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Sophisticated Program Planning Approaches Generate Large Benefits in High Risk Crop Farming

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Abstract

Agricultural production relies to a great extent on biological processes in natural environments. In addition to volatile prices, it is thus heavily exposed to risks caused by the variability of natural conditions such as rainfall, temperature and pests. With a view to the apparently lacking support of risky farm production program decisions through formal planning models, the objective of this paper is to examine whether, and eventually by how much, farmers' "intuitive" program decisions can be improved through formal statistical analyses and stochastic optimization models. In this performance comparison, we use the results of the formal planning approach that are generated in a quasi ex-ante analysis as a normative benchmark for the empirically observed ones. To avoid benchmark solutions that would possibly exceed the respective farmer's risk tolerance, we limit the formal search to a subset of solutions that are second-degree stochastically dominant compared to the farmer's own decision. We furthermore compare the suitability of different statistical (time series) models to forecast the uncertainty of single gross margins.

Keywords

stochastic optimization; stochastic processes; production risk; program planning; time series analysis

JEL codes

C1; C61; M11; Q12

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1. Introduction

Agricultural production relies to a great extent on biological processes in natural environments. It is thus heavily exposed to risks caused by the variability of natural conditions such as rainfall, temperature and pests. Due to climate change these risks are likely to increase in the future, especially in developing countries. While neither being able to avoid production nor market risks, farmers always have adapted, and always will adapt, to changing conditions. One important strategy is the adaptation of the production program. Hence, the question arises whether the decisions associated with such adjustment processes can be supported by formal statistical analysis and stochastic program optimization.

Optimization procedures have been receiving a lot of attention in agro-economic research and teaching for several decades (see e.g. Heady and Candler 1958; Brandes 1974; Dent et al. 1986; Rae 1994; Hardaker et al. 2004). Nonetheless, even in developed countries, where technological innovations in general have been readily adopted by farmers, formal optimization has scarcely found its way into the farm planning practice. With regard to teaching the question has even been raised whether valuable time at universities is wasted with linear optimization (Hanf 1991).

Questioning the extra value of formal risk programming models, a widespread opinion is that farmers - in particular with regard to the recurrent program decisions in crop farming - make near-optimal choices based on experience, incremental learning and simple heuristics (see also Gigerenzer and Selten 2001). This means that information such as the available acreage, the labour capacities, the potential cropping activities, the crop rotation requirements, the risk associated with various production activities and the individual risk aversion are considered implicitly in the mental model of the farmer. In contrast to that, the planning of the production program based on formal optimization procedures requires the explicit definition of the set of choices and restrictions and the objective function. Especially the difficulties arising from an explicit quantification of the uncertainty of single gross margins (see Adams et al. 1980) and the subjective risk attitude of the decision maker (see Hudson et al. 2005) are reasons why optimization has scarcely found its way into on-farm planning.

With a view to the apparently lacking support of risky farm decisions through formal planning models, the objective of this paper is to examine empirically whether, and eventually by how much, farmers' "intuitive" program decisions can be improved through adequate and manageable stochastic optimization procedures. A final answer to this question requires the consideration of two issues (see e.g. Verstegen et al. 1995): (i) the overall performance of the formal planning tool if adequately used, and (ii) its practical usefulness and de facto potential to increase profits given both the costs associated with using it and the cognitive constraints of potential users. This paper addresses only the former issue.[1] It compares farmers' empirical performance with the perform-

[1] See e.g. Brekke and Moxnes (2003) for an investigation of the practical usefulness of decision tools in a virtual setting.

ance that could have been realized if formally optimized programs had been implemented. We are the first, to our knowledge, to use the results of theoretical models generated in a quasi ex-ante analysis as a normative benchmark for empirically observed ones. From a methodical perspective, this requires the solution of two problems: firstly, the development of a manageable method to consider farmers' risk attitudes in the formal planning model. Secondly, the examination of the suitability of different statistical models to quantify the uncertainty of single gross margins.

The remainder of the paper is structured as follows: the second section describes the historical development of optimization approaches in the context of program planning decisions. The empirical database and the method are described in section three. This includes the statistical analysis of the single gross margin time series as well as the formal optimization model and the procedure of the benchmark comparison. Section four presents the results of the benchmark comparison. Finally, we provide conclusions in section five.

2. Historical development of optimization approaches

Soon after the simplex algorithm was found by Dantzig in 1947 (cf. Dantzig 1963), agricultural economists started to use linear programming for farm planning in the fifties and sixties of the last century. Early publications related to linear programming in agriculture either aimed at disseminating the mathematical knowledge by explaining the characteristics of the procedure (cf. e.g. Heady 1954; Boles 1955) or at pointing out its possible applications and general potential for farm management (cf. e.g. McCorkle 1955; Swanson 1961). As long as linear programming computer codes were not easily available, many efforts were made to reduce the structural complexity and size of the planning models. This concerned, for instance, the aggregation of production activities and the consideration of limited supplies and other constraints (cf. e.g. Huffman and Stanton 1969).

Soon, agro-economists aimed to improve optimization approaches by accommodating risk (synonymously: uncertainty) in their models, either in the objective function or in the constraints. The first approaches aimed at accounting for the two-dimensional objective "maximization of total gross margin" and "minimization of risk", include simple variant calculations and sensitivity analyses (cf. Dinkelbach 1969); they also include Chance Constrained Programming (CCP) where risk is considered in the constraints. That is, the amount of the available resources is reduced in the model from its expected value level to a lower level representing, for instance, the 5% percentile (cf. Charnes and Cooper 1959).

While these early approaches to risk do not explicitly take account of probabilistic information in terms of distributions, later programming approaches, following Freund (1956), explicitly resort to the variance, thus generating quadratic optimization problems. In the expected-value variance (EV-) model the decision-maker's risk attitude is endogenized into the analysis (cf. Arrow 1964; Pratt 1964). However, this is only true in that the objective function (preference function) of the formal optimization model includes the risk aversion coefficient. *If* the farmer's individual risk aversion coefficient could be ascertained, the combination of risk and income that maximizes his utility could indeed be identified endogenously. However, to elicit the individual risk aversion coefficients is an unsolved problem. Therefore, EV-models *factually* take the form of a set of variant calculations regarding the risk aversion coefficient. They thus provide a set of efficient combinations of variance and expected total gross margin, each of which is associated with a certain risk aversion coefficient. If one assumes that the total gross margin is normally distributed, the EV-efficient set is also risk efficient, meaning that the solutions are second-degree stochastically dominant (Hardaker et al. 1991). From a theoretical point of view, this assumption seems plausible as an approximation due to the central limit theorem. In conventional models the decision-maker is left with an exogenous choice to single out one combination of risk and income out of the many provided in the solution set. As alternatives to the EV-model, but equally without solving the problem of how to determine empirically the subjective risk attitude, Lambert and McCarl (1985) use direct maximization of the expected utility, and Patten et al. (1988) use utility efficient programming.

