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Changes in Regime and the Term Structure: A Note.

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Introduction

In a recent paper, James Hamilton (1988) has, with apparent success, modeled the term structure of interest rates in the US using a remarkably simple modification of the expectations model. He allows for stochastic change of regime in the model which determines the short rate of interest. The regime change affects the mean and variance of the short rate, and is governed by a Markov switching process. The long rate is governed by the expectations model in which future expected short rates are generated by the regime–switching model.

Hamilton argues that this structure is better able to account for the behaviour of long rates relative to short rates during the period between 1979 and 1982, in which the Federal Reserve changed its operating procedures, within the context of an expectations model of the term structure, than alternative models have been able to do. In particular, a linear term structure model with no regime shift would tend to under predict the increase in the mean and variability of the long rate which occurred in the period 1979 to 1982.

Hamilton's general approach to modelling time–series models with structural breaks or non–stationarity as log–linear with stochastic changes of regime (Hamilton, 1989) has been applied to a number of different phenomena, such as GDP growth and exchange rates as well as interest rates, with apparent success, and seems to be a promising approach to situations of this kind.

However, in the case of the term structure model (Hamilton, 1988), some of his empirical results do not survive scrutiny, because it appears that his data for the long term rate of interest, his variable R_t, the yield to maturity on 10–year US government bonds, which is listed in Table 1 of his paper, is actually the same as the raw data for the 3–month treasury bill rate (his variable rr_t discussed in his footnote 8 on page 401) divided by 2. As a consequence, his results for the long rate of interest are based on inappropriate data. His results for the regime change in the time–series model for the short rate, of course, are unaffected by this.

The purpose of this short note is to recalculate some of his estimates using appropriate data. It appears that while the regime shift in the process for the short rate between 1979 and 1982 is a robust result, the model for the long rate which Hamilton uses has a number of unsatisfactory features. It is not clear that the regime change can account adequately for important features of the data.

Data

The data used in this paper are taken from from Chase Econometrics' database for 3 month US treasury bill rates and the Federal Reserve Board's 10–year constant maturity government bonds, quarterly, for the period 1953 quarter 2 – 1989 quarter 2. Figure 1a plots the data and Figure 1B compares Hamilton's short rate data (his r_t multiplied by four to bring it approximately to an annual rate) with the Chase Econometrics data.

Results

A linear model for interest rates with no cross–equation restrictions, with two equations estimated jointly, for the period 1963 1 to 1987.3 yields the following results:

$$
\begin{array}{llllll}
r_t = & .5674 & + .8761 r_{t-1} & - .0342 r_{t-2} & + .1904 r_{t-3} & - .1127 r_{t-4} \\
 & (.2856) & (.1024) & (.1351) & (.1015) & (.1015) \\
 & + \hat{\epsilon}_t & & & &
\end{array}
$$

SE = 1.0812, $\overline{R}^2 = .8539$. LM tests: for serial correlation $\chi^2(4) = 4.7616$; for normality (the Jarque–Bera test) $\chi^2(2) = 131.0206$; for heteroskedasticity, $\chi^2(1) = 17.8259$. Standard errors are recorded in parentheses beneath parameter estimates.

$$R_t = \underset{(.2674)}{1.3472} + \underset{(.0948)}{.3792}r_t + \underset{(.1251)}{.1409}r_{t-1} + \underset{(.1251)}{.1277}r_{t-2} + \underset{(.0939)}{.3321}r_{t-3} + \hat{\epsilon}_{Rt}$$

SE = 1.0007, $\overline{R}^2 = .8767$. LM tests for: serial correlation, $\chi^2(4) = 71.085$; normality, $\chi^2(2) = 6.6105$; heteroskedasticity, $\chi^2(1) = 3.0076$; 1st order ARCH process in residuals, $\chi^2(1) = 40.9305$.

