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The Impacts of Fees and Taxes on Choices of Development Timing and Capital Intensity

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Abstract

This article compares the effects of various fiscal policies on choices of development timing and capital intensity when rents on housing follow geometric Brownian motion with those when rents follow arithmetic Brownian motion. These policy instruments include fees on capital, housing, and land, and taxes on urban income, and properties both before and after development. Regardless of the motion of rents, when one choice is fixed, the effects of these policy instruments on the other choice are qualitatively the same. When the two choices are determined endogenously, although these policy instruments exhibit the same qualitative effect on the choice of development timing, they may exhibit different effects on the choice of capital intensity if rents on housing follow different types of motions.

Keywords: Capital intensity, Development Timing, Fees, Taxation, Real Options

JEL Classification: G13, H21, H23, R52

I. Introduction

One central topic on urban economics is how fiscal policies affect choices of the timing and capital intensity of development.[1] In their seminal article, Arnott and Lewis (1979) abstract from the spatial factor and investigate how taxation before and after development affects these two choices in a deterministic environment. Turnbull (1988a; 1988b) investigates the same issue by incorporating the spatial factor into the Arnott-Lewis model. McFarlane (1999) extends Turnbull's model by providing a through analysis of the effects of various fees and taxes on these two choices.[2] All of the above articles have shed some insights, yet they abstract from the issue of uncertainty, which is thought to be a main characteristic of real estate markets.

The literature on real options typically assumes that the source of uncertainty comes from rents on housing. To capture the uncertainty, two types of motions are commonly employed: geometric Brownian motion (henceforth GBM) and aritmetric Brownian motion (henceforth ABM). As discussed in Capozza and Li (1994, p. 893), the empirical data shows that rents on housing look more like a normal distribution than a log-normal distribution, and thus they assume that these rents follow ABM. In addition to Capozza and Li (1994), the literature which assumes that rents on housing follow ABM include Capozza and Hesley (1990), and Capozza and Sick (1994). However, the shortcoming of this assumption is that rents can be negative. Consequently, it is more commonly for the real options literature (see, e.g., Bar-Ilan and Strange, 1996; Grenadier, 2000; Williams, 1991) to assume that rents on housing follow

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[1] Several earlier articles abstract from the choice of capital intensity. For example, Skouras (1978) provides an early analysis indicating that property taxation is non-neutral in its effect on land development timing. Mills (1983) introduces property taxation in a model of competitive equilibrium and investigates the timing effects of taxation on land development. Brueckner (1986) investigates how a shift to a graded tax system (where the improvements tax rate is lowered and the land tax rate is raised) affects the level of improvements, the value of land and the price of housing in the long-run. Finally, Anderson (1986) focuses on how property taxation affects the timing decision of land development.

[2] See also a thorough review by Anderson (2005).

GBM, which restricts rents to be positive. The purpose of this article is to investigate how the existence of uncertainty and how different types of uncertainty affect the results regarding the impacts of fiscal policies on choices of the timing and capital intensity of development.

The fiscal policies which we consider include three types of fees and three types of taxes: fees on capital, housing and land, and taxation on urban income and properties both before and after development. We find that, when the development timing is fixed, the effects of these policy instruments on the choice of capital intensity are qualitatively the same regardless of the type of evolution of rents on housing. When the capital intensity is fixed, the effects of these policy instruments on the choice of development timing are also qualitatively the same regardless of the type of evolution of rents on housing. When both choices are determined endogenously, although the effects of these policy instruments exhibit the same qualitative effect on the development timing, yet they may exhibit different effects on the choice of capital intensity for different types of evolution of rents on housing.

The remainder of this article is organized as follows. Section II presents the basic model. Section III solves choices regarding the timing and capital intensity of development when rents on housing follow GBM and ABM, respectively. Section IV investigates how various forms of fees and taxes on developed properties affect these two choices. Section V concludes and offers extension for future research.

II. The Model

Suppose that at date $t = 0$ a landowner has undeveloped land that is normalized at one unit. At any time, i.e., $t \geq 0$, the landowner is able to develop property. We assume that the rent on one unit of housing is given by

$$R(x(t), D) = x(t)e^{-D}, \tag{1}$$

where $x(t)$ denotes the macroeconomic shock from the demand side, and D is the distance from the CBD, which is the only characteristic that distinguishes parcels of land in a city that is monocentric. Equation (1) indicates that the rent on one unit of housing is decreasing convex in the distance from the CBD (i.e. $\partial R / \partial D < 0$, $\partial^2 R / \partial D^2 > 0$). This differs from the real options literature such as Capozza and Helsley (1990), Capozza and Li (1994), and Capozza and Sick (1994), where the rent on one unit of housing is assumed to decline linearly from the distance to the CBD, i.e., $\partial R / \partial D < 0$ and $\partial^2 R / \partial D^2 = 0$.

We assume that housing, Q, is produced with capital, K and land L. The production function is of a Cobb-Douglas type given by $Q(K, L) = K^{\gamma} L^{1-\gamma}$, where $0 < \gamma < 1$. Define $g = Q / L$, and $k = K / L$, thus the production function for the owner of one unit of vacant land is given by

$$g(k) = k^{\gamma}. \tag{2}$$

We assume that the cost of capital per unit of land is equal to a positive constant, denoted by c. We consider three forms of fees and three kinds of taxes. The former include the fee rate on capital, f_K, the fee rate on housing, f_Q, and the fee rate on land, f_L. As a result, the total costs per unit of land are given by

$$C(k) = (c + f_k)k + f_Q k^{\gamma} + f_L. \tag{3}$$

The tax instruments include the tax rate on urban income, τ_r, the tax rate on property before development, τ_b, and the tax rate on property after development, τ_a. As a result, the after-tax rent per unit of developed property is the product of

$(1-\tau_r)x(t)e^{-D}$ and k^{γ}, thus yielding

$$R_a(x(t),D)=(1-\tau_r)x(t)e^{-D}k^{\gamma}\,. \tag{4}$$

We assume that landowners are risk neutral and face a risk-less rate of interest, denoted by ρ, which is constant per unit of time. We also assume that undeveloped properties do not have any return, and thus landowners have no option value to abandon them. We also abstract from both the time-to-build problem that usually occurs in the real estate industry (see, e.g., Bar-Ilan and Strange, 1996; Grenadier, 2000), and the redevelopment problem addressed in Williams (1997). Consequently, in what follows, each landowner will make his development decision once and for all. Our simplified assumptions may be not so realistic, yet they help us gain insights regarding the impacts of optimal property taxation.[3]

III. Choices of the Date and Density of Development

Without risk of confusions, we denote $x(t)$ as x in what follows. Consider any $t\geq T$. Given that redevelopment is prohibited, the value of one unit of land after development is equal to the (time t) expected present value of the future cash flow given by

$$W_a(x,D,k)=E_t\int_t^{\infty}[(1-\tau_r)x(s)e^{-D}k^{\gamma}-\tau_a W_a(x(s),D,k)]e^{-\rho(s-t)}ds. \tag{5}$$

Equation (5) can be rewritten as

$$W_a(x,D,k)=E_t\int_t^{\infty}(1-\tau_r)x(s)e^{-D}k^{\gamma}e^{-(\rho+\tau_a)(s-t)}ds\,. \tag{6}$$

[3] We also assume that all lots are simultaneously developed and are finished instantly. These assumptions are usually adopted in the real options literature (see, e.g., Capozza and Li, 1994; Childs, Riddiough and Triantis, 1996; Williams, 1991). We thus neither allow lots to be developed sequentially nor allow the development of real estate to take the form of a sequential investment (see, e.g., Bar-Ilan and Strange, 1998).