Before non-linear programming computer codes became generally available in the early nineties, considerable efforts were made to find linear programming approximations to the quadratic risk programming problem (cf. Hazell 1971; Thomas et al. 1972; Chen and Baker 1974; Tauer 1983; Okunev and Dillon 1988), the best known of which is probably Hazell's Minimization-Of-Total-Absolute-Deviation (MOTAD) programming. Resorting to the empiric distribution of the single gross margins and their annual deviations from their mean, this model uses the total absolute deviation of the objective value (total gross margin) as a linear risk measure. For predefined total gross margin values, the total absolute deviation is minimized. While using a different risk measure, the MOTAD-model generates a risk efficient production frontier quite analogous to those provided by EV-models varying the risk aversion coefficient. That is, the decision-maker confronted with the frontier has to choose a combination of the total gross margin and the risk according to his personal risk attitude. However, contrary to the EV-model, the inability to specify a single solution is not caused by a data problem and consequential variant calculations. It is due to the very structure of the MOTAD-model itself which lacks any endogenous consideration of subjective risk attitudes. Table 1 systematizes the essential program optimization approaches according to the two problem dimensions that need to be considered when applying optimization to practical farm planning problems: firstly, the quantification of the uncertainty regarding stochastic variables, and secondly, the consideration of the risk attitude of the individual decision maker.

Table 1: **Essential modeling approaches to risky farm production planning decisions.**

	Consideration of risk attitude	
Consideration of probabilistic information	**Exogenous**	**Endogenous**
No explicit consideration of probabilistic information; cf. e.g. Dinkelbach (1969)	Simple variant calculations and sensitivity analyses (Linear Programming)	
A priori assumption: static, normal distribution, selected parameters; cf. Charnes and Cooper (1959)	Chance Constrained Programming (Linear Programming)	
A priori assumption: non-parametric (empiric) distribution (past values equal in weight); cf. Hazell (1971)	MOTAD (Linear Programming)	
A priori assumption: static, normal distribution (past values considered equal in weight); cf. Freund (1956)	in practice, a risk-efficient set of expectation value and variance combinations is provided ⟵	EV-Model (Quadratic Programming)

Planning results are very sensitive to how probabilistic information is considered, i.e. the statistical treatment of time series data (cf. Adams et al. 1980; Musshoff and Hirschauer 2004). Adams et al. (1980: 19) consequently point out that "if researchers intend to use the E,V approach in providing decision making information to producers, care should be exercised in the choice of income and risk measures used." Unfortunately, statistical analysis may be error-prone due to questionable a priori presumptions. The simple fact that time series may exhibit trends suffices to show that "static distribution models" - i.e. the recourse to past values considered equal in weight - do not represent good forecasting models. Nonetheless, some well-known concepts dealing with risk in agriculture use this assumption. Conventional MOTAD-models (cf. Hazell 1971) which resort to the empiric distribution are an example. The same applies to most expected-value-variance (EV-) models (cf. e.g. Robison and Barry 1987). In multivariate analysis underlying EV-models, static normally distributed variables are commonly assumed for computational convenience (cf. e.g. Hazell and Norton 1986), that is to say, to avoid computational difficulties that would arise if one considered the eventuality of multiple, non-normally distributed correlated variables (cf. Buccola 1986; Preckel and DeVuyst 1992; Sornette et al. 1999). In contrast to such questionable a priori presumptions that may not meet the real-life facts, explicit time series analysis can be seen as searching for the stochastic process (time series model) that reflects the true stochastic pattern of a variable's development over time (cf. Pindyck and Rubinfeld 1998).

Furthermore, planning results are very sensitive to how the subjective risk attitudes of decision makers are considered. With a view to the unsolved problem of quantifying empirical individual risk aversion (cf. Jolly 1983; Just and Pope 2003; Hudson et al. 2005) and the fact that conventional optimization approaches under uncertainty provide only *risk efficient* (or stochastically efficient) combinations of expected profits and risk, Hardaker et al. (1991: 22) describe the need for further research as "finding the best way of generating smaller and yet more relevant stochastically efficient solution sets."

Referring to the systematization in Table 1 one can state that, so far, there have been very few approaches to endogenize risk aversion into the decision model; and those who did had all problems to empirically elicit the risk aversion of the individual decision-maker under consideration. Furthermore, to our knowledge, there have been no approaches that really extract and use the available probabilistic information and carry out an open ended statistical analysis to identify the stochastic process (and thus the best forecasting model in terms of the future distribution of the random variable) that fits best to the particular time series analysed.

3. Data and method

Our exemplary empirical analysis is a case study looking at the performance of four German cash crop farms over a period of six years. In the following section 3.1 we explain the empirical database in detail. In section 3.2 we describe the statistical models that are alternatively used to quantify the volatility of the single gross margins of each crop. The optimization model is described in section 3.3. Unlike the optimization approaches proposed until now, our approach generates a single alternative production program which has the following characteristics:

a. The uncertainty of the single gross margins (of wheat, barley etc.) is quantified through a statistical analysis of individual farm data. The resulting probabilistic information is used for the required one-year-ahead forecasts that are fed into the risk programming model.

b. The farmer's subjective risk attitude is seen as being reflected by his choice of production program and his apparently accepted variance of the total gross margin. In the formal approach, this expression of his risk attitude is taken into account by the use of the observed variance as an upper bound in the optimization model.

Finally, for each of the 24 planning occasions, the total gross margins which could have been realized if the formally optimized programs had been implemented are then ex-post compared to those that were actually realized by the farmers.

3.1. Empirical database

We are investigating four cash crop farms in Brandenburg, North-East Germany. Farms 1, 2, and 3 are located about 50 km west of Berlin, and farm 4 is about 100 km north of Berlin. The acreage of these farms has virtually not changed over the last six years. The average farm size amounts to 729 ha for farm 1, 1 111 ha for farm 2, 1 210 ha for farm 3, and 175 ha for farm 4. With the farm owner/manager, three permanent workers are engaged on farm 1, five on farm 2, four on farm 3, and one on farm 4. The major production activities considered by farmers 1, 2, and 3 include winter and spring wheat, winter rye, winter and spring barley, winter canola, corn, and non-food canola or set-aside land. Having an otherwise similar crop mix, farmer 4 does not include spring crops in his repertoire.