The results for the short rate are very similar to those of Hamilton. Small discrepancies arise because of differences in the data. The equation betrays signs of non–normality in the residuals and heteroskedasticity, as would be expected if there was a regime shift of the kind suggested between 1979 and 1982. The results for the long rate are, as one would expect, considerably different from Hamilton's. In particular, the residuals display a lot of serial correlation, and there is evidence that they follow a first order ARCH process. This may of course result from the attempt to fit a constant linear structure to the entire sample period when in fact there have been structural changes as Hamilton argues.

When the above equations are estimated jointly, with the cross equation restrictions implied by the expectations model of the term structure imposed, as described in Hamilton (1988) on page 398 in equation (3.13), the following results are found:

$$r_t = \underset{(6.5299)}{.909159} + \underset{(8.3103)}{.763083}r_{t-1} - \underset{(1.0427)}{.134160}r_{t-2} + \underset{(.596)}{.076968}r_{t-3} + \underset{(1.994)}{.164772}r_{t-4}$$

T–statistics are given in brackets under the parameters. The estimated standard error of the equation, $\hat{\sigma}_0 = 1.122510$.

$$R_t = 1.347118 + .637014r_t + .0524288r_{t-1} + .146518r_{t-2} + .104809r_{t-3},$$

Estimated standard error of equation $\hat{\sigma} = 1.061738$.

The log–likelihood of the restricted model is -116.372 whereas the unrestricted model (with the equations estimated jointly) produces a log likelihood of -101.670. The reduction associated with the 4 restrictions is 14.702, so the $\chi^2(4)$ likelihood ratio test statistic for their validity is 29.404. Clearly the restrictions are rejected.

The alternative proposed by Hamilton is to include the possibility of a change of regime for the process driving the short rate. He proposes that the short rate be represented by

$$r_t = \alpha_0 + \alpha_1 S_t + z_t \tag{1}$$

where $z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \varphi_3 z_{t-3} + \varphi_4 z_{t-4} + [\omega_0 + \omega_1 S_t]\nu_t$

$\nu_t \sim N(0,1)$, S_t takes values zero or one, denoting the state of the world. The evolution of S_t is a markov process with constant transition probabilities, $\mathrm{Prob}[S_t=1 | S_{t-1}=1]=p$, and $\mathrm{Prob}[S_t=0 | S_{t-1}=0]=q$. The state determines the mean and variance of the short rate. Under the expectations model of the term structure, the long rate is a linear function of the current value and three lags of the short rate, and of the best currently available estimates of the state in the current and last three preceding periods. Viz,

$$\begin{aligned} R_t = {} & \kappa_R + \beta_0 r_t + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \beta_3 r_{t-3} + \\ & \gamma_0 P[S_t=1 | r_t, r_{t-1}, \ldots] + \gamma_1 P[S_{t-1}=1 | r_t, r_{t-1}, \ldots] + \\ & \gamma_2 P[S_{t-2}=1 | r_t, r_{t-1}, \ldots] + \gamma_3 P[S_{t-3}=1 | r_t, r_{t-1}, \ldots] + \epsilon_{Rt} \end{aligned} \tag{2}$$

where the coefficients of (2) are functions of the coefficients of (1) above. More details are given in Hamilton (1988). The most general model within this framework allows for the regime–switching in the short rate equation, and allows the long rate to depend on the inferred state of the world without any cross equation restrictions imposed. I. e., (1) and (2) are estimated with no restrictions on the coefficients of (2). A value of the log likelihood function of –65.813931 was achieved. There are 20 estimated parameters.

A more restrictive assumption is that the long rate is unaffected by the state of the world S_t as seen by agents. This involves estimating (1) and (2) jointly, with the restrictions imposed that $\gamma_i = 0$, $i = 0, 1, 2, 3$. This produces a log–likelihood of –79.430732, with 16 estimated parameters.

