The Political Economy of Shallow Lakes [1]

Gillian Salerno [2]
Rodney Beard [3]
Stuart McDonald [4]

March 9, 2007

[1]Paper presented at the 51st Annual Conference of the Australian Agricultural and Resource Economics Society, Queenstown, New Zealand, 2007

[2]Consulting

[3]CSERGE and School of Environmental Sciences, University of East Anglia

[4]Social and Information Systems Laboratory, California Institute of Technology and the School of economics, The University of Queensland

Abstract

Shallow lakes display hysteresis in their response to phosphorous loading. Gradual increases in the nutrient content of the lake can appear to have little effect on the oligotrophic state of the lake until a point at which the lake suddenly flips to a eutrophic state. Ecotaxes on phosphorous loading have been suggested as means to maintain the lake in the socially desirable state - oligotrophic or not - when society can agree on a common welfare function. In this paper, we consider the case where society is divided into two interest groups and is thus unable to agree. In particular, the communities that share the use of the lake disagree on the relative importance of the shallow lake acting as a waste sink for phosphorous run-off as opposed to other ecosystem service. A dynamic game in which communities maximize their use of the lake results in a Nash equilibrium where the lake is in a eutrophic state when in fact the Pareto-optimum would be for the lake to be in an oligotrophic state. The tax that would induce, in a non-cooperative context, all of society's members to behave in such a way as to achieve a Pareto-optimal outcome is derived. Further, both types of communities lobby to have their preferred level of tax applied based on their relative preferences for a clean lake and phosphorous loading. The effects of the lobbying on the application of the optimal tax are investigated for particular values of relative preferences and the relative size of each group.

1 Introduction

The artificial enrichment of lakes and rivers with residual nutrients from economic activity such as agriculture can lead to the transformation of these habitats from clear waters that provide a high level of ecosystem services into turbid waters containing an overabundance of aquatic plant life and often leads to the development of toxic algal blooms.

Certain bodies of water, and in particular shallow lakes, present a hysteresis in their response to phosphorous loading. That is, a lake will remain in an oligotrophic state over long periods of time with gradual increases in phosphorous loading to a point at which it suddenly flips to an altnernate, eutrophic state. Once the flip has occurred, the lake then remains eutrophic despite decreases in phosphorous loading. The threshold point at which the lake changes state is known as a Skiba point. It denotes the flip point between alternative basins of attraction and it is unique, as establised by Wagener (2003).

Several authors have integrated the dynamics of the shallow lake into economic analysis, in particular, S. R. Carpenter, D. Ludwig, W.A. Brock, W. D. Dechert, S. I. O'Donnell, L. Grüne, M. Kato, W. Semmler, K.-G. Mäler, A. Xepapadeas, A. De Zeeuw and F.O.O. Wagener. Carpenter et al. (1999) pose a lake dynamic equation with respect to phosphorous such that it can be used for economic analysis. They perform a dynamic stochastic analysis to show that models random shocks prescribe more conservative levels of phosphorous loading than deterministic models. Dechert and Brock (2000) first pose the problem as a dynamic game of communities each maximizing its welfare in its use of the lake and identify the presence of Skiba points when there are more than two communities around the lake. Grüne et al. (2005) use dynamic programming to solve the problem. O'Donnell and Dechert (2004); Dechert and O'Donnell (2005) use stochastic programming to derive Nash equilibrium results when the phosphorous loading into the lake is subject to rainfall as the random shock. Mäler et al. (2003) propose a tax as the optimal policy to induce a Pareto-optimal solution to the game.

It is assumed that society as a whole benefits from a body of water acting as a waste sink for agriculture but also providing a source of clean water for consumption, other production and recreational activities. Therefore, communities that share the use of a lake will often have the same relative preference for the lake as a waste sink to other uses that require a clean lake. In short, this means that each community around the lake will have the same welfare function for alternative uses of the lake. As shown by Mäler, Xepapadeas and De Zeeuw, a tax on phosphorous loading can achieve the Pareto-optimal state for the lake when the communities each seek to maximize their welfare in a non-cooperative manner.

When different communities benefit in different proportions from phosphorous loading to other benefits provided by the lake, however, it is not possible to model the welfare of all communities with the same function. In this case, the Pareto-optimal level of phosphorous loading will be different than when the welfare function is the same across all communitie. In addition, each community acting to maximize its welfare will lead to a different Nash equilibrium. This requires solving for a new tax to induce the Pareto-optimal level of phosphorous loading into the lake.

This paper therefore considers the case where society's preferences for the state of the lake are characterised by two distinct welfare functions. This can be thought of as two types of communities that share the use of the lake where one type of community is predominantly agricultural and benefits more from higher phosphorous loading, and the other, a green community, has a higher preference for an oligotrophic lake.

As noted above, the shallow lake presents a hysteresis in its response to phosphorous loading. Its biological function remains the same as in the case with a single welfare function. Under the constraint of the lake's response to phosphorous, each community will aim to optimize its welfare according to one of two welfare functions, thus giving rise to a Nash equilibrium different from the single-welfare function case. The Pareto-optimal amount of phosphorous loading will also be different in the two-function case, and therefore so will the tax required to induce each community to behave such a way as to attain a Pareto outcome. The first part of our analysis finds these results.

In the second part, lobbying by the two types of communities and its impact on the optimal tax policy is assessed. It is shown that as a result of lobbying, the optimal tax policy may not be implemented and further, that even when the tax falls only slightly short of the optimal tax, the hysteretic property of the lake may lead to its being in a eutrophic state.

Several articles explore the consequences of rent-seeking on environmental policy (Damania, 1999; Wilson and Damania, 2005) and on the optimal tax rate in particular (Lee, 1985; Brooks and Heijdra, 1987). However, there are currently no publications that combine the dynamics of the shallow lake, optimal taxation and the impact of rent-seeking behaviour on the application of this policy. The following work provides new results and insights into why socially optimal tax policy may often not be implemented.

We begin by describing the lake dynamics upon which this analysis is based.

2 Shallow Lake Dynamics

How are shallow lakes different from deep lakes? In summer months, the water of deep lakes stratifies into layers of different temperatures. During these months, nutrients are lost from the warm upper layers (*epilimnion*) to the colder deeper layers (*hypolimnion*), where they sink to the bottom into the sediments and remain segregated from the epilimnion until winter when the water column of the entire lake becomes mixed again.

Shallow lakes are different from deeper lakes because they tend to be *polymictic*, i.e. have a mixed water column most of the time. Shallow lakes can have a small or large surface, and the proportion of their water that is in contact with sediments makes them function differently from deep lakes. In particular, the rate of recycling of nutrients from sediments into the water is much higher. This means that more nutrients are available to consumers, including to microorganisms such as algae. As a result, contrary to deep lakes where vegetation is sparse and more present around the edges, shallow lakes are often filled with aquatic plants (Scheffer, 1998).

When the nutrient level of the lake is low, the plants tend to be small and the water clear. Increases in nutrient loading, however, encourage the development of larger plants and of phytoplankton. These plants and the surface layer of phytoplankton create shade and turbidity, which leads to the collapse of the vegetation that does not tolerate shade. This further favours the development of phytoplankton, and can result in the emergence of toxic algal bloom, cyanobacteria, which are shade tolerant (Scheffer, 1998).

This section presents the model of a shallow lake presented by Carpenter et al. (1999), which they claim accurately depicts long-term ecological data and the results of limnology and eutrophication studies. The lake equation they propose provides the constraint equation to the economic analysis that follows. In this model, the limiting factor for eutrophication is phosphorous. Lake eutrophication dynamics are based on total available phosphorous as the state variable, and phosphorous input as the control variable.

Although nitrogen is also used to stimulate plant growth, a model based on phosphorous makes sense because phosphorous is thought to be the limiting nutrient of plant growth in many cases (Ricklefs, 1979). In addition, cyanobacteria have the ability to fix nitrogen from the atmosphere, and therefore their growth will be limited by the phosphorous available (Alaouze, 1995).

Carpenter et al. (1999) identify three categories of lakes by their response to phosphorous input and reductions: fully reversible, hysteretic and irreversible. Our focus is an economic model of which the goal is to address eutrophication through policy aimed at mitigating phosphorous input alone.

2.1 The General Lake Model

The different functions of the lake with respect to phosphorous are used to build the lake-phosphorous dynamics equation as follows.

Phosphorous Sinks

Phosphorous is removed from the stock P available to algae via outflow, sequestration into biomass and sedimentation. Sedimentation is often the largest factor contributing to phosphorous loss and thus confers the lake its phosphorous waste sink function. The removal of phosphorous from the stock at time t is modelled as a linear function $-sP(t)$ where s is the rate of loss of phoshorous from the available stock.

