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Abstract

We consider a set of downstream firms each of which has a stochastic requirement for a particular input. Downstream firms can produce the input themselves yet do not trade it. Upstream firms produce the input to sell it through a Walrasian market to downstream firms. Efficient risk pooling requires that the input is produced by upstream firms and traded in the market. Yet, downstream firms will always vertically integrate. By producing some of its own input needs, a downstream firm cuts down aggregate input demand and thus depresses prices in the market.

Keywords: stochastic input demand, vertical upstream integration.
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I. Introduction

Uncertainty about the supply of inputs has often been given as a reason for vertical upstream integration. For example, Chandler (1969, p. 37) in his discussion of the history of the largest U.S. companies argues that the strategy for vertical integration had come from the desire to have a more certain supply of stocks, raw materials, and other supplies. In a similar spirit, the transactions cost literature (see, e.g., Coase (1937), Malmgren (1961), and Williamson (1971)) argues that uncertainty can make it difficult to deal in factor markets and thus creates an incentive for vertical upstream integration in order to bypass these problems by transferring goods internally.

Nevertheless, formal analyses on the effects of uncertainty about input supplies as an incentive for vertical upstream integration are rather rare. Arrow (1975) analyzes a model where it is assumed that vertically integrated firms obtain information about the input's supply conditions earlier than non-integrated firms. This information advantage creates a tendency towards complete vertical upstream integration.

Green (1986) considers a model where downstream firms face no uncertainty in their product market and sell all of their output at the exogenous market price. The input market is beset by exogenous stochastic demand. Input prices are fixed so that downstream firms may be rationed. To avoid rationing and to internalize the price system, downstream firms tend to fully integrate even though they are (slightly) less efficient than upstream firms. By additionally taking risk attitudes of the traders into account, Hendrikse and Peters (1989) obtain partial vertical integration as an equilibrium market structure in a setup in the spirit of Green.

Carlton (1979) analyzes a model where uncertainty from the product market transmits into the input market. Again fixed prices prevail on the input market so that rationing may occur. To rule out full integration, Carlton assumes that an integrated firm cannot sell its input on the market and may, therefore, be stuck with inputs for which it has no use. The equilibrium market structure is characterized by partial vertical upstream integration. Risk averse downstream firms wish to secure their high probability demand.

Closest to our analysis is an interesting paper by Bolton and Whinston (1990). They consider a setup where a single upstream firm produces an input that is used by two downstream firms. The upstream firm has random capacity so that supplies may be insufficient to meet both downstream firms' needs. As in Grossman and Hart (1986), ex ante contracts can only be written about the allocation of ownership over the productive assets. This leads to ex post bargaining over the procurement of the input. In this setup, Bolton and Whinston analyze different ownership structures. Perhaps the major difference to our analysis is that in Bolton and Whinston the input is transferred from the upstream firm to the downstream firms through a bargaining process in which the owner of the upstream firm gets a share of the surplus that the input generates for downstream firms. Since downstream firms do not appropriate the entire surplus, they engage in inefficiently low ex ante investments which in turn creates an incentive to vertically integrate. Instead, in our model the input is transferred through a market with flexible prices in which upstream firms have no market power.

We consider a finite set of downstream firms each of which has a stochastic requirement for a particular input—less in bad times than in good times. Downstream firms either produce the input themselves or purchase it through a Walrasian market from upstream firms that have no market power. Up- and downstream firms have access to the same input technology. To produce the input, a firm has to build up capacity at a fixed cost. If a firm has a certain capacity level, it can produce any quantity of the input that does not exceed capacity at a constant marginal cost. We assume, as is quite common in the literature (see, e.g., Williamson (1985)), that a downstream firm that produces the input itself does not sell it.[1] Up- and downstream firms simultaneously pick capacity levels. Nature then determines each downstream firm's input requirement. If a downstream firm's input requirement exceeds its own capacity, it shows up on the input market with positive demand. If market demand exceeds market supply, a high price prevails and vice versa so that the input market clears. Up- and downstream firms are risk neutral.

To begin with, we show that full integration always constitutes an equilibrium market structure. That is, all downstream firms have a capacity that allows to produce the maximum input requirement in good times and that is partly idle in bad times. However, this equilibrium is inefficient. Since downstream firms do not sell the input, they cannot pool their input requirements. Risk pooling can only be achieved if the input is produced by upstream firms and traded in the market. For example, the situation where downstream firms do not produce the input at all and purchase their needs on the market is more efficient than the full integration equilibrium. Nevertheless, this non-integration situation never constitutes a market equilibrium. If a downstream firm starts producing some of its own input needs, it cuts down aggregate demand and thus depresses prices in the input market. This favorable price effect outweighs the risk of idle capacity in bad times given that the vertically integrated capacity is not too large. It follows from this result that the input market will always be characterized by vertical upstream integration.

The following two questions then arise: when will we observe an efficient level of partial vertical upstream integration and under which conditions will the input market be characterized by too much vertical integration? The answer to the first question is fairly negative. If the model's parameters happen to be such that upstream firms make expected zero profits, then there exists an equilibrium with an efficient level of partial vertical upstream integration. However, the parameter constellation is unlikely to hold.

Our last result gives sufficient conditions for too much vertical upstream integration. If the zero profit condition fails to hold and the input requirement in good times is sufficiently high, downstream firms will have an inefficiently high level of vertically integrated capacity. The favorable effect of depressing market prices outweighs the loss from idle capacity in bad times.

We thus show that although an input market is characterized by flexible prices and no strategic power of upstream firms, downstream firms will always vertically integrate. Furthermore, the level of vertical upstream integration is often inefficiently high.

The paper is organized as follows. In the next section we describe the model. In section III we derive our results about the equilibrium market structures.

II. The Model

Consider a set of n identical downstream firms indexed by $a = 1, \ldots, n$. Each downstream firm faces stochastic demand for its output that can either be high or low. To produce the output, downstream firms need a particular input. If demand for its output is low, i.e., in bad times, a firm needs less of the input than in good times. Formally, denote a downstream firm's input requirement by

$$\tilde{x}_a = \begin{cases} \underline{x}, & \text{with probability } Pr(\underline{x}) \in (0,1); \\ \bar{x}, & \text{with probability } Pr(\bar{x}) = 1 - Pr(\underline{x}), \end{cases}$$

$a = 1, \ldots, n$. Let $\underline{x}, \bar{x} \in I\!N$, i.e., there is a smallest unit of account for the input and a downstream firm always needs some input. Moreover, $\bar{x} - \underline{x} \leq \underline{x} < \bar{x}$. Measured in input terms, good times are not more than twice as good as bad times. Let $\pi > 0$ denote the downstream firms' reservation price per unit of the input. Accordingly, at price π downstream firms are indifferent between obtaining and forgoing a unit of the input. As an interpretation think of π as the price of a backstop substitute for the input, i.e., an expensive yet still profitable substitute that is available in abundant supply.[2]