For all farms, minor crops such as alfalfa, oil flax or peas are excluded from the performance comparison. These crops represent something close to hobbies, rather than serious production activities. Proportionately they are almost irrelevant on all farms. On the contrary, sugar beets are very relevant and profitable without question. They are left out of the analysis in that their acreage is a priori presumed to amount to the maximum level defined by the production quota allocated to each farm. Excluding both hobby activities and the most competitive crop from the model does not impede the insight to be gained from the analysis. It only leads to a shift in the level of both the empirical and the optimized total gross margins and is therefore irrelevant for the performance comparison.

The farmers were interviewed with respect to their factor endowment (human resources and farm land) over the last six years. We also inquired about the number of field working days, the maximum working hours per day, the time required for the various activities in the critical seasons (March/April, May/June, mid-July/mid-September, and mid-September/mid-November) and possible purchasing activities such as hiring seasonal labor. Furthermore, crop rotation constraints (minimum and maximum proportions of the particular crops) were taken into consideration.

Both production and market risks are embedded in the single gross margins of each crop. Thus, farm-specific time series of single gross margins are needed to specify the probabilistic information relevant to each farmer for his program decision. In principle, data sets should be as large as possible for time series analysis. However, due to the structural discontinuity at the beginning of the nineties (collapse of the centralized economy and transition to market economy), farm-specific gross margins of before 1992/1993 for farms located in the new federal states of Germany often contain ambiguous information or are not available at all. Hence, while using individual farm data after 1992, we construct proxies for the years 1980 to 1992. These proxies are site-specific single gross margins which are based on yields obtained on comparable soils and under comparable natural conditions in the old federal states of Germany and on West German price data (ZMP, several years; LSD Brandenburg 2003).

3.2. Time series analysis

The time series available at the respective planning dates $t^* - 1$ comprise the years $t = 1980$ to $t^* - 1$. The considered planning dates $t^* - 1$ are 1998, 1999, 2000, 2001, 2002 and 2003. Thus, a farm-specific time series of 19 data is available for each crop at the first planning date “fall 1998”.

For each of the following planning dates, the available time series increases by one year.

According to the concept of "stochastic process", the realization of a random (time series) variable at any one point in time is seen as a function of a systematic component as well as a random component. The systematic component - contrary to causal models - describes the functional relationship linking prior realizations with the actual one. In other words: the systematic component describes the "pattern" of the stochastic development of a time series such as a trend, seasonal fluctuations, a combination of the two, a zig-zag-pattern, etc. In a stochastic world, the systematic component can only explain a part of the development of a time series. The unexplained residual is white noise which is taken into consideration by a random component.

The complexity of the stochastic process (stochastic pattern) depends on the number of prior realizations and prior random effects that are to be considered in the systematic component according to the statistical analysis which searches for the best fit of the model to the observed time series. In the simplest case, the systematic component is a constant. This is equivalent to a static distribution. In more complex cases, the best fit can be a constant plus one or several of past observed values in different weights and of past random effects, also in different weights. This is equivalent to a linear functional relationship linking past values to the actual one. The simplest example is a linear trend, a more complex example would be the combination of a trend with a zig-zag-pattern. The complexity increases still further if the functional relationship is non-linear. It should be noted that, while the real pattern cannot be known before the analysis, the results of different classes of statistical models are limited by their respective presumptions: static distribution models are less flexible (and less costly to handle) than linear models and the latter less flexible (and less costly) than non-linear models. This implies that it is possible to identify a static distribution (to identify a linear relationship) by using a linear model (by using a non-linear model) in statistical analysis but not vice versa. In other words, the less flexible models will fail to identify the true stochastic pattern if the particular time series exhibits more complexities than what the statistical model allows for. Because of the "antagonism" between costs and flexibility in the application, for each single gross margin and farm/year combination (1) static distributions, (2) linear time series models, and (3) unbiased (non-linear) time series models are tested as alternative statistical methods and thus forecasting models.

3.2.1. Forecasting model 1: static distributions

With forecasting model 1 one determines a static parametric distribution for each of the single gross margins. According to the Chi-Square, Kolmogorov-Smirnov and Anderson-Darling tests carried out using the MS-EXCEL Add-In BEST FIT, the normal distribution cannot be rejected for any of the considered single gross margin time series (at a significance level of 5%). However, when compared to the normal distribution, Beta, logistic and/or triangular distributions show a slightly better match with the empirical distributions in some cases. In agreement with the standard approach (cf. Hazell and Norton 1986: 81), we nonetheless assume a normal distribution for all single gross margins.

Let GM_t^j denote the gross margin per unit of production activity j observed at time t. Then the assumption of a static normal distribution for a single gross margin can be described as follows:

$$GM_{t*}^{j} = E\left(GM_{t*}^{j}\right) + \chi_{t*}^{j} = \frac{1}{N} \cdot \sum_{t=1980}^{1980+N-1} GM_t^j + \chi_{t*}^{j}, \text{ with } N = t^* - 1980 \tag{1}$$

The future gross margin GM_{t*}^{j} results from its expected value $E\left(GM_{t*}^{j}\right)$ and a $N\left[0,\sigma_{t*}^{j}\right]$-normally distributed random component χ_{t*}^{j} (error term, white noise). For a static distribution, the expected value corresponds to the mean. The standard deviation of the error term σ_{t*}^{j} simply reflects the standard deviation of all observed values. We introduce notation (1) - even though it is rather unusual for simple distributions - on grounds of consistency with the notation for the stochastic processes described below.

3.2.2. Forecasting model 2: linear time series models

Allowing for stochastic processes implies that one examines the time-dependent pattern of random variables through time series analysis. Abstracting from discontinuities, a stochastic process represents the best estimation with regard to the variable's distribution at future points in time. Auto-Regressive-Integrated-Moving-Average models of the order *p*, *d* and *q* (ARIMA(*p*,*d*,*q*)-models) are linear time series models. Due to their flexibility, ARIMA(*p*,*d*,*q*)-models are used to represent a multitude of stochastic economic processes (cf. e.g. Pindyck and Rubinfeld 1998).

The ARIMA(*p*,*d*,*q*)-model that fits best to a particular time series can be determined with the Box-Jenkins test procedure (Box and Jenkins 1976). According to this test, an AR(*p*)-process results for all single gross margin time series in our illustrative analysis:

$$GM_{t*}^{j} = E\left(GM_{t*}^{j}\right) + \chi_{t*}^{j} = \alpha_0^j + \sum_{u=1}^{p} \alpha_u^j \cdot GM_{t*-u}^{j} + \chi_{t*}^{j}, \text{ with } \sum_{u=1}^{p} \left| \alpha_u^j \right| < 1 \qquad (2)$$

α_0^j denotes a constant, α_u^j denote the weight factors that need to be estimated for the last p observation values GM_{t*-u}^{j}, and χ_{t*}^{j} describes a $N\left[0,\sigma_{t*}^{j}\right]$-normally distributed error term. It should be noted that the expected value as well as the error term can, and mostly will, differ from the ones obtained with the static distribution model even if the same data are analyzed.

