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THE ESTIMATION OF MIXED DEMAND SYSTEMS

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Abstract

In a mixed demand system one treats the prices of some goods as exogenous and the corresponding quantities as endogenous, while for the other goods the situation is reversed. The coefficients of such a system are related to those of a regular or of an inverse demand system. Estimating a regular demand system, taking into account the endogenous nature of some of the prices yields indirectly estimates of the mixed system, in a much more convenient way than direct estimation would have done in view of the theoretical constraints on the coefficients. The paper proposes a maximum likelihood estimation procedure which it applies to the market of fresh and preserved vegetables in Belgium 1975 - 1984.

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1. Introduction

In applied demand analysis the quantities demanded are usually explained as a function of prices and total expenditure. Complete systems of such demand functions describe in this way all (groups of) commodities in the budget of the consumer. They reflect the basic assumptions about utility maximizing behaviour of the consumer by the parametrization of the functions. The Rotterdam demand system of Theil (1965) may serve as an example of such a regular system.

Actually, the first law of demand as formulated by Davenant in 1699 explained the price of corn as a function of the available quantity of corn. Such "inverse" demand functions underly the work of Antonelli (1886). For more recent theoretical treatment see Katzner (1970) or Anderson (1980). Theil (1976) estimated an inverse demand system under its mode as a regular system. Salvas-Bronsard *et al.* (1977) estimated such a system directly. A complete system of inverse demand functions displays properties analogous to those of a regular demand system. These properties are of considerable use in estimation. Inverse demand functions appear to be specifically suitable for the explanation of the price formation of quickly perishable goods, like fish - see e.g. Barten and Bettendorf (1989).

Regular and inverse demand systems are both extreme cases with either all quantities or all prices endogenous. One can think of a situation where of some commodities the prices are endogenous and the corresponding quantities exogenous while the reverse holds for the other commodities. Such a mixed system has been put forward by Samuelson (1965). Bronsard and Salvas-Bronsard (1980) and Chavas (1984) analyze it extensively from a theoretical point of view. Bronsard and Salvas-Bronsard also provide estimates of a mixed system for 7 Canadian consumer categories with Food and Clothing as the price endogenous goods.

A natural way to estimate a mixed demand system it to first write it in reduced form and then estimate the equations. This is usually done for regular

regular and inverse demand systems. One starts from the first-order conditions for a maximum of the utility function subject to the budget constraint and solves these for the quantities in the regular case and for the prices in the inverse case. In the mixed context one would solve for the endogenous prices and the endogenous quantities. The disturbance terms of such "reduced forms" can taken to be independent of the right-hand side variables. There is then no problem of inconsistency of estimation due to "simultaneity".

In the case of the regular and inverse system it is fairly simple to select a parametrization which reflects the constraints implied by the structural formulation in a way which is easy to take into account when estimating the system. This property does not hold to the same extent for mixed systems.

Another approach is possible: estimate either a regular or an inverse demand system taking into account the endogenous nature of some of the right-hand side variables. One can then fully benefit from the simple parametrization properties of those systems without being inconsistent in estimation. For estimation one can use an instrumental variables approach with the exogenous quantities and prices as instruments. Theil (1976) estimated an inverse demand system for meat (U.S.A.) in its mode as a regular system with the quantities as instruments. Meyermans (s.a.) used such an approach for the estimation of a mixed system for meat and vegetables (Belgium) with the exogenous quantities and prices as the instruments. It is not so simple to impose the negativity condition of demand in this way.

One can also apply a maximum likelihood estimation procedure with a properly specified Jacobian transformation determinant. This is the approach taken here. It builds upon the maximum likelihood approach to estimate demand systems put forward in Barten (1969) and Barten and Geyskens (1975). The parametrization used is one of a regular Rotterdam system.

The approach is applied to the demand for vegetables. Under the assumption of weak separability of preferences the group of vegetables can be isolated from the rest of the consumer choice problem. Total real expenditure on vegetables is taken to be exogenously determined at a higher allocation level. Our data is quarterly. For the type of vegetables considered, a quarter is too short to let supply adjust. Price will adjust to let demand equal supply. For canned or otherwise preserved vegetables the prices are taken to be set by the producer and the quantity will adjust.