Downstream firms' input requirements are stochastically independent.[3] Once nature picked each downstream firm's input requirement, a state of the world can be described by the $n \times 1$ vector whose ath component denotes downstream firm a's input requirement

$\underline{x}$ or $\bar{x}$. Denote the possible states by Ξ_{ij}, $i = 0, \ldots, n$, $j = 1, \ldots, \binom{n}{n-i}$. The index i denotes the number of components of Ξ_{ij} that equal $\bar{x}$. Call a state where i firms have input requirement $\bar{x}$ of order i. The index j denotes the number of states of order i. Accordingly, the state of the world is a random variable $\tilde{\Xi}$ that is distributed according to

$$\tilde{\Xi} = \Xi_{ij} \quad \text{with} \quad Pr(\Xi_{ij}) = Pr(\underline{x})^{(n-i)} Pr(\bar{x})^{i} \quad i = 0, \ldots, n,\ j = 1, \ldots, \binom{n}{n-i}.$$

Let e be the $1 \times n$ unit vector. Define $X_i = e\Xi_{ij} = (n-i)\underline{x} + i\bar{x}$, $i = 0, \ldots, n$. The number X_i denotes aggregate input requirement in a state of order i. Aggregate input requirement is thus a random variable $\tilde{X} = \sum_{a=1}^{n} \tilde{x}_a$ that is distributed according to

$$\tilde{X} = X_i \quad \text{with} \quad Pr(X_i) = \binom{n}{n-i} Pr(\Xi_{ij}), \quad i = 0, \ldots, n.$$

There are m identical upstream firms indexed by $b = 1, \ldots, m$. Upstream firms produce the input to sell it through a market to downstream firms. Upstream firms produce the input according to the following technology. Since downstream firms use the same technology, we will not distinguish between up- and downstream when we describe the technological setup and simply talk about firms.

To produce the input, a firm has to build up capacity $y \in I\!N_0$ at a unit cost $f > 0$. If a firm has capacity y, it can produce any quantity of the input $v \le y$, $v \in I\!N_0$ at a constant marginal cost $c > 0$. It is not possible to produce $v > y$. The input technology is thus given by the cost function

$$C(v, y) = \begin{cases} fy + cv, & \text{if } v \le y; \\ \infty, & \text{otherwise.} \end{cases} \tag{1}$$

We assume that upstream firm b chooses capacity $y^b \in \{0, 1\}$, $b = 1, \ldots, m$. Under this assumption upstream firms have no strategic power which in turn makes market exchange as attractive as possible for downstream firms. Specifically, this assumption ensures the existence of equilibria where upstream firms make expected zero profits. Let $m \geq X_n = n\bar{x}$, i.e., there are enough upstream firms to be able to produce the maximum aggregate input requirement.

Downstream firms can produce the input themselves according to the technology (1). Downstream firm a chooses capacity $y_a \in \mathbb{N}_0$, $a = 1, \ldots, n$. Downstream firms that produce the input do not sell it. It might not be profitable for downstream firms to digress from marketing their output by additionally selling the input. The input market might also be regulated such that downstream firms may produce their own input requirement but are not allowed to become sellers of the input.[4]

Let

$$f \leq (\pi - c)Pr(\bar{x}). \tag{2}$$

Assumption (2) implies the following. If there is no market for the input, a downstream firm holds capacity $\bar{x}$ to be able to produce its maximum requirement itself rather than forgo some input.

Let us now turn to the formulation of the game. Downstream firms strategically pick capacity $y_a \in \mathbb{N}_0$, $a = 1, \ldots, n$. All downstream firms exhaust their own input capacity and purchase the remaining requirement on the market should this be necessary. Downstream firm a's input demand is thus a random variable $\tilde{d}_a = \max[0, \tilde{x}_a - y_a]$, $a = 1, \ldots, n$. Let the random variable of $n \times 1$ vectors of individual demands be $\tilde{\Delta}$. Finally, denote aggregate input demand by $\tilde{D} = \sum_{a=1}^{n} \tilde{d}_a$.

Simultaneously with downstream firms, upstream firm b chooses as a strategic variable the capacity $y^b \in \{0, 1\}$, $b = 1, \ldots, m$. Call an upstream firm that picks capacity 1 active and inactive if it chooses capacity 0. Upstream firms tell an auctioneer their capacity y^b, $b = 1, \ldots, m$. Denote upstream firms' aggregate capacity by $Y = \sum_{b=1}^{m} y^b$. Call Y market capacity.

Nature then determines each downstream firm's input requirement. Downstream firms tell the auctioneer their input demand. The auctioneer clears the market by the following pricing rule

$$p = \begin{cases} c, & \text{if } \tilde{D} \leq Y; \\ \pi, & \text{otherwise.} \end{cases}$$

If aggregate demand does not exceed market capacity, price equals marginal production cost. At the rockbottom price $p = c$ active upstream firms are indifferent between producing and not producing the input. Accordingly, in a buyers' market each active upstream firm makes a loss f.

If aggregate demand exceeds market capacity, price equals downstream firms' reservation price. At the sky-high price $p = \pi$ downstream firms are indifferent between obtaining and forgoing the input. In a sellers' market active upstream firms make a profit $(\pi - c - f)$. Note that the pricing rule clears the market. Thus, no rationing prevails.[5]

Up- and downstream firms are risk neutral. Upstream firm b, $b = 1, \ldots, m$ maximizes with respect to y^b expected profits

$$G^b(y^1, \ldots, y^m, y_1, \ldots, y_n) = \begin{cases} (\pi - c) \sum_{\tilde{D} > Y} Pr(\tilde{D}) - f, & \text{if } y^b = 1; \\ 0, & \text{otherwise.} \end{cases}$$

Downstream firm $a, a = 1, \ldots, n$ minimizes with respect to y_a the expected costs of obtaining the input

$$K_a(y_1, \ldots, y_n, Y) =$$

$$f y_a + c\underline{x} Pr(\underline{x}) + c\bar{x} Pr(\bar{x}) + (\pi - c) \max[0, \underline{x} - y_a] \sum_{\substack{e\tilde{\Delta} > Y \wedge \\ \tilde{d}_a = \max[0, \underline{x} - y_a]}} Pr(\tilde{\Delta}) +$$

$$(\pi - c) \max[0, \bar{x} - y_a] \sum_{\substack{e\tilde{\Delta} > Y \wedge \\ \tilde{d}_a = \max[0, \bar{x} - y_a]}} Pr(\tilde{\Delta}).$$

A downstream firm has to pay marginal cost c in any case whether it produces the input itself or purchases the input on the market. It pays the amount $(\pi - c)$ in excess of marginal cost on the market if aggregate demand exceeds market capacity, i.e., if $\tilde{D} > Y$ or equivalently if $e\tilde{\Delta} > Y$. We focus on Nash-equilibria of the simultaneous move game.