3.2.3. Forecasting model 3: non-linear time series models

Recent research emphasizes the importance of non-linear dependencies in time series (e.g. Chavas and Holt 1991; Wei and Leuthold 1998). Non-linearity cannot be identified with standard statistical procedures such as the Box-Jenkins test (cf. Box and Jenkins 1976) or the Dickey-Fuller test (cf. Dickey and Fuller 1981). These "conventional" tests presume a priori linearity. Hence, more sophisticated statistical methods, which do not predispose linearity but facilitate the unbiased identification of both linear and non-linear processes, are needed. Artificial neural networks, increasingly applied in the field of financial analysis for predicting stock prices and interest rates (cf. e.g. Bishop 1995; Haykin 1999), have the capacity to approximate both linear and non-linear relationships. The result of a neural network analysis, however, is always an "implicit" model; i.e. no mathematical formulation of the time series is provided.

The method of "heuristic self-organizing time series models" originally described by Ivachnenko (1983) offers an alternative for the identification and specification of non-linear stochastic relationships (cf. Farlow 1984; Mueller and Lemke 2003). Compared to neural networks, the big advantage of this method is that it provides a mathematical formulation for the time series. It facilitates an automatic specification of a time series model of "optimal complexity", without the need for any a priori assumptions regarding the structure (dimension of the polynomial).

A special class of self-organizing algorithms is the so-called Group Method of Data Handling (GMDH). GMDH-algorithms combine the connectionistic approach to artificial neural networks with the classical method of regression. They generate general polynomial process models (cf. Mueller and Lemke 2003: 77):

$$GM_{t^*}^{j} = \gamma_0^j + \sum_{u=1}^{p} \gamma_u^j \cdot GM_{t^*-u}^{j} + \chi_{t^*}^{j}$$
$$+ \sum_{u=1}^{p}\sum_{v=1}^{p} \gamma_{u,v}^j \cdot GM_{t^*-u}^{j} \cdot GM_{t^*-v}^{j} + \sum_{u=1}^{p}\sum_{v=1}^{p}\sum_{w=1}^{p} \gamma_{u,v,w}^j \cdot GM_{t^*-u}^{j} \cdot GM_{t^*-v}^{j} \cdot GM_{t^*-w}^{j} + ... \tag{3}$$

These polynomials take into account up to p preceding values with different weights as well as non-linear terms, the potential number of which exponentially increases with the number of the considered preceding values. Thus, the functional form of the polynomial may easily get quite large. Its general structure, however, can be understood as an AR(p)-process (upper line) which is extended by a non-linear component (lower line). While the GMDH-model has the principal capacity to account for any kind of distribution, we assume normally distributed error terms since the results of the Chi-Square, the Kolmogorov-Smirnov and the Anderson-Darlings tests show that the normal distribution cannot be rejected at a significance level of 5%.

3.3. Optimization model and performance analysis

Fig. 1 illustrates and outlines the methodical steps of data collection and processing that are needed to optimize the production program.

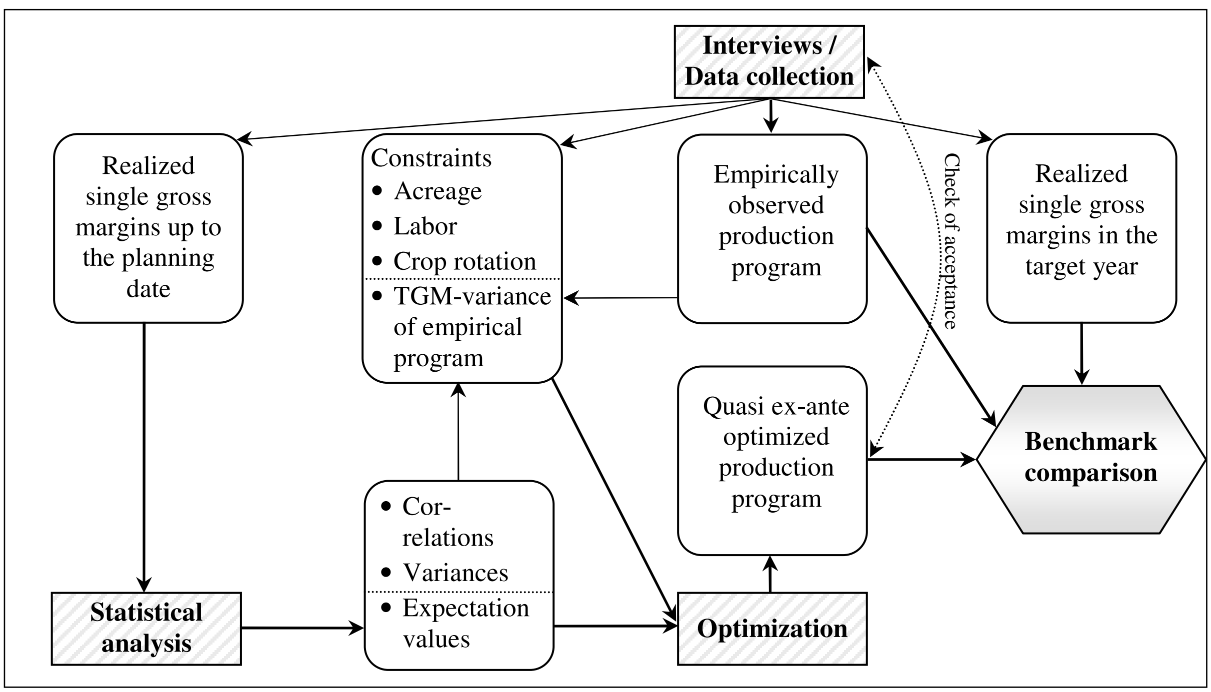

Fig. 1. Synopsis of the methodological procedure.

As described above, data are collected for each year and farm. This concerns the realized single gross margins up to the planning date, the constraints such as acreage, labor and crop rotation requirements, the empirically observed production program, and the achieved single gross margins realized in the target year. In the next step the single gross margin time series up to the respective planning date are statistically analyzed. The resulting probabilistic information includes the correlations, variances, and expectation values of the single gross margins. These values are specified separately in each of the three variants of statistical analysis. Depending on the consequently differing model inputs, the optimization model, of which the general structure is described below, will thus provide three alternative planning variants for each target year and farm.

