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Abstract

In this paper an adjustment process for an exchange economy with linear production technologies is given. The process reaches an equilibrium by simultaneous adaptations of prices and activity levels. Till thusfar no such process exists.

An important feature of the process is that it keeps track of the location of the starting price vector throughout its operation. Furthermore, the choice of the starting price vector is only limited by economic considerations. Besides, the process converges under rather weak conditions and possesses an appealing economic interpretation.

Keywords: Adjustment, Exchange economy, Linear production, Equilibrium.

1. Introduction

In this paper we propose a process that reaches an equilibrium in an exchange economy with linear production via adaptations of prices and activity levels. Till thusfar no such process exists. This is contrary to the special case of a pure exchange economy for which already several adjustment procedures have been introduced.

The standard adjustment procedure for a pure exchange economy is the Walrasian tatonnement process. In this process the prices of goods in excess demand are increased whereas prices of goods in excess supply are decreased. Another process is the Newton-like method of Smale [3]. That process adapts the prices in such a way that excess supplies and excess demands are diminished simultaneously. The major drawback of both iterative price adjustment processes is that their convergence is only guaranteed under quite strong conditions. The Walras process needs a revealed preference condition to hold, whereas Smale needs a condition on the demand behaviour when one or more prices are zero. Besides, Smale's method has to start with some of the prices equal to zero. An economy with production is much more complicated and both these methods seem not easily to be generalized for such an economy.

Van der Laan and Talman [1] have presented another type of price adjustment processes for a pure exchange economy. These processes converge under weak regularity assumptions due to the fact that at a generated price vector the adjustments are governed by both the excess demand vector and the location of the price vector with respect to the initial price vector. One of these processes follows a path of prices such that the ratio of the current price and the initial price of goods in excess demand is always maximal whereas the same price ratio of goods in excess supply is always minimal. As soon as a good becomes in equilibrium the price ratio is allowed to vary between these bounds and the good is kept in equilibrium. Here we generalize this process to handle economies with linear production technologies.

This paper consists of five sections. In Section 2 we present the model and introduce the process. In Section 3 we give a mathematical description of the path followed by the process and prove its existence and convergence. In Section 4 we give an economic interpretation. Finally, in Section 5 we consider two examples.

2. The model

We consider a standard model of an exchange economy with linear production technologies. There are a finite number of consumers, m production activities, also called firms, and n+1 commodities. The firms are indexed by i, $i \in \{1,\dots,m\}$, and the commodities or goods by j, $j \in \{1,\dots,n+1\}$. Consumers are assumed to be endowed with some of the goods, such as labour and capital. More precisely, the nonnegative vector w in $\mathbb{R}^{n+1}$ denotes the (aggregated) initial endowments of the consumers, with w_j the amount of commodity j, $j \in \{1,\dots,n+1\}$. At a price vector p in $\mathbb{R}^{n+1}_{+}\backslash\{0\}$, the (n+1)-vector d(p) denotes the aggregate demand of the consumers for the commodities. We assume that demand is homogeneous of degree zero in the prices and that it satisfies Walras' law, i.e. $d(\lambda p) = d(p)$ for all $\lambda > 0$ and $p^{\mathsf{T}}d(p) = p^{\mathsf{T}}w$, respectively, for every price vector p. We also assume that the function d is smooth on $\mathbb{R}^{n+1}_{++}$.

An activity of a firm is represented by an (n+1)-vector whose negative components correspond to the inputs and whose positive components to outputs. More precisely, the (n+1)-vector a^i denotes the activity vector of firm i, $i \in \{1,\dots,m\}$, with $a^i_j \geqq 0$ the (net) amount of output of commodity j and $-a^i_j \geqq 0$ the amount of input of commodity j, $j \in \{1,\dots,n+1\}$, when the activity level is equal to one. A vector y in $\mathbb{R}^m_+$ denotes an activity level vector with y_i the activity level of firm i, $i \in \{1,\dots,m\}$. So, the (n+1)-vector Ay, with A the $(n+1)\times m$ matrix with i-th column a^i for $i \in \{1,\dots,m\}$, is the aggregate net input-output vector for activity level vector y. We assume that there can be no production without input, i.e. $Ay \geqq 0$ and $y \geqq 0$ implies $y = 0$.

The m-vector $A^{\mathsf{T}}p$ is the unit level profit vector at price vector p, with $(A^{\mathsf{T}}p)_i$ the profit of firm i when it operates at unit level. We call a price vector p^* and an activity level vector y^* an equilibrium if for each commodity demand is at most equal to endowment plus net production and if no activity makes profit.

Definition 2.1. A pair $(p^*,y^*) \in (\mathbb{R}_+^{n+1}\backslash\{0\}) \times \mathbb{R}_+^m$ is an equilibrium if

i) $d(p^*) - Ay^* \leqq w$

ii) $A^\top p^* \leqq 0$.

From the definition and Walras' law we can derive some properties holding at an equilibrium. Multiplying i) with p^* and ii) with y^* delivers respectively $-p^{*\top}Ay^* \leqq 0$ and $p^{*\top}Ay^* \leqq 0$. Thus, $p^{*\top}Ay^* = 0$ and therefore with Walras' law also $p^{*\top}(d(p^*) - Ay^* - w) = 0$. Because $A^\top p^* \leqq 0$ and $y^* \geqq 0$, $p^{*\top}Ay^* = 0$ means that in equilibrium a firm can only operate at a positive production level if it makes zero profit, i.e. if $y_i^* > 0$ then $(A^\top p^*)_i = 0$. Similarly, because $d(p^*) - Ay^* - w \leqq 0$ holds, $(p^*)^\top(d(p^*) - Ay^* - w) = 0$ means that in equilibrium the consumers demand for a commodity can only be less than its endowments plus net production if the price of that commodity is equal to zero.