Within the nonlinear framework, the most restrictive hypothesis is that the restrictions on the parameters of (2) implied by the expectations model of the term structure are imposed. Hamilton (1988) describes these in detail in section 3.3, pages 398–400 of his paper. When these restrictions are imposed, the results are as follows:

Parameter	Estimate	S. E.
α_0	5.922321	.718084
α_1	–.754550	0.542263
p	.963121	.024163
q	.991127	.008659
ω_0	0.682035	0.060875
ω_1	1.808523	.537403
φ_1	.893882	.094030
φ_2	–.193094	.134809
φ_3	–.011355	.128689
φ_4	.195085	.078944
σ	1.060250	.081365
κ	1.060023	.480676

Log likelihood = –94.597159. 12 parameters estimated.

A summary of results for this exercise, equivalent to Hamilton's table 2 is as follows:

Table 1

Model	Short Rate r_t	Long Rate R_t	Cross Equation Restrictions	No.of parameters	Log likelihood
1	linear	linear	no	12	–101.670
2	linear	linear	yes	8	–116.372
3	non–lin	linear	no	16	– 79.430732
4	non–lin	non–lin	no	20	– 65.813931
5	non–lin	non–lin	yes	12	– 94.597159

The restrictions in the term structure model within the non–linear framework are clearly rejected. Testing model 5 against 4 gives an LR test statistic of 57.6 (distributed as $\chi^2(8)$). Thus it appears that the non–linear model put forward by Hamilton does not rescue the expectations model of the term structure for this US data.

On further inspection of the data, the term–structure model with regime switching appears to perform very poorly largely because the long–term interest rate does not behave as predicted by the model. Figure 2 shows actual and predicted values of the long term interest rate obtained from the restricted non–linear model above (model 5 in Table 1). The estimated residuals clearly contain a lot of autocorrelation. Introducing the possibility of regime–switching has clearly not corrected the shortcomings of the linear model reported above. This suggests that equation (2) above may not be a good representation of the long rate. A possible explanation for the poor performance of (2) is that (1) is not an appropriate model for the short rate. However this does not seem to be the case,as can be seen by looking at OLS estimates of the linear model for the period up to and including 1978(3) in which there appears to have been no switch in regime.

Hamilton's result that the world appears to have been in state 0 during the sample period except in the period 1979 (4) to 1982 (3) seems to be robust. Figure 3 shows the smoothed probability that the world is in state zero in each period, using the Chase econometrics data, and using the restricted non–linear model (model 5 in Table 1 above). There are substantial departures from one only in the period 1979 (3) to 1983 (3). The estimated probability of a switch from regime zero to regime one is 0.031. During the period up to and including 1978 (3), there appears ex post to have been no switch of regime. For that period of time, the short rate might be satisfactorily modeled by an ordinary AR(4) process without any possibility of a regime change. OLS estimation for the period 1963 (1) – 1978 (4) gave the following results:

$$\underset{(2.49)}{r_t = .7835} + \underset{(8.08)}{1.0261r_{t-1}} - \underset{(1.05)}{0.1894r_{t-2}} + \underset{(1.71)}{.3097r_{t-3}} - \underset{(2.26)}{.2870r_{t-4}}$$

$R^2 = .8191$; $\chi^2(4)$ test for serial correlation: 2.2774; $\chi^2(1)$ test for heteroskedasticity: 6.4083; $\chi^2(5)$ test of constancy of regression coefficients: 4.0952; $\chi^2(2)$ test for normality of residuals .6411; $\chi^2(1)$ test for functional form 3.3445. Thus there is evidence of heteroskedasticity, but the parameter estimates seem stable and the residuals are not serially correlated. However, the corresponding equation for the long rate is not so successful. OLS regression for the same sample period gives:

$$R_t = \underset{(3.68)}{1.5377} + \underset{(3.05)}{.4977r_t} - \underset{(.37)}{.0869r_{t-1}} + \underset{(.07)}{.0162r_{t-2}} + \underset{(2.93)}{.4839r_{t-3}}$$

$R^2 = .6971$; DW = .2467; $\chi^2(4)$ LM test for serial correlation: 52.76; $\chi^2(1)$ LM test for 1st order ARCH process in the residuals: 22.096. T–statistics are in parentheses under estimated parameters. There is very strong evidence of serial correlation in the residuals of this equation. Since it is estimated over a period for which the short rate had followed a stable AR(4) process, with no apparent shift in regime, it would have been stable also, had the expectations model of the term structure been valid. It may be possible that time–varying risk premia explain this serial correlation. It may reflect departures from efficiency in the bond market, in the form of excess volatility of long rates, or fads, though this seems unlikely, *a priori*.