III. Market Structure

Let us now derive market structures for our input market. We will identify the degree of vertical upstream integration by the amount of capacity downstream firms have. Let us start with the full integration equilibrium that always exists.

Proposition 1: *There exists an equilibrium where downstream firms pick capacity* $\hat{y}_a = \bar{x},\ a = 1, \ldots, n$ *and upstream firms choose capacity* $\hat{y}^b = 0,\ b = 1, \ldots, m$.

Proof: Suppose all upstream firms pick $\hat{y}^b = 0$ so that market capacity $\hat{Y} = 0$. This implies $p = \pi \ \ \forall \tilde{D} > 0$. Take $z \in I\!N$ and consider w.l.o.g. downstream firm 1. Downstream firm 1 will never deviate with capacity $y_1 = \bar{x} + z$. It incurs a fixed cost fz without ever using the additional capacity. Now take $\bar{x} \geq z > 0$. If downstream firm 1 deviates with capacity $y_1 = \bar{x} - z$ we have

$$K_1(\bar{x} - z, \hat{y}_2, \ldots, \hat{y}_n, \hat{Y}) = f(\bar{x} - z) + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) +$$
$$(\pi - c)\max[0, \underline{x} - (\bar{x} - z)]Pr(\underline{x}) + (\pi - c)zPr(\bar{x}) \geq$$
$$f\bar{x} + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)zPr(\bar{x}) - fz \geq f\bar{x} + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) =$$
$$K_1(\hat{y}_1, \ldots, \hat{y}_n, \hat{Y})$$

where the last inequality follows from assumption (2). Thus, given $\hat{Y} = 0$, it is optimal for downstream firms to have capacity $\hat{y}_a = \bar{x},\ a = 1, \ldots, n$.

Conversely, if all downstream firms have capacity $\hat{y}_a = \bar{x}$, aggregate demand $\tilde{D} = 0$. Upstream firms thus never sell any input. Building up capacity $y^b = 1$ yields losses for an upstream firm because it incurs a fixed cost f without obtaining any revenue.

Q.E.D.

The existence of the full integration equilibrium is an immediate consequence of (2). Assumption (2) says that the probability of good times is sufficiently high so that it is worthwhile to have capacity $y_a = \bar{x}$. The capacity $\bar{x}$ is only partly used in bad times. Yet, the cost to forgo some input in good times outweighs the cost of idle capacity in bad times.

However, the full integration equilibrium typically is inefficient. Downstream firms do not sell the input. Accordingly, they cannot pool their input requirements.[6] Risk pooling can only be achieved if the input is produced by upstream firms and traded in the market. Typically, such a market structure is more efficient than the full integration equilibrium.

To be more specific, consider an example. Suppose $n = 3$, $\underline{x} = 1$, $\bar{x} = 2$, $Pr(\underline{x}) = 1/2$, $f = 1$, $c = 1$, and $\pi = 3$. In the full integration equilibrium downstream firm a's cost $K_a(2,2,2,0) = 3.5$, $a = 1,2,3$. Now consider the non-integration situation where all downstream firms have capacity $y_a = 0$ so that $\tilde{\Delta} = \tilde{\Xi}$ and $\tilde{D} = \tilde{X}$. Suppose the first 4 upstream firms are active so that market capacity $Y = 4$. Thus, if $\tilde{D} \in \{3,4\}$, $p = c$, and if $\tilde{D} \in \{5,6\}$, $p = \pi$. Active upstream firms' aggregate expected profits

$$\sum_{b=1}^{4} G^b(\cdot) = cD_1 1/8 + cD_2 3/8 + \pi Y 1/2 - [cD_1 1/8 + cD_2 3/8 + cY 1/2 + fY] = 0.$$

An active upstream firm has fixed cost f. In a buyers' market $p = c$ and active upstream firms do not recover fixed costs. Yet, in a sellers' market $p = \pi$ and each active upstream firm earns a contribution margin $(\pi - c) > 0$. If $Y = 4$, $f = (\pi - c)\sum_{\tilde{D}>Y} Pr(\tilde{D})$, i.e., active upstream firms make expected zero profits. Accordingly, all upstream firms are as well off as in the full integration equilibrium.

A downstream firm's cost of obtaining the input in the non-integration situation is

$$K_a(0,0,0,4) = c\underline{x}1/2 + c\bar{x}1/2 + (\pi - c)\underline{x}1/8 + (\pi - c)\bar{x}3/8 = 3.25, \quad a = 1,2,3,$$

i.e., all downstream firms are better off in the non-integration situation than in the full integration equilibrium.

This improvement may be explained as follows. Downstream firms pay the markup $(\pi - c)$ with probability $\sum_{\tilde{D}>Y} Pr(\tilde{D})$. Suppose whenever there is a sellers' market, downstream firm a has high demand $\bar{x}$, i.e., $\sum_{\substack{e\tilde{\Delta}>Y \wedge \\ \tilde{d}_a = \underline{x}}} Pr(\tilde{\Delta}) = 0$. The downstream firm then always pays the markup for the high quantity, i.e., it pays $(\pi - c)\bar{x} \sum_{\tilde{D}>Y} Pr(\tilde{D}) = f\bar{x}$ in excess of marginal costs. Accordingly, purchasing the input in the market or having capacity $y_a = \bar{x}$ amounts to the same in this case. If, however, as in our example, a sellers' market coincides with $\tilde{d}_a = \underline{x}$, i.e., $\sum_{\substack{e\tilde{\Delta}>Y \wedge \\ \tilde{d}_a = \underline{x}}} Pr(\tilde{\Delta}) > 0$, the downstream firm pays in excess of marginal costs

$$(\pi - c)\underline{x} \sum_{\substack{e\tilde{\Delta}>Y \wedge \\ \tilde{d}_a = \underline{x}}} Pr(\tilde{\Delta}) + (\pi - c)\bar{x} \sum_{\substack{e\tilde{\Delta}>Y \wedge \\ \tilde{d}_a = \bar{x}}} Pr(\tilde{\Delta}) < (\pi - c)\bar{x} \sum_{\tilde{D}>Y} Pr(\tilde{D}) = f\bar{x}.$$

The market charges the fair expected price $(\pi - c) \sum_{\tilde{D}>Y} Pr(\tilde{D}) = f$ to recover active upstream firms' fixed costs. If a downstream firm is lucky and has low input demand $\tilde{d}_a = \underline{x}$ in a sellers' market, it is strictly better off in the non-integration situation than in the full integration equilibrium. Notice that $(\pi - c) \sum_{\tilde{D}>Y} Pr(\tilde{D}) = f$ and

$$\sum_{\substack{e\tilde{\Delta}>Y \wedge \\ \tilde{d}_a = \underline{x}}} Pr(\tilde{\Delta}) > 0 \Leftrightarrow Y < D_{n-1}$$

are sufficient conditions for the non-integration situation to be more efficient than the full integration equilibrium.