Due to the homogeneity of degree zero of the demand function d we have that if (p^*,y^*) is an equilibrium, then $(\lambda p^*,y^*)$ is an equilibrium for any $\lambda > 0$. This allows us to normalize the price vectors to the n-dimensional unit simplex S^n defined by

$$S^n = \{p \in \mathbb{R}_+^{n+1} \mid \Sigma_{j=1}^{n+1} p_j = 1\}.$$

Assuming that d is a smooth function on the interior of S^n, it is well-known from a fixed-point argument that an equilibrium always exists in $S^n \times \mathbb{R}_+^m$. Since at an equilibrium $(p^*,y^*) \in S^n \times \mathbb{R}_+^m$ it holds that $A^\top p^* \leqq 0$, p^* lies in S_A^n, where $S_A^n := \{p \in S^n \mid A^\top p \leqq 0\}$. To find an equilibrium vector pair (p^*,y^*) in $S_A^n \times \mathbb{R}_+^m$, we propose to follow a piecewise smooth path, denoted P, in $S_A^n \times \mathbb{R}_+^m$. Let p^0 be an arbitrarily chosen point in the interior of S_A^n, i.e. $p_j^0 > 0$ for all j and $(p^0)^\top a^i < 0$ for all i. According to Farkas' lemma the relative interior of S_A^n is nonempty, since $Ay \geqq 0$ and $y \geqq 0$ implies $y = 0$. The path connects the pair $(p^0,0)$ and an equilibrium pair (p^*,y^*). The path can be interpreted as the path generated by an adjustment process in which prices and activity levels simultaneously adjust. All points (p,y) along the path P in $S_A^n \times \mathbb{R}_+^m$ satisfy for $j \in \{1,\dots,n+1\}$

$$p_j/p_j^0 = \min_h p_h/p_h^0 \qquad \text{if } d_j(p) - (Ay)_j < w_j,$$

$$\min_h p_h/p_h^0 \leqq p_j/p_j^0 \leqq \max_h p_h/p_h^0 \qquad \text{if } d_j(p) - (Ay)_j = w_j,$$

$$p_j/p_j^0 = \max_h p_h/p_h^0 \qquad \text{if } d_j(p) - (Ay)_j > w_j,$$

and for $i \in \{1,...,m\}$ (2.1)

$$y_i = 0 \qquad \text{if } p^\top a^i < 0$$

$$y_i \geqq 0 \qquad \text{if } p^\top a^i = 0.$$

3. Existence of the path

An arbitrary price vector p^0 in $\mathrm{int}(S_A^n)$ can be determined by solving the Linear Programming problem

$$c = \min(p_1 + \ldots + p_{n+1})$$

$$\text{such that } A^\top p \leqq -e^m \text{ and } p \geqq e^{n+1},$$

where e^k is the k-dimensional vector of ones. Clearly, this L.P. problem has a solution $\bar{p}$. After dividing $\bar{p}$ by c, the sum of its components, a price vector p^0 in the interior of S_A^n is obtained.

Given p^0, the set of points $(p,y) \in S_A^n \times \mathbb{R}_+^m$ satisfying (2.1) is denoted by B, i.e. B is the set of points (p,y) in $S_A^n \times \mathbb{R}_+^m$ such that

i) for $j \in \{1,...,n+1\}$,

$$p_j/p_j^0 = \min_h p_h/p_h^0 \qquad \text{if } d_j(p) - (Ay)_j < w_j$$

and

$$p_j/p_j^0 = \max_h p_h/p_h^0 \qquad \text{if } d_j(p) - (Ay)_j > w_j,$$

ii) $y_i = 0 \qquad$ if $p^\top a^i < 0, \quad i \in \{1,...,m\}$.

Clearly, the point $(p^0,0)$ satisfies i) and ii) with both the minimum and the maximum equal to one. Also, all equilibria lie in the set B. We now show that under standard nondegeneracy and transversality conditions the set B consists of piecewise smooth paths and loops. One of these paths is the path P, connecting $(p^0,0)$ and an equilibrium (p^*,y^*).

The points (p,y) in the set B are determined by the location of p with respect to p^0 and the sign pattern of the net excess demand vector $d(p) - Ay - w$. Observe that at all $(p,y) \in B$ condition ii) of Definition 2.1 is fulfilled. If also $d(p) - Ay - w \leqq 0$ then (p,y) is an equilibrium. Furthermore, for all $(p,y) \in B$ we have because $y_i = 0$ if $p^\top a^i < 0$, $i \in \{1,\ldots,m\}$, that $p^\top Ay = 0$ and hence according to Walras' law that $p^\top(d(p) - Ay - w) = 0$. Suppose now that there exists an index j for which $d_j(p) - (Ay)_j - w_j > 0$. Since $(p,y) \in B$ then $p_j = p_j^0.\max_h p_h/p_h^0 > 0$. Therefore, $d(p) - Ay - w$ must also contain at least one negative component with corresponding price positive. Let $s \in \{-1,0,+1\}^{n+1}$ denote a sign vector in $\mathbb{R}^{n+1}$. If s contains at least one +1 and one -1 we say that s is a <u>feasible</u> sign vector.