3.3.1. Optimization model

The farm-specific constraints and the expectation values, variances and correlations of the single gross margins are fed into a quadratic optimization model. While considering the probabilistic information derived from the complete time series of single gross margins up to each planning date, we do not model the actual program decisions as dynamic or multi-period optimization problems. Instead, the program for each year is optimized subject to the constraints that are effective in the respective year as indicated by the individual farmer. Interdependencies between the activities of subsequent years are considered through crop rotation constraints (i.e. maximum and minimum percentage shares of crops) that may vary from year to year. For each farm and planning date $t^* - 1$ (1998 to 2003), formally optimized alternative programs are determined according to expectations regarding the respective target year t^* (1999 to 2004). The optimization model can be described as follows:

$$\max_{x_{t*}^j} E(TGM) = \sum_{j=1}^{J} E\left(GM_{t*}^j\right) \cdot x_{t*}^j$$

s.t.

$$\sum_{j=1}^{J} a_{t*}^{i,j} \cdot x_{t*}^j \le b_{t*}^i, \quad i = 1,2,...,I \tag{4}$$

$$V \le V_{emp}$$

$$x_{t^*}^j \ge 0, \quad j = 1,2,...,J$$

$E(TGM)$ denotes the expected total gross margin in the respective target year of the optimized production program. The objective function coefficients $E\left(GM_{t*}^j\right)$ represent the expected gross margins per unit of production activity j. x_{t*}^j describe the levels of the production activities. b_{t*}^i denote the capacities (restrictions), and $a_{t*}^{i,j}$ represent the capacity requirements per unit of production activity. V_{emp} is the total gross margin (TGM-) variance inherent to the empirically observed production program. V, in contrast, denotes the TGM-variance of the optimized program. Using V_{emp} as an upper bound ensures that the reflection of the farmer's risk attitude as observed in his own choice of production program is taken into account in the optimization.[2]

The calculation of the variance is based on the results of the statistical analysis; i.e. the standard deviations and the correlation coefficients of the single gross margins (in the case of static distributions), and the standard deviations and the correlation coefficients of their error terms (in the case of stochastic processes). Since the J random variables (single gross margins) are normally distributed and additively combined, the TGM-variance of the *empirical* production program V_{emp} can be

[2] For the sake of convenience, we omit the subscript t^* when referring to $E(TGM)$, V_{emp} and V. Nonetheless, they always correspond to the respective target year.

calculated in a way analogous to a portfolio consisting of J asset positions (cf. Jorion 1997: 150):

$$V_{emp} = \sum_{j=1}^{J} \left(x_{t^*,emp}^{j} \cdot \sigma^{j} \right)^2 + 2 \cdot \sum_{j=1}^{J} \sum_{k<j}^{J} x_{t^*,emp}^{j} \cdot \sigma^{j} \cdot x_{t^*,emp}^{k} \cdot \sigma^{k} \cdot \rho^{j,k} \tag{5}$$

$\rho^{j,k}$ denote the correlation coefficients between the single gross margins j and k (in the case of static distributions), or between their error terms (in the case of stochastic processes). σ^j and σ^k describe the respective standard deviations. The correlation coefficient and the standard deviation are determined on the basis of the empirical data collected up to the particular planning date. $x_{t^*,emp}^{j}$ and $x_{t^*,emp}^{k}$ respectively, represent the weight (acreage) of the production activities in the farmer's empirically observed program. The TGM-variance of the *optimized* program V is to be determined in a way analogous to (5). One merely needs to replace the observed production levels $x_{t^*,emp}^{j}$ and $x_{t^*,emp}^{k}$ by the optimized production levels $x_{t^*}^{j}$ and $x_{t^*}^{k}$.

Crop rotation requirements justifying the model's linearity assumption (i.e. the assumption that the resources used and the revenues derived are linearly linked with the activity level) are represented in each year's optimization model as upper and lower bounds on the crop acreages. In a first optimization run, we use the maximum and minimum crop proportions as indicated by the farmers in the interviews as constraints. We then cross-check with farmers whether they accept the resulting programs and whether they consider them feasible and consistent both with their preferences and the specific farm situation. If necessary, constraints are changed or added to the model. This includes changes of the feasible minimum and maximum crop proportions which may vary depending on the production program(s) of the previous year(s). This feasibility check is repeated until no further modifications are needed. This step by step procedure ensures that the optimized programs, while differing from the farmers' realized programs, are both feasible and acceptable for the real decision-makers.

3.3.2. Benchmark comparison

We finally calculate the normative benchmark, i.e. the hypothetical total gross margin TGM which would have been realized in the target year *if* the optimized production program had been implemented:

$$TGM = \sum_{j=1}^{J} GM_{t^*}^{j} \cdot x_{t^*}^{j} \tag{6}$$

$GM_{t^*}^{j}$ indicate the actually realized single gross margins in the target year, and $x_{t^*}^{j}$ denote the planned proportions of crops as derived from the formal planning model. An ex-post comparison between the normative benchmark TGM and the farmer's actually realized total gross margin TGM_{emp} reveals whether an extra value could have been derived from using the formal planning model. TGM_{emp} is to be determined in a way analogous to (6). One merely needs to replace the optimized production level $x_{t^*}^{j}$ by the farmer's observed production levels $x_{t^*,emp}^{j}$. Regarding the validity of the performance comparison, it should be emphasized that no informational advantage was accorded to the formal planner. The single gross margins realized in the target year are exclusively used for the final benchmark comparison.

3.3.3. A brief comparison with conventional quadratic risk programming

While bearing resemblance to conventional EV-models the proposed optimization approach under uncertainty exhibits some particularities: as has been mentioned above, EV-models factually handle the problem of unknown risk attitudes by carrying out variant calculations regarding the risk aversion coefficient, thus providing a set of efficient combinations of variance and expected total gross margin (cf. Fig. 2, left).