Phosphorous Sources

Nutrients are retained by the lake and recycled between the physical environment and made available to living organisms such as fish or benthic plants (Ricklefs, 1979). Therefore external phosphorous loading from the catchment, such as run-off from agricultural activities or sewage effluents, contribute to the total stock of phosphorous available to consumers. The external input of phosphorous at time t is represented by $L(t)$.

Within the lake, phosphorous is recycled and made available from living organisms and sediment. Phosphorous release from sediment is a major source of recycled phosphorous and the cause of the hysteresis phenomenon. Lake studies have shown that long after external loading of phosphorous has ceased, the available stock of phosphorous remains high due to the release of phosphorous from compounds in the sediment. Scheffer (1998) gives an example of a Danish lake studied over a period of eight years following reduction in external loading. During this time, phosphorous continued to be released from the sediments. At the end of the study, it was concluded that the lake could continue to release phosphorous for another ten years.

The recycling equation is a sigmoid function, that is, it has an inflection point at which the curve switches from convex to concave. Phosphorous recycling is given by

$$\frac{rP^q(t)}{m^q + P^q(t)}$$

where

r is the maximum rate of recycling of P.

m is the value of P at which recycling reaches half the maximum rate r.

q is dimensionless and determines the steepness of the curve at the point of inflection. $q \geq 2$ and the larger q, the steeper the curve.

Adding the sources of phosphorous, i.e. loading and recycling, gives us the total phosphorous made available for consumption at time t:

$$L(t) + \frac{rP^q(t)}{m^q + P^q(t)}$$

Lake-Phosphorous Dynamics

The change in phosphorous stock in the lake at time t is given by the sum of the phosphorous sinks and source at time t and the lake-phosphorous dynamics are therefore modelled as:

$$\frac{dP}{dt} = L(t) - sP(t) + \frac{rP^q(t)}{m^q + P^q(t)} \tag{1}$$

The plots of the sinks and sources of phosphorous can be overlayed to show how controlling phosphorous loading can be used to affect the state of the lake. Refer to Appendix A for the graph and its analysis.

2.2 The Shallow Lake Model

The focus of this paper is the case where the eutrophic state of the lake is reversible by reductions of phosphorous alone. This case lends itself to the application of policy that induces agents to modify the activities that result in external phosphorous input. Because of the possibility of hysteresis in the response of the lake to phosphorous loading, however, policy implementations must allow for reduction in loading not having an immediate effect on restoring the lake to an oligotrophic state. Lakes that are deep and cold, or benefit from rapid flushing or rates of sedimentation or that have been eutrophied for only a short time will be easier to reverse (Carpenter et al., 1999). Shallow lakes, however, will tend to be hysteretic due to characteristics such as warmer water and higher ratio of water in contact with the bottom of the lake that confer them higher rates of recycling.

The parameter values that characterize shallow lakes are identified as follows. Consider the following initial value problem given by the differential equation (1) of section 2.1:

$$\dot{P}(t) = L(t) - sP(t) + r\frac{P^2(t)}{P^2(t) + m^2}, \quad P(0) = P_0, \tag{2}$$

As in several economic analyses around the shallow lake, $q = 2$ is chosen as the parameter for the steepness of the recycling response to the stock of phosphorous (Mäler et al., 2003; Grüne et al., 2005; Wagener, 2003).

To make the problem scale invariant, the following substitutions are made[1]:

$$\begin{aligned} P &= x/m, \\ L &= ar, \\ s &= br/m \end{aligned}$$

and by changing the time scale to tr/m, one obtains the following equations for the shallow lake dynamics:

$$a(t) = \dot{x}(t) + bx(t) - \frac{x^2}{x^2 + 1}$$

This can be interpreted as the external loading of phosphorous as a function of the stock of phosphorous. It may seem strange from a biological point of view to present loading as a function of the stock of phosphorous. However, the objective of policy is to manage phosphorous content in the lake and, as pointed out by Grüne et al. (2005), "[t]he management can measure the stock and can control the loading as a function of the stock".

Assuming steady-state conditions where the stock of phosphorous is constant, i.e. $\dot{x}(t) = 0$, we have:

$$a(t) = bx - \frac{x^2}{x^2 + 1}$$

Furthermore, if the phosphorous loading is constant, i.e. $da/dt = 0$, we have:

$$\dot{a}(t) = b - \frac{2x}{(x^2 + 1)^2} \tag{3}$$

[1]Refer to Murray (1989) pp. 5 and 652 for a more detailed description of this technique. Carpenter et al. (1999) also make use of it in Appendix A of their article, as do Mäler et al. (2003)

Similarly to Mäler et al. (2003), by analysing this equation, we find that for $q = 2$ and $\frac{1}{2} \leq b \leq \frac{3}{8}\sqrt{3}$, the lake displays a reversible hysteresis in its response to phosphorous loading [2]. These are the parameters that will be used to model the shallow lake that can be reversed from a eutrophic back to an oligotrophic state. Note that in this case, eutrophication is reversible by simple control of external phosphorous input. In this case, we can assume that all that is needed to keep the lake in an oligotrophic state is to manage levels of external phosphorous loading without any requirement for policy that alters the rates of phosphorous sedimentation or recycling.

3 Optimal Tax with Two Types of Communities

In their article, Mäler et al. (2003) consider the case where all communities are able to agree on a common welfare function. They note, however, that it is possible that different interest groups may in fact not be able to agree on a common welfare function. In the following, the case where the communities that share the lake are divided into two interest groups with different welfare function is considered.

The results from this section will be used in the subsequent section to extend the dynamic shallow lake-communities' model and consider the impact of lobbying by the interest groups on the optimal tax, that is, on the tax that would induce a Pareto-optimal state of the lake.

Consider that society is made up of two groups with conflicting interests: agricultural communitites and green communities. The agricultural communities are predominantly made up of farmers who privately benefit from applying fertilizer and, by proxy, from phosphorous loading into the lake. The green communities are predominantly made up of people who, although they benefit from the application of fertilizer to crops because they consume agricultural products, have a high preference for an oligotrophic lake.

We adopt the welfare function used by Mäler et al. (2003) and modify it to create two welfare functions that each represents the preferences of the two groups. In this scenario, the farmers attach very low importance c_1 to the ecosystem services provided by the lake, and the green communities attach a relatively high importance c_2 to ecosystem services and so $c_1 < c_2$. The total n communities previously considered can be divided into n_1 agricultural communities and n_2 green communities.
Each agricultural community i's welfare function is thus given by

$$W_i = \ln a_i - c_1 x^2, \;\; i = 1, \ldots, n_1$$

while each green community j's welfare function is given by

$$W_j = \ln a_j - c_2 x^2 \;\; j = 1, \ldots, n_2$$

where

$c_1 < c_2$ i.e., the agricultural communities have a lower relative preference

[2]Refer to Appendix B for the derivation os these parameters.

for an oligotrophic lake than do the green communities

n_1 is the number of communities with a majority in favour of a low tax rate

n_2 is the number of communities with a majority in favour of a high tax rate

$n_1 + n_2 = n$ is the total number of communities that share the use of the lake

3.1 Pareto-optimal Phosphorous Loading

A benevolent politcian wishing to act on behalf of citizens will want to implement a tax that optimizes social welfare. To achieve a Pareto-optimal solution, he needs to first find the total amount of phosphorous loading a that will maximize social welfare subject to the lake remaining in steady state. He may choose to do this by maximizing the sum of the communities' welfares, i.e. by solving:

$$\begin{aligned}\max_a \left(\sum_{i=1}^{n_1} W_i + \sum_{j=1}^{n_2} W_j \right) &= \sum_{i=1}^{n_1} \int_0^\infty e^{-\rho t} \left[\ln a_i(t) - c_1 x^2(t) \right] dt \\ &+ \sum_{j=1}^{n_2} \int_0^\infty e^{-\rho t} \left[\ln a_j(t) - c_2 x^2(t) \right] dt,\end{aligned}$$

$$= \int_0^\infty e^{-\rho t} \left[\sum_{i=1}^{n_1} \ln a_i(t) - n_1 c_1 x^2(t) + \sum_{j=1}^{n_2} \ln a_j(t) - n_2 c_2 x^2(t) \right] dt, \tag{4}$$

$$\tag{5}$$

$$i = 1, \ldots, n_1,\ j = 1, \ldots, n_2$$

$$\text{s.t. } \dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t) + 1} = 0,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=2}^{n_2} a_j(t), \quad i = 1, \ldots, n_1,\ j = 1, \ldots, n_2$$