For a feasible sign vector s, let $I^0(s) := \{j | s_j = 0\}$ be the index set of zero-components of s and let U be a subset of $\{1,\ldots,m\}$. The number of elements in $I^0(s)$ and U is denoted by $|I^0(s)|$ and $|U|$ respectively. We want to split up B into subsets related to a feasible sign vector s and a subset U of $\{1,\ldots,m\}$. For that we first define $A(s,U)$ by

$$A(s,U) = \{(p,y) \in S_A^n \times \mathbb{R}_+^m | (a^i)^\top p = 0 \text{ for } i \in U,\ y_i = 0 \text{ for } i \notin U,$$

$$p_j/p_j^0 = \min_h p_h/p_h^0 \quad \text{when } s_j = -1$$

$$p_j/p_j^0 = \max_h p_h/p_h^0 \quad \text{when } s_j = +1\}.$$

Note that there are $n - 1 - |I^0(s)| + |U|$ constraints on $p \in S_A^n$ and $m - |U|$ constraints on y. Thus, if $|I^0(s)| = |U|$, there are n-1 constraints on p and because $\dim(S_A^n)$ equals n, one degree of freedom is left. Indeed, the p-components in $A(s,U)$ then form a line segment and so a set of dimension 1. Clearly, if $|U| > |I^0(s)| + 1$ there are more than n constraints on p and in general no price vector in S_A^n satisfies all conditions, i.e. $A(s,U) = \emptyset$. If $|U| \leqq |I^0(s)| + 1$ the set $A(s,U)$ is well-defined and its dimension is equal to $|I^0(s)| + 1$.

We now consider the set of points (p,y) satisfying $(p,y) \in A(s,U)$ for $s = \mathrm{sgn}(d(p) - Ay - w)$, i.e. $s_j > 0$ (< 0) if $(d(p) - Ay - w)_j > 0$ (< 0) and $s_j = 0$ if $(d(p) - Ay - w)_j = 0$. We denote the closure of this set by $B(s,U)$. Clearly, every point (p,y) satisfying (2.1) lies in some set $B(s,U)$. More precisely, B is the union of $B(s,U)$ over all feasible sign vectors s and subsets U of $\{1,...,m\}$ with $|U| \leqq |I^0(s)| + 1$. In particular the point $(p^0,0)$ lies in $A(s^0,\emptyset)$, where $s^0 = \mathrm{sgn}(d(p^0) - w)$. Without loss of generality we assume that the excess demand vector $d(p^0) - w$ does not contain zero components so that the dimension of $A(s^0,\emptyset)$ equals 1. Observe that in the definition of each $B(s,U)$ one degree of freedom is left because $\dim(A(s,U)) = |I^0(s)| + 1$ whereas the condition that $\mathrm{sgn}(d(p) - Ay - w) = s$ imposes $|I^0(s)|$ conditions on (p,y). Thus, assuming standard nondegeneracy and transversality conditions, a nonempty $B(s,U)$ forms a collection of disjoint smooth paths and loops. An end point (p,y) of a path in $B(s,U)$ is characterized by one of the following cases:

$$
\begin{array}{ll}
\text{o)} & (p,y) = (p^0,0); \\
\text{i)} & \min_h p_h/p_h^0 = 0; \\
\text{ii)} & p_j/p_j^0 = \min_h p_h/p_h^0 \text{ for some } j \in I^0(s); \\
\text{iii)} & p_j/p_j^0 = \max_h p_h/p_h^0 \text{ for some } j \in I^0(s); \\
\text{iv)} & (a^i)^\top p = 0 \text{ for some } i \notin U; \\
\text{v)} & y_i = 0 \text{ for some } i \in U; \\
\text{vi)} & d_j(p) - (Ay)_j = w_j \text{ for some } j \notin I^0(s).
\end{array}
\tag{3.1}
$$

We argue that an end point of a path in $B(s,U)$ is either $(p^0,0)$, or an equilibrium, or an end point of a unique path in some other set $B(\bar{s},\bar{U})$, so that the paths in different sets $B(s,U)$ can be linked to form piecewise smooth paths and loops.

Suppose that case o) occurs. Since $d(p^0) - w$ is assumed not to contain zeros and $(A^{\top}p^0)_i < 0$ for all i, s must be equal to s^0 and $U = \emptyset$. Thus, $(p^0,0)$ is only an end point of a path in $B(s^0,\emptyset)$.

In case i) we must have that $p_j = 0$ for all j for which $s_j = -1$. Hence, $d_j(p) - (Ay)_j \leqq w_j$ for all indices j for which $p_j = 0$, whereas $d_j(p) - (Ay)_j \geqq w_j$ whenever $p_j > 0$. Since, $p^{\top}(d(p) - Ay - w) = 0$ for all $(p,y) \in B$, this implies $d_j(p) - (Ay)_j = w_j$ if $p_j > 0$. Therefore (p,y) is an equilibrium.

Next suppose that case ii) or case iii) holds. Then (p,y) must be also an end point of a path in $B(\bar{s},U)$ where $\bar{s}_h = s_h$ for all $h \neq j$ and $\bar{s}_j = -1$ in case ii), $\bar{s}_j = +1$ in case iii). Clearly, this path is uniquely determined.

In case iv) the point (p,y) is also an end point of a path in $B(s,U \cup \{i\})$ and in case v) of a path in $B(s,U\backslash\{i\})$. In both cases these paths are uniquely determined.

Finally, we consider case vi). If $s_j = +1$ (-1) and s has no other positive (negative) components, then according to the fact that $p^{\top}(d(p) - Ay - w) = 0$, (p,y) must be an equilibrium. Otherwise (p,y) is also an end point of a path in $B(s',U)$ where $s'_j = 0$ and $s'_h = s_h$ for all $h \neq j$. Again, this path is uniquely determined.

Consequently, for different s and U, the paths in the sets $B(s,U)$ can be linked to form disjoint piecewise smooth paths and loops. Exactly one path P has $(p^0,0)$ as an end point. All other end points of these paths are equilibria. To prove that the path P has another end point, which must be then an equilibrium, we show in the next lemma that the set B is bounded.

Lemma 3.1. The set B is bounded.