Define $W_b(x,T,D,k)$ as the value of one unit of vacant land, which is given by

$$W_b(x,T,D,k) = E_t\{\int_t^T (-\tau_b W_b(x(s),T,D,k)e^{-\rho(s-t)}ds \tag{7}$$

$$+\int_T^\infty ((1-\tau_r)x(s)e^{-D}k^\gamma - \tau_a W_a(x(s),D,k))e^{-\rho(s-t)}ds - [(c+f_K)k + f_Q k^\gamma + f_L]e^{-\rho(T-t)}\}$$

Equation (7) indicates that the expected present value of returns to one unit of vacant land is the sum of the negative expected present value of taxation on vacant land until time T and the after-tax expected present value of land rent beginning at the time of development, less the expected present value of the developing costs. Following Anderson (1993), Capozza and Li (1994), and McFarlane (1999), a more tractable expression for the value of one unit of vacant land is given by

$$\begin{aligned} W_b(x,T,D,k) = E_t\{e^{-(\rho+\tau_b)(T-t)}[\int_T^\infty (1-\tau_r)x(s)e^{-D}k^\gamma e^{-(\rho+\tau_a)(s-T)}ds \\ -((c+f_K)k + f_Q k^\gamma + f_L)]\}. \end{aligned} \tag{8}$$

A landowner needs to choose an appropriate timing T and capital intensity k to maximize the value of one unit of vacant land. This is defined as $Z(x) = \max_{T,k} W_b(x,T,D,k)$. In the following, we respectively assume that rents on one unit of housing follow GBM and ABM.

A. Geometric Brownian Motion

Suppose that the factor that shifts the rent on one unit of housing follows GBM given by

$$\frac{dx(t)}{x(t)} = \alpha dt + \sigma d\Omega(t), \quad \sigma > 0, \tag{9}$$

where the growth rate of $x(t)$ has a constant expected growth rate α and a constant variance of the growth rate σ^2, and $d\Omega(t)$ is an increment to a standard Wiezer process. The value of one unit of land after development given by equation (6) is thus equal to

$$W_a(x, D, k) = \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a-\alpha)}. \tag{10}$$

Using equation (9) and applying Itô's lemma yields (see Appendix A)

$$Z(x) = A_1 x^{\beta_1} + A_2 x^{\beta_2} + \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a-\alpha)} - [(c+f_K)k + f_Q k^{\gamma} + f_L], \tag{11}$$

where A_1 and A_2 are constants to be determined, and β_1 and β_2 are respectively given by

$$\begin{aligned} \beta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2(\rho+\tau_b)}{\sigma^2}} > 1, \\ \beta_2 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2(\rho+\tau_b)}{\sigma^2}} < 0. \end{aligned} \tag{12}$$

A landowner simultaneously chooses the timing and capital intensity in the presence of uncertainty. As indicated by Dixit and Pindyck (1994, p.139), when uncertainty arises, we are unable to determine a non-stochastic timing. Instead, the development rule takes the form where the landowner will not develop vacant land until x rises to a level, denoted by x^*. When this trigger level is reached, the landowner will develop vacant land at the capital intensity, denoted by k^*. Both x^* and k^*, together with A_1 and A_2 in equation (11), are solved from the equations given by:

$$\lim_{x\to 0} Z(x) = -[(c+f_K)k + f_Q k^{\gamma} + f_L], \tag{13}$$

$$Z(x^*) = 0, \tag{14}$$

$$\frac{\partial Z(x^*)}{\partial x} = 0, \tag{15}$$

$$\frac{\partial\{W_a(x^*,D,k^*)-[(c+f_k)k^*+f_Q k^{*\gamma}+f_L]\}}{\partial k}=0. \tag{16}$$

Equation (13) is the boundary condition which states that the value of the option to develop one unit of vacant land, the first two terms on the right-hand side of equation (11), is worthless as the factor that shifts the rent on one unit of housing approaches its minimum permissible value of zero. Equation (14) is the value-matching condition which states that, at the optimal timing of development, a landowner should be indifferent as to whether vacant land is developed or not. Equation (15) is the smooth-pasting condition, which requires that the landowner not obtain any arbitrage profits from deviating from the optimal timing of development. Equation (16) indicates that the chosen capital intensity should maximize the net value of developing one unit of vacant land immediately, which in turn, requires that the marginal value of capital be equal to the marginal cost of capital.

To solve a landowner's choice of timing, we can solve A_1 and A_2 from equations (13) and (15), respectively. Substituting these values into equation (14), and referring to its result as $M_g(x^*,k^*)$ yields

$$M_g(x^*,k^*)=-(1-\frac{1}{\beta_1})\frac{(1-\tau_r)e^{-D}k^{*\gamma}x^*}{(\rho+\tau_a-\alpha)}+(c+f_K)k^*+f_Q k^{*\gamma}+f_L=0, \tag{17}$$

where the notation $M_g(\cdot)$ represents the condition for the choice of development timing when the rent on one unit of housing follows GBM. Referring to the result of equation (16) as $S_g(x^*,k^*)$ yields

$$S_g(x^*,k^*)=\frac{\gamma(1-\tau_r)e^{-D}k^{*\gamma-1}x^*}{(\rho+\tau_a-\alpha)}-(c+f_K)-\gamma f_Q k^{*\gamma-1}=0, \tag{18}$$

where the notation $S_g(\cdot)$ represents the condition for the choice of capital intensity when the rent on one unit housing follows GBM.

B. Arithmetic Brownian Motion

Suppose that the factor that shifts the rent on one unit of housing follows ABM given by

$$x(t) = \alpha dt + \sigma d\Omega(t), \quad \sigma > 0, \tag{19}$$

where $x(t)$ has a constant expected growth rate α and a constant variance of the growth rate σ^2, and $d\Omega(t)$ is an increment to a standard Wiener process. The value of one unit of land after development given by equation (6) is thus equal to

$$W_a(x, D, k) = \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)} + \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)^2}. \tag{20}$$

Using equation (19) and applying Itô's lemma yields (see Appendix B)

$$Z(x) = B_1 e^{\beta_1 x} + B_2 e^{\beta_2 x} + \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)} + \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)^2}, \tag{21}$$

where B_1 and B_2 are constants to be determined, and β_1 and β_2 are defined in equation (12). The counterparts of equations (17) and (18), which respectively show the conditions for choices of development timing and capital intensity when the rent on one unit of housing follows ABM, are respectively, given by

$$M_a(x^*, k^*) = (\frac{1}{\beta_1} - x^*)\frac{(1-\tau_r)e^{-D}k^{*\gamma}}{(\rho+\tau_a)} - \frac{(1-\tau_r)e^{-D}k^{*\gamma}\alpha}{(\rho+\tau_a)^2} + (c + f_K)k^* + f_Q k^{*\gamma} + f_L = 0, \tag{22}$$

$$S_a(x^*,k^*)=\frac{(1-\tau_r)\gamma e^{-D}k^{*\gamma-1}x^*}{(\rho+\tau_a)}+\frac{(1-\tau_r)\gamma e^{-D}k^{*\gamma-1}\alpha}{(\rho+\tau_a)^2}-(c+f_K)-\gamma f_Q k^{*\gamma-1}=0. \qquad (23)$$

We have not obtained analytically tractable solutions for both the choice of date of development, x^*, and that of capital density, k^* for both cases in which rents on housing follow GBM and ABM, respectively. To gain insights regarding how the underlying exogenous forces affect x^* and k^*, we must focus on both the condition for deriving x^* given by equation (17), and that for deriving k^* given by equation (18) for the case where rents on housing to follow GBM. Equation (17) implicitly defines the positive dependence of x^* on k^*, and equation (18) implicitly defines the positive dependence of k^* on x^*. We derive these two relationships in equations (C1)-(C7) in Appendix C. Similar arguments can be applied to equations (22) and (23) for the case where rents on housing follow ABM. We derive their relationships in equations (D1)-(D7) in Appendix D.

IV. Comparative Statics

Propositions 1 and 2 hold regardless of whether the rent on one unit of housing follows GBM or ABM.

Proposition 1: *Suppose that the capital intensity is given. A landowner will wait for a better state to develop* (x^* *is increased*) *if* (*i*) *the landowner's parcel of land is located farther from the CBD* (D *is increased*)*, or the landowner pays more taxes on either urban rental income* (τ_r *is increased*)*, or property after development* (τ_a *is increased*)*;* (*ii*) *the landowner pays more fees on capital* (f_K *is increased*)*, on housing* (f_Q *is increased*) *or on land* (f_L *is increased*)*;* (*iii*) *the landowner pays less tax on property before development* (τ_b *is decreased*).