In the next section the theory of mixed demand systems is briefly reviewed. The issue of the parametrization of such systems is taken up in Section 3. It prepares the way for the formulation of the likelihood function. The application to the market for vegetables in Belgium, first quarter 1975 - last quarter 1984, follows in section 5. The paper ends with some concluding remarks.

2. Some theory of mixed demand functions

Let $q \in \mathbb{R}^n_+$ be the vector of quantities of n commodities. We assume that a preference order is defined on all admissible q-vectors that can be represented by a well-behaved real-valued utility function.

$$u(q) \tag{2.1}$$

i.e. a strongly quasi-concave, monotone increasing function, at least twice differentiable with the second-order derivatives being continuous functions of q.

The n commodities have positive prices per unit: p. The product of prices and quantities adds up to the given budget m:

$$p'q = m \tag{2.2}$$

Defining $\pi = (1/m)p$ one can write (2.2) also as

$$\pi'q = 1 \tag{2.3}$$

with π being the positive vector of "normalized" prices.

The consumer's optimum is the vector q^* that maximizes $u(q)$ among all q satisfying (2.2) and (2.3). Under the assumptions made the q^* will satisfy (2.3) and the Second Law of Gossen:

$$\frac{\partial u(q^*)}{\partial q} = \lambda\pi \tag{2.4}$$

On the left-hand side one has the vector of first-order derivatives of (2.1) - the marginal utilities - evaluated for $q = q^*$. The scalar λ is the Lagrange multiplier associated with constraint (2.3).

Solution of (2.3) and (2.4) with respect to q^* and λ for given π yields the Marshallian demand functions

(2.5) $$q^* = f(\pi)$$

Such a system of regular demand functions indicates the quantities consumed for given prices π. It satisfies (2.3). Hence $\pi' f(\pi) = 1$ and the Cournot aggregation condition

(2.6) $$\pi' \frac{\partial f}{\partial \pi'} = -f(\pi)'$$

Inserting $f(\pi)$ for q in (2.1) yields the indirect utility function

(2.7) $$v(\pi) = u(f(\pi)) = \max_q (u(q) \,|\, \pi' q = 1)$$

This is a monotone decreasing function in π. One can maximize $v(\pi)$ with respect to π subject to $\pi'\bar{q} = 1$. Here $\bar{q}$ is taken to be fixed. One has

(2.8) $$\frac{\partial v(\pi^*)}{\partial \pi} = \mu \bar{q}$$

as first-order conditions from which the inverse demand functions

(2.9) $$\pi^* = g(\bar{q})$$

can be obtained. These express the (normalized) prices one is willing to pay for a given bundle $\bar{q}$.

To show that inverse system (2.9) is indeed the inverse of (2.5) one can proceed as follows. One has from (2.7), (2.4) and (2.6)

(2.10) $$\frac{\partial v}{\partial \pi'} = \frac{\partial u}{\partial q'} \frac{\partial f}{\partial \pi'} = \lambda \pi' \frac{\partial f}{\partial \pi'} = -\lambda f(\pi)'$$

holding for any π, so also for $\pi = \pi^*$. In that case also (2.8) applies. Because of (2.3)

$$\mu = \pi^{*\prime} \frac{\partial v(\pi^*)}{\partial \pi} = -\lambda$$

and therefore $f(\pi^*) = \bar{q}$, which is the inverse of (2.9). Otherwise said, π^* are the prices which induce a consumer to purchase the vector $\bar{q}$ out of free choice.

In view of this relationship between the regular and inverse demand systems they both can be derived from the solution of (2.3) and (2.4), in the regular case with respect to the quantities given prices and in the inverse case with respect to the prices given quantities.