Proof. Suppose that the set B is unbounded. Then without loss of generality there is some feasible sign vector s and set U such that $B(s,U)$ contains a sequence $\{(p^k,y^k)\}_{k=1}^{\infty}$, with some of the components of (p^k,y^k) going to infinity. Since S_A^n is compact the sequence $\{p^k\}_{k=1}^{\infty}$ is bounded and has a cluster point, $\hat{p}$, which lies in $A(s,U)$. Hence, some of the components of y^k must go to infinity. Because $(p^k,y^k) \in B(s,U)$ for each k, there exist $\mu_h^k \geqq 0$, $h \notin I^0(s)$, such that

$$d(p^k) - \sum_{i\in U} a^i y_i^k - \sum_{s_h\neq 0} \mu_h^k s_h e(h) = w,$$

where $e(h)$ denotes the h-th unit vector in $\mathbb{R}^{n+1}$. Since p^k converges to $\hat{p}$, the latter system can only have a solution for all k if the homogeneous system of linear equations

$$\sum_{i\in U} a^i y_i + \sum_{s_h\neq 0} \mu_h s_h e(h) = 0 \tag{3.2}$$

has a nonzero solution $\bar{y}_i \geqq 0$ for $i \in U$ and $\bar{\mu}_h \geqq 0$ for $h \notin I^0(s)$. Since $\hat{p} \in A(s,U)$ there exist a number b, $0 \leqq b \leqq 1$, and a vector $q \in \mathbb{R}_+^{n+1}$ with $q_j > 0$ if $s_j = +1$, $q_j \geqq 0$ if $s_j = 0$, and $q_j = 0$ if $s_j = -1$ such that $\hat{p} = bp^0 + q$. Clearly, $q^\top a^i > 0$ for $i \in U$ since $(\hat{p})^\top a^i = 0$ and $(p^0)^\top a^i < 0$ for $i \in U$. Premultiplying (3.2) with $q^\top$ yields

$$\sum_{i\in U} (q^\top a^i) y_i + \sum_{s_h=+1} \mu_h q_h = 0.$$

Since $(q^\top a^i) > 0$ for $i \in U$ and $q_h > 0$ for $s_h = +1$, this can only hold when all the y_i's and all μ_h's for which $s_h = +1$ are equal to zero. But then according to (3.2) all other μ_h's must also be equal to zero. Hence, system (3.2) has no nonzero nonnegative solution, which completes the proof.

□

Lemma 3.1 implies that the path P is bounded and therefore has another end point which must be an equilibrium.

4. The adjustment process

In this section we provide an economic interpretation of the adjustments of prices and activity levels along the path P as defined in Section 3. The references made in the text are to the cases listed in

(3.1). In the sequel we mean p_j/p_j^0, $j \in \{1,\dots,n+1\}$, when we speak about the price ratio of commodity j at a price vector p. Further, we use the following notation. By z(p) we denote the consumers excess demand at price vector p, i.e. $z(p) := d(p) - w$, whereas $\tilde{z}(p,y)$ denotes the (total) excess demand at price vector p and activity level vector y, i.e. $\tilde{z}(p,y) := z(p) - Ay$.

The process starts in (p^0,y^0) with $y^0 = 0$ and price vector p^0 such that all prices are positive and all activities make losses. Moreover, at p^0, the excess demand $\tilde{z}(p^0,0)$ is equal to the consumers excess demand $z(p^0)$ and is assumed not to contain zeros. Now, the process leaves $(p^0,0)$ by increasing proportionally the prices of the commodities in excess demand $(z_j(p^0) > 0)$ and decreasing proportionally the prices of the commodities in excess supply $(z_j(p^0) < 0)$. The process continues in this way until a price vector p is reached at which either a price becomes zero (case i)) or one of the goods becomes in equilibrium, i.e. $z_j(p) = 0$ for some j (case vi)), or one of the activities makes zero profit, i.e. $p^\top a^i = 0$ for some i (case iv)). In case i) an equilibrium has been reached because in this case all the prices of the goods in excess supply have become zero simultaneously. Walras' law then implies that there cannot be a good in excess demand anymore. In case vi) the process continues by keeping the price ratios of the goods in excess demand (excess supply) maximal (minimal) while the good j is kept in equilibrium by varying its price ratio. Finally, in case iv) the activity level y_i of activity i is increased from zero. This activity level is increased until the excess demand or supply of some commodity becomes zero. Lemma 3.1, saying that the set of points (p,y) in B is bounded, guarantees that this will occur as can also be seen as follows. Let $s^0 = \operatorname{sgn} z(p^0)$ then also at p we must have $\operatorname{sgn} z(p) = s^0$. Since $p^\top a^i = 0$, $(p^0)^\top a^i < 0$, $p_j > p_j^0$ if $s_j^0 = +1$, and $p_j < p_j^0$ if $s_j^0 = -1$, there exists at least one index j such that $\operatorname{sgn} a_j^i = s_j^0$. From this it immediately follows that $d_h(p) - a_h^i y_i - w_h$ must go to zero for at least one index h if y_i is increased from zero.

In general, the process generates a path of price vectors p in S_A^n and activity level vectors y in $\mathbb{R}_+^m$. At such pair (p,y) the price ratio p_j/p_j^0 of a good in excess demand $(\tilde{z}_j(p,y) > 0)$ is maximal and the price ratio of a good in excess supply $(\tilde{z}_j(p,y) < 0)$ is minimal. The price ratio of a good in equilibrium ($\tilde{z}_j(p,y) = 0$) lies between this minimum and maximum. An activity level can only be positive ($y_i > 0$) if the corresponding activity makes zero profit ($p^T a^i = 0$). Finally, an activity not operating ($y_i = 0$) makes negative or zero profit ($p^T a^i \leqq 0$).

As soon as a pair (p,y) is reached at which a good becomes in equilibrium, that good is kept in equilibrium and its price ratio is allowed to vary between the minimal and maximal price ratios. On the other hand, if the price ratio of a good in equilibrium becomes equal to the maximal (or minimal) price ratio, then it is kept equal to the maximal (minimal) price ratio and the good becomes in excess demand (excess supply).

