X}\beta + \mathbf{u}, \qquad \mathbf{u} \sim \mathrm{IID}(\mathbf{0}, \sigma^2\mathbf{I}),$$

where the dimensions of the various components are $\mathbf{y} \sim (T \times 1)$, $\mathbf{u} \sim (T \times 1)$, $\beta \sim (K \times 1)$ and $\mathbf{X} \sim (T \times K)$. The vector $\mathbf{y}$ and the matrix $\mathbf{X}$ constitute sample information while the vector β represents parameters to estimate and the vector $\mathbf{u}$ contains random disturbances. The principal assumption of the GME formalism is that a parameter β_k is regarded as the mathematical expectation of some discrete support values Z_{km}, $m = 1,...,M$, such that

$$\text{(2)} \qquad \beta_k = \sum_{m=1}^{M} Z_{km} p_{km}$$

where $p_{km} \geq 0$, $k = 1,...,K$, $m = 1,...,M$, are probabilities and $\sum_{m=1}^{M} p_{km} = 1$ for $k = 1,...,K$. The element Z_{km} constitutes *a priori* information provided by the researcher, while p_{km} is an unknown probability whose value must be determined by solving a maximum entropy problem. An analogous reparametrization of the random errors u_t, $t = 1,...,T$, is also assumed for the GME estimator. In particular, let

(3) $$u_t = \sum_{j=1}^{J} V_{tj} w_{tj},$$

where $w_{tj} \geq 0,\ t = 1,...,T,\ j = 1,..., J$, are probabilities, $\sum_{j=1}^{J} w_{tj} = 1$ for $t = 1,...,T$, and V_{tj} are the support values of the random errors. They too constitute *a priori* information provided by the researcher. Then, the GME estimator can be stated as

(4) $$\max_{p_{km}, w_{tj}} H(\mathbf{p}, \mathbf{w}) \overset{def}{=} -\sum_{k=1}^{K} \sum_{m=1}^{M} p_{km} \ln(p_{km}) - \sum_{t=1}^{T} \sum_{j=1}^{J} w_{tj} \ln(w_{tj})$$

subject to $$y_t = \sum_{k=1}^{K} X_{tk} \sum_{m=1}^{M} Z_{km} p_{km} + \sum_{j=1}^{J} V_{tj} w_{tj}, \quad t = 1,...,T$$

$$\sum_{m=1}^{M} p_{km} = 1, \quad k = 1,...,K,$$

$$\sum_{j=1}^{J} w_{tj} = 1, \quad t = 1,...,T.$$

The two extreme support values for each parameter and error term constitute the support's bounds that are the subject of our paper. For future reference, let us denote by $\mathbf{Z}$ and $\mathbf{V}$ the matrices of parameter and error supports, respectively.

The choice of support's bounds, whether for parameters or errors, has important implications for the parameter estimates and the estimated variance of the error term. For example, if the parameter estimates are sensitive to variations of support bounds, then it is probable that policy implications will also be affected by the subjective choice of such *a priori* information. It is rather disappointing, therefore, that the analysis of the same sample data performed by two different researchers will produce different estimates and different testing results. Motivated by such concerns, Caputo and Paris have carried out a complete comparative statics analysis of the GME estimator for the general linear model. They showed that nothing can be said, *a priori*, about the estimates' direct response to changes in either parameter or error bounds. They demonstrated however that, in general, there exists a symmetric and negative semidefinite comparative statics matrix, each individual element of which consists of a linear combination of $T+1$ Slutsky-like forms.

Their Theorem 1 thus shows that it is the *compensated* changes in the support bounds that result in unequivocal comparative statics for the GME problem (4). This implies that it is not possible to derive unequivocal comparative statics results for the effects of the support bounds on the *individual* parameter and errors. In spite of this pitfall of the GME estimator, the econometric literature seems reluctant to acknowledge it, thereby leaving the researcher with the wrong impression that support bounds do not matter much as determinant of the estimates and their concomitant policy implications.