Given that the non-integration situation is typically more efficient than the full integration equilibrium, it seems worthwhile to investigate under which conditions the non-integration situation constitutes an equilibrium. In the next Proposition we will show that the non-integration situation never constitutes a market equilibrium. It follows from this result that in any equilibrium market structure we observe vertical upstream integration.

Proposition 2: *There does not exist an equilibrium where downstream firms pick capacity $y_a = y$, $a = 1, \ldots, n$ with $y \in [0, 2\underline{x} - \bar{x}]$, $y \in I\!N_0$.*

<u>Proof:</u> Suppose on the contrary that there exists an equilibrium where downstream firms pick capacity $\hat{y}_a = y$, $a = 1, \ldots, n$, $y \in [0, 2\underline{x} - \bar{x}]$, $y \in I\!N_0$. Then $\tilde{d}_a = \tilde{d}_a(y) = \tilde{x}_a - y$, $a = 1, \ldots, n$, $\tilde{\Delta} = \tilde{\Delta}(y) = \tilde{\Xi} - e'y$, and $\tilde{D} = \tilde{D}(y) = \tilde{X} - ny$.

Let $\bar{Y} = \bar{Y}(y) = \max\{Y \in I\!N_0 | f \leq (\pi - c) \sum_{\tilde{D} > \bar{Y}} Pr(\tilde{D})\}$. The upstream firms' equilibrium aggregate capacity $\hat{Y} = \hat{Y}(y)$ satisfies $\hat{Y} \leq \bar{Y}$. If $Y > \bar{Y}$, $Y \in I\!N$, $f > (\pi - c) \sum_{\tilde{D} > Y} Pr(\tilde{D})$. All active upstream firms make expected losses and are better off if they become inactive.

If $f > (\pi - c)\big(1 - Pr(\underline{x})^n\big)$, $\hat{Y} < D_0$. Market capacity is so low that there is always a sellers' market. By assumption (2) downstream firms pick $y_a = \bar{x}$, $a = 1, \ldots, n$. Accordingly, $\hat{y}_a = y$, $a = 1, \ldots, n$, $y \in [0, 2\underline{x} - \bar{x}]$ does not constitute an equilibrium in this case.

Now consider the converse case where $f < (\pi - c)\big(1 - Pr(\underline{x})^n\big)$ so that $\hat{Y} > D_0$. That is, there is a buyers' market with positive probability. Suppose downstream firm 1 deviates with $y_1 = \underline{x} > y$ while $\hat{y}_a = y$, $a = 2, \ldots, n$. Downstream firm 1's input demand is thus $\tilde{d}'_1 = \tilde{x}_1 - \underline{x}$, the vector of individual demands $\tilde{\Delta}' = \tilde{\Delta} - \omega(\underline{x} - y)$ where ω is the $n \times 1$ vector whose first component is 1 and the others are 0, and $\tilde{D}' = \tilde{D} - (\underline{x} - y)$.

Since $\bar{x} - \underline{x} \leq \underline{x}$, we have $D'_i \leq D_{i-1}$, $i = 1, \ldots, n$ and thus $\sum_{\tilde{D} \leq \hat{Y}} Pr(\tilde{D}) < \sum_{\tilde{D}' \leq \hat{Y}} Pr(\tilde{D}')$. That is, if downstream firm 1 switches to $y_1 = \underline{x}$ there is more often a glut on the input market than if it sticks to $\hat{y}_1 = y$. Moreover, we have $\sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \underline{x} - y}} Pr(\tilde{\Delta}) >$

$\sum_{\substack{e\tilde{\Delta}'>\hat{Y}\wedge \\ \tilde{d}_1'=\bar{x}-\underline{x}}} Pr(\tilde{\Delta}')$. Thus,

$$K_1(\hat{y}_1,\ldots,\hat{y}_n,\hat{Y}) = fy + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) +$$

$$(\pi - c)(\underline{x} - y)\sum_{\substack{e\tilde{\Delta}>\hat{Y}\wedge \\ \tilde{d}_1=\underline{x}-y}} Pr(\tilde{\Delta}) + (\pi - c)(\bar{x} - y)\sum_{\substack{e\tilde{\Delta}>\hat{Y}\wedge \\ \tilde{d}_1=\bar{x}-y}} Pr(\tilde{\Delta}) \geq$$

$$f\underline{x} + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)(\bar{x} - \underline{x})\sum_{\substack{e\tilde{\Delta}>\hat{Y}\wedge \\ \tilde{d}_1=\bar{x}-y}} Pr(\tilde{\Delta}) >$$

$$f\underline{x} + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)(\bar{x} - \underline{x})\sum_{\substack{e\tilde{\Delta}'>\hat{Y}\wedge \\ \tilde{d}_1'=\bar{x}-\underline{x}}} Pr(\tilde{\Delta}') =$$

$$K_1(\underline{x}, \hat{y}_2, \ldots, \hat{y}_n, \hat{Y}),$$

where the first inequality follows from algebraic manipulations and the observation that $\hat{Y} \leq \bar{Y}$. Consequently, downstream firm 1 is strictly better off if it picks $y_1 = \underline{x}$ instead of $\hat{y}_1 = y \in [0, 2\underline{x} - \bar{x}]$.

Q.E.D.

Let us explain this result by means of the example. If $\hat{y}_a = 0$, $a = 1, 2, 3$, an upstream firms' best response entails $\hat{Y} \in \{3, 4\}$. Suppose $\hat{Y} = 4$ so that 4 upstream firms are active. This upstream firms' best response is most favorable one for downstream firms because active upstream firms make expected zero profits, i.e., $f = (\pi - c)\sum_{\tilde{D}>\hat{Y}} Pr(\tilde{D})$.

Suppose downstream firm 1 switches from $\hat{y}_1 = 0$ to $y_1 = 1$ so that $\tilde{d}_1' = \tilde{x}_1 - \underline{x}$ and $\tilde{D}' = \tilde{X} - \underline{x}$. Then we have $p = c$ if $\tilde{D}' \in \{2, 3, 4\}$ and $p = \pi$ if $\tilde{D}' = 5$. Accordingly, the probability of a sellers' market decreases from 1/2 to 1/8. If downstream firm 1 unilaterally deviates with $y_1 = 1$ we have $K_1(1, 0, 0, 4) = f\underline{x} + c\underline{x}1/2 + c\bar{x}1/2 + (\pi - c)0 \cdot 3/8 + (\pi - c)(\bar{x} - \underline{x})1/8 = 2.75 < 3.25 = K_1(0, 0, 0, 4)$. Consequently, downstream firm 1 is strictly better off if it picks capacity $y_1 = 1$ instead of capacity $\hat{y}_1 = 0$.