When a pair (p,y) is reached at which an activity not producing makes zero profit, then the activity level of this activity is allowed to become positive and its profit is kept equal to zero. Also, when the process reaches a (p,y) at which the level of an activity making zero profit becomes zero, then the process continues by keeping this activity level equal to zero while the price adaptations are allowed to bring this activity into a loss situation.

The process stops as soon as each market is in equilibrium or in excess supply. Lemma 3.1 guarantees that the process indeed will reach an equilibrium.

In the interpretation given above, the adaptations of prices and activity levels were treated more or less separately. In fact, they adjust simultaneously. Changes in activity levels do have an impact on excess demands, whereas changes in price levels influence both all the excess demands and profits. We illustrate this with a general example. The path generated by the process is depicted in Figure 4.1. This figure shows the projection of the path on the price space. The reader should keep in mind that the process adjusts prices as well as activity levels and in fact operates in $S^n_A \times \mathbb{R}^m_+$ and not in S^n_A.

We consider an economy with three goods and two activities. All relevant information of the economy is given in Figure 4.1. The simplex is the price set. Activity 1 uses the first commodity as input whereas the other goods are outputs. We denote this as $a^1 = (-,+,+)^T$. Similarly, we have $a^2 = (-,+,-)^T$. The line segments $[v,f]$ and $[\ell,r]$ denote the price vectors at which the profit of activity 1 and 2 respectively, is zero. Because at $p = e(1)$ both activities make losses the set S_A^2 is the convex hull of $e(1)$, v, p^* and ℓ. Also drawn are the curves $\{p|z_j(p) = 0\}$, $j \in \{1,2,3\}$. We assume that $z_j(p)$ is positive whenever $p_j = 0$. Because of Walras' law the curves $\{p|z_j(p) = 0\}$ and $\{p|z_h(p) = 0\}$, $j \neq h$, meet each other on the edge $\{p|p_i = 0, i \neq j,h\}$. Moreover, the three curves intersect each other in q^* which is the equilibrium of the corresponding pure exchange economy. Note that q^* lies outside the region S_A^2, so that the economy with production activities has no equilibrium for zero activity levels.

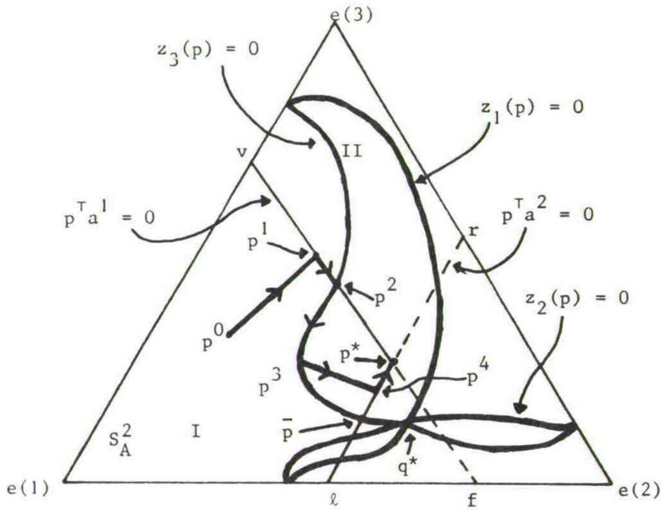

Figure 4.1. Exchange economy with three goods and two activities. At prices p in region I, $\text{sgn } z(p) = (-1,+1,+1)^\top$. For region II holds $\text{sgn } z(p) = (-1,+1,-1)^\top$.

Let us consider now what happens along the path of the process starting from $(p^0,0)$. Since p^0 lies in the interior of S_A^2 both activities make losses. At $(p^0,0)$ commodity 1 is in excess supply, and the commodities 2 and 3 are in excess demand. The process leaves $(p^0,0)$ by proportionally increasing the prices of goods 2 and 3 and decreasing the price of good 1. Thus, the path of prices leaves p^0 in the direction opposite to $e(1)$. Meanwhile, the activity levels remain zero.

At $(p^1,0)$ the profit of activity 1 becomes zero and its activity level y_1 is increased. Because $\mathrm{sgn}\ \tilde{z}(p^1,0) = \mathrm{sgn}\ z(p^1) = (-1,+1,+1)^\top$ and $a^1 = (-,+,+)^\top$, this increase tends to offset the imbalances on all markets. The level y_1 is increased till say $\bar{y}_1$, at which one of the markets becomes in equilibrium. In case commodity 1 becomes in equilibrium the other markets must also be in equilibrium and hence $(p^1,(\bar{y}_1,0)^\top)$ is an equilibrium. This because $p^\top\tilde{z}(p,y) = 0$ for all (p,y) on the path whereas $\tilde{z}_2(p^1,0) > 0$ and $\tilde{z}_3(p^1,0) > 0$. If commodity 2 becomes in equilibrium, i.e. $\tilde{z}_2(p^1,(\bar{y}_1,0)^\top) = 0$, then the prices and y_1 are adjusted simultaneously such that commodity 2 is kept in equilibrium, the profit of activity 1 remains zero, and the price ratio of commodity 2 lies between the minimum price ratio (p_1/p_1^0) and the maximum price ratio (p_3/p_3^0). In the figure the path of prices then moves in the direction of v. It can be shown that then there must be an equilibrium $(\tilde{p},(\tilde{y}_1,0)^\top)$ with $\tilde{p}$ on the line segment $[p^1,v]$. The more complicated case occurs when at $(p^1,(\bar{y}^1,0)^\top)$ commodity 3 becomes in equilibrium $(\tilde{z}_3(p^1,(\bar{y}^1,0)^\top) = 0)$. Then the prices and the activity level y_1 are adjusted simultaneously such as to keep commodity 3 in equilibrium whereas the profit of activity 1 remains zero and the price ratio of good 3 (p_3/p_3^0) varies between the minimum price ratio (p_1/p_1^0) and the maximum price ratio (p_2/p_2^0). In the figure the path of prices moves towards f. If between p^1 and p^2 the market of commodity 1 becomes in equilibrium then also the market of commodity 2 must become in equilibrium and an equilibrium of the economy is reached. Again this is due to the fact that $p^\top\tilde{z}(p,y) = 0$ along the path.