This paper's objective is threefold: First, we will scrutinize and provide evidence to counter the assertions about the impact of support bounds' variations made by the original proponents of the GME estimator. This examination will be accomplished by means of Monte Carlo experiments. We will show that the assertions of Golan *et al*. are unwarranted in general, and may be valid only within the limited confine of the Monte Carlo studies which accompany them. Second, we will use the GME estimator in order to attempt the extraction of econometric inference from the famous sample of US manufacturing data that was used in the original analysis of production functions carried out by Cobb and Douglas in 1928. In this section of the paper we will show the difficulty of deciding which sets of support bounds ought to be selected in order to verify the economic implications of Cobb and Douglas' hypotheses. It is a case of the proverbial chicken-and-egg dilemma. Third, we will summarize the general findings of Caputo and Paris regarding the lack of any unequivocal comparative statics results for the impact of the support bounds on the individual parameters and errors, briefly alluded to above.

The Impact of Variations of the Support Bounds on Parameter Estimates

Golan *et al*. said relatively little about this aspect of their estimator, but what they said seems to have had a lasting influence. In their seminal book they state (p. 138): "The restrictions imposed on the parameter space through $\mathbf{Z}$ reflect prior knowledge about unknown parameters. However, such knowledge is not always available, and the researcher may want to entertain a variety of plausible bounds on β." Reporting the result of a Monte Carlo experiment where three sets of alternative parameter bounds were examined, they say (p. 138): "As the parameter supports are widened, the GME risk functions (a loss function called MSEL and defined as the trace of the mean squared error matrix of parameter estimates) modestly shift upward reflecting the reduced constraints on the parameter space. Hence, wide bounds may be used without extreme risk consequences if our knowledge is minimal and we want to ensure that $\mathbf{Z}$ contains β. Intuitively, increasing the bounds increases the impact of the data and decreases the impact of the support." With respect to the support bounds of the error terms, Golan *et al*. say simply (p. 88) that this selection should be made according to the 3σ rule, by which it is meant that the error bounds should be set at three times the standard deviation from the origin, under the assumption that the error terms are distributed with mean zero and variance σ^2.

Essentially the same conclusions have been reiterated by Lence and Miller, but with a notable variant. In two similar papers they stated that, within the scope of their Monte Carlo study, a variation of support bounds has little or no appreciable impact on sample estimates. They write (1998a, p. 860): "The most important pattern observed in table 2 is that doubling (or halving) the parameter and error bounds has little impact on the fit of the auxilliary regressions. … The impact of changes in the parameter bounds are also slight but do not exhibit a consistent pattern." Similarly, in their other paper Lence and Miller write (1998b, p. 195): "GME results are not sensitive to changes in the width of the error supports, and the changes in the parameter supports must be relatively large to have an impact on the parameter and input estimates." This last statement contradicts the previous assertion by Golan *et al*. according to which: "… increasing the (parameter) bounds … decreases the impact of the support."

The aggregate message of these studies suggests that widening the parameter supports has little impact on the estimates as measured by the risk function (MSEL) and, similarly, that widening the error supports has also little effect on the estimates. To examine these implicit generalizations about the GME estimator we performed three Monte Carlo experiments, two of which deal with well-posed models and one with an ill-posed specification.

The first Monte Carlo experiment is characterized by the following data generating process (DGP): There are ten parameters β_{0k}, $k = 1,...,10$, to estimate. With the exception of β_{01}, each parameter β_{0k} was drawn from a uniform distribution $U[-2, 2$. The parameter β_{01} was defined as the model's intercept with a value of 58. Each element of the matrix of regressors $\mathbf{X}$ was drawn from a uniform distribution $U[1,5]$ except for the first regressor. All the regressors, except the first one, were measured in natural units. The condition number of the matrix of regressors is equal to 29. The values of the dependent variable range from 50 to 80. Finally, each component of the disturbance vector $\mathbf{u}$ was drawn from a normal distribution $N(0,\sigma_0^2) = N(0,16$. One hundred samples of 50 observations were drawn using the pseudo-random routines available in the nonlinear programming application GAMS by Brooke et al.

The second Monte Carlo experiment is characterized by the same DGP as described above, except that the regressors (but not the intercept regressor) and the dependent variable are now defined in natural logarithms.