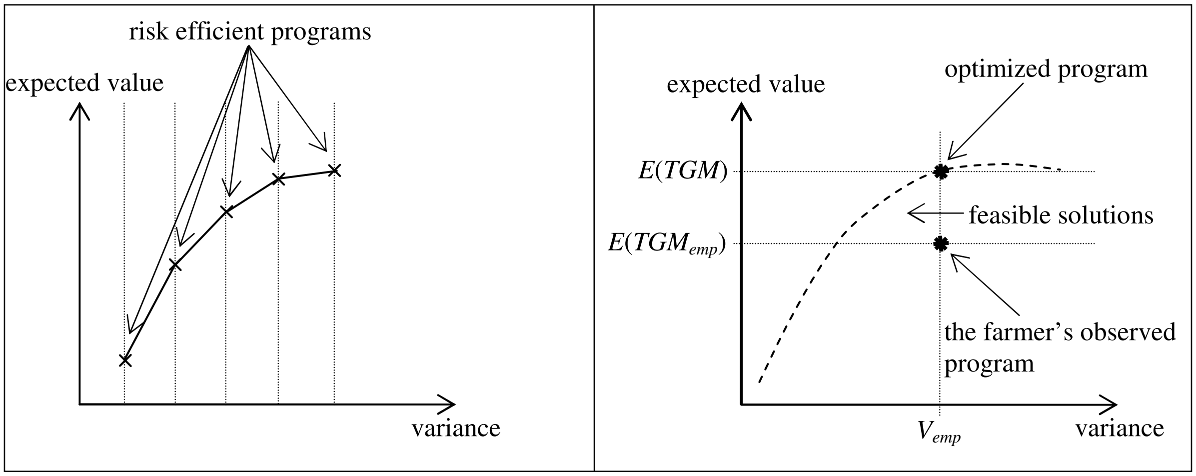

Fig. 2. Classical procedure in the EV-model (left) vs. practical decision support (right).

Aiming at supporting practical decision-making, we maximize the expected total gross margin subject to the constraint of not exceeding the empirically observed willingness to accept risk. This means taking the variance of the total gross margin V_{emp} , inherent to the production program chosen by the farmer, as an observable, albeit incomplete, *reflection* of his subjective risk attitude (cf. Fig. 2, right). We do not argue that this reflection represents the farmer's risk attitude completely. Nor do we presume that our procedure necessarily ensures the identification of the production program that maximizes the farmers' utility. We rather focus on the manageability and applied usefulness of the approach which allows for a clear endogenous recommendation by reducing the efficient solution *set* to one *single* combination of variance and expected total gross margin. Technically speaking, we limit the set of feasible solutions to those yielding a higher or identical expected total gross margin at a lower or identical variance as the one previously accepted by the farmer. This is equivalent to limiting the formal search to an identifiable subset of solutions that are second-degree stochastically dominant compared to the farmer's own decision. We are thus sure to increase (or at least meet) the farmer's *expected* utility compared to the one resulting from his own program.

4. Results

4.1. Profitability comparison

Until now, none of the decision-makers of the considered farms has been using formal optimization procedures to determine the production program. In fact, production planning is completely based on non-formal planning and intuition. Table 2 compares the average annual total gross margins realized by the farmers with those that could have been realized *if* the formally optimized programs had been implemented.

As an additional point of reference, the first row of Table 2 shows the hypothetical room for improvement (extra value) that would be generated if one had a "perfect forecasting model", i.e. a model, which exactly predicts the single gross margins in the respective target years. If such perfect information was available, the total gross margin could be improved by 15.9% on average over all farms and all years. When interpreting this figure one should note that a perfect prognosis is never possible in reality because any time series contains unsystematic and unpredictable random errors. Hence, the figures depicted in the first row of Table 2 are only a first hint that it is worthwhile examining the potential for improvement that might be generated by using formal procedures of statistical analysis and stochastic optimization.

Rows 2 to 4 of Table 2 depict the extra value that is added by formal planning based on the three different forecasting models described above. Only farm 2 could have improved its average annual performance considerably through the formal planning approach 1. The increase would have been quite low in farm 4, and on farm 1 and 3 the average annual performance would even have been inferior to that of the actual programs. Although not shown in Table 2, a look at the crop mix of farm 2 reveals that the optimized program suggests a sharp decrease of the proportion of corn. Farmer 2 has in reality planted corn on 15% of his acreage on an average whereas it is virtually irrelevant in the programs of the other three considered farms. Its comparative competitiveness being low, the 15% proportion of corn on farm 2 must be interpreted as a serious planning mistake. It seems that the mistake is so serious that it could even have been alleviated through an optimization approach based on an ill-founded forecasting model. Despite its positive effect on farm 2, the overall change of performance caused by approach 1 over all farms and all years is nearly zero. We must therefore conclude that a "standard" risk programming approach which incorporates risk through static distributions may even be inferior to informal decision-making of reasonably good farm managers.

On the contrary, very encouraging results were found for the other two planning approaches (see the third and fourth row of Table 2). The average total gross margin could have been improved noticeably on all four farms if farmers had used formal optimization based on probabilistic information derived from systematic time series analysis: farmer 1, for instance, achieved in reality an average total gross margin of € 323 002. Optimized production programs based on forecasting model 3 would have increased that amount to € 349 028. In other words: the average annual total gross margin for farm 1 could have been increased by 8.1% (or € 26 026 per annum). For farm 2 the respective figures amount to 15.0% (or € 48 749 per annum), in the case of farm 3 to 3.2% (or € 13 943 per annum), and for farm 4 to 4.7% (or € 3 725 per annum).

Averaged over all farms, the potential for improvement compared to the farmers' actual programs amounts to nearly 8% (7.9% for approach 2, and 8.0% for approach 3).[3] That is, even though much more effort went into the statistical analysis of approach 3, the results are only slightly superior to those derived from linear time series models. This might be interpreted as preliminary evidence that, with GMDH models which allow for non-linear time series, we have arrived at a stage of model sophistication where the marginal returns of increasing planning efforts sharply decrease.

[3] Additionally one could mention that the average total gross margin over all farms and years could be increased by 4.4% if the formal planning approach had been based on a (linear) trend regression of the single gross margin time series (at a significance level of 5%). In other words: optimization based on such a "rudimentary time series model" would still be inferior to optimization based on explicit time series analysis because a linear trend may be only *one* out of the many systematic components of a time series that should be considered.