Proof: See Appendix E.

The intuition behind Proposition 1 is as follows. Consider the premises stated in Proposition 1(i), i.e., an increase in either the distance from the CBD, the tax rate on urban rental income, or the tax rate on property after development. The value of one unit of developed property, given by either equation (10) for the case of GBM or equation (20) for the case of ABM, will then be reduced. Given the premises stated in Proposition 1(ii), i.e., an increase in the fee on either capital, housing, or land, the costs of development given by equation (3) will be increased. All of these decrease the net value of developing one unit of vacant land immediately. To compensate this loss, a landowner will thus wait for a better state to develop vacant land. By contrast, given the premise stated in Proposition 1(iii), i.e., a fall in the tax rate before development, the after-tax value of property before development will be raised. The opportunity cost for a landowner to develop vacant land will thus be increased, forcing the owner to develop vacant land at a better state. The results of Proposition 1 is also shown by an upward shift from line MM to line $M'M'$ as shown by Figures 1-4.

Proposition 2: *Suppose that the timing of development is given. A landowner will employ capital less intensively* (k^* *is decreased*) *if* (*i*) *the landowner's parcel of land is located farther from the CBD, or the landowner pays more taxes on either urban rental income or property after development;* (*ii*) *the landowner pays more fees on either capital or housing;* (*iii*) *The landowner's choice of capital intensity is independent on either the fee rate on land or the tax on property before development.*

Proof: See Appendix F.

The intuition behind Propositions 2 is as follows. Given the premises stated in

Proposition 2(i), i.e., an increase in either the distance from the CBD, the tax rate on urban rental income, or the tax rate on property after development, the marginal benefit of capital will be reduced. Given the premises stated in Proposition 2(ii), i.e., an increase in the fee rate on either capital or housing, the marginal cost of capital will be raised. All of these induce a landowner to employ less capital. Propositions 2(i) and 2(ii) are also shown by an leftward shift from line SS to line $S'S'$ as shown by Figures 2-4. By contrast, a change in either the fee rate on land or the tax rate on property before development will affect neither the marginal benefit nor the marginal cost of capital, and is thus unrelated to the choice of capital intensity. This explains the reason for Proposition 2(iii).

Propositions 3 and 4 stated below (see also Table 1) indicate how changes in various forms of fees and taxes affect a landowner's choices of development timing and capital intensity when these two choices are jointly determined. They respectively examine the cases where rents on housing follow GBM and ABM.

Proposition 3: *Suppose that the rent on one unit of housing follows GBM.* (*i*) *A landowner will wait for a better state to develop, while employing more capital if he pays more fees on land or less tax on property before development.* (*ii*) *A landowner will wait for a better state to develop, while leaving his choice of capital intensity unchanged if the landowner either owns a parcel of land that is located farther from the CBD, or pays more taxes on urban rental income or on property after development.* (*iii*) *A landowner will wait for a better state to develop vacant land, while employing more capital if he pays more fees on housing.* (*iv*) *A landowner will wait for a better to develop, while employing less capital if he pays more fees on capital.* (*v*) *When properties before and after development are taxed at the same rate, an increase in this rate will reduce the choice of capital intensity, while exhibiting an indeterminate effect* *on the choice of development timing.*

Proof: See Appendix G.

We explain the intuition behind Propositions 3(i) by using Figure 1. Let us consider a rise in the fee rate on land or a fall in the tax rate on property before development. Suppose that the initial equilibrium may be represented by point A, the intersection of lines SS and MM, where a landowner develops vacant land at the date x_1^* and at the capital intensity k_1^*. As indicated by Propositions 1(ii) and 1(iii), respectively, when the capital intensity is fixed, the landowner will develop later (as shown by the line MM that shifts upward to line $M'M'$). We call this the "direct" effect on the choice development timing. This, in turn, will induce the landowner to employ more capital to develop vacant land, as indicated by equation (C4). We call this the "indirect" effect on the choice of capital intensity. The equilibrium point thus moves from point A along line SS to point B, the intersection of the lines SS and $M'M'$. As compared to the initial equilibrium point A, at point B, the landowner develops vacant land at a later date x_2^* ($> x_1^*$), and also employs more capital k_2^* ($> k_1^*$).

For the other cases, we must also take the following effects into account: As shown by Proposition 2, when the timing of development is fixed, given the premises stated in Propositions 3(ii)-(iv), a landowner perceives that the net marginal benefit from developing land will be reduced such that the landowner will employ less capital. We call this the "direct" effect on the choice of capital intensity. This is shown by the line SS that shifts leftward to the line $S'S'$ in Figures 2-4. This, in turn, will induce the landowner to develop earlier because waiting will then be less valuable when he develops less intensively, as indicated in equation (C1). We call this the "indirect" effect on the choice of development timing. The equilibrium point thus moves from point B along line $S'S'$ to point C in these figures. Summing these two effects may give rise to ambiguous results with regard to the choices of development timing and capital intensity. For all the cases stated in Propositions 3(ii)-(iv), a landowner will delay developing vacant land because the direct (delaying) effect more than offsets the indirect (hastening) effect. Nevertheless, the impacts of the factors stated in Propositions 3(ii)-(iv) on capital intensity can be different. As shown by Figure 2, the landowner will choose the same capital intensity when the distance from the CBD is lengthened, and the tax rate on either urban income or property after development is increased because the indirect (more capital intensive) effect exactly offsets the direct effect. Consequently, the landowner will choose the same capital intensity as compared to the initial equilibrium point A. Furthermore, as shown by Figure 3, the landowner will employ more capital when the fee rate on housing is increased because the indirect effect more than offsets the direct effect. By contrast, as shown by Figure 4, the landowner will employ less capital when the fee rate on capital is increased because the indirect effect less than offsets the direct effect.

Table 1 also shows the results of Proposition 3(v). It is clear that if properties after and before development are taxed at the same rate, i.e., $\tau_a = \tau_b$, an increase in this rate will affect the choice of development timing, x^* ambiguously as an increase in τ_a raises x^*, while an increase in τ_b reduces x^*. An increase in this rate will reduce the choice of capital intensity, k^*, as an increase in τ_a does not affect k^*, while an increase in τ_b reduces k^*. We thus explain the reasoning for Proposition 3(iv).

Proposition 4: *Suppose that the rent on one unit of housing follows ABM.* (*i*) *A* *landowner will wait for a better state to develop, while employing more capital if he pays either more fees on land or less tax on property before development.* (*ii*) *A landowner will wait for a better state to develop, while leaving his choice of capital intensity unchanged if he pays more fees on housing.* (*iii*) *A landowner will wait for a better state to develop, while employing less capital if he either owns a parcel of land that is located farther from the CBD, pays more fees on capital, or pays more taxes on urban rental income and property after development.* (*iv*) *When properties before and after development are taxed at the same rate, an increase in this rate will reduce the choice of capital intensity, while exhibiting an indeterminate effect on the choice of development timing.*

Proof: See Appendix H.

The reason for Proposition 4(i) is exactly the same as that for Proposition 3(i). Consider the premises stated in Proposition 4(i), i.e., a rise in the fee rate on land or a fall in the tax rate on property before development. Both of which do not exhibit direct impacts on the choice of capital intensity and will thus exhibit the same qualitative indirect effect on the choice of development timing and capital intensity as for the case where rents on housing follow GBM. This is also shown by Figure 1. For factors other than these two instruments, both lines that characterize choices of the development timing and capital intensity, i.e., lines MM and SS, respectively in Figures 2-4, will shift when these factors are changed. We can compare how these two lines shift when the rent on one unit housing follows ABM with those when it follows GBM. When x(t) follows ABM, it will be normally distributed. As a result, the negative impacts of these factors on the choice of capital intensity will be greater as compared to the case where x(t) follows GBM, which implies a log-normal distribution of $x(t)$ that is skew to the right. Consequently, a rise in the fee rate on housing, which raises the choice of capital intensity when x(t) follows GBM as shown by Figure 3, will instead leave the choice of capital intensity unchanged when x(t) follows ABM, as shown by Figure 2. This explains the reason for Proposition 4(ii).