While it may be true that for some commodities the quantities adjust to the prices, it is very much possible that for other commodities the prices adjust to the supplied quantities. Think of quickly perishable goods like fresh fish or fresh vegetables. By definition these cannot be stored without losing some of their quality. Partition the set of commodities into two subsets such that

$$q = (q_1', \bar{q}_2')' \qquad \pi = (\bar{\pi}_1', \pi_2')'$$

with q_1 being endogenous and $\bar{q}_2$ exogenous while π_2 is endogenous and $\bar{\pi}_1$ exogenous. (The exogenous variables are barred.) The counterpart of (2.3) is now

$$\bar{\pi}_1' q_1 + \pi_2' \bar{q}_2 = 1 \tag{2.11}$$

To derive a mixed demand system explaining the endogenous prices and quantities in terms of the exogenous quantities and prices Samuelson (1965) formulates the <u>utility potential</u>

$$z(q_1, \bar{q}_2, \bar{\pi}_1, \pi_2) = u(q_1, \bar{q}_2) - v(\bar{\pi}_1, \pi_2) \tag{2.12}$$

with u(.) being the direct utility function and v(.) the indirect one. Clearly, z is a monotone increasing function of its arguments. For prices and quantities satisfying (2.11) its maximum is clearly zero. At this maximum

$$\frac{\partial z(q_1^*, \bar{q}_2, \bar{\pi}_1, \pi_2^*)}{\partial q_1} = \frac{\partial u(q_1^*, \bar{q}_2)}{\partial q_1} = \nu \bar{\pi}_1 \tag{2.13}$$

$$(2.14) \qquad \frac{\partial z(q_1^*,\bar{q}_2,\bar{\pi}_1,\pi_2^*)}{\partial \pi_2} = - \frac{\partial v(\bar{\pi}_1,\pi_2^*)}{\partial \pi_2} = \nu \bar{q}_2$$

These conditions can be solved for q_1^* and π_2^* yielding the system of mixed demand equations

$$(2.15) \qquad q_1^* = h_1(\bar{\pi}_1,\bar{q}_2)$$

$$(2.16) \qquad \pi_2^* = h_2(\bar{\pi}_1,\bar{q}_2)$$

Obviously, the case of all prices exogenous is a special case of (2.12) with (2.5) as the result, while the case of all quantities exogenous is the opposite extreme with inverse system (2.9) as the outcome. However, the relation between a mixed demand system and the regular and inverse system goes deeper.

Let $f_1(.)$ and $f_2(.)$ be the obvious partitions of $f(.)$ and let $g_1(.)$ and $g_2(.)$ be the corresponding subvectors of $g(.)$. Then one can state

$$(2.17) \qquad q_1^* = h_1(\bar{\pi}_1,\bar{q}_2) = f_1(\bar{\pi}_1,\pi_2^*)$$

$$(2.18) \qquad \bar{q}_2 = f_2(\bar{\pi}_1,\pi_2^*)$$

$$(2.19) \qquad \bar{\pi}_1 = g_1(q_1^*,\bar{q}_2)$$

$$(2.20) \qquad \pi_2^* = h_2(\bar{\pi}_1,\bar{q}_2) = g_2(q_1^*,\bar{q}_2)$$

To see this one may start from

$$\frac{\partial v(\bar{\pi}_1,\pi_2^*)}{\partial \pi_2'} = \frac{\partial u(q_1^*,\bar{q}_2)}{\partial q_1'} \frac{\partial f_1(\bar{\pi}_1,\pi_2^*)}{\partial \pi_2'} + \frac{\partial u(q_1^*,\bar{q}_2)}{\partial q_2'} \frac{\partial f_2(\bar{\pi}_1,\pi_2^*)}{\partial \pi_2'}$$

$$= \nu\bar{\pi}_1' \frac{\partial f_1}{\partial \pi_2'} + \frac{\partial u}{\partial q_2'} \frac{\partial f_2}{\partial \pi_2'}$$

$$= \left[\frac{\partial u}{\partial q_2'} - \nu\pi_2^{*\prime}\right]\frac{\partial f_2}{\partial \pi_2'} - \nu f_2 = -\nu\bar{q}_2$$

where use is made of (2.13), (2.6) and (2.14). Otherwise said

$$\left[\frac{\partial u(q_1^*,\bar{q}_2)}{\partial q_2'} - \nu\pi_2^{*\prime}\right]\frac{\partial f_2}{\partial \pi_2'} = \nu(f_2(\bar{\pi}_1,\pi_2^*) - \bar{q}_2)$$