The current value Hamiltonian for this equation is:

$$H^c = \sum_{i=1}^{n_1} ln a_i(t) - c_1 n_1 x^2(t) + \sum_{j=1}^{n_2} ln a_j(t) - c_2 n_2 x^2(t) + \lambda(t) \left[a(t) - bx(t) + \frac{x^2(t)}{x^2(t) + 1} \right],$$

$$\lambda = e^{\rho t} \mu,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=2}^{n_2} a_j(t),$$

$$i = 1, \ldots, n_1,\ j = 1, \ldots, n_2$$

The first order conditions are:

$$\frac{dH^c}{da_i(t)} = \frac{1}{a_i(t)} + \lambda(t) = 0,\ i = 1, \ldots, n_1 \tag{6}$$

$$\begin{aligned}
\frac{dH^c}{da_j(t)} &= \frac{1}{a_j(t)} + \lambda(t) = 0,\ j = 1, \ldots, n_2 && (7)\\
\frac{d\lambda}{dt} - \rho\lambda &= -\frac{dH^c}{dx} = 2n_1c_1x(t) + 2n_2c_2x(t) + \lambda(t)\left[b - \frac{2x(t)}{(x^2(t)+1)^2}\right] && (8)\\
\frac{dH^c}{d\lambda(t)} &= a(t) - bx(t) + \frac{x^2(t)}{(x^2(t)+1)^2} && (9)
\end{aligned}$$

From (6) and (7),

$$\lambda(t) = -\frac{1}{a_i(t)} = -\frac{1}{a_j(t)} \quad i = 1, \ldots, n_1,\ j = 1, \ldots, n_2, \tag{10}$$

which implies

$$a_i(t) = a_j(t) \quad \forall i = 1, \ldots, n_1,\ j = 1, \ldots, n_2 \tag{11}$$

Moreover:

$$\lambda(t) = -\frac{1}{a_i(t)} \quad i = 1, \ldots, n_1 \text{ implies } n_1a_i(t) = -\frac{n_1}{\lambda(t)}$$

and therefore

$$\sum_{i=1}^{n_1} = n_1a_i(t), \text{ and by similar argument } \sum_{j=1}^{n_2} = n_2a_j(t),$$

which means that

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=2}^{n_2} a_j(t) \quad i = 1, \ldots, n_1,\ j = 1, \ldots, n_2 \tag{12}$$

and

$$a(t) = n_1a_i(t) + n_2a_j(t)\ i = 1, \ldots, n_1,\ j = 1, \ldots, n_2 \tag{13}$$

Note that λ can be expressed as a function of a via the following reasoning:

$$\begin{aligned}
\lambda(t) &= -\frac{1}{a_i(t)} \quad i = 1, \ldots, n_1 \text{ and } \lambda(t) = -\frac{1}{a_j(t)} \quad j = 1, \ldots, n_2\\
\Rightarrow \qquad a_i(t)\lambda(t) &= -1 \quad i = 1, \ldots, n_1 \quad \text{and} \quad a_j(t)\lambda(t) = -1 \quad j = 1, \ldots, n_2\\
\Rightarrow \lambda(t)\sum_{i=1}^{n_1} a_i(t) &= -n_1 \quad \text{and} \quad \lambda(t)\sum_{j=1}^{n_2} a_j(t) = -n_2
\end{aligned}$$

Adding the two together:

$$\lambda(t)\sum_{i=1}^{n_1} a_i(t) + \lambda(t)\sum_{j=1}^{n_2} a_j(t) = -(n_1 + n_2)$$

$$\Rightarrow \lambda(t)a(t) = -(n_1 + n_2)$$

and therefore

$$\Rightarrow \quad \lambda(t) = -\frac{(n_1 + n_2)}{a(t)} \tag{14}$$

From (8)

$$\dot{\lambda}(t) = 2x(t)(n_1c_1 + n_2c_2) + \lambda(t)\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right] \tag{15}$$

From (10):

$$\dot{\lambda}(t) = \frac{\dot{a}_i(t)}{{a_i}^2} \tag{16}$$

Combined with (15) gives:

$$\frac{\dot{a}_i(t)}{{a_i}^2} = 2x(t)(n_1c_1 + n_2c_2) - \frac{1}{a_i(t)}\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right] \tag{17}$$

and because $a_i(t) = a_j(t)$, from equation (11),

$$\frac{\dot{a}_i(t)}{a_i(t)a_j(t)} = 2x(t)(n_1c_1 + n_2c_2) - \frac{1}{a_i(t)}\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right] \tag{18}$$

Multiplying by $(n_1 + n_2)a_i(t)a_j(t)$, one obtains:

$$\begin{aligned} (n_1a_i(t) + n_2a_j(t)) &= 2a_i(t)(a_i(t)n_1 + a_j(t)n_2)(n_1c_1 + n_2c_2)x(t) \\ &- (a_i(t)n_1 + a_i(t)n_2)\left[b + \rho + \frac{2x(t)}{(x^2(t)+1)^2}\right] \end{aligned}$$

Using results (11) and (13), the above is equivalent to:

$$\dot{a}(t) = 2a(t)a_i(t)(n_1c_1 + n_2c_2)x(t) - a(t)\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right]$$

And noting that $a_i(t) = \frac{a(t)}{n_1+n_2}$ from equations (14) and (10) gives:

$$\dot{a}(t) = 2a^2(t)x(t)\frac{(c_1n_1 + c_2n_2)}{n_1 + n_2} - a(t)\left[b\rho - \frac{2x(t)}{(x^2(t)+1)^2}\right]$$

With constant loading $da/dt = 0$, and so:

$$\dot{a}(t) = \frac{2a^{*2}(c_1n_1 + c_2n_2)}{n_1 + n_2}x(t) - a^*\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right] = 0$$

which means $a^* = 0$ or:

$$\dot{a}(t) = \frac{2a^*}{n_1 + n_2} x(t) \left(c_1 n_1 + c_2 n_2\right) - \left[b + \rho - \frac{2x(t)}{\left(x^2(t) + 1\right)^2}\right] = 0$$

And so the steady-state Pareto-optimal solution is:

$$a^* = \frac{(n_1 + n_2)\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]}{2x(t)\left(c_1 n_1 + n_2 c_2\right)} \tag{19}$$

3.2 Graph and Analysis of the Steady-state Dynamic Pareto-optimal Solution

This solution can be plotted in the (x,a)-plane together with the phase plot for the steady-states of the lake when $dx/dt = 0$, given by equation (3). The intersection of the two curves gives society's optimal phosphorous loading solution. Using the hysteretic lake value $b = 0.6$, $\rho = 0.03$, $n_1 = 2$, $n_2 = 2$, $c_1 = 0.2$ and $c_2 = 2$, the graphs intersect at $(x^*, a^*) = (0.3472, 0.1007)$, as shown in Figure 1 below, thus giving us the Pareto-optimal steady-state equilibrium. Note that this result is below the point at which the lake flips from an oligotrophic to a eutrophic state, i.e. for the selected constants, society prefers an oligotrophic lake.

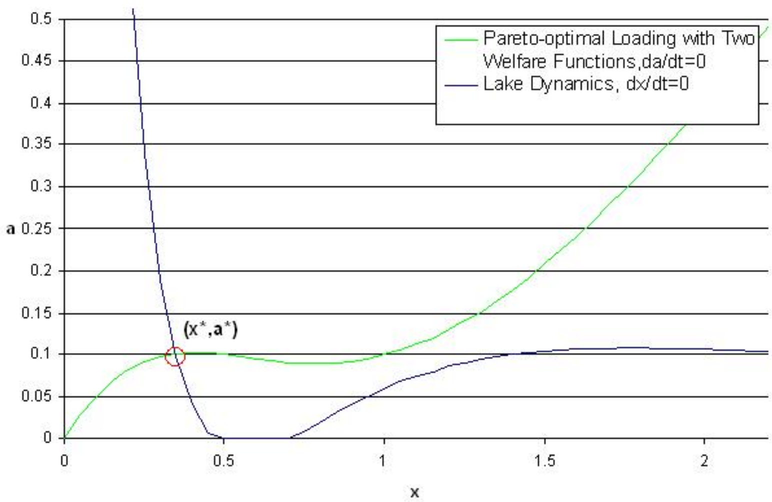

Figure 1: Pareto-optimal Loading with Two Welfare Functions

3.3 Non-cooperative Equilibria

In the absence of management, however, each community maximizes its own utility according to its welfare function, and therefore the agricultural communities each

$$\max_a \int_0^\infty e^{-\rho t}\left[\ln a_i(t) - c_1 x^2(t)\right] dt,\ i = 1,\ldots,n_1$$

$$\text{s.t. } \dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1} = 0,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=1}^{n_2} a_j(t)$$

and the green communities each

$$\max_a \int_0^\infty e^{-\rho t}\left[\ln a_j(t) - c_2 x^2(t)\right] dt,\ j = 1,\ldots,n_2$$

$$\text{s.t. } \dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1} = 0,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=1}^{n_2} a_j(t)$$