Table 2: **Average annual total gross margins (in €) of realized and optimized production programs.**

	Farm 1			Farm 2			Farm 3			Farm 4			Sum over all farms		
	Empirical	Optimized	Change	Empirical	Optimized	Change	Empirical	Optimized	Change	Empirical	Optimized	Change	Empirical	Optimized	Change
"Perfect forecasting model"	323 002	360 063	11.5%	325 636	413 944	27.1%	432 118	482 390	11.6%	78 894	87 826	11.3%	1 159 650	1 344 224	15.9%
Approach 1: assuming static distributions	323 002	307 874	-4.7%	325 636	360 656	10.8%	432 118	420 384	-2.7%	78 894	80 286	1.8%	1 159 650	1 169 200	0.8%
Approach 2: assuming linear time series	323 002	344 679	6.7%	325 636	373 466	14.7%	432 118	450 237	4.2%	78 894	82 878	5.1%	1 159 650	1 251 259	7.9%
Approach 3: allowing for non-linear time series	323 002	349 028	8.1%	325 636	374 385	15.0%	432 118	446 061	3.2%	78 894	82 619	4.7%	1 159 650	1 252 093	8.0%

Table 3: **Annual total gross margins (in €) of realized and optimized production programs based on the superior planning approach 3.**

	Farm 1			Farm 2			Farm 3			Farm 4			Sum over all farms		
Target year t^*	Empirical	Optimized	Change	Empirical	Optimized	Change	Empirical	Optimized	Change	Empirical	Optimized	Change	Empirical	Optimized	Change
1999	480 466	511 087	6.4%	370 455	506 530	36.7%	609 084	622 857	2.3%	66 239	66 636	0.6%	1 526 245	1 707 110	11.9%
2000	339 544	347 890	2.5%	295 934	315 797	6.7%	462 911	479 697	3.6%	76 887	80 250	4.4%	1 175 276	1 223 634	4.1%
2001	434 064	436 099	0.5%	410 929	434 361	5.7%	593 912	577 690	-2.7%	74 546	77 211	3.6%	1 513 451	1 525 360	0.8%
2002	240 468	243 935	1.4%	303 739	320 128	5.4%	308 538	321 348	4.2%	59 748	65 338	9.4%	912 493	950 749	4.2%
2003	168 913	274 723	62.6%	255 150	326 454	27.9%	263 096	285 644	8.6%	59 389	63 460	6.9%	746 548	950 282	27.3%
2004	274 556	280 436	2.1%	317 608	343 042	8.0%	355 169	389 127	9.6%	136 556	142 820	4.6%	1 083 889	1 155 425	6.6%
Average over all years	323 002	349 028	8.1%	325 636	374 385	15.0%	432 118	446 061	3.2%	78 894	82 619	4.7%	1 159 650	1 252 093	8.0%

Going beyond the consideration of averages, Table 3 informs of the details of the performance of the most competitive planning approach 3 for each of the 24 cases modeled. The most essential results can be summarized as follows: firstly, the optimized programs derived from approach 3 surpasses in performance the empirical ones in 23 out of the 24 cases. Although not being depicted, it can be added that the same applies to approach 2. Approach 1, in contrast, outperforms the realized programs in merely 11 out of the 24 cases. Second, being the odd exception, the total gross margin realized in the year 2001 by farmer 3 is higher than the one that would have been achieved with approach 3. At the planning date in the year 2000, however, the *expected* total gross margin of the optimized production program was 1.5% higher than the one of the farmer's program. This underlines the well-known fact that in an uncertain environment a faulty decision may by chance result in higher profits, but uninformed choices will not be superior in the long run.

4.2. Comparison of production programs

Table 4 provides a rough characterization of production programs by comparing the farmers' crop mix with the optimized and more profitable mix that would have been derived from approach 3. While only commenting on the benchmark comparison with approach 3, identical conclusions are to be drawn from the comparison with approach 2. Comparing the programs and identifying the main divergences provides first evidence for systematic planning mistakes made by farmers.

The most noticeable result of the comparison is that, according to the superior planning approach 3, the share of (winter) rye should have been increased considerably in all considered farms and in 22 out of the 24 cases modeled. Rye is very drought-resistant. From a crop science perspective it is thus especially well suited for the natural conditions of Brandenburg which are characterized by low and uncertain rainfalls as well as by poor and quickly draining soils. Searching for the most common and apparent change suggested by the formal planning approach 3, we may conclude that, besides farmer's 2 particular planning mistake regarding corn, all farmers should reduce their proportion of winter barley in favor of rye.

This result is an indication for systematic planning errors made by the farmers in the past. Evolutionary economics (cf. e.g. Nelson and Winter 1982) could be called upon to explain this finding: even important economic decisions are often not supported by formal decision models. Rather, they are based on the decision-maker's experience gathered in the course of the past decades and on simple heuristics such as "never make any decision that differs very much from past ones" (cf. e.g. Gigerenzer and Selten 2001). In an environment where the relative competitiveness of different crops, for instance, changes quickly, decision-makers may thus not be quick enough to adapt to changed conditions. In other words: we might ask the question whether (boundedly rational) farmers making routine production program decisions learn too slowly. In the considered context it seems reasonable to speculate that farmers have not yet adapted their routines to account fully for two major changes of their relevant environment: on the one hand, the enormous progress in rye breeding over the last years which brought rye production up to competitive levels, and, on the other hand, the increasingly precarious rainfalls in Brandenburg, possibly caused by climatic change. Slow learning and adaptation, in turn, justifies the use of formal decision aids by farmers and management consultants.

Table 4: **Average crop proportions (in %) empirically realized by farmers compared to those derived from the superior planning approach 3.**

	Farm 1			Farm 2			Farm 3			Farm 4			Sum over all farms		
	Diffe-rence	Increase*	Decrease*	Diffe-rence	Increase*	Decrease*	Diffe-rence	Increase*	Decrease*	Diffe-rence	Increase*	Decrease*	Diffe-rence	Increase*	Decrease*
Winter wheat	-1.8	3	3	3.8	3	3	8.7	3	2	-1.6	2	4	0.8	11	12
Spring wheat	0.9	1	0	0.0	0	0	–	0	0	0.6	1	0	0.4	2	0
Winter rye	16.6	5	0	22.1	6	0	7.3	5	1	11.4	6	0	16.1	22	1
Winter barley	-13.6	1	5	-5.6	0	5	-17.5	0	6	-7.2	0	6	-8.6	1	22
Spring barley	-1.2	0	1	2.5	3	0	–	1	0	1.5	1	0	1.2	5	1
Winter canola	-3.1	3	3	-6.8	0	6	3.7	2	4	-3.5	1	5	-4.2	6	18
Corn	0.0	0	0	-14.9	0	6	–	0	0	-1.7	0	3	-5.8	0	9
Non-food canola	2.8	5	1	0.1	1	0	-3.1	3	3	2.0	5	1	1.3	14	5
Set-aside	-0.7	1	4	-1.3	1	5	0.8	2	4	-1.5	1	5	-1.1	5	18

* Number of years - out of the six planning years considered for each farm - in which changes should have been made according to formal modeling. Cases in which the crop proportion remains unchanged can be calculated as residuals.

4.3. *Exploratory research regarding of the model's robustness*

Using a very cautious wording, the above-presented results provide evidence that cases exist where an improvement of farm production program planning can be provided through formal stochastic optimization. However, the variability of circumstances as well as the small number (24) of analyzed cases does not allow for statistical generalizations. Aiming to identify directions for relevant future research we carried out a preliminary exploration regarding the influence of differing soil qualities on the model's robustness to provide superior results.