Suppose there is no equilibrium between p^1 and p^2. Then the process reaches p^2 where, because $z_3(p^2) = 0$, the level of activity 1 needed to keep commodity 3 in equilibrium becomes zero. Now, y_1 is fixed at 0, prices are allowed to bring activity 1 into a loss situation whereas good 3 is still kept in equilibrium. Thus, the process follows a path of vectors $(p,0)$ with p in $\mathrm{int}(S_A^2)$ and $z_3(p) = 0$. In the figure this piece of the path is represented by the curve connecting p^2 and p^3.

Then, at p^3 the price ratio of commodity 3 (p_3^3/p_3^0) becomes equal to the minimum price ratio (p_1^3/p_1^0). This because p^3 lies on the line segment connecting p^0 and $e(2)$. The process continues from $(p^3,0)$ by keeping the price ratios of commodities 1 and 3 equal to each other $(p_3/p_3^0 = p_1/p_1^0)$ whereas commodity 3 is allowed to become into excess supply. In Figure 4.1, the path of prices enters region II in the direction of $e(2)$.

At $(p^4,0)$ the profit of activity 2 becomes zero and its activity level y_2 is increased. This situation is similar to that at $(p^1,0)$. Because sgn $\tilde{z}(p^4,0) = (-1,+1,-1)^\top$ whereas $a^2 = (-,+,-)^\top$, the increase of y_2 diminishes both the excess demand for good 2 and the excess supplies of commodities 1 and 3. Thus, there is a level $\bar{y}_2$ at which one of the commodities becomes in equilibrium. If $\tilde{z}_2(p^4,(0,\bar{y}_2)^\top) = 0$ then $(p^4,(0,\bar{y}_2)^\top)$ is an equilibrium because of Walras' law. In case $\tilde{z}_1(p^4,(0,\bar{y}_2)^\top) = 0$ then prices and y_2 are adjusted such that commodity 1 is kept in equilibrium, the profit of activity 2 remains zero whereas the price ratio p_1/p_1^0 varies between the maximum price ratio (p_2/p_2^0) and the minimum price ratio (p_3/p_3^0). The corresponding path of prices in Figure 4.1 goes in the direction of ℓ. We argue that in that case there must be an equilibrium on the open segment $(p^4,\bar{p})$. Assume the contrary. Then the process must reach $\bar{p}$ with corresponding $\hat{y}_2$ such that $\tilde{z}_1(\bar{p},(0,\hat{y}_2)^\top) = 0$. Note that $\hat{y}_2 > 0$ because $z_1(\bar{p}) < 0$. However, because $z_3(\bar{p}) = 0$, we then have that $\tilde{z}_3(\bar{p},(0,\hat{y}_2)^\top) > 0$. Thus, the sign of $\tilde{z}_3$ changes from -1 at $(p^4,(0,\bar{y}_2)^\top)$ into +1 at $(\bar{p},(0,\hat{y}_2)^\top)$. But then there must be a point $(p,(0,y_2)^\top)$ with $p \in (p^4,\bar{p})$, at which $\tilde{z}_3(p,(0,y_2)^\top) = \tilde{z}_1(p,(0,y_2)^\top) = 0$, and hence $\tilde{z}_2(p,(0,y_2)^\top) = 0$, i.e. $(p,(0,y_2)^\top)$ is an equilibrium. Finally, we consider the case when at $(p^4,(0,\bar{y}_2)^\top)$ commodity 3 becomes in equilibrium. Then the process keeps commodity 3 in equilibrium and fixes the profit of activity 2 at zero while varying the price ratio of commodity 3 between the minimum price ratio (p_1/p_1^0) and the maximum price ratio (p_2/p_2^0). In Figure 4.1 the path projected on the price space moves from p^4 towards r. If the process reaches a price vector p in $(p^4,p^*]$ with corresponding activity level y_2 such that $\tilde{z}_3(p,(0,y_2)^\top) = 0$ while besides $\tilde{z}_1(p,(0,y_2)^\top) = 0$ then, due to Walras' law, also $\tilde{z}_2(p,(0,y_2)^\top) = 0$ and $(p,(0,y_2)^\top)$ is an equilibrium. Otherwise, the process reaches the price vector p^* with corresponding activity level vector $\tilde{y} = (0,\tilde{y}_2)^\top$ being such that $\tilde{z}_3(p^*,\tilde{y}) = 0$ whereas $\tilde{z}_1(p^*,\tilde{y}) < 0$ and $\tilde{z}_2(p^*,\tilde{y}) > 0$. At p^* also activity 1 makes zero profit, so that its activity level y_1 is increased. Since commodity 3 is an input for activity 2 whereas it is an output of activity 1, an increase in y_1 must be matched by an increase in y_2 to keep commodity 3 in equilibrium. Lemma 3.1 then guarantees that y_1 eventually reaches a level y_1^* with corresponding y_2^* for which commodity 1 or commodity 2 becomes in equilibrium. But then all markets are in equilibrium and $(p^*,(y_1^*,y_2^*)^\top)$ is an equilibrium.

5. Examples

In this section we apply the adjustment process to two specific examples. In both examples we consider an economy with three goods and two activities.