Conclusion

James Hamilton has made a bold attempt to fit an explicit model for the time–series properties of short and long term interest rates in the US, and thereby to rehabilitate the expectations model of the term structure. His regime–switching model is intuitively very appealing for situations like that in the US where a distinct change of the policy rule occurred, and its success is borne out by the results for the short term rate of interest. His conclusions are confirmed in this paper which has used a slightly different data set.

However, it does not appear to have solved the empirical puzzles generated by US term structure data. His results were obtained with erroneous data for the 10 year bond yield. Using more appropriate data, the weakness of the model is manifested in two straightforward ways. Firstly, the cross–equation restrictions implied by the expectations model are rejected by the data. Secondly, the residuals produced by the model for the 10 year bond yield are highly serially correlated. The results raise questions as to how these data might be modeled, and whether some modified form of the expectations model might account for them. It may be, for example, that the serial correlation reflects time–varying and serially correlated risk premia. An alternative possibility might be that bond markets are inefficient, given to excess volatility, fads, and predictable excess holding yields on long bonds. However these questions lie beyond the scope of this brief note.

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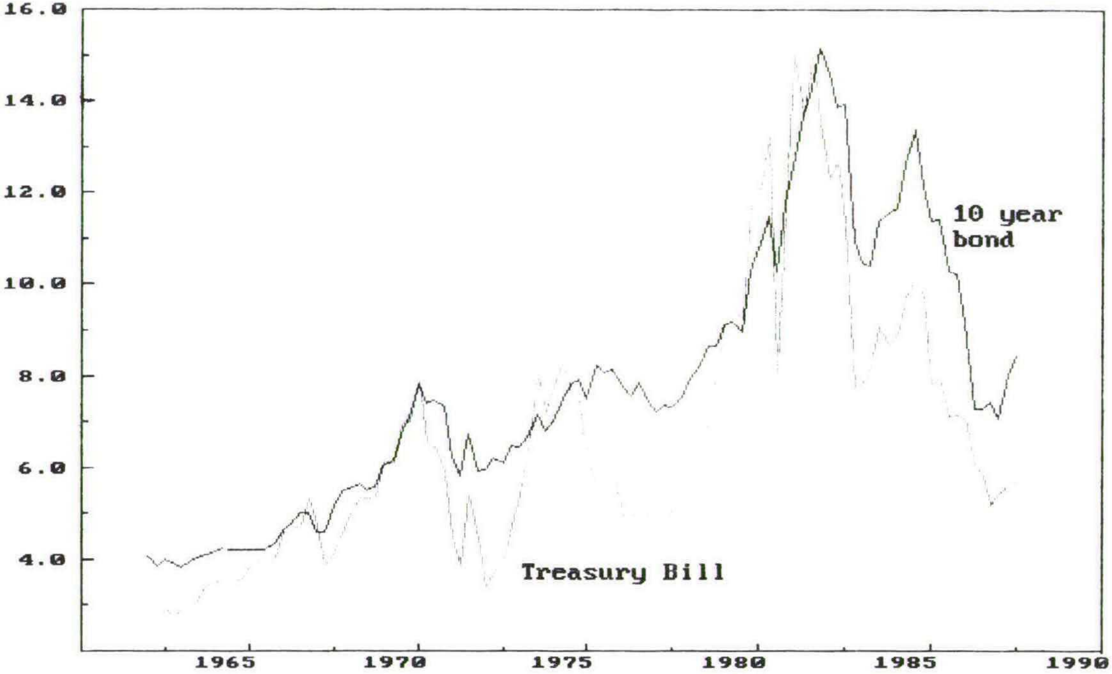
____________________________, 1989, "A New Approach to the Analysis of Non–Stationary Time–Series and the Business Cycle," **Econometrica**, vol 57, no 2, 357–384.