Setting up a current value Hamiltonian and solving first order conditions for these two problems yields:

$$\dot{a}_i(t) = 2a_i(t)^2 n_1 c_1 x(t) - n_1 a_i(t)\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]$$

and

$$\dot{a}_j(t) = 2a_j(t)^2 n_2 c_2 x(t) - n_2 a_j(t)\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]$$

Solving for constant phosphorous loading, that is, $da_i/dt = 0$ and $da_j/dt = 0$, one obtains the steady-state open-loop Nash equilibrium for total loading a:

$$a = \frac{\left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2}\right]}{2x(t)}\left[\frac{n_1}{c_1} + \frac{n_2}{c_2}\right]$$

3.4 Graph and Analysis of the Nash Equilibrium Solution

Again, this solution is plotted in the (x,a)-plane together with the phase plot for the steady-states of the lake when $dx/dt = 0$, given by equation (3). The intersection of the two curves gives the Nash equilibrium phosphorous loading solutions for the state of the lake. Using the hysteretic lake value $b = 0.6$, $\rho = 0.03$ and $n_1 = 2$, $n_2 = 2$, $c_1 = 0.2$ and $c_2 = 2$, the curves intersect at $(0.4485, 0.1016)$, $(0.7402, 0.0902)$ and $(3.1832, 0.9976)$, as show in Figure 2. Point $(0.7402, 0.0902)$ is an unstable skiba point, i.e., a small variation in loading will cause the equilibrium to shift to either the lower equilibrium at $(0.4485, 0.1016)$ or the higher equilibrium at $(0.7402, 0.0902)$. Note that both of these points are above the point at which the lake flips to a eutrophic state. This means that when the green communities and agricultural communities do not cooperate, the lake will be in a eutrophic state.

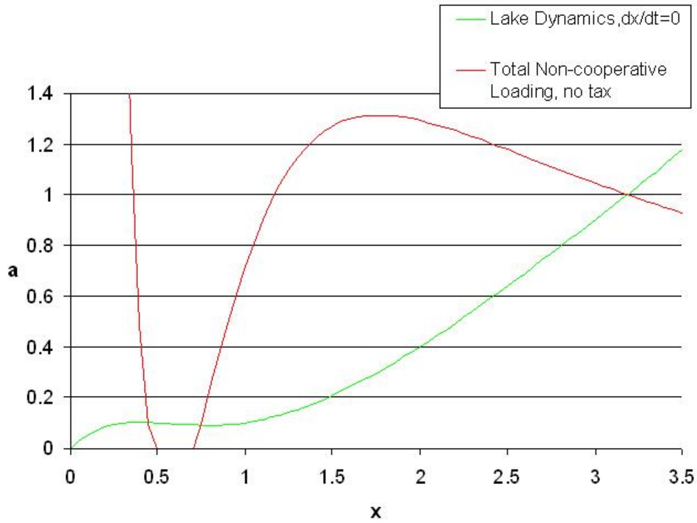

Figure 2: Nash Equilibrium Loading with Two Welfare Functions

3.5 Optimal Taxation with Two Interest Groups

Our benevolent politician therefore wants to find the tax rate that will achieve the optimal social welfare outcome derived in section 3.1 without the need for management. The effect of the tax will be to modify each community's welfare function and induce each one to modify its phosphorous loading accordingly. With the tax, the agricultural and green communities will each

$$\max_{a_i} \int_0^\infty e^{-\rho t} \Big[\ln a_i(t) - \tau(t) a_i(t) - c_1 x^2(t) \Big]\, dt, \ i = 1, \ldots, n_1$$

$$\text{s.t. } \dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1} = 0,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=1}^{n_2} a_j(t)$$

and

$$\max_{a_j} \int_0^\infty e^{-\rho t} \Big[\ln a_j(t) - \tau(t) a_j(t) - c_2 x^2(t) \Big]\, dt, \ j = 1, \ldots, n_2$$

$$\text{s.t. } \dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1} = 0,$$

$$a(t) = \sum_{i=1}^{n_1} a_i(t) + \sum_{j=1}^{n_2} a_j(t)$$

Setting up a current value Hamiltonian and solving first order conditions for each of these two problems yields:

$$\tau(t) = \frac{1}{a_i(t)} + \lambda_i(t),\ i = 1, \ldots, n_1 \tag{20}$$

and

$$\tau(t) = \frac{1}{a_j(t)} + \lambda_j(t),\ j = 1, \ldots, n_2 \tag{21}$$

From these two results, the following holds:

$$n_1\tau(t) = \sum_{i=1}^{n_1} \frac{1}{a_i(t)} + \sum_{i=1}^{n_1} \lambda_i(t)$$

and

$$n_2\tau(t) = \sum_{j=1}^{n_2} \frac{1}{a_j(t)} + \sum_{j=1}^{n_2} \lambda_j(t)$$

which added together gives

$$(n_1 + n_2)\tau(t) = \sum_{i=1}^{n_1} \frac{1}{a_i(t)} + \sum_{j=1}^{n_2} \frac{1}{a_j(t)} + \sum_{i=1}^{n_1} \lambda_i(t) + \sum_{j=1}^{n_2} \lambda_j(t)$$

Recognizing the first two terms on the right hand side as $-(n_1 + n_2)\lambda(t)$ from Section 3.1 results in

$$\begin{aligned}(n_1 + n_2)\tau(t) &= -(n_1 + n_2)\lambda(t) + \sum_{i=1}^{n_1} \lambda_i(t) + \sum_{j=1}^{n_2} \lambda_j(t) \\ \Rightarrow \tau(t) &= -\lambda(t) + \frac{1}{(n_1 + n_2)} \sum_{i=1}^{n_1} \lambda_i(t) + \frac{1}{(n_1 + n_2)} \sum_{j=1}^{n_2} \lambda_j(t) \qquad (22)\end{aligned}$$

Note that in parallel with Mäler et al.'s result in the single welfare function case, the optimal tax bridges the gap between society's shadow cost of phosphorous loading and each community's private cost.

3.6 Optimal Constant Tax Rate

As noted by Mäler et al., it is not practical to implement a time-variable tax and a constant tax is preferable, i.e., a tax such that $d\tau/dt = 0$. Using this condition and combining with equation (22), one can solve for constant λ^* as follows to then derive the constant tax rate.

$$\begin{aligned}\frac{d\tau}{dt} &= -\frac{d\lambda(t)}{dt} + \frac{d}{dt}\left(\frac{1}{(n_1 + n_2)} \sum_{i=1}^{n_1} \lambda_i(t)\right) + \frac{d}{dt}\left(\frac{1}{(n_1 + n_2)} \sum_{j=1}^{n_2} \lambda_j(t)\right) = 0 \\ \Rightarrow \frac{d\lambda(t)}{dt} &= \frac{1}{(n_1 + n_2)} \sum_{i=1}^{n_1} \left(\frac{d\lambda_i(t)}{dt}\right) + \frac{1}{(n_1 + n_2)} \sum_{j=1}^{n_2} \left(\frac{d\lambda_j(t)}{dt}\right) \qquad (23)\end{aligned}$$

From the Hamiltonian first order conditions for this problem,

$$\dot{\lambda}_i(t) = 2c_1x(t) + \lambda_i(t)\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right] \tag{24}$$

and

$$\dot{\lambda}_j(t) = 2c_2x(t) + \lambda_j(t)\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right] \tag{25}$$

The optimal steady-state tax rate is found by setting each of $\dot{\lambda}(t)$, $\dot{\lambda}_i(t)$ and $\dot{\lambda}_j(t)$ equal to zero and solving to find λ^*, λ_i^* and λ_j^*. From optimal management equation (14):

$$\lambda(t) = -\frac{(n_1+n_2)}{a(t)}$$

This holds for all t, therefore this is also true for steady-state λ^* and a^*, i.e.

$$\lambda^* = -\frac{(n_1+n_2)}{a^*}$$

To find λ_i^*, one solves

$$\begin{aligned}
\dot{\lambda}_i(t) &= 2c_1x(t) + \lambda_i(t)\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right] = 0 \\
\Rightarrow \lambda_i^* &= \frac{-2c_1x(t)}{\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]}
\end{aligned}$$

Similarly, one finds that

$$\lambda_j^* = \frac{-2c_2x(t)}{\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]} \tag{26}$$

Substituting back into (22), gives the optimal constant tax:

$$\begin{aligned}
\tau^* &= \frac{(n_1+n_2)}{a^*} + \frac{n_1}{(n_1+n_2)}\left(\frac{-2c_1x(t)}{\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]}\right) + \frac{n_2}{(n_1+n_2)}\left(\frac{-2c_2x(t)}{\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]}\right) \\
\Rightarrow \tau^* &= \frac{(n_1+n_2)}{a^*} - \frac{1}{(n_1+n_2)}\left(\frac{2x(t)(n_1c_1+n_2c_2)}{\left[b + \rho - \frac{2x(t)}{\left(x^2(t)+1\right)^2}\right]}\right) \\
\Rightarrow \tau^* &= \frac{(n_1+n_2)}{a^*} - \frac{1}{a^*}
\end{aligned}$$

Therefore:

$$\Rightarrow \tau^* = \frac{(n_1 + n_2 - 1)}{a^*} \tag{27}$$

is the constant tax that will achieve the Pareto-optimal amount of phosphorous loading when each community acts to maximize its welfare in a non-cooperative way.