The reason for Proposition 4(iii) is as follows: a rise in either the distance from the CBD or the tax rate on either urban income or property after development does not exhibit any impact on the choice of capital intensity when x(t) follows GBM, as shown by Figure 2. However, it will reduce the choice of capital intensity when x(t) follows ABM, as shown by Figure 4. A rise in the fee rate on capital already reduces the choice of capital intensity when x(t) follows GBM, and thus also exhibits the same qualitative effect when x(t) follows ABM. This is also shown in Figure 4.

Table 1 also shows the results of Proposition 4(iv). It is clear that if properties after and before development are taxed at the same, i.e., $\tau_a = \tau_b$, an increase in this rate will affect the choice of development timing, x^*, ambiguously as an increase in τ_a raises x^*, while an increase in τ_b reduces x^*. An increase in this rate will reduce the choice of capital intensity, k^*, as increases in both τ_a and τ_b reduce k^*. We thus explain the reason for Proposition 4(iv).

Insert Figures 1, 2, 3 and 4 here

McFarlane (1999) is closely related to our article, but departs from ours as follows. First, McFarlane does not impose any functional form on the rent on housing. His result thus hinges on a ratio which captures the growth rate of the rent on housing.

Second, McFarlane assumes that the rent on housing is increasing over time, thus precluding the possibility of negative rents on housing as captured by the ABM of $x(t)$. Table 1 indicates that we yield the same qualitative results regarding the impacts on choices of development timing and capital intensity as McFarlane's for two policy instruments, i.e., the fee rate on land and the tax rate on land before development. We yield the same qualitative result regarding the impact on the choice of development timing as McFarlane's for two exogenous variables, i.e., the distance from the CBD and the fee rate on housing. Finally, we yield different qualitative results regarding the impact on the timing of development for two policy instruments, i.e., the fee rate on capital and the tax rate on urban income; We find that the imposing of these two instruments delays development, while McFarlane finds that it exhibits no effect on the timing of development.

We can also compare our results with those of Arnott and Lewis (1979) and Capozza and Li (1994) regarding how taxation on property before and after development affects choices of development timing and capital intensity. Given that the rent on housing follows GBM, our results are qualitative the same as those of Arnott and Lewis (1979). Our model can be viewed as an extension of their model as they abstract from the spatial factor and assume that the rent on housing is increasing over time at a constant rate. We find that their results continue to hold even when we relax their assumptions. Finally, given that the rent on housing follows ABM, we get the same qualitative result as Capozza and Li's as our assumptions are exactly the same as theirs.

V. Conclusion

This article compares the effects of various fiscal policies on choices of development timing and capital intensity when rents on housing follow geometric Brownian motion with those when rents follow arithmetic Brownian motion. These policy instruments include fees on capital, housing, and land, and taxes on urban income, and properties both before and after development. Regardless of the motion of rents, when one choice is fixed, the effects of these policy instruments on the other choice are qualitatively the same. When the two choices are determined endogenously, although these policy instruments exhibit the same qualitative effect on the choice of development timing, they may exhibit different effects on the choice of capital intensity if rents on housing follow different types of motions.

Our article can be extended as follows. In a recent article, Arnott (2005) investigates the issue regarding how to choose a set of policy instruments, which include time-invariant tax rates on predevelopment land value, post development residual site value, and structure value such that the three objectives are achieved: neutrality with respect to development timing and density, and expropriation of a desired fraction of value. Within our framework, we can also investigate how these sets of policy investments are affected by the existence of uncertainty, and by different assumptions regarding the types of uncertainty.

Appendix A:

Consider the problem of choosing the appropriate time of development and capital intensity such that equation (8) is maximized. Given that $x(t)$ follows GBM as shown by equation (9), the Bellman-Hamilton-Jacobi value function for this problem can be written as

$$(\rho+\tau_b)Z(x)= \max_{T,k}\left\{\frac{(\rho+\tau_b-\alpha)}{(\rho+\tau_a-\alpha)}(1-\tau_r)xe^{-D}k^{\gamma}-(\rho+\tau_b)[(c+f_K)k+f_Qk^{\gamma}+f_L]+E_t\frac{dZ(x)}{dt}\right\}. \quad \text{(A1)}$$

Treating $Z(\cdot)$ as an asset value, using equation (9), and applying Itô's lemma yields its expected capital gain as

$$E_t\frac{dZ(\cdot)}{dt}=\frac{1}{2}\sigma^2x^2\frac{\partial^2Z(\cdot)}{\partial x^2}+\alpha x\frac{\partial Z(\cdot)}{\partial x}. \quad \text{(A2)}$$

Substituting equation (A2) into equation (A1) yields the differential equation

$$\frac{1}{2}\sigma^2x^2\frac{\partial^2Z(\cdot)}{\partial x^2}+\alpha x\frac{\partial Z(\cdot)}{\partial x}-(\rho+\tau_b)Z(\cdot)+\frac{(\rho+\tau_b-\alpha)(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a-\alpha)} -(\rho+\tau_b)[(c+f_K)k+f_Qk^{\gamma}+f_L]=0. \quad \text{(A3)}$$

Equation (A3) indicates that the expected capital gain of the asset plus the cash flow, $\frac{(\rho+\tau_b-\alpha)(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a-\alpha)}-(\rho+\tau_b)[(c+f_K)k+f_Qk^{\gamma}+f_L]$, the flow-equivalent rents from developing vacant land minus the flow equivalent costs of development, should be equal to the normal return $(\rho+\tau_b)Z(\cdot)$ to prevent any arbitrage profits from arising. The term x^{β} solves the homogeneous part of the equation (A3). Substituting this into equation (A3) yields the quadratic equation given by

$$-\frac{\sigma^2}{2}\beta(\beta-1)-\alpha\beta+(\rho+\tau_b)=0. \quad \text{(A4)}$$

Define β_1 and β_2 as the larger and smaller roots of equation (A4). One particular solution from the non-homogeneous part of equation (A3) is:

$$Z_p(x_1, x_2) = \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_b-\alpha)} - [(c+f_K)k + f_Q k^{\gamma} + f_L]. \tag{A5}$$

The general solution of equation (A3), which is composed of solutions from both the homogeneous and non-homogeneous parts of equation (A3), is shown in equation (11).

Appendix B:

Consider the problem of choosing the appropriate time and density of development such that equation (8) is maximized. Given that $x(t)$ follows ABM as shown by equation (19), the Bellman-Hamilton-Jacobi value function for this problem can be written as

$$\begin{aligned}(\rho+\tau_b)Z(x) = \max_{T,k}\Bigg\{ &\frac{(\rho+\tau_b)}{(\rho+\tau_a)}(1-\tau_r)xe^{-D}k^{\gamma} + \frac{(1-\tau_r)e^{-D}k^{\gamma}\alpha(\tau_b-\tau_a)}{(\rho+\tau_a)^2} \\ &-(\rho+\tau_b)[(c+f_K)k + f_Q k^{\gamma} + f_L] + E_t\frac{dZ(x)}{dt}\Bigg\}.\end{aligned} \tag{B1}$$

Treating $Z(\cdot)$ as an asset value, using equation (19), and applying Itô's lemma yields its expected capital gain as

$$E_t\frac{dZ(\cdot)}{dt} = \frac{1}{2}\sigma^2\frac{\partial^2 Z(\cdot)}{\partial x^2} + \alpha\frac{\partial Z(\cdot)}{\partial x}. \tag{B2}$$

Substituting equation (B2) into equation (B1) yields the differential equation

$$\begin{aligned}&\frac{1}{2}\sigma^2\frac{\partial^2 Z(\cdot)}{\partial x^2} + \alpha\frac{\partial Z(\cdot)}{\partial x} - (\rho+\tau_b)Z(\cdot) \\ &+\frac{(\rho+\tau_b)(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)} + \frac{(1-\tau_r)e^{-D}k^{\gamma}\alpha(\tau_b-\tau_a)}{(\rho+\tau_a)^2} - (\rho+\tau_b)[(c+f_K)k + f_Q k^{\gamma} + f_L] = 0.\end{aligned} \tag{B3}$$

Equation (B3) indicates that the expected capital gain of the asset plus the cash flow,

$$\frac{(\rho+\tau_b)(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)} + \frac{(1-\tau_r)e^{-D}k^{\gamma}\alpha(\tau_b-\tau_a)}{(\rho+\tau_a)^2} - (\rho+\tau_b)[(c+f_K)k + f_Q k^{\gamma} + f_L],$$

should be equal to the normal return $(\rho + \tau_b)Z(\cdot)$ to prevent any arbitrage profits from arising. The term x^{β} solves the homogeneous part of the equation (B3). Substituting this into equation (B3) also yields the quadratic equation given by equation (A4). One particular solution from the non-homogeneous part of equation (B3) is:

$$Z(x) = \frac{(1-\tau_r)xe^{-D}k^{\gamma}}{(\rho+\tau_a)} + \frac{(1-\tau_r)e^{-D}k^{\gamma}\alpha}{(\rho+\tau_a)^2} - [(c + f_K)k + f_Q k^{\gamma} + f_L]. \tag{B4}$$

The general solution of equation (B3), which is composed of solutions from both the homogeneous and non-homogeneous parts of equation (B3), is shown in equation (21).