Downstream firm 1 pays the fair expected markup $f = (\pi - c)\sum_{\tilde{D}>\hat{Y}} Pr(\tilde{D})$ on the market. Downstream firm 1 needs $\underline{x}$ for sure and an additional $(\bar{x} - \underline{x})$ when it faces good times. If downstream firm 1 purchases its entire input needs on the market, the amount it pays in excess of marginal costs equals $(\pi - c)\underline{x}\sum_{\tilde{D} > \hat{Y}} Pr(\tilde{D}) + (\pi - c)(\bar{x} - \underline{x})\sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \bar{x}}} Pr(\tilde{\Delta}) = f\underline{x} + (\pi - c)(\bar{x} - \underline{x})\sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \bar{x}}} Pr(\tilde{\Delta})$. Accordingly, if the probability of a sellers' market coinciding with downstream firm 1's high demand would not change by the switch to $y_1 = \underline{x}$, downstream firm 1 were indifferent between $\hat{y}_1 = 0$ and $y_1 = \underline{x}$. But this probability decreases. Recall that $D_i - D_{i-1} = \bar{x} - \underline{x}$, $i = 1, \ldots, n$. Since $\underline{x} \geq \bar{x} - \underline{x}$, we have $D_i' \leq D_{i-1}$, $i = 1, \ldots, n$ and therefore $\sum_{\tilde{D}' > \hat{Y}} Pr(\tilde{D}') < \sum_{\tilde{D} > \hat{Y}} Pr(\tilde{D})$. By building up capacity $y_1 = \underline{x}$, downstream firm 1 cuts down aggregate demand by an amount large enough to increase the probability of a glut. Let $k^* = \min\{k \in \{0, \ldots, n\} | D_k > \hat{Y}\}$. Since $k^* \geq 1$, there exists $\Delta_{k^* j}$, $e\Delta_{k^* j} = D_{k^*} > \hat{Y}$, with $\tilde{d}_1 = \bar{x}$. If downstream firm 1 switches to $y_1 = \underline{x}$, $\Delta'_{k^* j} = \Delta_{k^* j} - (\underline{x}, 0, \ldots, 0)'$ and $e\Delta'_{k^* j} = D'_{k^*} < \hat{Y}$. Accordingly, $\sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \bar{x}}} Pr(\tilde{\Delta}) > \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \bar{x} - \underline{x}}} Pr(\tilde{\Delta}')$. That is, by the switch to $y_1 = \underline{x}$ downstream firm 1 decreases the probability of a seller's market coinciding with its own high demand. Consequently, if downstream firm 1 unilaterally deviates with $y_1 = \underline{x}$, it is strictly better off and the non-integration situation cannot be an equilibrium.

We have thus shown that we will always observe some vertical upstream integration. A downstream firm that builds up capacity has to take into account the following effects. The first unfavorable effect is that building up capacity has a fixed cost f. The second unfavorable effect is that the downstream firm may run the risk of idle capacity. The first favorable effect is that the downstream firm can produce the input itself at marginal cost c. The second favorable effect is that the downstream firm cuts down aggregate demand and may thus decrease the probability that high prices prevail on the input market.

Now consider a downstream firm that switches from capacity $y \in [0, 2\underline{x} - \bar{x}]$ to capacity $\underline{x}$. If upstream firms make expected zero profits given that all downstream firms pick capacity y, the first unfavorable and first favorable effect cancel.[7] The second favorable effect is zero because the downstream firm needs $\underline{x}$ for sure. Yet, the second favorable effect is strictly positive. The switch to $\underline{x}$ decreases aggregate input demand by an amount large enough to decrease the probability that high prices prevail on the input market. Overall then, it is attractive for downstream firms to build up positive capacity.

Note that the downstream firms' incentive to build up capacity is strict. Accordingly, our non-existence result still holds if downstream firms have a 'slightly worse' technology than upstream firms, i.e., if, for example, downstream firms' fixed cost is $f' = f + \epsilon$ with $\epsilon > 0$ and small. However, if downstream firms' fixed cost is f', any vertical upstream integration is inefficient. Upstream firms can build up the same capacity at a lower cost. Thus, in this case we may immediately conclude that we observe an inefficiently high level of vertical upstream integration on the input market.

This last discussion raises the following two questions: when will we observe an efficient level of vertical upstream integration and under which conditions will the input market be characterized by too much vertical integration? These questions are dealt with in the next two Propositions. In Proposition 3 we show that given parameters are such that upstream firms make expected zero profits, there exists an equilibrium with an efficient level of partial vertical upstream integration.

Proposition 3: *Let $y \in (2\underline{x} - \bar{x}, \underline{x}]$, $y \in I\!N$, and $\tilde{D}(y) = \tilde{X} - ny$. If there exists $\hat{Y}(\underline{x}) = \min\{Y \in I\!N_0 | (\pi - c) \sum_{\tilde{D}(\underline{x}) > \hat{Y}} Pr(\tilde{D}(\underline{x})) = f\}$, there exists an equilibrium where downstream firms choose capacity $\hat{y}_a = y$, $a = 1, \ldots, n$ and upstream firms pick $\hat{y}^b = 1$, $b = 1, \ldots, \hat{Y}(y)$ and $\hat{y}^b = 0$, $b = \hat{Y}(y) + 1, \ldots, m$, $\forall y \in (2\underline{x} - \bar{x}, \underline{x}]$, $y \in I\!N$.*

<u>Proof:</u> First notice that the existence of $\hat{Y}(\underline{x})$ implies the existence of $\hat{Y}(y)$ $\forall y \in (2\underline{x} - \bar{x}, \underline{x}]$, $y \in I\!N$. If all downstream firms decrease capacity by $(\underline{x} - y)$, $\tilde{D}(y) = \tilde{D}(\underline{x}) - n(\underline{x} - y)$. Then $\hat{Y}(y) = \hat{Y}(\underline{x}) + n(\underline{x} - y)$ satisfies $(\pi - c) \sum_{\tilde{D}(y) > \hat{Y}(y)} Pr(\tilde{D}(y)) = f$. Next note that $\hat{Y}(y) = D_k(y)$ for some $k = 0, \ldots, n-1$. All active upstream firms make expected zero profits. Consequently, no inactive firm wishes to become active and vice versa.