FIGURE 1A
US Government Instruments -- Chase Econometrics Data



Sample Period is 1962(1) - 1987(3)

FIGURE 1B

Comparison of Treasury Bill data from two different sources

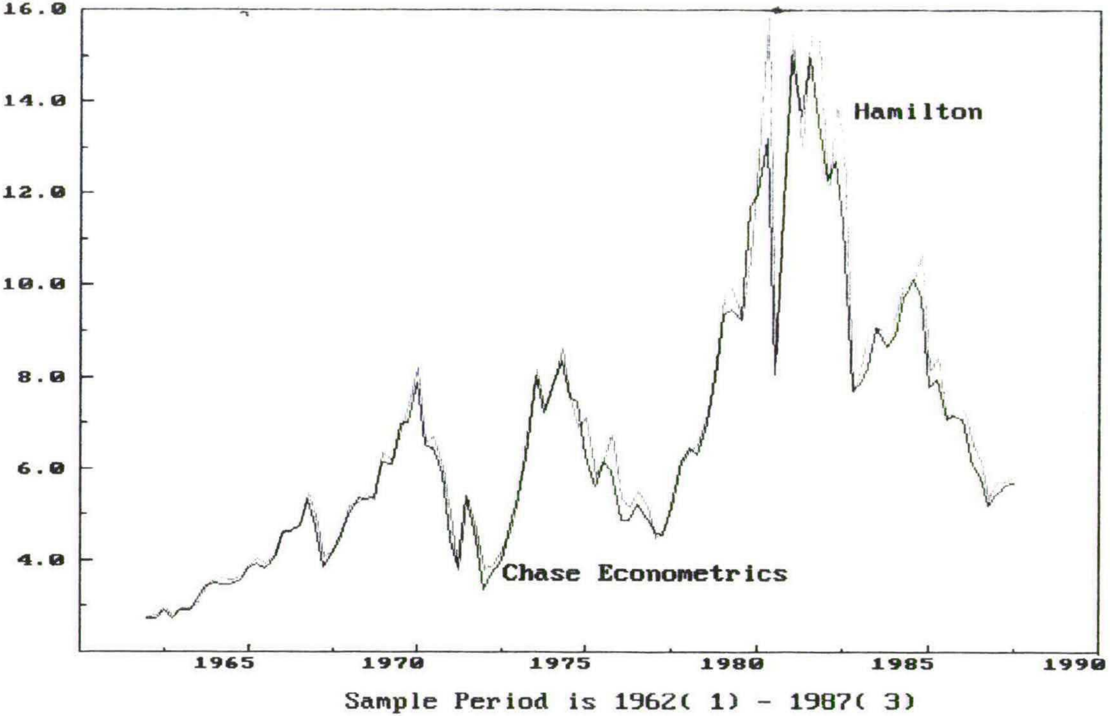
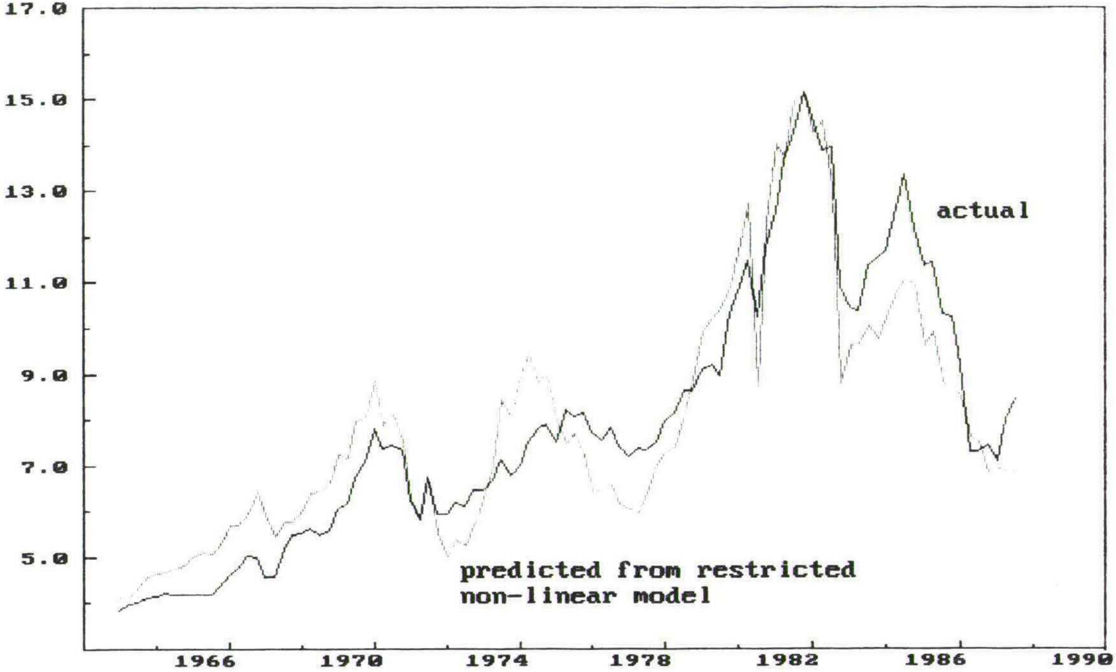