3.7 Private Equilibrium with Tax

To determine the impact of the tax on the non-cooperative phosphorous loading in steady state and on the state of the lake, $\dot{\lambda}(t)$, $\dot{\lambda}_i(t)$ and $\dot{\lambda}_j(t)$ is substituted into the steady-state tax equation. Therefore substituting (14), (24) and (25) back into (23) one obtains:

$$\begin{aligned} \frac{-(n_1+n_2)\dot{a}(t)}{a^2(t)} = \frac{1}{(n_1+n_2)} & \left[\sum_{i=1}^{n_1} \left(2c_1 x(t) + \lambda_i(t) \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2} \right] \right) \right. \\ & \left. + \sum_{j=1}^{n_2} \left(2c_2 x(t) + \lambda_j(t) \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2} \right] \right) \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \dot{a}(t) &= -2x(t)a^2(t)\frac{(n_1c_1 + n_2c_2)}{(n_1+n_2)^2} \\ &- \frac{a^2(t)}{(n_1+n_2)^2} \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2} \right] \left(\sum_{i=1}^{n_1} \lambda_i(t) + \sum_{j=1}^{n_2} \lambda_j(t) \right) \end{aligned} \tag{28}$$

From (20)

$$\lambda_i(t) = \tau - \frac{1}{a_i(t)} \quad \text{and} \quad \lambda_j(t) = \tau - \frac{1}{a_j(t)t} \tag{29}$$

By substituting into (28) one obtains:

$$\begin{aligned} \dot{a}(t) &= -2x(t)a^2(t)\frac{(n_1c_1 + n_2c_2)}{(n_1+n_2)^2} \\ &- \frac{a^2(t)}{(n_1+n_2)^2} \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2} \right] \left[\sum_{i=1}^{n_1} \left(\tau - \frac{1}{a_i(t)} \right) + \sum_{j=1}^{n_2} \left(\tau - \frac{1}{a_j(t)} \right) \right] \\ \Rightarrow \dot{a}(t) &= -2x(t)a^2(t)\frac{(n_1c_1 + n_2c_2)}{(n_1+n_2)^2} \\ &- \frac{a^2(t)}{(n_1+n_2)^2} \left[b + \rho - \frac{2x(t)}{(x^2(t)+1)^2} \right] \left[(n_1+n_2)\tau - \left(\sum_{i=1}^{n_1} \frac{1}{a_i(t)} + \sum_{j=1}^{n_2} \frac{1}{a_j(t)t} \right) \right] \end{aligned}$$

Recognizing the last two terms as $-\lambda(n_1+n_2)$ and substituting $-(n_1+n_2)/a(t)$

for λ yields

$$\begin{aligned}
\dot{a}(t) &= -2x(t)a^2(t)\frac{(n_1c_1+n_2c_2)}{(n_1+n_2)^2} \\
&- \frac{a^2(t)}{(n_1+n_2)^2}\left[b+\rho-\frac{2x(t)}{(x^2(t)+1)^2}\right]\left[(n_1+n_2)\tau-\frac{(n_1+n_2)^2}{a(t)}\right] \\
\Rightarrow \dot{a}(t) &= -2x(t)a^2(t)\frac{(n_1c_1+n_2c_2)}{(n_1+n_2)^2} \\
&- \frac{a^2(t)}{(n_1+n_2)}\left[b+\rho-\frac{2x(t)}{(x^2(t)+1)^2}\right][a(t)\tau-(n_1+n_2)]
\end{aligned}$$

Assuming steady-state loading, i.e. $\dot{a}(t)=0$:

$$a^*\left[-2a^*x(t)\frac{(n_1c_1+n_2c_2)}{(n_1+n_2)^2}-\frac{1}{(n_1+n_2)}\left[b+\rho-\frac{2x(t)}{(x^2(t)+1)^2}\right][a^*\tau-(n_1+n_2)]\right]=0$$

which implies $a^*=0$ or

$$2a^*x(t)\frac{(n_1c_1+n_2c_2)}{(n_1+n_2)^2}+\left[b+\rho-\frac{2x(t)}{(x^2(t)+1)^2}\right]\left[\frac{a^*\tau}{(n_1+n_2)}-1\right]=0$$

Solving for a^* gives:

$$a^*=\frac{\left[b+\rho-\frac{2x(t)}{(x^2(t)+1)^2}\right]}{2x(t)\frac{(c_1n_1+c_2n_2)}{(n_1+n_2)^2}+\frac{\tau^*}{(n_1+n_2)}\left[b+\rho-\frac{2x(t)}{(x^2(t)+1)^2}\right]} \tag{30}$$

which is the Nash equilibrium phosphorous loading when the tax is applied.

3.8 Graph and Analysis of the New Nash Equilibrium Solution

The plot of the solution is overlayed onto the Pareto-optimal steady-state loading curve from section 3.1 and shown in Figure 3. Note that when the optimal tax is applied, the Nash equilibrium loading intersects the lake dynamics in exactly the same point $(x^*, a^*) = (0.3472, 0.1007)$ as the Pareto-optimal loading. Moreover, there is now only one Nash equilibrium, and it is oligotrophic in accordance with society's preferences. This is not surprising given that the objective of the tax is to bring phosphorous loading to the same level as that which would be achieved under optimal management.

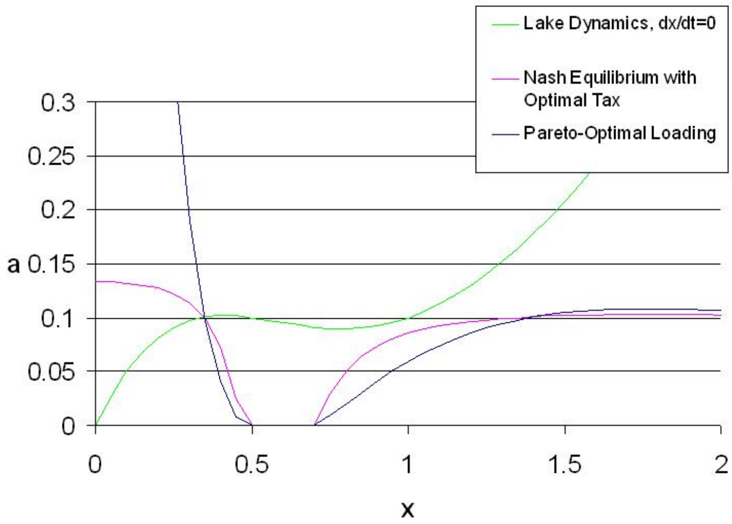

Figure 3: Nash Equilibrium Loading with Two Welfare Functions and Tax

4 Lobbying, Political Ambition and their Impact on the Optimal Tax

Consider the case where the lake is in a eutrophic state in spite of a current tax on phosphorous loading. The green communities have a preference for an oligotrophic lake and consider that the current tax rate is too low. The n communities face an election to elect a single politician. The politician favoured by the green communities promises to implement a higher tax on phosphorous that will bring phosphorous loading down so as to reverse the lake to an oligotrophic state. The farming communities favour a politician who promises to maintain the tax at its current low level.

Figure 4 shows the Nash equilibrium loading with a low tax, $\tau = 1$, together with the curve of the Nash equilibrium loading with the optimal tax $\tau^* = 29.78$, as given by equation (27). Note that in the context of a low tax, the high eutrophic Nash equilibrium could prevail, whereas in the case of the optimal tax, only one Nash equilibrium is possible.

4.1 Lobbying to Influence Policy Outcomes

Each group applies lobbying effort to influence the policy outcome. The green communities lobby in favour of a high tax and the farmers lobby for a low tax on phosphorous loading. The lobbying effort of each agricultural community i is denoted by l_i and the lobbying effort of each green community j is denoted by m_j.