To do so, in addition to the four farms analyzed above we looked at five more crop farms in North-East Germany. Since farm-specific single gross margin data had only been recorded on these five farms since 1998, we only considered the target years 2002, 2003 and 2004. Forming equal-sized groups, we attributed three farms to category I (low-quality soil: below 30 points according to relative the German soil quality classification scheme from zero to 100 points), three to category II (medium-quality soil: between 31 and 44 points), and three to category III (high-quality soil: between 45 and 52 points).

As shown in Table 5 in the three analysed years the largest performance improvement of 12% on average could have been realized in the three farms of the medium-quality soil category II. This figure is down to 3.5% in the high-quality soil category, and to 7% in the low-quality soil category. We are not able to fully explain this finding. However, there are some facts upon which to base reasonable speculations that might be used as hypotheses or research questions in further studies.

Let us first look for a possible explanation for the lower improvement potential in the farms of high-quality soil category compared to those on medium-quality soils: the number of crops to be considered by farmers on high-quality soils may be very small and include only very few high-yield cropping activities. In other words: if the relative competitiveness of different crops is clearly differing and if the farmers' respective knowledge is adequate to allow for a significant reduction of the planning complexity through an a priori exclusion of those crops that are "out of question", little support is needed, and can be given, by a sophisticated planning approach.

Table 5: **Potential for improvement (in % of the total gross margin) in differing soil categories based on planning approach 3.**

	Average change	Standard deviation
category I (low-quality soil; three farms and years)	7.1	6.3
category II (medium-quality soil; three farms and years)	12.0	19.8
category III (high-quality soil; three farms and years)	3.5	4.1

Let us now search for an explanation for the lower improvement potential in the farms of the low-quality soil category compared to those on medium-quality soils: according to their empirically observed programs the latter also accept a higher total gross margin variance, which, in turn, can be attributed partly to a lower risk aversion and partly to a steeper risk efficient production frontier. The higher performance potential can thus be seen as an indication that the use of the production factor risk generates decreasing marginal returns, not only due to the decreasing slope of the production function (frontier), but also to the decreasing ability of boundedly rational decision-makers to realize the potential returns. This can again be attributed to the higher complexity of the planning problem.

We may thus formulate as a plausible hypothesis that the room for improvement opened up by formal optimization approaches increases with an increasing complexity (for whatever reason) of the planning problem.

5. Conclusion

At first view, this paper seems to revive the discussion about the benefits of optimization models in applied agricultural program planning - a discussion that has been virtually closed in the past for the so-called good reason that agricultural practitioners have found neither need nor want to use formal approaches in on-farm planning. Our illustrative analysis of 24 farm/year combinations, however, suggests that a different attitude may be needed.

The formal approach used in this study provides *practical* assistance for dealing with the problem of individual risk attitudes. It includes the variance of the total gross margin inherent to the production program chosen by the farmer as an additional restriction in the stochastic optimization model. Any practical decision support procedure which relies on this approach thus requires that farmers first specify their "own" production programs without the formal planning aid, thus providing an observable reflection of their risk attitude. Afterwards, one can search for alternative programs which - with the same or even less variance - lead to superior or at least equal expected total gross margins. With regard to decision theory it must be recognized that the recommended alternative does *not* necessarily represent the solution that maximizes the farmer's utility. Instead the search is limited to second-degree stochastically dominant alternatives, thus providing decision support without needing to elicit the farmer's risk aversion.

Our methodical comparison of different variants of statistical analysis (static distributions vs. stochastic processes) indicates that the extra value to be derived from formal optimization methods depends on the available data being *adequately* processed and used. Data may be time-dependent and exhibit a trend. This simple fact suffices to show that approaches which prima facie resort to the mean and variance of past values considered equal in weight may not represent good forecasting models in many cases. Inserting too simplistic assumptions into formal planning models may well cause their performance to be inferior to that of planning on a rule of thumb basis. This could be an explanation why some of the formal optimization approaches proposed in the past have not been accepted by farmers, and rightly so.

While the analyzed sample is small, the identified capacity to outperform farmers' informal decisions provides first evidence that the efficiency of on-farm decision-making might be improved through formal optimization. However, this needs to be investigated through further research. The identified dimension and the continuity of the efficiency gains found in the case study warrant the effort to do so. In other words, the model's robustness to provide superior results should be tested by applying it to a larger number of farms in different regions and with different sizes, production structures and operating figures. In this context the following extensions of the model may prove valuable:

- As long as one is concerned with program planning on large farms, basically any crop rotation requirement can be translated into a respective (maximum or minimum) proportion of crops in any one year. This may be different on small farms where one needs to consider that fields, being of smaller and differing size, are not to be further subdivided for different crops. Furthermore, soil quality may differ from one field to the other. Extending the model to include such field-specific information requires additional effort but could be implemented in principal.

- We included the total gross margin variance inherent to the farmer's observed program as a fixed restriction in the formal optimization model. Thus, we did not consider that farmers might be prepared, for instance, to accept some additional volatility if the expected increase of the total gross margin covers their risk premium. That is, we cannot be sure to have found the utility maximizing production program. This does not impede the insights and clear-cut results of the analysis. In fact, the consequence is just that there may be even more room for improvement. One could investigate this by letting farmers choose from alternatives derived from a stepwise relaxation of the variance restriction. However, this means abandoning the model-endogenous recommendation of a single superior solution.
- In our case study, extending the time series model to include non-linear structures did not add much extra value compared to simple linear models. This can be seen as evidence that increasing planning efforts and further model sophistication are not feasible due to decreasing marginal returns. It might nonetheless be worthwhile to search for models that perform better still. Such a search could include models which allow for process parameters that are variable over time. Explicit GARCH-models could, for instance, be used in the case of a time-variable variance (cf. Bollerslev 1986; Koekebakker and Lien 2004).

Decisions regarding the resources to be spent for planning are, like all economic choices, subject to efficiency considerations. Thus, additional efforts such as the introduction of formal and more sophisticated planning models need to be justified by additional benefits. Before plunging into any of the above-mentioned activities, it should be checked whether the gain in information justifies the additional costs. We can assume that the critical farm size allowing for sufficient economies of scale depends on the costs (including learning costs) associated with the introduction of optimization models. These costs, in turn, depend on the knowledge and the skills of farm managers and thus, amongst other things, on the quality of their training. Better trained agricultural managers and consultants will need less time and effort to adopt more sophisticated approaches because they have less learning costs.

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