(table 1)

The results of the two experiments are reported in table 1, where the MSE loss function (MSEL) is presented together with the estimate of the error variance. In both experiments, the values of the MSEL function indicate a large variation when the error bounds are widened, more than 140 percent in experiment 1 and more than 40 percent in experiment 2, as measured from the value of the MSEL function corresponding to error bounds of $3\sigma = 12$ and parameter bounds [-5,5]. The more interesting observation, however, is that these values first decrease and then increase, denying a monotonic response of the MSEL function to variations of the support bounds. This result conflicts with the repeated suggestion by Golan *et al*. that the error bounds could be chosen without too much risk according to the 3σ rule. In the two Monte Carlo experiments reported here, this rule produces error bounds of [-12,12] but table 1 shows that, in the first experiment, the lowest value of the MSEL function occurs in correspondence of the [-40,40] bounds with almost a 40 percent decrease from the tighter bounds of the 3σ rule. In addition, the estimated error variance is closer to the true value in correspondence of the [-20,20] error bounds. In the second experiment, the lowest value of the MSEL function occurs in correspondence of the [-20,20] error bounds, with more than a 40 percent decrease from the 3σ rule. Similarly, the widening of the parameter bounds produces a significant 33 percent increase of the MSEL function in experiment 1, and more than 200 percent in experiment 2, as measured from the value of the MSEL function corresponding to error bounds of $3\sigma = 12$ and parameter bounds [-5,5]. Hence, the results of table 1 contrast sharply with the "modest shift upward" reported by Golan *et al*.

(table 2)

One of the most appealing aspects of the GME estimator is that it can easily produce unique estimates of the parameters belonging to an ill-posed model. The third Monte Carlo experiment, therefore, is devoted to examining the behavior of the MSEL function of an ill-posed model characterized by the following DGP. There are 10 observations and 20 parameters. Each parameter β_{0k}, $k = 2,\ldots,2$, was drawn from a uniform distribution $U[-2,2$. The parameter β_{01} was defined as the model's intercept with a value of 58. Each element of the matrix of regressors $\mathbf{X}$ was drawn from a uniform distribution $U[1,5]$ except for the first regressor. All the regressors, except the first one, were measured in natural units. The values of the dependent variable varied from 100 to

400. Finally, each component of the disturbance vector $\mathbf{u}$ was drawn from a normal distribution $N(0,\sigma_0^2) = N(0,16.$

From the information of table 2, variations of both parameter and error bounds induce extremely large changes in the MSEL function and error variance. For example, the variation in the error bounds resulted in more than 700 percent difference in the value of the MSEL function and more than 4000 percent difference in the error variance. Similarly, the variation in parameter bounds results in more than 5700 percent difference in the value of the MSEL function and more than a 100 percent difference in the error variance, as measured from parameter bounds [-5,5] and error bounds [-10,10]. It is interesting to note that, also in this case, an estimate of the error variance which is close to the true variance corresponds to error bounds between [-20,20] and [-40,40], well above the 3σ rule. These results reinforce the previous conclusion that variations of either parameter and/or error bounds induce complex and unpredictable patterns of response on the parameter estimates and on the statistical performance functions.

The conclusion gleaned from the three Monte Carlo experiments is anything but encouraging for the practical application of GME in the context of flexible functional forms. The reason for this pessimistic assessment is that the individual parameters of these functions have no direct economic interpretation, as in the translog, generalized Leontief, and asymptotically ideal model (Barnett and Jonas) functional forms, for example. As a consequence, no *a priori* economic information can be brought to bear on the parameter support bounds in such instances, leaving the applied researcher with little knowledge on which to base her choice of **Z**. Because the use of flexible functional forms is the rule in applied demand and production analysis, the application of the GME estimator in these contexts must be accompanied by an extensive exploration of the parameter space and an informative reporting of all the results in order for the conclusion of the empirical work to be of some predictive policy value and to convince the reader of its reliability. A few trials are not sufficient and may reflect only the personal bias of the researcher.

The Cobb-Douglas Sample of US Manufacturing Data

Monte Carlo experiments are useful to gain some information about limited aspects of an estimator but, in general, prove nothing. On the contrary, they can disprove a conjectural belief no matter how firmly held, as in this case. When this event happens, the econometric researcher who desires to use the GME approach is left without guidelines on estimation and inference and, therefore, must decide how to proceed only on the basis of the available information that is confined to one sample of data. This endeavor may not always be feasible.