We apply the Tullock model of rent-seeking (Tullock, 1980) to obtain the probability of each groups' lobbying efforts being successful at having their de-

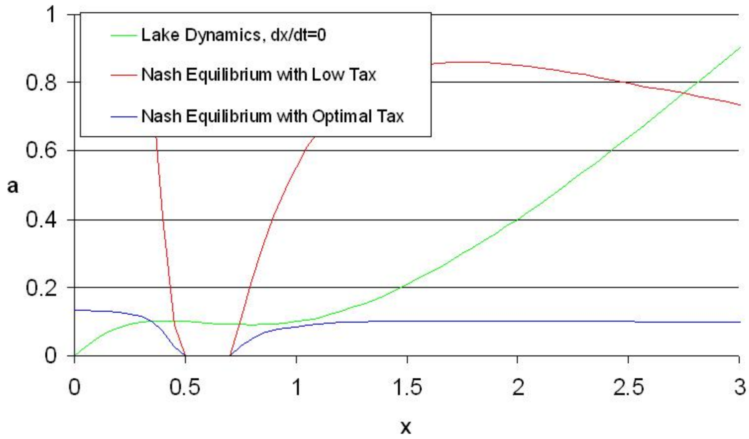

Figure 4: Nash Equilibrium Loading Low Tax and Optimal Tax

sired policy applied. The simplest, linear, form of the model is applied where the expected payoff of each player is defined as the ratio of his investment divided by the sum of the investments of all the players, multiplied by the reward if he wins. Here, the investments are the lobbying efforts and the rewards are the communities' net benefit of having their desired policy applied, that is, their welfare function with their preferred tax rate. Note that the probability of having their preferred policy implemented is synonymous with the probability of having their preferred politician elected.

The probability of the agricultural communities having their preferred policy applied is:

$$P_f = \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j} \tag{31}$$

The probability of the green communities having their preferred policy applied is:

$$P_g = \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j} \tag{32}$$

The expected payoff of each community is the probability of having its preferred policy applied, minus the probability of it not being applied, minus its own initial lobbying investment. The problem thus becomes for each community to maximize its expected return by applying the correct amount of lobbying effort, i.e.

$$\begin{aligned} \max_{l} E\left\{\Pi_F\right\} &= \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j}\left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2\right) \\ &+ \left(1 - \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j}\right)\left(\ln a_{H\,\tau} - c_1 x_{H\,\tau}^2\right) - l_i \end{aligned} \tag{33}$$

and

$$\max_m E\{\Pi_G\} = \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j}\left(\ln a_{H\,\tau} - c_2 x_{H\,\tau}^2\right) + \left(1 - \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j}\right)\left(\ln a_{L\,\tau} - c_2 x_{L\,\tau}^2\right) - m_j \tag{34}$$

where

τ The tax rate applied to phosphorous loading.
$L\,\tau$ denotes the low tax rate and $H\,\tau$ denotes the high tax rate.
l The lobbying effort of farmers in favour of a low tax rate.
m The lobbying effort of greens in favour of a high tax rate.

To find the optimum lobbying efforts l_i^* and m_j^*, one finds the values of l_i and m_j for which the first derivative is equal to zero, i.e. $dE\{\Pi_F\}/dl_i = 0$ and $dE\{\Pi_G\}/dm_j = 0$. (These values will maximize the expected pay-offs provided the profit functions are concave, i.e. if their second derivatives are negative.) Therefore we begin by solving:

$$\begin{aligned}
\frac{dE\{\Pi_F\}}{dl_i} &= \frac{d}{dl_i}\left[\frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j}\left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2\right)\right.\\
&+ \left.\left(1 - \frac{\sum_{i=1}^{n_1} l_i}{\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j}\right)\left(\ln a_{H\,\tau} - c_1 x_{H\,\tau}^2\right) - l_i\right] = 0\\
\Leftrightarrow \frac{dE\{\Pi_F\}}{dl_i} &= \frac{d}{dl_i}\left[\frac{\sum_{i=1}^{n_1} l_i}{\left(\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j\right)}\left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2\right)\right.\\
&+ \left.\frac{\sum_{j=1}^{n_2} m_j}{\left(\sum_{i=1}^{n_1} l_i + \sum_{j=1}^{n_2} m_j\right)}\left(\ln a_{H\,\tau} - c_1 x_{H\,\tau}{}^2\right) - l_i\right] = 0
\end{aligned} \tag{35}$$

For simplicity, we assume that all the communities are approximately the same size, that is, they contribute an equal amount of lobbying effort, so that $\sum_{i=1}^{n_1} l_i = n_1 l$ and $\sum_{j=1}^{n_2} m_j = n_2 m$. Equation (35) then becomes

$$\frac{dE\{\Pi_F\}}{dl_i} = \frac{d}{dl}\left[\frac{n_1 l}{(n_1 l + n_2 m)}\left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2\right) + \frac{n_2 m}{(n_1 l + n_2 m)}\left(\ln a_{H\,\tau} - c_1 x_{H\,\tau}^2\right) - l\right] = 0$$

$$\Leftrightarrow n_1 n_2 m\left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2 - \ln a_{H\,\tau} + c_1 x_{H\,\tau}^2\right) = (n_1 l + n_2 m)^2 \tag{36}$$

which expanded is

$$(n_1 l)^2 + 2 n_2 m n_1 l + (n_2 m)^2 - n_1 n_2 m\left(\ln a_{L\,\tau} - c_1 x_{L\,\tau}^2 - \ln a_{H\,\tau} + c_1 x_{H\,\tau}^2\right) = 0$$

We recognise this as a second-order polynomial of the form $dX^2 + eX + f = 0$, where

$X = n_1 l$
$d = 1$,
$e = 2n_2 m$ and
$f = (n_2 m)^2 - n_1 n_2 m \left(lna_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right)$,

which has two roots X of the form

$$X = \frac{-e \overset{+}{-} \sqrt{e^2 - 4df}}{2d}$$

Substituting for d, e and f, and keeping only the positive root of the polynomial, we obtain the farmers' optimal lobbying effort:

$$l = \frac{-n_2 m}{n_1} + \frac{1}{n_1} \sqrt{n_1 n_2 m \left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right)} \tag{37}$$

Similarly, from equation (34) we obtain:

$$n_2 n_1 l \left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + x^2_{L\,\tau} \right) = \left(n_1 l + n_2 m \right)^2, \tag{38}$$

of which the positive real root gives us:

$$m = \frac{-n_1 l}{n_2} + \frac{1}{n_2} \sqrt{n_2 n_1 l \left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau} \right)} \tag{39}$$

We note that l and m are dependent on each other. By noticing, from equations (36) and (38), that

$$l \left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau} \right) = m \left(\ln a_{l\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right)$$

and by substituting back into equations (37) and (39), we can express the respective optmimal lobbying effort of farmers and 'greens', l^* and m^* as

$$l^* = \frac{n_1 n_2 \left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right)^2 \left(\ln a_{h\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau} \right)}{\left[n_1 \left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right) + n_2 \left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau} \right) \right]^2} \tag{40}$$

and

$$m^* = \frac{n_1 n_2 \left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right) \left(\ln a_{H\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau} \right)^2}{\left[n_1 \left(\ln a_{L\,\tau} - c_1 x^2_{L\,\tau} - \ln a_{H\,\tau} + c_1 x^2_{H\,\tau} \right) + n_2 \left(\ln a_{h\,\tau} - c_2 x^2_{H\,\tau} - \ln a_{L\,\tau} + c_2 x^2_{L\,\tau} \right) \right]^2} \tag{41}$$

In the following section, we use these results to study the impact of the lobbying efforts on the optimal tax policy derived in section 3.6 and depicted in section 3.8.

4.2 Probability of the Optimal Tax Being Implemented: A Numerical Analysis

Recall that in our scenario, the lake is in a eutrophic state in spite of an existing tax on phosphorous loading. Either the current state of the lake reflects the preferences of all of the communities around the lake or the tax is too low to keep it in an oligotrophic state. Knowing that the Pareto-optimal state of the lake is as shown in Section 3.1, the benevolent politician promises if he is elected to implement the optimal tax rate.

What is the likelihood of the tax policy being implemented, that is, of this politician being elected, given the relative preferences of the green communities and farming communities and their resulting lobbying efforts?

The following constant values are used in the Nash equilibrium with tax equation (30) and evaluated for values of x between 0 and 3.5.

$b = 0.6$ - recall from Section 2.2 that this is the phosphorous recycling value that gave rise to a hysteresis in the lake dynamics.

$c_1 = 0.2$ - thus denoting the farming communities' low relative preference for lake ecosystem services.

$c_2 = 2$ - thus denoting the green communities' high relative preference for a clean lake.

$L\,\tau = 1$ - is selected as the current taxation that results in high phosphorous loading and thus a eutrophic state of the lake.

The dynamic socially optimal equilibrium level of phosphorous loading is given by the intersection of the optimal a^* equation (19) and the lake dynamics equation (3), as depicted in Figure 1. The optimal tax is such that the dynamic non-cooperative equilibrium intersects the lake dynamics equation for the same optimal (x^*, a^*) coordinates, as shown in Figure 3.