Appendix C:

Totally differentiating equation (17) with respect to k^*, and using equations (17) and (18) yields

$$\frac{\partial x^*}{\partial k^*} = \frac{\Delta_{12}}{-\Delta_{11}} > 0, \tag{C1}$$

where

$$\Delta_{11} = \frac{\partial M_g(x^*, k^*)}{\partial x^*} = -(1 - \frac{1}{\beta_1})\frac{(1-\tau_r)e^{-D}k^{*\gamma}}{(\rho+\tau_a-\alpha)} < 0, \tag{C2}$$

$$\Delta_{12} = \frac{\partial M_g(x^*, k^*)}{\partial k^*} = \frac{(1-\tau_r)e^{-D}k^{*\gamma-1}x^*}{\beta_1(\rho+\tau_a-\alpha)} > 0. \tag{C3}$$

Totally differentiating equation (18) with respect to x^*, and using equations (17) and (18) yields

$$\frac{\partial k^*}{\partial x^*} = \frac{\Delta_{21}}{-\Delta_{22}} > 0, \tag{C4}$$

where

$$\Delta_{22} = \frac{\partial S_g(x^*, k^*)}{\partial k^*} = \frac{-(1-\gamma)(c + f_K)}{k^*} < 0, \tag{C5}$$

$$\Delta_{21} = \frac{\partial S_g(x^*, k^*)}{\partial x^*} = \frac{\gamma(1-\tau_r)e^{-D}k^{*\gamma-1}}{(\rho + \tau_a - \alpha)} > 0. \tag{C6}$$

The Jacobian condition also requires that

$$\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21} > 0. \tag{C7}$$

We depict the impact of k^* on x^* in equation (C1), and that of x^* on k^* in equation (C4) by line MM and line SS in Figures 1-4, respectively. Equation (C7) requires that the slope of SS be steeper than that of MM, and we find that this requirement is satisfied.

This completes the proof.

Appendix D:

Totally differentiating equation (22) with respect to k^* and using equations (22) and (23) yields

$$\frac{\partial x^*}{\partial k^*} = \frac{\Delta'_{12}}{-\Delta'_{11}} > 0, \tag{D1}$$

$$\Delta'_{11} = \frac{\partial M_a(x^*, k^*)}{\partial x^*} = -\frac{(1-\tau_r)e^{-D}k^{*\gamma}}{(\rho + \tau_a)} < 0, \tag{D2}$$

where

$$\Delta'_{12} = \frac{\partial M_a(x^*, k^*)}{\partial k^*} = (1-\gamma)(c + f_K) - \frac{\gamma f_L}{k^*} > 0. \tag{D3}$$

Totally differentiating equation (23) with respect to x^*, and using equations (22) and

(23) yields

$$\frac{\partial k^*}{\partial x^*} = \frac{\Delta'_{21}}{-\Delta'_{22}} > 0, \tag{D4}$$

where

$$\Delta'_{22} = \frac{\partial S_a(x^*, k^*)}{\partial k^*} = \frac{-(1-\gamma)(c + f_K)}{k^*} < 0, \tag{D5}$$

$$\Delta'_{21} = \frac{\partial S_a(x^*, k^*)}{\partial x^*} = \frac{(1-\tau_r)\gamma e^{-D} k^{*\gamma-1}}{(\rho + \tau_a)} > 0. \tag{D6}$$

The Jacobian condition also requires that

$$\Delta'_{11}\Delta'_{22} - \Delta'_{12}\Delta'_{21} > 0. \tag{D7}$$

We depict the impact of k^* on x^* in equation (D1), and that of x^* on k^* in equation (D4) by line MM and line SS in Figures 1-4, respectively. Equation (D7) requires that the slope of line SS be steeper than that of line MM in Figure 1, and we find that this condition is satisfied.

This completes the proof.

Appendix E:

Differentiating $M_g(x^*, k^*)$ in equation (17) with respect to D, f_K, f_Q, f_L, τ_r, τ_a, and τ_b yields

$$\frac{\partial x^*}{\partial D} = \frac{\Delta_{13}}{-\Delta_{11}} > 0, \tag{E1}$$

$$\frac{\partial x^*}{\partial f_K} = \frac{\Delta_{14}}{-\Delta_{11}} > 0, \tag{E2}$$

$$\frac{\partial x^*}{\partial f_Q} = \frac{\Delta_{15}}{-\Delta_{11}} > 0, \tag{E3}$$

$$\frac{\partial x^*}{\partial f_L} = \frac{\Delta_{16}}{-\Delta_{11}} > 0, \tag{E4}$$

$$\frac{\partial x^*}{\partial \tau_r} = \frac{\Delta_{17}}{-\Delta_{11}} > 0, \tag{E5}$$

$$\frac{\partial x^*}{\partial \tau_a} = \frac{\Delta_{18}}{-\Delta_{11}} > 0, \tag{E6}$$

$$\frac{\partial x^*}{\partial \tau_b} = \frac{\Delta_{19}}{-\Delta_{11}} < 0, \tag{E7}$$

where

$$\Delta_{13} = \frac{\partial M_g(x^*, k^*)}{\partial D} = (1 - \frac{1}{\beta_1}) \frac{(1-\tau_r)e^{-D} k^{*\gamma} x^*}{(\rho + \tau_a - \alpha)} > 0,$$

$$\Delta_{14} = \frac{\partial M_g(x^*, k^*)}{\partial f_K} = k^*,$$

$$\Delta_{15} = \frac{\partial M_g(x^*, k^*)}{\partial f_Q} = k^{*\gamma},$$

$$\Delta_{16} = \frac{\partial M_g(x^*, k^*)}{\partial f_L} = 1,$$

$$\Delta_{17} = \frac{\partial M_g(x^*, k^*)}{\partial \tau_r} = (1 - \frac{1}{\beta_1}) \frac{e^{-D} k^{*\gamma} x^*}{(\rho + \tau_a - \alpha)},$$

$$\Delta_{18} = \frac{\partial M_g(x^*, k^*)}{\partial \tau_a} = (1 - \frac{1}{\beta_1}) \frac{(1-\tau_r)e^{-D} k^{*\gamma} x^*}{(\rho + \tau_a - \alpha)^2},$$

$$\Delta_{19} = \frac{\partial M_g(x^*, k^*)}{\partial \tau_b} = -\frac{1}{\beta_1^2} \frac{\partial \beta_1}{\partial \tau_b} \frac{(1-\tau_r)e^{-D} k^{*\gamma} x^*}{(\rho + \tau_a - \alpha)} < 0.$$

since $d\beta_1 / d\tau_b > 0$.