Let $\tilde{d}_a = \tilde{d}_a(y) = \tilde{x}_a - y$, $a = 1, \ldots, n$ and $\tilde{\Delta} = \tilde{\Delta}(y) = \tilde{\Xi} - e'y, \tilde{D} = \tilde{D}(y)$, and $\hat{Y} = \hat{Y}(y)$. Take $0 < z < (\bar{x} - y)$, $z \in I\!N$. Consider downstream firm 1 that deviates with $y_1 = y + z$. Accordingly, $\tilde{d}'_1 = \max[0, \tilde{d}_1 - z]$, $\tilde{\Delta}' = \tilde{\Delta} - \omega(\tilde{d}_1 - \tilde{d}'_1)$, and $\tilde{D}' = \tilde{D} - (\tilde{d}_1 - \tilde{d}'_1)$. We then have

$$\sum_{\tilde{D} > \hat{Y}} Pr(\tilde{D}) = \sum_{\tilde{D}' > \hat{Y}} Pr(\tilde{D}') \quad \text{and} \quad \sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \bar{x} - y}} Pr(\tilde{\Delta}) = \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}'_1 = \bar{x} - y - z}} Pr(\tilde{\Delta}').$$

If downstream firm 1 increases capacity by z, it does not cut down aggregate demand by an amount large enough to change the probability of a glut or the probability of a sellers' market coinciding with its own high demand. This implies

$$\begin{aligned}
& K_1(y + z, \hat{y}_2, \ldots, \hat{y}_n, \hat{Y}) = f(y + z) + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + \\
& (\pi - c)\max[0, \underline{x} - y - z] \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}'_1 = \max[0, \underline{x} - y - z]}} Pr(\tilde{\Delta}') + \\
& (\pi - c)(\bar{x} - y - z) \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}'_1 = \bar{x} - y - z}} Pr(\tilde{\Delta}') \geq \\
& fy + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)\max[0, \underline{x} - y] \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}'_1 = \max[0, \underline{x} - y - z]}} Pr(\tilde{\Delta}') + \\
& (\pi - c)(\bar{x} - y) \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}'_1 = \bar{x} - y - z}} Pr(\tilde{\Delta}') + fz - (\pi - c)z \sum_{\tilde{D}' > \hat{Y}} Pr(\tilde{D}') = \\
& K_1(\hat{y}_1, \ldots, \hat{y}_n, \hat{Y}),
\end{aligned}$$

where the last equality follows from the definition of $\hat{Y}$ and the observation that firm 1's switch to $y + z$ leaves the probabilities of the market contingencies unchanged. Consequently, downstream firm 1 is not better off by the switch from $\hat{y}_1 = y$ to $y_1 = y + z$.

Now take $z = \bar{x} - y$ and suppose downstream firm 1 deviates with $y_1 = y + z = \bar{x}$.

Then we have

$$K_1(\bar{x}, \hat{y}_2, \ldots, \hat{y}_n, \hat{Y}) = f\bar{x} + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) =$$

$$fy + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)(\bar{x} - y)\sum_{\tilde{D} > \hat{Y}} Pr(\tilde{D}) \geq$$

$$fy + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)\max[0, \underline{x} - y]\sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \max[0, \underline{x} - y]}} Pr(\tilde{\Delta}) +$$

$$(\pi - c)(\bar{x} - y)\sum_{\substack{e\tilde{\Delta} > \hat{y} \wedge \\ \tilde{d}_1 = \bar{x} - y}} Pr(\tilde{\Delta}) = K_1(\hat{y}_1, \ldots, \hat{y}_n, \hat{Y}).$$

Accordingly, downstream firm 1 is not better off by the switch from $\hat{y}_1 = y$ to $y_1 = \bar{x}$.

Finally, take $z \in (0, y]$, $z \in I\!N$. Suppose downstream firm 1 deviates with $y_1 = y - z$ so that $\tilde{d}_1' = \tilde{d}_1 + z$, $\tilde{\Delta}' = \tilde{\Delta} + \omega z$, and $\tilde{D}' = \tilde{D} + z$. We then have

$$\sum_{\tilde{D} > \hat{Y}} Pr(\tilde{D}) < \sum_{\tilde{D}' > \hat{Y}} Pr(\tilde{D}') \quad \text{and} \quad \sum_{\substack{e\tilde{\Delta} > \hat{Y} \wedge \\ \tilde{d}_1 = \bar{x} - y}} Pr(\tilde{\Delta}) \leq \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \bar{x} - y + z}} Pr(\tilde{\Delta}').$$

Accordingly, if downstream 1 switches from $\hat{y}_1 = y$ to $y_1 = y - z$ it increases market demand and thus the probability of a sellers' market. This implies

$$K_1(y - z, \hat{y}_2, \ldots, \hat{y}_n, \hat{Y}) = f(y - z) + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) +$$

$$(\pi - c)(\underline{x} - y + z)\sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \underline{x} - y + z}} Pr(\tilde{\Delta}') + (\pi - c)(\bar{x} - y + z)\sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \bar{x} - y + z}} Pr(\tilde{\Delta}') =$$

$$fy + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)(\underline{x} - y)\sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \underline{x} - y + z}} Pr(\tilde{\Delta}') +$$

$$(\pi - c)(\bar{x} - y)\sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \bar{x} - y + z}} Pr(\tilde{\Delta}') - fz + (\pi - c)z\sum_{\tilde{D}' > \hat{Y}} Pr(\tilde{D}') >$$

$$K_1(\hat{y}_1, \ldots, \hat{y}_n, \hat{Y}),$$

where the last inequality follows from the observation that downstream firm 1's switch to $y - z$ increases the probability that high prices prevail. Consequently, downstream firm 1 is strictly worse off by having capacity $y_1 = y - z$ instead of $\hat{y}_1 = y$.

Q.E.D.

We have thus shown that partial vertical upstream integration constitutes an equilibrium given that upstream firms make expected zero profits. Downstream firms pick a capacity level that is positive but does not exceed $\underline{x}$. That is, downstream firms pool the risky part of their input requirements in the market. Consequently, such a market structure is efficient. In the following discussion of this result we focus on the case where all downstream firms pick capacity $y = \underline{x}$.

The zero profit assumption is important for the following two reasons. Suppose upstream firms make positive expected profits, i.e., $f < (\pi - c) \sum_{\tilde{D} > Y} Pr(\tilde{D})$. Unless there are no advantages from risk pooling, a downstream firm is better off by having its own capacity instead of relying on the market. If upstream firms make zero profits, downstream firms face no tradeoff between paying an unfair expected price and the advantages from risk pooling.

The second reason is more subtle. If parameters are such that upstream firms make zero profits, any market capacity $Y \in [D_k, D_{k+1})$ for some $k \in \{0, \ldots, n-1\}$ satisfies $f = (\pi - c) \sum_{\tilde{D} > \hat{Y}} Pr(\tilde{D})$. Accordingly, upstream firms are indifferent between these capacity levels. Yet, for a partial vertical upstream equilibrium to exist, upstream firms have to pick $\hat{Y} = D_k$, i.e., the lowest market capacity that allows zero profits. If market capacity $\hat{Y} = D_k$, a small decrease of market demand does not decrease the probability that high prices prevail. That is, the incentive for downstream firms to cut down aggregate demand to decrease the probability of a sellers' market is zero at the margin. If upstream firms pick the highest market capacity that allows zero profits, a small decrease in market demand decreases the probability of a sellers' market. Accordingly, the downstream firms' incentive to cut down aggregate demand at the margin is strictly positive in this case. If the expected zero profit premise fails to hold, i.e., if $f < (\pi - c) \sum_{\tilde{D} > Y} Pr(\tilde{D})$, $Y \in [D_k, D_{k+1})$

for some $k \in \{0, \ldots, n-1\}$ and $f > (\pi - c) \sum_{\tilde{D} > Y} Pr(\tilde{D})$, $Y \geq D_{k+1}$, upstream firms will build up the highest market capacity that allows positive profits. Consequently, we may conclude that the expected zero profit assumption is necessary to drive down to zero the downstream firms' incentive to cut down aggregate demand at the margin.