Varying values of n_1 and n_2 results in different (x^*, a^*) coordinates and affects the amount of lobbying applied by the different communities to obtain their desired outcome with respect to the proposed tax increase versus keeping the current low tax. This in turn affects the probability of the benevolent politician being elected and thus of the optimal tax policy being implemented. Substituting these values back into the equations derived earlier in the chapter, namely, equations (40), (41), (31) and (32) gives us the lobbying efforts of the green and agricultural communities as well as the probabilities of the optimal tax policy being implemented.

The results are summarized in Table 1 below.

Table 1: Summary of Results.

	$n_1=5,\ n_2=1$	$n_1=4,\ n_2=1$	$n_1=3,\ n_2=1$	$n_1=2,\ n_2=2$
P_i	1	0.0021	0.0492	0.0511
P_g	0	0.9979	0.9508	0.9488
(x^*,a^*)	(0.3932,0.102)	(0.3934,0.102)	(0.3795,0.1018)	(0.3472,0.1007)
τ^*	37.74	33.82	29.44	29.7792

We interpret these results as follows. For the selected preference rations, even with as little as 1 in 6 communities with a high preference for a clean lake, the Pareto-optimal outcome is for a level of phosphorous loading that results in an oligotrophic lake. And yet, for these same prefences, when the ratio of n_2 to n_1 is less than or equal to one to five, the probability of the green politician being elected is 0.

On the other hand, when n_2 to n_1 is one to four or greater, the probability of the politician being elected increases to very high levels, i.e. relatively close to 1. This means that a relatively small proportion of the population can gain enough power to influence policy when their preferences are strong enough. This result can be attributed to the amount of lobbying effort that is expended when communities attach a relatively high value to ecosystem services. Moreover, as the proportion of green communities increases, the tax rate required to bring the lake back to oligotrophic levels is lower, which also explains a lower lobbying effort against a higher tax by farming communities.

In addition, the following can be derived with regard to the probability of the optimal policy being applied. An ambitious politician will want to implement the policy that will ensure that he is elected. To do this he will propose a tax so as to maximize the probability of being elected, that is he will:

$$\max P_g = \frac{\sum_{j=1}^{n_2} m_j}{\sum_{i=1}^{n_1} l_i + \sum_{i=j}^{n_2} m_j} \tag{42}$$

by changing τ.

We find that for $n_1 = 2$ and $n_2 = 2$, to maximize his probability of being elected, the benevolent politician would have to set the tax rate at $\tau = 11.40$. This tax increases the probability of election to one, that is, by proposing this tax he is certain of being elected. Unfortunately, this tax will result in an insufficient reduction in phosphorous loading levels and the lake will remain in its eutrophic state. Recall that the skiba point is at $(x_{F1}, a_{F1}) = (0.4084, 0.1021)$, c.f. Section 2.2. For $n_1 = 2$ and $n_2 = 2$, the optimal levels of phosphorous are $(x^*, a^*) = (0.3472, 0.1007)$, which denotes an oligotrophic state of the lake. To achieve the optimal level of phosphorous loading, the required tax rate is $\tau^* = 29.78$. Therefore a proposed tax policy of $\tau = 11.40$ would be far inferior to that required to achieve the socially desirable level of phosphorous loading. This may be an example that illustrates the observation by Lee (1985) that "political objectives can be realized by establishing "acceptable" pollution standards and many of them have little to do with protecting the environment."

5 Conclusion

In summary, we have found that lobbying and the composition of the electorate have an effect on the implementation of the socially optimal tax policy. When a portion of the communities have a strong preference for a clean lake, as little as one fifth, the probability of the politician being elected increases to very high levels, i.e. relatively close to 1. This is an interesting result because it implies that the number of green communities need not be very high, only sufficiently high, for the environmental policy to have a very high chance of being implemented.

A perhaps more interesting result is that by proposing a tax level below the one required to bring the lake back to a socially optimal oligotrophic state, a politician can ensure that he is elected. This shows that political ambition can indeed prevent socially desirable policy from being implemented.

Appendices

A Phosphorous Sources and Sinks

With the stock of phosphorous held constant, the phosphorous sink and source equations can each be plotted as in Figure 5 and by Carpenter et al. (1999) to show the rates of flux of phosphorous against the quantity of available P. Superimposing the phosphorous sinks' straight line and the phosphorous recycling's sigmoid shows the domains of attraction of oligotrophic and eutrophic states. The upper point of instersection between the two curves is an attractor toward oligotrophy, whereas the lower intersection is an attractor toward eutrophy. The intersection point in the middle is an unstable repeller and represents a Skiba point. This means that at this level of phosphorous stock, a small change in the stock could precipitate the lake into either an oligotrophic or eutrophic state. This illustrates the possible existence of a hysteresis in the lake's response to phosphorous input.

The graph illustrates that a eutrophied lake can be restored in several ways:

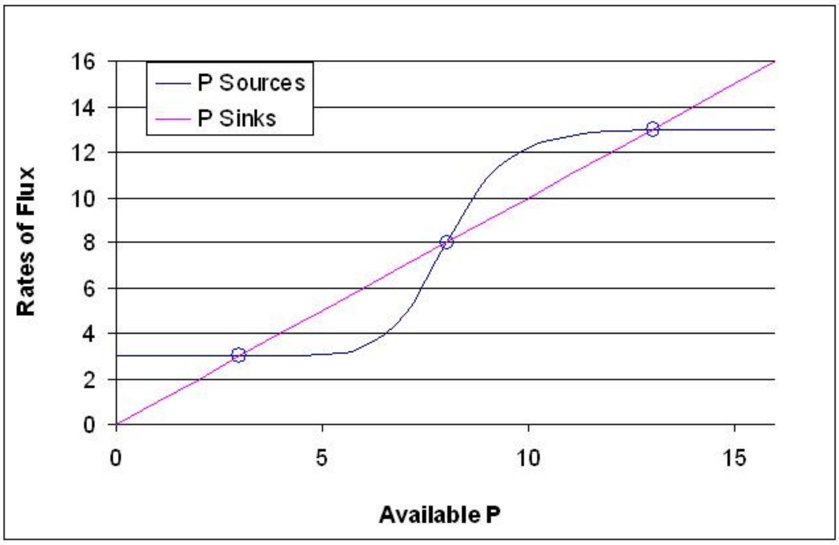

Figure 5: Phosphorous Sources and Sinks

- By increasing the sinks, i.e., by affecting s and thus altering the slope of the 'sinks' line thereby changing the points of intersection with the 'sources' curve;

- By decreasing recycling r to push the sigmoid down such that the only point of intersection with the 'sinks' curve lies in the oligotrophic basin of attraction;
- By lowering external phosphorous input L; or
- via a combination of the above.

Increasing the sinks and decreasing recycling requires measures that are independent of phosphorous input. These measures include sediment treatment, such as the addition of aluminum sulfate to precipitate the sediments, injections of oxygen into the hypolimnion, and biomanipulation, such as the introduction of consumers of phosphorous, e.g. fish, or large aquatic plants. These methods tend to be costly and therefore reductions in phosphorous input will tend to be preferable as a measure to restore lakes to an oligotrophic state.

Lowering external loading to an effective level may not always be possible however, because the minimum phosphorous input may not be controllable by human intervention when it is due to factors such as soil chemistry and airborne phosphorous deposition. This is likely to be the case for lakes in phosphorous-rich regions, for lakes that have been subject to a high level of external loading for extended periods of time, and for shallow lakes (Carpenter et al., 1999).

B Parameters that Determine the Type of Lake

As done by Mäler et al. (2003), analysis of equation (3) give rises to the following results.

- For high values of a, the equation has one stable equilibrium.
- For low values of a, that is $a < 0.3$, three situations occur depending on the value of b.

By solving the equation for values of x between 0 and 2, the curve for equation (3) can be used to plot $a(t)$ against $x(t)$ and to recreate the graphs provided in Mäler et al. for different values of b. The value of b is what affects the lake's reversibility from a state of eutrophication.