Differentiating $M_a(x^*, k^*)$ in equation (23) with respect to D, f_K, f_Q, f_L, τ_r, τ_a, and τ_b yields

$$\frac{\partial x^*}{\partial D} = \frac{\Delta'_{13}}{-\Delta'_{11}} > 0, \tag{E8}$$

$$\frac{\partial x^*}{\partial f_K} = \frac{\Delta'_{14}}{-\Delta'_{11}} > 0, \tag{E9}$$

$$\frac{\partial x^*}{\partial f_Q} = \frac{\Delta'_{15}}{-\Delta'_{11}} > 0, \tag{E10}$$

$$\frac{\partial x^*}{\partial f_L} = \frac{\Delta'_{16}}{-\Delta'_{11}} > 0, \tag{E11}$$

$$\frac{\partial x^*}{\partial \tau_r} = \frac{\Delta'_{17}}{-\Delta'_{11}} > 0, \tag{E12}$$

$$\frac{\partial x^*}{\partial \tau_a} = \frac{\Delta'_{18}}{-\Delta'_{11}} > 0, \tag{E13}$$

$$\frac{\partial x^*}{\partial \tau_b} = \frac{\Delta'_{19}}{-\Delta'_{11}} < 0, \tag{E14}$$

where

$$\Delta'_{13} = \frac{\partial M_a(x^*, k^*)}{\partial D} = (c + f_K)k^* + f_Q k^{*\gamma} + f_L,$$

$$\Delta'_{14} = \frac{\partial M_a(x^*, k^*)}{\partial f_K} = k^*,$$

$$\Delta'_{15} = \frac{\partial M_a(x^*, k^*)}{\partial f_Q} = k^{*\gamma},$$

$$\Delta'_{16} = \frac{\partial M_a(x^*, k^*)}{\partial f_L} = 1,$$

$$\Delta_{17}' = \frac{\partial M_a(x^*, k^*)}{\partial \tau_r} = \frac{1}{(1-\tau_r)}[(c + f_K)k^* + f_Q k^{*\gamma} + f_L],$$

$$\Delta_{18}' = \frac{\partial M_a(x^*, k^*)}{\partial \tau_a} = \frac{(1-\tau_r)e^{-D}k^{*\gamma}\alpha}{(\rho+\tau_a)^3} + \frac{1}{(\rho+\tau_a)}[(c + f_K)k^* + f_Q k^{*\gamma} + f_L],$$

$$\Delta_{19}' = \frac{\partial M_a(x^*, k^*)}{\partial \tau_b} = -\frac{1}{\beta_1^2}\frac{\partial \beta_1}{\partial \tau_b}\frac{(1-\tau_r)e^{-D}k^{*\gamma}}{(\rho+\tau_a-\alpha)} < 0\,.$$

This completes the proof.

Appendix F:

Differentiating $S_g^*(x^*, k^*)$ in equation (18) with respect to D, f_K, f_Q, f_L, τ_r, τ_a, and τ_b yields

$$\frac{\partial k^*}{\partial D} = \frac{\Delta_{23}}{-\Delta_{22}} < 0, \qquad \text{(F1)}$$

$$\frac{\partial k^*}{\partial f_K} = \frac{\Delta_{24}}{-\Delta_{22}} < 0, \qquad \text{(F2)}$$

$$\frac{\partial k^*}{\partial f_Q} = \frac{\Delta_{25}}{-\Delta_{22}} < 0, \qquad \text{(F3)}$$

$$\frac{\partial k^*}{\partial f_L} = \frac{\Delta_{26}}{-\Delta_{22}} = 0, \qquad \text{(F4)}$$

$$\frac{\partial k^*}{\partial \tau_r} = \frac{\Delta_{27}}{-\Delta_{22}} < 0, \qquad \text{(F5)}$$

$$\frac{\partial k^*}{\partial \tau_a} = \frac{\Delta_{28}}{-\Delta_{22}} < 0, \qquad \text{(F6)}$$

$$\frac{\partial k^*}{\partial \tau_b} = \frac{\Delta_{29}}{-\Delta_{22}} = 0, \qquad \text{(F7)}$$

where

$$\Delta_{23} = \frac{\partial S_g(x^*, k^*)}{\partial D} = \frac{-(1-\tau_r)e^{-D}k^{*\gamma-1}x^*}{(\rho+\tau_a-\alpha)},$$

$$\Delta_{24} = \frac{\partial S_g(x^*, k^*)}{\partial f_K} = -1,$$

$$\Delta_{25} = \frac{\partial S_g(x^*, k^*)}{\partial f_Q} = -\gamma k^{*\gamma-1},$$

$$\Delta_{26} = \frac{\partial S_g(x^*, k^*)}{\partial f_L} = 0,$$

$$\Delta_{27} = \frac{\partial S_g(x^*, k^*)}{\partial \tau_r} = \frac{-e^{-D}\gamma k^{*\gamma-1}x^*}{(\rho+\tau_a-\alpha)},$$

$$\Delta_{28} = \frac{\partial S_g(x^*, k^*)}{\partial \tau_a} = \frac{-(1-\tau_r)e^{-D}\gamma k^{*\gamma-1}x^*}{(\rho+\tau_a-\alpha)^2},$$

$$\Delta_{29} = \frac{\partial S_g(x^*, k^*)}{\partial \tau_b} = 0.$$

Differentiating $S_a^*(x^*, k^*)$ in equation (24) with respect to D, f_K, f_Q, f_L, τ_r, τ_a, and τ_b yields

$$\frac{\partial k^*}{\partial D} = \frac{\Delta'_{23}}{-\Delta'_{22}} < 0, \tag{F8}$$

$$\frac{\partial k^*}{\partial f_K} = \frac{\Delta'_{24}}{-\Delta'_{22}} < 0, \tag{F9}$$

$$\frac{\partial k^*}{\partial f_Q} = \frac{\Delta'_{25}}{-\Delta'_{22}} < 0, \tag{F10}$$

$$\frac{\partial k^*}{\partial f_L} = \frac{\Delta'_{26}}{-\Delta'_{22}} = 0, \tag{F11}$$

$$\frac{\partial k^*}{\partial \tau_r} = \frac{\Delta'_{27}}{-\Delta'_{22}} < 0, \tag{F12}$$

$$\frac{\partial k^*}{\partial \tau_a} = \frac{\Delta'_{28}}{-\Delta'_{22}} < 0, \tag{F13}$$

$$\frac{\partial k^*}{\partial \tau_b} = \frac{\Delta'_{29}}{-\Delta'_{22}} = 0, \tag{F14}$$

where

$$\Delta'_{23} = \frac{\partial S_a(x^*, k^*)}{\partial D} = -[(c + f_K) + \gamma f_Q k^{*\gamma-1}],$$

$$\Delta'_{24} = \frac{\partial S_a(x^*, k^*)}{\partial f_K} = -1,$$

$$\Delta'_{25} = \frac{\partial S_a(x^*, k^*)}{\partial f_Q} = -\gamma k^{*\gamma-1},$$

$$\Delta'_{26} = \frac{\partial S_a(x^*, k^*)}{\partial f_L} = 0,$$

$$\Delta'_{27} = \frac{\partial S_a(x^*, k^*)}{\partial \tau_r} = \frac{-1}{(1-\tau_r)}[(c + f_K) + \gamma f_Q k^{*\gamma-1}],$$

$$\Delta'_{28} = \frac{\partial S_a(x^*, k^*)}{\partial \tau_a} = \frac{-1}{(\rho+\tau_a)}[(c + f_K) + \gamma f_Q k^{*\gamma-1}] - \frac{(1-\tau_r)\gamma e^{-D} k^{*\gamma-1}\alpha}{(\rho+\tau_a)^3},$$

$$\Delta'_{29} = \frac{\partial S_a(x^*, k^*)}{\partial \tau_b} = 0.$$

This completes the proof.