Nevertheless, the zero profit condition holds only for special parameter values. What can be said about the efficiency properties of market structures if the zero profit condition fails to hold? When will downstream firms have inefficiently high capacity, i.e., a capacity level in excess of $\underline{x}$? We have just seen that if the zero profit condition does not hold, upstream firms build up the highest market capacity that allows positive profits. A small decrease of aggregate demand is thus sufficient to decrease the probability of a sellers' market. Accordingly, on the one hand, a downstream firm is tempted to increase its capacity slightly beyond $\underline{x}$ to cut down aggregate demand. On the other hand, if a downstream firm has capacity in excess of $\underline{x}$, it has idle capacity in bad times. In the next Proposition we provide a sufficient condition for the second favorable effect to outweigh the second unfavorable effect so that we observe too much vertical integration.

Proposition 4: *Let $y \in (2\underline{x} - \bar{x}, \bar{x}]$, $y \in \mathbb{N}$, and $\tilde{D}(y) = \tilde{X} - ny$. Suppose there does not exist $Y(\underline{x}) = \min\{Y \in \mathbb{N}_0 | (\pi - c) \sum_{\tilde{D}(\underline{x}) > Y} Pr(\tilde{D}(\underline{x})) = f\}$. If $\bar{x}$ is sufficiently large, there does not exist an equilibrium where downstream firms choose capacity $y_a = y$, $a = 1, \ldots, n$, $y \in (2\underline{x} - \bar{x}, \bar{x}]$, $y \in \mathbb{N}$.*

<u>Proof:</u> Suppose on the contrary that there exists an equilibrium where downstream firms pick capacity $\hat{y}_a = y$, $a = 1, \ldots, n$, $y \in (2\underline{x} - \bar{x}, \bar{x}]$, $y \in \mathbb{N}$. Then $\tilde{d}_a = \tilde{d}_a(y) = \tilde{x}_a - y$, $a = 1, \ldots, n$, and $\tilde{\Delta} = \tilde{\Delta}(y) = \tilde{\Xi} - e'y$.

First notice that the non-existence of $Y(\underline{x})$ implies the non-existence of $Y(y) = \min\{Y \in \mathbb{N}_0 | (\pi - c) \sum_{\tilde{D}(y) > Y} Pr(\tilde{D}(y)) = f\}$ $\forall y \in (2\underline{x} - \bar{x}, \bar{x}]$, $y \in \mathbb{N}$. Suppose on the contrary that there exists $Y(y)$ for some $y < \underline{x}$ but not $Y(\underline{x})$. Then $Y = Y(y) - n(\underline{x} - y)$ satisfies $(\pi - c) \sum_{\tilde{D}(\underline{x}) > Y} Pr(\tilde{D}(\underline{x})) = f$, contradicting the non-existence of $Y(\underline{x})$.

Let $\bar{Y} = \bar{Y}(y) = \max\{Y \in I\!N_0 | f < (\pi - c)\sum_{\tilde{D}>Y} Pr(\tilde{D})\}$. Since there does not exist a market capacity $Y(y)$ where upstream firms make zero profits, the equilibrium market capacity $\hat{Y} = \hat{Y}(y) = \bar{Y}$. If $Y > \bar{Y}$, $Y \in I\!N$, $f > (\pi - c)\sum_{\tilde{D}>Y} Pr(\tilde{D})$. All active upstream firms make expected losses and are better off if they become inactive.

If $f > (\pi - c)(1 - Pr(\underline{x})^n)$, $\hat{Y} < D_0$. Market capacity is so low that there is always a sellers' market. By assumption (2) downstream firms pick capacity $y_a = \bar{x}$, $a = 1, \ldots, n$. Accordingly, $\hat{y} = y$, $a = 1, \ldots, n$, $y \in (2\underline{x} - \bar{x}, \bar{x}]$ does not constitute an equilibrium in this case.

Now consider the converse case where $f < (\pi - c)(1 - Pr(\underline{x})^n)$ so that $\hat{Y} > D_0$. That is, we observe a buyers' market with positive probability. Suppose downstream firm 1 deviates with $y_1 = y + 1$ while $\hat{y}_a = y$, $a = 2, \ldots, n$. Then we have

$$\sum_{\tilde{D}>\hat{Y}} Pr(\tilde{D}) > \sum_{\tilde{D}'>\hat{Y}} Pr(\tilde{D}'), \qquad \sum_{\substack{e\tilde{\Delta}>\hat{Y}\wedge \\ \tilde{d}_1 = \underline{x}-y}} Pr(\tilde{\Delta}) \geq \sum_{\substack{e\tilde{\Delta}'>\hat{Y}\wedge \\ \tilde{d}_1' = \max[0,\underline{x}-y-1]}} Pr(\tilde{\Delta}'),$$

and $$\sum_{\substack{e\tilde{\Delta}>\hat{Y}\wedge \\ \tilde{d}_1 = \bar{x}-y}} Pr(\tilde{\Delta}) > \sum_{\substack{e\tilde{\Delta}'>\hat{Y}\wedge \\ \tilde{d}_1' = \bar{x}-y-1}} Pr(\tilde{\Delta}').$$

Upstream firms have the highest possible capacity that allows positive profits. An increase of one unit in downstream firms' capacity is thus sufficient to decrease the probability of a sellers' market.

We then have

$$K_1(y+1, \hat{y}_2, \ldots, \hat{y}_n, \hat{Y}) = f(y+1) + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) +$$

$$(\pi - c)\max[0, \underline{x} - y - 1] \sum_{\substack{e\tilde{\Delta}'>\hat{Y}\wedge \\ \tilde{d}_1' = \max[0,\underline{x}-y-1]}} Pr(\tilde{\Delta}') +$$

$$(\pi - c)(\bar{x} - y - 1) \sum_{\substack{e\tilde{\Delta}'>\hat{Y}\wedge \\ \tilde{d}_1' = \bar{x}-y-1}} Pr(\tilde{\Delta}') \leq$$

$$fy + c\underline{x}Pr(\underline{x}) + c\bar{x}Pr(\bar{x}) + (\pi - c)\max[0, \underline{x} - y] \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \max[0, \underline{x} - y - 1]}} Pr(\tilde{\Delta}') +$$

$$(\pi - c)(\bar{x} - y) \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \bar{x} - y - 1}} Pr(\tilde{\Delta}') +$$

$$(\pi - c)\Big(f - \sum_{\substack{e\tilde{\Delta}' > \hat{Y} \wedge \\ \tilde{d}_1' = \bar{x} - y - 1}} Pr(\tilde{\Delta}')\Big) < K_1(\hat{y}_1, \ldots, \hat{y}_n, \hat{Y}),$$

where the last inequality holds if $\bar{x}$ is sufficiently large.