In this section we will illustrate this point by analyzing a famous data sample originally used by Cobb and Douglas in 1928 to estimate the first aggregate production function. That study introduced and popularized the Cobb-Douglas functional form. The objective of the two famous authors was actually very ambitious, as they intended to use available accounting data of the US manufacturing economy to reconcile the marginal theory of production with the marginal theory of income distribution. This reconciliation required that the input coefficients–representing the marginal contribution of capital and labor to production–be positive and sum to unity in order to validate a long-run equilibrium between production and income distribution. On an accounting basis, the income share of labor was measured at 0.75 while that of capital was measured at 0.25.

Although this study has been criticized–with hindsight's wisdom–on both theoretical and empirical grounds, it remains a path-breaking example of econometric analysis. We, thus, will use the same data and model to trace the intellectual itinerary that could have been undertaken by Cobb and Douglas if the GME estimator had been available to them. The econometric model of interest is specified by the following Cobb-Douglas production function

(5) $$\log(P_t) = \alpha + \delta t + \beta_1 \ln(C_t) + \beta_2 \ln(L_t) + u_t$$

where, P_t, C_t and L_t are indices for production, capital and labor, and δt represents technical progress at time *t*. The economic hypotheses of Cobb and Douglas require that $\beta_1 > 0$ and $\beta_2 > 0$ and, furthermore, $\beta_1 + \beta_2 = 1$. The technical progress coefficient δ may be either positive or negative without jeopardizing the hypotheses. The time series of production, capital and labor indices span a 24-year period from 1899 to 1922. The actual series of data were taken from Cobb and Douglas, p. 152, table VI, p. 145, table II, and p. 148, table III, respectively.

For reference, the ordinary least-squares (OLS) estimates of relation (5) are: $\hat{\beta}_1 = -0.5262$, $\hat{\beta}_2 = 0.9060$, $\hat{\delta} = 0.0469$, and $\hat{\alpha} = 2.8132$. With $\hat{\beta}_1 < 0$ and $\hat{\beta}_1 + \hat{\beta}_2 = 0.4798$, these OLS results clearly do not support the economic hypotheses of the marginal theory of value. Can the GME estimator produce results that support these hypotheses?

(table 3)

Table 3 reports the GME estimates of relation (5). Before examining this table, however, it is convenient to recall that Golan *et al*. (p. 138) asserted that "…wide (parameter) bounds may be used without extreme risk consequences if our knowledge is minimal and we want to ensure that $\mathbf{Z}$ contains β. Intuitively, increasing the bounds increases the impact of the data and *decreases* the impact of the support," and that Lence and Miller (p. 195) also wrote that "GME results are not sensitive to changes in the width of the error supports,…"

The results of table 3 tell a different story. Widening error bounds, while keeping wide parameter bounds constant, has the effect of changing the sign of the capital coefficient in the direction of Cobb and Douglas' expectations. Furthermore, for the theory of production to match the theory of income distribution, the production elasticities of capital and labor should be close to 0.25 and 0.75, respectively. Hence, it would appear that a combination of bounds in the proximity of [-100,0,100] for the parameters and [-30,0,30] for the errors would fulfill Cobb and Douglas' expectations.

Of course, this way of looking at the results is not an admissible process of statistical inference since it would appear that the GME estimator, when properly massaged, is capable of telling almost any story, including the story that Cobb and Douglas desired. The fatal impropriety lies in using Cobb and Douglas' expectations for exploring the *a priori* information and mining the data until they produced the desired results. There may be always unexplored corners of the parameter and error spaces that could have revealed the true story or, at least, a more sensible one. But with this process we will never know. There is nothing that the GME estimator can do to break this circular reasoning and "let the data speak" on their own.

Comparative Statics of the GME Estimator

As remarked earlier, Monte Carlo results are not general and thus their conclusions are not typically robust. The conclusions derived from a limited set of Monte Carlo experiments are at best correct within the specific confines of those experiments. There are, however, general results available concerning the effects of perturbations in the parameter and error support bounds on the GME parameter estimates. To put the above Monte Carlo and empirical results into proper perspective, therefore, we summarize the general comparative statics results of Caputo and Paris, which were briefly alluded to in the introduction.

To that end, and for the sake of keeping the present paper self contained, we present the central theorem of Caputo and Paris. Note that the ensuing theorem applies to a version of the GME problem (4) in which each parameter and error has a pair of symmetrically placed support values about the origin, given by $[-Z_k, Z_k]$, $k = 1, \ldots, K$, and $[-V_s, V_s]$, $s = 1, \ldots, T$, respectively.