Appendix G:

Define $d_0 = 1 - \frac{\gamma f_L}{(1-\gamma)(c + f_K)k^*} > 0$. Totally differentiating x^* and k^* with respect to D, f_K, f_Q, f_L, τ_r, τ_a, and τ_b yields

$$\frac{dx^*}{dD} = \frac{\partial x^*}{\partial D} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial D} = \alpha x^*[1 - \frac{\beta_1 d_0}{(\beta_1 - 1)}] > 0, \qquad \text{(G1)}$$

$$(>0)\,(>0)\,(<0)$$

$$\frac{dk^*}{dD} = \frac{\partial k^*}{\partial D} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial D} = 0, \qquad \text{(G2)}$$

$$(<0)\,(>0)\,(>0)$$

$$\frac{dx^*}{df_K} = \frac{\partial x^*}{\partial f_K} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial f_K} = \frac{\gamma x^*(d_0^{-1} - 1)}{(\beta_1 - 1)(1-\gamma)(c + f_K)}(\frac{1}{d_0}) > 0, \qquad \text{(G3)}$$

$$(>0)\,(>0)\,(<0)$$

$$\frac{dk^*}{df_K} = \frac{\partial k^*}{\partial f_K} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial f_K} = \frac{-k^*}{(1-\gamma)(c + f_K)}[1 - \frac{\gamma}{(1 - 1/\beta_1)}] < 0, \qquad \text{(G4)}$$

$$(<0)\,(>0)\,(>0)$$

$$\frac{dk^*}{df_Q} = \frac{\partial x^*}{\partial f_Q} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial f_Q} = \frac{\gamma x^* k^{*\gamma-1}(d_0^{-1} - \gamma)}{(\beta_1 - 1)(1-\gamma)(c + f_K)} > 0, \qquad \text{(G5)}$$

$$(>0)\,(>0)\,(<0)$$

$$\frac{dk^*}{df_Q} = \frac{\partial k^*}{\partial f_Q} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial f_Q} = \frac{\gamma k^{*\gamma}}{(\beta_1 - 1)(1-\gamma)(c + f_K)} > 0, \qquad \text{(G6)}$$

$$(<0)\,(>0)\,(>0)$$

$$\frac{dx^*}{df_L} = \frac{\partial x^*}{\partial f_L} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial f_L} > 0, \qquad \text{(G7)}$$

$$(>0)\,(>0)\,(=0)$$

$$\frac{dk^*}{df_L} = \frac{\partial k^*}{\partial f_L} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial f_L} > 0, \qquad \text{(G8)}$$

$$(=0)\,(>0)\,(>0)$$

$$\frac{dx^*}{d\tau_r} = \frac{\partial x^*}{\partial \tau_r} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial \tau_r} = \frac{1}{(1-\tau_r)}\frac{dx^*}{dD} > 0, \qquad \text{(G9)}$$

$$(>0)\,(>0)\,(<0)$$

$$\frac{dk^*}{d\tau_r} = \frac{\partial k^*}{\partial \tau_r} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_r} = 0, \quad \text{(G10)}$$
$$(<0)(>0)(>0)$$

$$\frac{dx^*}{d\tau_a} = \frac{\partial x^*}{\partial \tau_a} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial \tau_a} = \frac{x^*}{(\rho+\tau_a-\alpha)}[1-\frac{\gamma\beta_1 d_0}{(\beta_1-1)}] > 0, \quad \text{(G11)}$$
$$(>0)(>0)(<0)$$

$$\frac{dk^*}{d\tau_a} = \frac{\partial k^*}{\partial \tau_a} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_a} = 0, \quad \text{(G12)}$$

$$\frac{dx^*}{d\tau_b} = \frac{\partial x^*}{\partial \tau_b} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial \tau_b} < 0, \quad \text{(G13)}$$
$$(<0)(>0)(=0)$$

$$\frac{dk^*}{d\tau_b} = \frac{\partial k^*}{\partial \tau_b} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_b} < 0. \quad \text{(G14)}$$

This completes the proof.

Appendix H:

Totally differentiating x^* and k^* with respect to D, f_K, f_Q, f_L, τ_r, τ_a, and τ_b yields

$$\frac{dx^*}{dD} = \frac{\partial x^*}{\partial D} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial D}$$
$$(>0)(>0)(<0) \quad \text{(G15)}$$
$$= \frac{\gamma}{\beta_1(1-\gamma)(c+f_K)k^*}[\frac{(c+f_K)k^* + f_Q k^{*\gamma} + f_L}{d_0} - (c+f_K)k^* - \gamma f_Q k^{*\gamma}] > 0,$$

$$\frac{dk^*}{dD} = \frac{\partial k^*}{\partial D} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial D} = \frac{-(1-\tau_r)e^{-aD}k^*}{\beta_1(1-\gamma)(c+f_K)(\rho+\tau_a)} < 0, \quad \text{(G16)}$$
$$(<0)(>0)(>0)$$

$$\frac{dx^*}{df_K} = \frac{\partial x^*}{\partial f_K} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial f_K} = \frac{\gamma(d_0^{-1}-1)}{\beta_1(1-\gamma)(c+f_K)} > 0, \tag{G17}$$
$$(>0)\,(>0)\,(<0)$$

$$\frac{dk^*}{df_K} = \frac{\partial k^*}{\partial f_K} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial f_K} = \frac{-k^*}{(c+f_K)} < 0, \tag{G18}$$
$$(<0)\,(>0)\,(>0)$$

$$\frac{dx^*}{df_Q} = \frac{\partial x^*}{\partial f_Q} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial f_Q} = \frac{\gamma k^{*\gamma-1}(d_0^{-1}-1)}{\beta_1(1-\gamma)(c+f_K)} > 0, \tag{G19}$$
$$(<0)\,(>0)\,(>0)$$

$$\frac{dk^*}{df_Q} = \frac{\partial k^*}{\partial f_Q} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial f_Q} = 0, \tag{G20}$$
$$(>0)\,(>0)\,(<0)$$

$$\frac{dx^*}{df_L} = \frac{\partial x^*}{\partial f_L} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial f_L} > 0, \tag{G21}$$
$$(>0)\,(>0)\,(=0)$$

$$\frac{dk^*}{df_L} = \frac{\partial k^*}{\partial f_L} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial f_L} > 0, \tag{G22}$$
$$(=0)\,(>0)\,(>0)$$

$$\frac{dx^*}{d\tau_r} = \frac{\partial x^*}{\partial \tau_r} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial \tau_r} = \frac{1}{(1-\tau_r)}\frac{dx^*}{dD} > 0, \tag{G23}$$
$$(>0)\,(>0)\,(<0)$$

$$\frac{dk^*}{d\tau_r} = \frac{\partial k^*}{\partial \tau_r} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_r} = \frac{1}{(1-\tau_r)}\frac{dk^*}{dD} > 0, \tag{G24}$$
$$(<0)\,(>0)\,(>0)$$

$$\frac{dx^*}{d\tau_a} = \frac{\partial x^*}{\partial \tau_a} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial \tau_a} \tag{G25}$$
$$(>0)\,(>0)\,(<0)$$

$$= \frac{\gamma}{\beta_1(1-\gamma)(c+f_K)k^*}\left[\left(\frac{(1-\tau_r)e^{-D}k^{*\gamma}}{(\rho+\tau_a)^3} + \frac{f_Q k^{*\gamma}}{(\rho+\tau_a)}\right)(d_0^{-1}-\gamma)\frac{(c+f_K)}{(\rho+\tau_a)}(d_0^{-1}-1) + \frac{f_L}{(\rho+\tau_a)}\right] > 0,$$

$$\frac{dk^*}{d\tau_a} = \frac{\partial k^*}{\partial \tau_a} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_a} = \frac{-d_0}{(\rho + \tau_a)} < 0, \tag{G26}$$
$$(<0)\,(>0)\,(>0)$$

$$\frac{dx^*}{d\tau_b} = \frac{\partial x^*}{\partial \tau_b} + \frac{\partial x^*}{\partial k^*}\frac{\partial k^*}{\partial \tau_b} < 0, \tag{G27}$$
$$(<0)\,(>0)\,(=0)$$

$$\frac{dk^*}{d\tau_b} = \frac{\partial k^*}{\partial \tau_b} + \frac{\partial k^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_b} < 0, \tag{G28}$$
$$(=0)\,(>0)\,(<0)$$

This completes the proof.