- $\mathbf{b \geq \frac{3}{8}\sqrt{3} \equiv b \approx 0.6495}$
  For these values of b, all values of a lead to one stable equilibrium. This implies that the sedimentation and recycling rates are such that the lake can continue to be used as a waste-sink without regard to the amount of phosphorous loading a. This further implies that the lake requires no management as far as controlling phosphorous loading is concerned. (See Figure 6.)
- $\mathbf{b \leq \frac{1}{2}}$
  For $b \leq \frac{1}{2}$, values of a below the local maximum (where $da/dx = 0$) yield

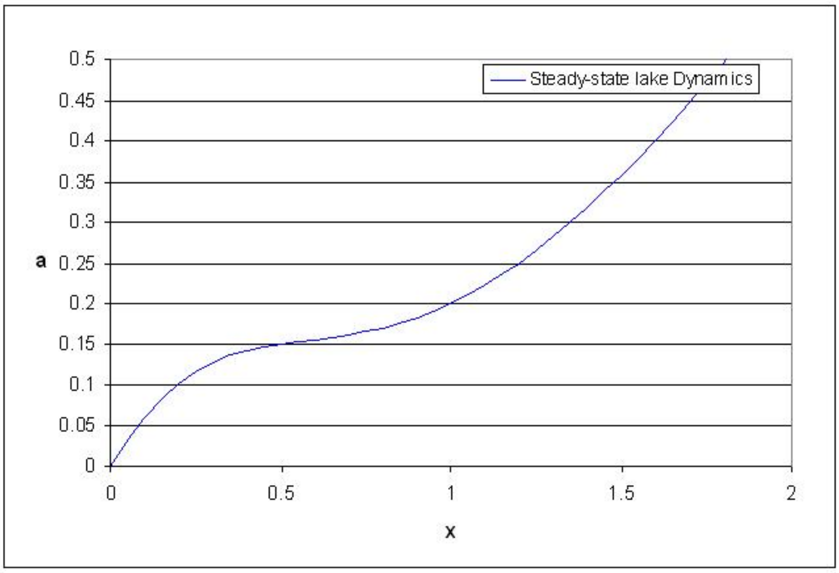

Figure 6: Lake Dynamics for b=0.7

two equilibria. The third root, i.e. the highest value of x where the curve intersects the x-axis, gives the basins of attraction: above this point, the high equilibrium will result and below this point the low equilibrium. This means that if the lake flips to the eutrophic state and reaches that level of phosphorous stock, reducing levels of phosphorous loading will not be sufficient to bring the lake back to an oligotrophic state. In that case, only a change in the parameter b can restore the lake because the hysteresis is irreversible. Altering b means resorting to such costly measures as biomanipulation and oxygenation as discussed earlier in the chapter. (See Figure 7)

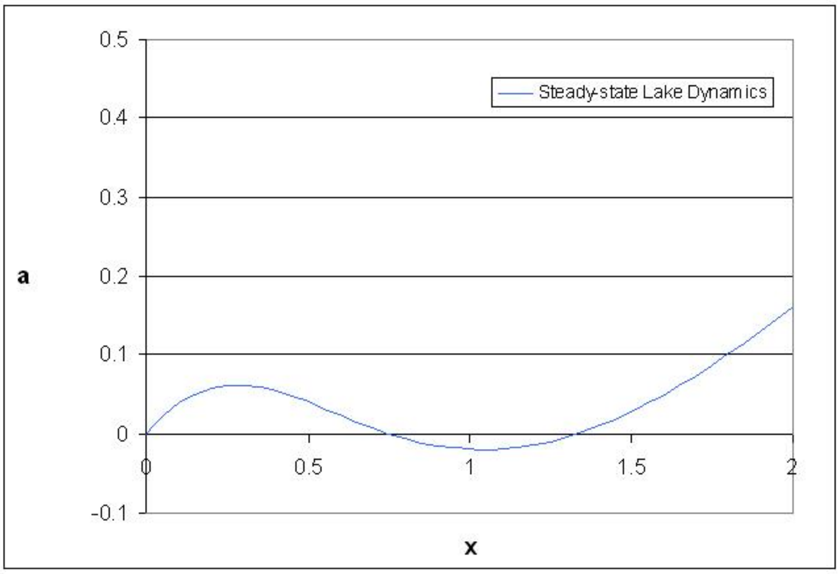

Figure 7: Lake Dynamics for b=0.48

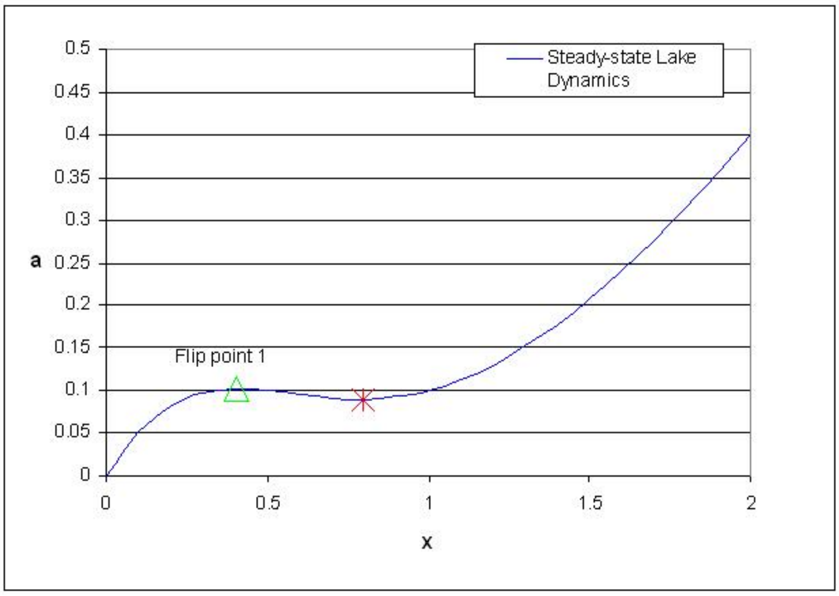

Figure 8: Lake Dynamics for b=0.6

- $\mathbf{\frac{1}{2} \leq b \leq \frac{3}{8}\sqrt{3}}$
  For values of b in this range, a has three equilibria. (See Figure 8) This is the hysteresis effect: the lake remains in an oligotrophic state up to a certain point at which it flips to a eutrophic state. The point at which the lake flips from oligotrophic to eutrophic is where $da/dx = 0$ at $(x_{F1}, a_{F1}) = (0.4084, 0.1021)$ and is an unstable steady-state. The point $(x, a) = (0.7876, 0.0897)$ is a stable eutrophic state. This hysteresis allows eutrophication to be reversible, however, as lowering external loading be-

low this second point will restore the lake to an oligotrophic state.

One sees that for $q = 2$ and $\frac{1}{2} \leq b \leq \frac{3}{8}\sqrt{3}$, the lake displays a reversible hysteresis in its response to phosphorous loading. In this case, eutrophication is reversible by the control of external phosphorous input alone.

References

C. Alaouze. A short run economic analysis of the eutrophication problem of the barwon and darling rivers in new south wales. Technical report, Discussion Paper, The University of New South Wales, 1995.

M. A. Brooks and B. J. Heijdra. Rent-seeking and pollution taxation: An extension. *Southern Economic Journal*, 54(2):335–442, 1987.

S. Carpenter, D. Ludwig, and W. Brock. Management of eutrophication for lakes subject to potentially irreversible change. *Ecological Applications*, 9(3): 751–771, 1999.

R. Damania. Political competition, rent seeking and the choice of environmental policy instruments. *Environmental and Resource Economics*, 13:415–433, 1999.

W. D. Dechert and W. A. Brock. The lake game, 2000. URL `http://www.ssc.wisc.edu/econ/archive/wp2003-24.pdf`.

W. D. Dechert and S. I. O'Donnell. The stochastic lake game: A numerical solution. *Journal of Economic Dynamics and Control*, 30:1569–1587, 2005.

L. Grüne, M. Kato, and W. Semmler. Solving ecological management problems using dynamic programming. *Journal of Economic Behavior and Organization*, 57(4):448–473, 2005.

D. R. Lee. Rent-seeking and its implications for pollution taxation. *Southern Economic Journal*, 51(3):731–744, 1985.

K.-G. Mäler, A. Xepapadeas, and A. D. Zeeuw. The economics of shallow lakes. *Environmental and Resource Economics*, 26:603–624, 2003.

J. D. Murray. *Mathematical Biology*. Biomathematics. Springer-Verlag, 1989.

S. I. O'Donnell and W. D. Dechert. A stochastic lake game, 2004. URL `http://panda.econ.uh.edu/research/stochastic.lake.ps`.

R. E. Ricklefs. *Ecology*. Chiron Press, 2nd edition, 1979.

M. Scheffer. *Ecology of Shallow Lakes*. Chapman & Hall, 1998.

G. Tullock. Efficient rent seeking. In J. M. Buchanan, R. D. Tollison, and G. Tullock, editors, *Toward a Theory of the Rent-Seeking Society*, pages 97–112. Texax A & M University Press, 1980.

F. Wagener. Skiba points and heteroclinic bifurcations, with applications to the shallow lake system. *Journal of Economic Dynamics & Control*, 27: 1533–1561, 2003.

J. K. Wilson and R. Damania. Corruption, political competition and environmental policy. *Journal of Environmental Economics and Management*, 49: 516–535, 2005.