Theorem 1 (Complete Comparative Statics): *The* $K \times K$ *comparative statics matrix* $\boldsymbol{\Psi}(\mathbf{a})$ *for the GME problem (4) is symmetric and negative semidefinite, where*

$$\begin{aligned}\Psi_{kk'}(\mathbf{a}) \overset{\text{def}}{=} & \left[\sum_{t=1}^{T}\hat{\lambda}_t(\mathbf{a})X_{tk}\right]\left[\frac{\partial\left[\hat{\beta}_k(\mathbf{a})/Z_k\right]}{\partial Z_{k'}} - \left[\frac{\hat{\beta}_{k'}(\mathbf{a})}{Z_{k'}}\right]\sum_{s=1}^{T}\frac{X_{sk'}}{\left[\hat{u}_s(\mathbf{a})/V_s\right]}\frac{\partial\left[\hat{\beta}_k(\mathbf{a})/Z_k\right]}{\partial V_s}\right] \\ & - \left[\frac{\hat{\beta}_k(\mathbf{a})}{Z_k}\right]\sum_{j=1}^{T}\frac{\hat{\lambda}_j(\mathbf{a})X_{jk}}{\left[\hat{u}_j(\mathbf{a})/V_j\right]}\left[\frac{\partial\left[\hat{u}_j(\mathbf{a})/V_j\right]}{\partial Z_{k'}} - \left[\frac{\hat{\beta}_{k'}(\mathbf{a})}{Z_{k'}}\right]\sum_{s=1}^{T}\frac{X_{sk'}}{\left[\hat{u}_s(\mathbf{a})/V_s\right]}\frac{\partial\left[\hat{u}_j(\mathbf{a})/V_j\right]}{\partial V_s}\right],\end{aligned}$$

$k,k' = 1, \ldots, K$. *Moreover, the rank of* $\boldsymbol{\Psi}(\mathbf{a})$ *is no larger than K.*

The vector $\mathbf{a}$ contains all the parameter and error support bounds. The elements $\hat{\lambda}_t(\mathbf{a})$, $t = 1,...,T$, are the Lagrange multipliers of the T sample observations in the linear model of problem (4).

Theorem 1 contains *all* the qualitative information derivable from the GME problem (4) without imposing additional assumptions on its structure. That is, Theorem 1 gives the fundamental comparative statics properties of the GME problem (4). Each element $\Psi_{kk'}(\mathbf{a})$ of the comparative statics matrix $\boldsymbol{\Psi}(\mathbf{a})$ consists of a linear combination of $T+1$ Slutsky-like forms. The Slutsky-like forms consist of a parameter support-bound effect, given by the expression $\partial\left[\hat{\beta}_k(\mathbf{a})/Z_k\right]/\partial Z_{k'}$ or $\partial\left[\hat{u}_j(\mathbf{a})/V_j\right]/\partial Z_{k'}$, and a linear combination of T error support-bound effects, the latter given by the expressions $\partial\left[\hat{\beta}_k(\mathbf{a})/Z_k\right]/\partial V_s$ or $\partial\left[\hat{u}_j(\mathbf{a})/V_j\right]/\partial V_s$.

To acquire some understanding of the complex relations exhibited by the $\boldsymbol{\Psi}(\mathbf{a})$ matrix, it is important to recognize that the form of the comparative statics given in Theorem 1 applies not to the estimates of parameters and residuals, but to their values *relative* to the endpoint of their own support interval. The kth diagonal element of the $\boldsymbol{\Psi}(\mathbf{a})$ matrix, for example, expresses the direct and indirect impacts of the variation of the support bound Z_k on the own kth parameter estimate measured in relative terms by reference to its own support bound $\hat{\beta}_k(\mathbf{a})/Z_k$. But because the parameter β_k enters in each of the T

observations, its *relative* estimate $\hat{\beta}_k(\mathbf{a})/Z_k$ is also related to each of the T residuals measured in *relation* to its own endpoint $\hat{u}_s(\mathbf{a})/V_s$, $s = 1,...,T$. In other words, by changing the support bound Z_k of parameter β_k, an unequivocal response is detected only through a linear combination of all direct and indirect impacts generated by that change through all the T sample observations.