FIGURE 2

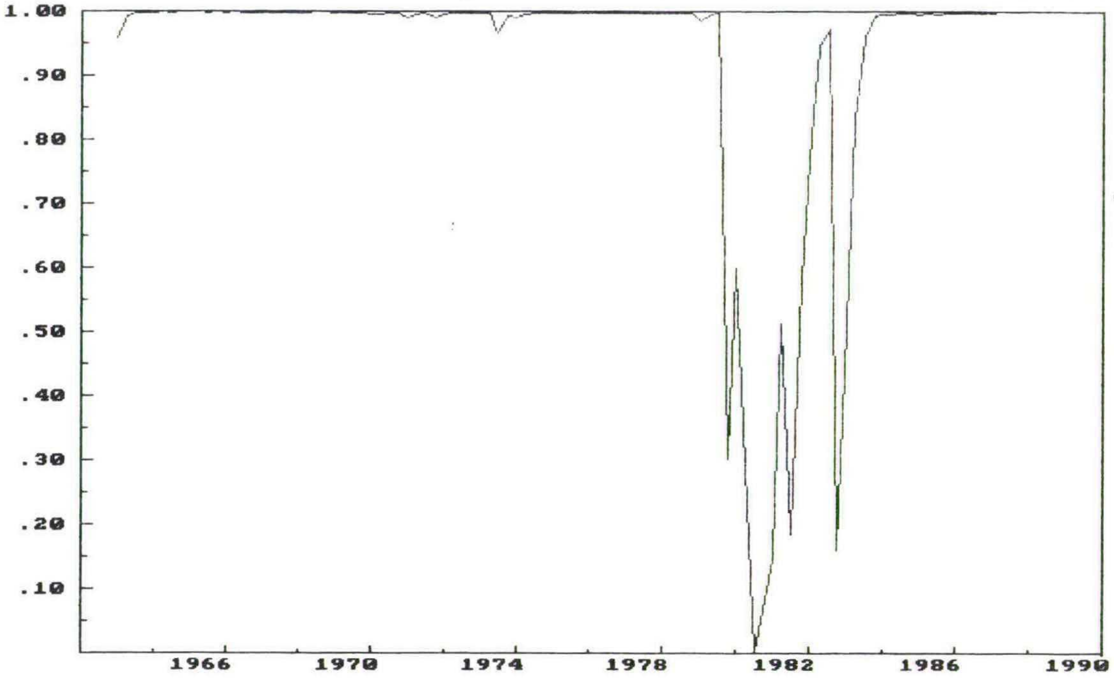
10 year US Government Bond yields



Sample Period is 1963(1) - 1987(3)

FIGURE 3

Probability of being in state 0. Restricted non-linear model



Sample Period is 1963(1) - 1987(3)

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