Example 5.1. We assume that the consumers can be represented by one consumer having a Cobb-Douglas utility function $u(x_1,x_2,x_3) = x_1^{1/10}x_2^{2/5}x_3^{1/2}$, with x_i the amount of good i consumed. The initial endowments are $w = (1,1,1)^\top$. Straightforward calculations yield that the consumer excess demand at price p equals $z(p) = ((1/10p_1) - 1,(2/5p_2) - 1, (1/2p_3) - 1)^\top$. The activity vectors are given by $a^1 = (-3/2,1,1)^\top$ and $a^2 = (-1,-77/27,11/9)^\top$. This economy is depicted in Figure 5.1. Observe that $\{p|z_j(p) = 0\} = \{p|p_j = \alpha_j\}$ with $\alpha_1 = 1/10$, $\alpha_2 = 2/5$, $\alpha_3 = 1/2$ the budget shares of the consumer. Furthermore, note that $\{p|z_j(p) = 0\}$ and $\{p|z_k(p) = 0\}$ do not intersect at a price vector p with $p_i = 0$, $i \neq j,k$. (cf. Figure 4.1). This because z_j is not continuous at price vectors p with $p_j = 0$.

Again, the equilibrium price vector $q^* = (1/10,2/5,1/2)^\top$ of the corresponding pure exchange economy lies outside the set S_A^2 being the convex hull of the vectors ℓ, q, f and e(1). Thus, in the economy with production there is no equilibrium with zero activity levels. We consider how the process reaches an equilibrium when starting from $(p^0,0)$ with $p^0 = (1/2,1/20,9/20)^\top$. The excess demand vector at p^0 is equal to $z(p^0) = (-4/5,7,1/9)^\top$. Thus, initially commodity 1 is in excess supply, whereas the goods 2 and 3 are in excess demand. Besides, activity 1 makes a loss of 1/4 whereas activity 2 makes a loss of 5/54. The process leaves $(p^0,0)$ by proportionally increasing the prices of goods 2 and 3, decreasing the price of good 1, and keeping both activity levels equal to zero. In the price set the process goes from p^0 into the direction opposite of e(1). At $(p^1,0)$, with $p^1 = (22/49,27/490,243/490)^\top$, the profit of activity 2 becomes zero. Then the activity level y_2 of activity 2 is increased until

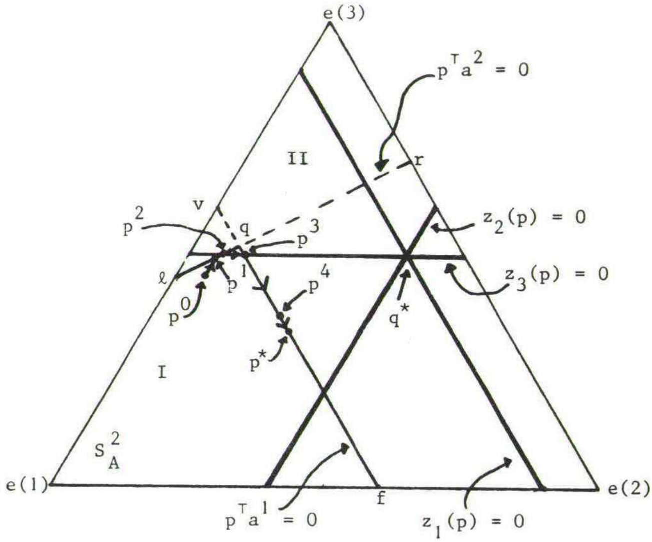

Figure 5.1. The set S_A^2 for the economy of Example 5.1. For a price vector p in region I holds that sgn $z(p) = (-1,+1,+1)^\top$. For region II, sgn $z(p) = (-1,+1,-1)^\top$.

at $y_2 = 2/297$ commodity 3 becomes in equilibrium, i.e. $\tilde{z}_3(p^1,(0,2/297)^\top) = 0$. Then the process continues by keeping commodity 3 in equilibrium and the profit of activity 2 equal to zero, whereas the price ratio of commodity 3 varies between the minimum price ratio p_1/p_1^0 and the maximum price ratio p_2/p_2^0. In the figure the process goes towards r. At $p^2 = (11/25,3/50,1/2)^\top$, $z_3(p^2) = 0$ and hence the level of activity 2 needed to keep commodity 3 in equilibrium becomes zero. Then the process continues by keeping y_2 equal to zero and commodity 3 still in equilibrium. Activity 2 is allowed to become in a deficit situation. Thus, the process generates points $(p,0)$ with p going from p^2 towards q^* such that $\text{sgn}\,\tilde{z}(p,0) = \text{sgn}\,z(p) = (-1,+1,0)^\top$ and $p^\top a^i < 0$, $i \in \{1,2\}$. At $(p^3,0)$ with $p^3 = (2/5,1/10,1/2)^\top$, activity 1 makes zero profit. Then the process increases y_1. Without simultaneously changing the prices this would give an excess supply on the market for commodity 3. Thus, the process has to adjust the prices such that the consumer excess demand for commodity 3 becomes positive. Projected on the price set, the process goes from p^3 towards f, i.e. it generates price vectors p at which $p^\top a^1 = 0$ and $z_3(p) > 0$. At $p^4 = (2/5,6/25,9/25)^\top$ the corresponding activity level vector is $y = (7/18,0)^\top$ with $\tilde{z}(p^4,y) = (-1/6,5/18,0)^\top$. Observe that at p^4 the price ratio of commodity 3 is equal to the minimum price ratio, i.e. $p_3^4/p_3^0 = p_1^4/p_1^0 = 4/5$. Then, the adjustment process keeps both price ratios equal and minimal while bringing commodity 3 in excess supply by a further increase of y_1. At $y_1 = 1/2$ commodity 1 becomes in equilibrium $(\tilde{z}(p^4,(1/2,0)^\top) = (0,1/6,-1/9)^\top)$. Thus, at p^4, the adaptation of the activity level changes $\text{sgn}\,\tilde{z}$ from $(-1,+1,0)^\top$ into $(0,+1,-1)^\top$. Then the process continues by adjusting the prices and y_1 such as to keep commodity 1 in equilibrium, while the profit of activity 1 is still equal to zero. The price ratio of good 1 is allowed to vary between the minimal price ratio p_3/p_3^0 and the maximal price ratio p_2/p_2^0. In the figure the process moves from p^4 towards f. At $p^* = (2/5,4/15,1/3)^\top$ the corresponding activity level vector $y^* = (1/2,0)^\top$ is such that $\tilde{z}(p^*,y^*) = 0$. Hence (p^*,y^*) is an equilibrium of this economy.