The comparative statics matrix $\Psi(\mathbf{a})$, therefore, shows that it is the *compensated changes* in the support values, scilicet, a parameter support-bound effect compensated with every error support-bound effect, that results in unequivocal comparative statics for the GME problem (4). Therefore, the fundamental comparative statics properties of the GME problem (4) consist of compensated derivatives rather than simple partial derivatives, and apply to the values of the parameters estimates and residuals relative to the endpoint of their support interval. This implies that, in general, one cannot hope to derive unequivocal comparative statics results in the form of direct partial derivatives for the GME problem (4), say, of the form $\partial\hat{\beta}_k(\mathbf{a})\big/\partial Z_{k'}$.

For a better illustration of Theorem 1, we now assume an extreme ill-posed situation by letting $K = 2$ and $T = 1$. In this case the typical element of the (2×2) $\Psi(\mathbf{a})$ matrix takes on the form

$$\Psi_{kk'}(\mathbf{a}) \stackrel{\text{def}}{=} \left[\hat{\lambda}_1(\mathbf{a})X_{1k}\right]\left\{\left[\frac{\partial\left[\hat{\beta}_k(\mathbf{a})\big/Z_k\right]}{\partial Z_{k'}} - X_{1k'}\left[\frac{\left[\hat{\beta}_{k'}(\mathbf{a})\big/Z_{k'}\right]}{\left[\hat{u}_1(\mathbf{a})\big/V_1\right]}\right]\frac{\partial\left[\hat{\beta}_k(\mathbf{a})\big/Z_k\right]}{\partial V_1}\right]\right.$$
$$\left. - \left[\frac{\left[\hat{\beta}_k(\mathbf{a})\big/Z_k\right]}{\left[\hat{u}_1(\mathbf{a})\big/V_1\right]}\right]\left[\frac{\partial\left[\hat{u}_1(\mathbf{a})\big/V_1\right]}{\partial Z_{k'}} - X_{1k'}\left[\frac{\left[\hat{\beta}_{k'}(\mathbf{a})\big/Z_{k'}\right]}{\left[\hat{u}_1(\mathbf{a})\big/V_1\right]}\right]\frac{\partial\left[\hat{u}_1(\mathbf{a})\big/V_1\right]}{\partial V_1}\right]\right\},\quad k,k' = 1,2.$$

The Slutsky-like nature of the comparative statics matrix $\Psi(\mathbf{a})$ is now even more self-evident. It consists of a linear combination of two Slutsky-like terms under the simplifying assumptions $K = 2$ and $T = 1$. The form of $\Psi(\mathbf{a})$ shows that even in this very special case it is not possible to derive unequivocal comparative statics results in the form of partial derivatives for the GME problem (4), that is, comparative statics of the form $\partial \hat{\beta}_k(\mathbf{a}) / \partial Z_{k'}$, exactly as recognized above. Thus, in general, no simple definitive relationship exists between changes in the support bounds and the values of the parameters and residuals in GME problem (4).

Conclusion

The GME estimates of a linear statistical model are sensitive, in general, to variations of either parameter and/or error support bounds. Without a precise *a priori* knowledge of the true range of parameter variations, the implementation of the GME estimator depends heavily upon the subjective information provided by the researcher. In this paper we have shown by empirical evidence and demonstrated by comparative statics analysis that the impact of variations of parameter and error support bounds is unpredictable. Three non trivial Monte Carlo experiments have produced a large risk associated with such bounds' variations. When dealing with a single data sample, it is difficult to decide which support bounds ought to be selected to verify the model's hypotheses. These results are the Achilles heel of the GME estimator.

References

Barnett, W.A. and A. Jonas. "The Müntz-Szatz Demand System: An Application of a Globally Well Behaved Series Expansion." *Economics Letters*, 11(March 1983):337:342.

Brooke, A., D. Kendrick, and A. Meeraus. *GAMS: A User's Guide*. The Scientific Press, 1988.

Caputo, M. R. and Q. Paris, "Comparative Statics of the Generalized Maximum Entropy Estimator of the General Linear Model," Department of Agricultural and Resource Economics, University of California, Davis, June 2000, 25 pages.