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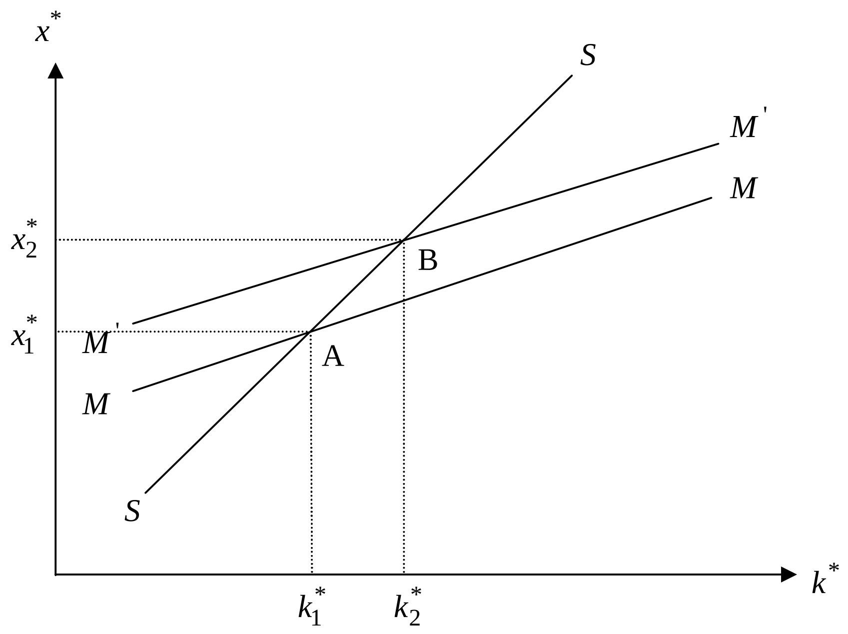

Figure 1: A rise in f_L or a fall in τ_b when the rent follows GBM or ABM.

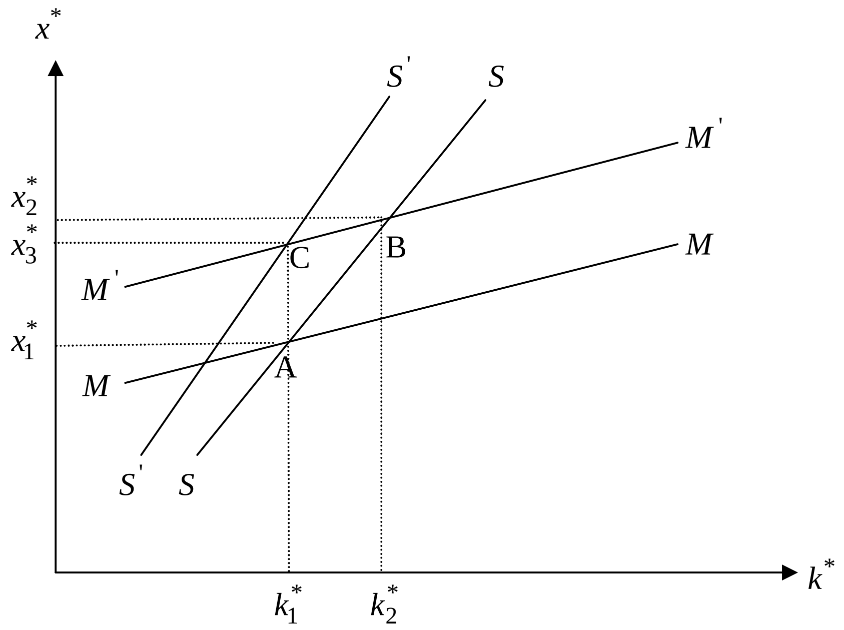

Figure 2: A rise in D, τ_r, or τ_a when the rent follows GBM, or a rise in f_Q when the rent follows ABM.

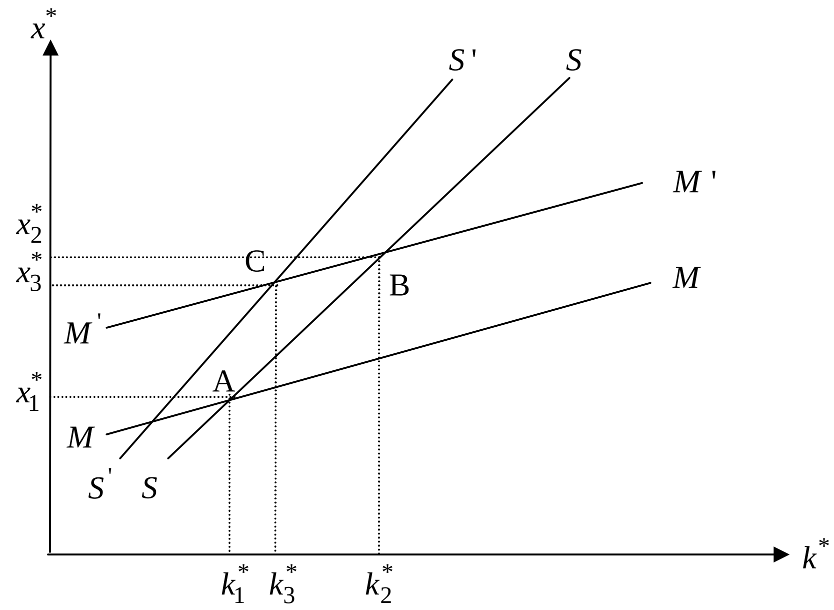

Figure 3: A rise in f_Q when the rent follows GBM

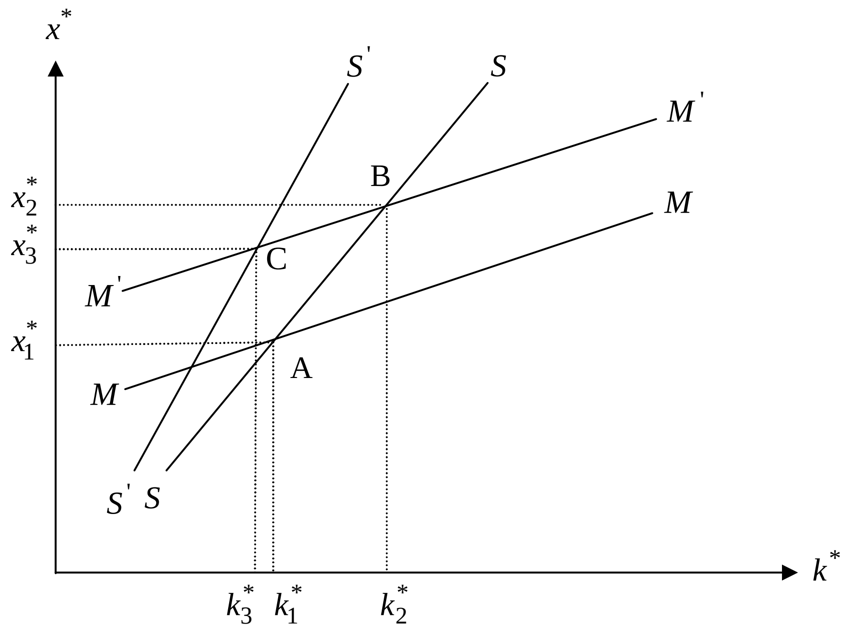

Figure 4: A rise in f_K when the rent follows GBM
or a rise in D, f_K, τ_r, or τ_a when the rent follows ABM.

Table 1: Comparative Statics Results

	k^* fixed	x^* fixed	$x(t)$ follows GBM		$x(t)$ follows ABM		McFarlane (1999)	
	x^*	k^*	x^*	k^*	x^*	k^*	x^*	k^*
D	>0	<0	>0	=0	>0	<0	>0	$\gtrless 0$
f_K	>0	<0	>0	<0	>0	<0	=0	$\gtrless 0$
f_Q	>0	<0	>0	>0	>0	=0	>0	$\gtrless 0$
f_L	>0	=0	>0	>0	>0	>0	>0	>0
τ_r	>0	<0	>0	=0	>0	<0	=0	$\gtrless 0$
τ_a	>0	<0	>0	=0	>0	<0	$\gtrless 0$	$\gtrless 0$
τ_b	<0	=0	<0	<0	<0	<0	<0	<0
$\tau_a = \tau_b$			$\gtrless 0$	<0	$\gtrless 0$	<0	$\gtrless 0$	$\gtrless 0$