Q.E.D.

We have thus shown that we will observe an inefficiently high level of vertical upstream integration if upstream firms make positive expected profits and $\bar{x}$ is large. To explain this 'too much vertical integration' result, consider the situation where all downstream firms pick capacity $y = \underline{x}$, i.e., where they pool the risky part of their input requirements in the market. Upstream firms then have the highest market capacity that allows positive profits. If market capacity increases by one unit, all active upstream firms make expected losses. This in turn implies that a downstream firm's switch from $y = \underline{x}$ to $y = \underline{x} + 1$ is sufficient to decrease the probability of a sellers' market.

In terms of our four effects such a deviation implies the following. The sum of the first unfavorable and the first favorable effects is strictly favorable for a downstream firm. The market charges a price that is too high to recover upstream firms' fixed costs. The second unfavorable effect is strictly unfavorable for a downstream firm. The downstream firm has capacity in excess of $\underline{x}$. The excess capacity is idle in bad times. The second favorable effect is strictly favorable. By the switch to $y = \underline{x} + 1$, a downstream firm decreases the probability of a sellers' market. This means that it gets the risky part of its input requirement $\bar{x} - \underline{x}$ cheaper. If $\bar{x}$ is sufficiently large, the second favorable effect outweighs the second unfavorable effect. Consequently, a downstream firm will deviate with the inefficiently high capacity level $y = \underline{x} + 1$.

IV. Conclusions

We have thus shown that although an input market is characterized by flexible prices and upstream firms have no strategic power, downstream firms will always vertically integrate. If a downstream firm starts producing some of its own input needs, it cuts down aggregate input demand and thus depresses prices in the market. This favorable price effect outweighs the risk of idle capacity in bad times given that the vertically integrated capacity is not too large. This result does not hinge on the assumption that downstream firms do not sell the input. Moreover, we have shown that the incentive to depress prices on the input market often leads to an inefficiently high level of vertically integrated capacity.

If we allowed for a dual market structure with downstream firms negotiating advance contracts and spot markets taking care of random fluctuations, the incentive for vertical upstream integration might disappear. Nevertheless, such a dual market structure raises the issue of breach of contract. If high prices prevail on the spot market, upstream firms are tempted to break the advance contract and vice versa for downstream firms. These problems shed doubt on the conjecture that a dual market structure works smoothly.

Footnotes

1) See, e.g., Ordover, Saloner, and Salop (1988) for a setup where vertical foreclosure may arise as an equilibrium phenomenon.

2) Alternatively, the reservation price may be interpreted as follows. Let the units of account for the input be normalized such that a downstream firm needs one unit of the input to produce one unit of the output. The price per unit of the output is independent of the demand realization. Then π is a downstream firms' profit per unit of the output plus the cost of one unit of the input.

3) The assumption that input requirements are i.i.d. is made for notational convenience. All of our results also apply to the case where downstream firms differ in the probability of bad times, i.e., $Pr_a(\underline{x}) \neq Pr_{a'}(\underline{x})$, and the input requirements of two firms, respectively, are not perfectly correlated, i.e., $\mathrm{cor}(\tilde{x}_a, \tilde{x}_{a'}) \in (-1, 1)$, $a, a' = 1, \ldots, n$, $a \neq a'$. Accordingly, all we need is that the distribution of the states of the world has full support.

4) See Carlton (1979) for a more elaborate discussion of the first argument. The Motor Carrier Act of 1935 created the conditions given in the second argument. A license is needed to enter the common carrier truck business. Private carriage, i.e., carriage in trucks owned and operated by the shipper are exempted from regulation. See Kahn (1971,p. 14-21). Note that we do not need this assumption to prove Proposition 2.

5) If $\tilde{D} = Y$, any price $p \in [c, \pi]$ clears the market. The reason why we choose $p = c$ is as follows. Suppose up- and downstream firms' capacity choices are such that $\tilde{D} = Y$ occurs with positive probability and moreover that $p > c$ in this case. If a downstream firm increases its capacity by a small amount, it cuts down aggregate demand by the same quantity. Thereby, it turns the event $\tilde{D} = Y$ and $p > c$ into an event where $p = c$. Accordingly, the downstream firm gains $(p - c) > 0$ with positive probability. To rule out this incentive to cut down aggregate demand at the margin we set $p = c$ if $\tilde{D} = Y$.

6) Notice that we rule out horizontal mergers. If downstream firms merge horizontally,

they can pool their input requirements.

7) If upstream firms make positive expected profits, the net-effect is even favorable for the downstream firm.

References

1. K. J. ARROW: Vertical Integration and Communication, *Bell Journal of Economics* **6** (1975), 175 - 183.
2. P. BOLTON and M. D. WHINSTON: "Incomplete Contracts, Vertical Integration, and Supply Assurance", Ecole Polytechnique Working Paper (1990), forthcoming.
3. D. W. CARLTON: Vertical Integration in Competitive Markets under Uncertainty, *Journal of Industrial Economics* **27** (1979), 189 - 209.
4. A. D. CHANDLER Jr.: "Strategy and Structure: Chapters in the History of the American Enterprise", M.I.T. Press, Cambridge, Mass. (paperback ed.) (1969).
5. R. H. COASE: The Nature of the Firm, *Economica* **4** (1937), 386 - 405.
6. J. R. GREEN: Vertical Integration and the Assurance of Markets, *in* "New Developments in the Analysis of Market Structure" (J. E. Stiglitz and G. F. Mathewson, eds.), MacMillan, New York (1986).
7. S. J. GROSSMAN and O. D. HART: The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration, *Journal of Political Economy* **94** (1986), 691 - 719.
8. G. HENDRIKSE and H. PETERS: A Note on Partial Vertical Integration, *Economics Letters* **30** (1989), 249 - 252.
9. A. E. KAHN: "The Economics of Regulation", Vol 2, Wiley, New York (1971).
10. H. B. MALMGREN: Information, Expectations, and the Theory of the Firm, *Quarterly Journal of Economics* **75** (1961), 399 - 421.
11. J. A. ORDOVER, G. SALONER, and S. C. SALOP: "Equilibrium Vertical Foreclosure", NYU Research Report #88-02 (1988).
12. O. E. WILLIAMSON: The Vertical Integration of Production: Market Failure Considerations, *American Economic Review* **61** (1971), 112 - 123.
13. O. E. WILLIAMSON: "The Economic Institutions of Capitalism", Free Press, New York (1985).

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