Cobb, C. W. and P.H. Douglas. "A Theory of Production." *Am. Econ Rev.*, *Supplement* 18(March 1928):139-165.

Golan, A., G.G. Judge and D. Miller. *Maximum Entropy Econometrics*, Wiley and Sons, Chichester, UK, 1996.

Lence, S. H. and D. J. Miller. "Recovering Output-Specific Input from Aggregate Input Data: A Generalized Cross-Entropy Approach." *Am J. Agr. Econ.* 80(November 1998a):852-867.

Lence, S. H. and D. J. Miller. "Estimation of Multi-Output Production Functions with Incomplete Data: A Generalized Maximum Entropy Approach." *Eur. Rev. Agr. Econ.* 25(1998b): 188-209.

Miller, D. J. and A. J. Plantinga. "Modeling Land Use decisions with Aggregate Data." *Am J. Agr. Econ.* 81(February 1999):180-194.

Paris, Q. and R. E. Howitt. "Analysis of Ill-Posed Production Problems Using Maximum Entropy." *Am. J. Agr. Econ.*, 80(August 1998): 124-138.

Zhang, X. and S. Fan. "Estimating Crop-Specific Production Technologies in Chinese Agriculture: A Generalized Maximum Entropy Approach." *Am J. Agr. Econ.* 83(May 2001):378-388.

Table 1. MSEL and error variance, well-posed models. T=50, K=10, $\sigma^2 = 16$. 100 **samples.**

Bounds		Natural Units Experiment 1		Logarithmic Units Experiment 2	
Parameter /	Error	MSEL	Error Variance	MSEL	Error Variance
[-5,0,5]	[-10,0,10]	26.684	15.359	20.384	15.792
[-5,0,5]	[-12,0,12]	24.459	15.376	15.394	16.143
[-5,0,5]	[-20,0,20]	17.769	16.133	8.633	17.824
[-5,0,5]	[-40,0,40]	14.981	20.401	8.700	20.169
[-5,0,5]	[-100,0,100]	27.103	29.373	9.358	21.437
[-5,0,5]	[-200,0,200]	60.093	32.592	21.978	21.667
[-10,0,10]	[-12,0,12]	30.171	15.257	31.308	15.297
[-20,0,20]	[-12,0,12]	31.933	15.254	41.817	15.195
[-40,0,40]	[-12,0,12]	32.396	15.255	45.389	15.192
[-100,0,100]	[-12,0,12]	32.527	15.256	46.420	15.194
[-200,0,200]	[-12,0,12]	31.687	15.227	45.911	15.169

Table 2. MSEL and error variance, ill-posed model. T=10, K=20, $\sigma^2 = 16$, 100 **samples.**

Bounds Parameter	Bounds Error	MSEL	Error Variance
[-5,0,5]	[-10,0,10]	56.881	2.469
[-5,0,5]	[-20,0,20]	86.664	12.294
[-5,0,5]	[-40,0,40]	131.405	41.102
[-5,0,5]	[-100,0,100]	223.442	88.149
[-5,0,5]	[-200,0,200]	490.988	102.561
[-10,0,10]	[-10,0,10]	584.328	0.382
[-20,0,20]	[-10,0,10]	1928.708	0.041
[-40,0,40]	[-10,0,10]	2927.637	0.003
[-100,0,100]	[-10,0,10]	3334.252	0.00009
[-200,0,200]	[-10,0,10]	3334.320	0.000006

Table 3. GME estimates of a Cobb-Douglas production function for the US manufacturing economy, 1899-1922.

Bounds Parameter / Error		Capital $\hat{\beta}_1$	Labor $\hat{\beta}_2$	Technical Progress	Intercept	$\hat{\beta}_1 + \hat{\beta}_2$
[-100,0,100]	[-3,0,3]	-0.2746	0.8940	0.0309	1.7245	0.6194
[-100,0,100]	[-5,0,5]	-0.1086	0.8814	0.0205	1.0279	0.7728
[-100,0,100]	[-10,0,10]	0.0746	0.8428	0.0097	0.3739	0.9274
[-100,0,100]	[-30,0,30]	0.3066	0.6722	-0.0009	0.1103	0.9788
[-100,0,100]	[-100,0,100]	0.4577	0.5219	-0.0059	0.1046	0.9796