Example 5.2. In this example the excess demand function is based on Scarf [2] and is given by $z(p) = (p_2-p_3,p_3-p_1,p_1-p_2)^\top$. The activities are $a^1 = (-3/2,1,1)^\top$ and $a^2 = (-1,-1,3/2)^\top$. All information is graphically presented in Figure 5.2. The unique equilibrium price vector of the related pure exchange economy is $q^* = (1/3,1/3,1/3)^\top$. Also in this example q^* lies outside S_A^2 being the convex hull of the vectors ℓ,p^*,f and $e(1)$.

Now, let the process start at $(p^0,0)$, with $p^0 = (1/2,2/5,1/10)^\top$. The excess demand at that price vector equals $z(p^0) = (3/10,-2/5,1/10)^\top$. Thus, commodity 2 is in excess supply while the goods 1 and 3 are in excess demand. The process leaves $(p^0,0)$ by proportionally increasing the prices of the commodities 1 and 3, whereas the price of 2 is decreased. The activity levels remain zero because the process generates prices in

$\mathrm{int}(S_A^2)$, i.e. the profits are negative. Projected on the price set, the process moves from p^0 in the direction opposite of e(2) (see Figure 5.2).

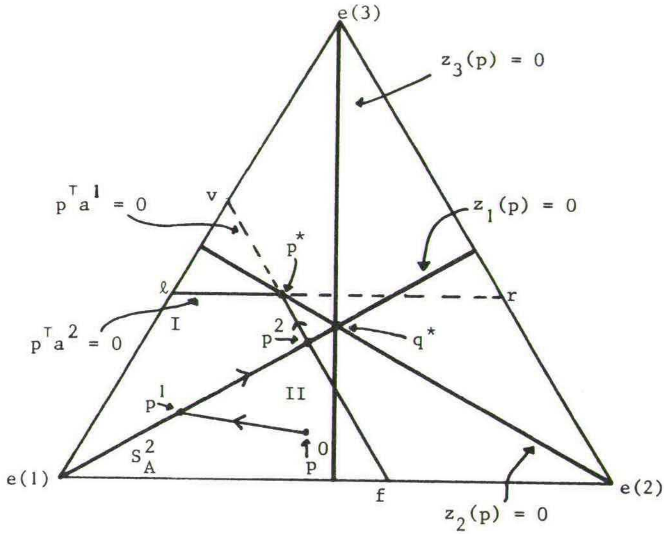

Figure 5.2. The set S_A^2 for the economy of Example 5.2. For a price vector in region I, II the sign of the vector z(p) equals $(-1,-1,+1)^T$, $(+1,-1,+1)^T$, respectively.

At $(p^1,0)$, with $p^1 = (5/7,1/7,1/7)^T$, the market of good 1 becomes in equilibrium ($z(p^1) = (0,-4/7,4/7)^T$). Now, the process adapts the prices such that the market of good 1 remains in equilibrium while the price ratio of good 1 is allowed to vary between the minimum price ratio p_2/p_2^0 and the maximum price ratio p_3/p_3^0. In the figure the process goes from p^1 into the direction of q^*. At $p^2 = (2/5,3/10,3/10)^T$ where $z(p^2) = (0,-1/10,1/10)^T$, activity 1 makes zero profit so that its activity level y_1 is increased from zero. However, an increase of y_1 at constant prices immediately yields an excess demand situation on the market for good 1

$(\tilde{z}_1(p^2,(y_1,0)^\top) = z_1(p^2) - y_1.a_1^1 = 0 + y_1 3/2 > 0)$. Thus, to keep good 1 in equilibrium the prices have to be adjusted simultaneously making the consumers excess demand for good 1 negative. Thus, projected on S_A^2 the process moves from p^2 towards v keeping the profit of activity 1 equal to zero. At $p^* = (2/5,1/5,2/5)^\top$ the consumers excess demand $z(p^*)$ equals $(-1/5,0,1/5)^\top$ and the level $\hat{y}_1$ making $\tilde{z}_1(p^*,(\hat{y}_1,0)^\top) = 0$ is equal to 2/15. It holds that $\tilde{z}(p^*,(\hat{y}_1,0)^\top) = (0,-2/15,1/15)^\top$. Besides, at p^* the profit level of activity 2 becomes zero so that the activity level y_2 is increased from zero. Because good 1 is an input for both activities, an increase in y_2 must be met by a decrease in y_1 to keep commodity 1 in equilibrium. Furthermore, it cannot happen that y_1 becomes zero. This because then y_2 must be equal to 1/5 in order to keep good 1 in equilibrium whereas $\tilde{z}(p^*,(0,1/5)^\top) = (0,1/5,-1/10)^\top$. Thus, when going from $y = (2/15,0)^\top$ to $y = (0,1/5)^\top$ the market for good 2 turns from an excess supply situation into an excess demand. The opposite occurs for good 3. Thus, the increase in y_2 leads to a decrease in y_1 but before y_1 becomes zero the market for good 2 becomes in equilibrium at $y^* = (2/25,2/25)^\top$. But then also good 3 must be in equilibrium and (p^*,y^*) is an equilibrium of this economy.

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