Here $\partial f_2/\partial \pi_2'$ is a square and generally speaking nonsingular matrix. Thus (2.18) holds if and only if

$$\frac{\partial u(q_1^*,\bar{q}_2)}{\partial q_2'} = \nu\pi_2^{*\prime}$$

Combining this condition with (2.13) one has the Second Law of Gossen. Together with (2.15) and (2.16) this implies (2.17) through (2.20).

The conclusion of these various equivalences is that the Second Law of Gossen can serve as an unifying starting point for the derivation of regular, inverse and mixed demand systems. One simply solves this condition (and $\pi'q = 1$) for the relevant endogenous variables (and λ) in terms of the exogenous variables. Depending on the particular choice of what is endogenous and what exogenous one obtains the desired system. Because of this common basis one can easily switch from one system to the other by simply relabeling an exogenous price as endogenous and the corresponding endogenous quantity as exogenous or vice versa. This switching property is employed in the next section.

3. Functional Specification

The previous section applies to any type of parametrization. Here we will use one in particular: the Rotterdam specification first proposed by Theil (1965) and used in many applications. A regular demand equation of the Rotterdam variety for time series is written here as

$$(3.1) \qquad \bar{w}_{it}\Delta \ln q_{it} = b_i \sum_k \bar{w}_{kt}\Delta \ln q_{kt} + \sum_j s_{ij}\Delta \ln p_{jt} + u_{it}$$

where Δ is the operator of taking first backward differences

q_{it} is the quantity of good i

p_{jt} is the price of good j

u_{it} is a disturbance term

$\bar{w}_{it} = (w_{it} + w_{i,t-1})/2$ with

$w_{it} = p_{it}q_{it}/m_t$, the budget share of good i

$m_t = \sum_k p_{kt}q_{kt}$, total expenditure

b_i, s_{ij} are constants

$i, j, k = 1, \ldots, n$

t is time subscript

We will not go into the details of this specification here but simply note that for the marginal propensities to consume b and the Slutsky substitution coefficients s_{ij} the following properties hold:

$$(3.2) \qquad \sum_i b_i = 1, \ \sum_i s_{ij} = 0 \qquad \text{(adding-up)}$$

$$(3.3) \qquad \sum_j s_{ij} = 0 \qquad \text{(homogeneity)}$$

$$(3.4) \qquad s_{ij} = s_{ji} \qquad \text{(symmetry)}$$

$$(3.5) \qquad \sum_i \sum_j x_i s_{ij} x_j < 0 \qquad \text{(negativity)}$$

if at least one x_i is different from the other x's.

To simplify notation we will use

(3.6) $\quad \Delta \ln y_{it} = \bar{w}_{it} \Delta \ln q_{it}$

We will define

(3.7) $\quad \Delta \ln Q_t = \sum_k \bar{w}_{kt} \Delta \ln q_{kt} = \sum_k \Delta \ln y_{kt}$

which may be seen as the relative change in real total expenditure, or in the average quantity or in the quantity index. In what follows $\Delta \ln Q_t$ is taken to be exogenously determined. In view of homogeneity condition (3.3) one may replace the p_{jt} in (3.1) by $\pi_{jt} = p_{jt}/m_t$.

With all these notational conventions we rewrite (3.1) as

(3.8) $\quad \Delta \ln y_t = b_i \Delta \ln Q_t + \sum_j s_{ij} \Delta \ln \pi_{jt} + u_{it}$

In obvious matrix notation we write the full system as

(3.9) $\quad \Delta \ln y_t = b \Delta \ln Q_t + S \Delta \ln \pi_t + u_t$

with the following properties following from (3.2) - (3.5):

(3.10) $\quad \iota' b = 1 \qquad \iota' S = 0'$ (adding-up)

(3.11) $\quad S\iota = 0$ (homogeneity)

(3.12) $\quad S = S'$ (symmetry)

(3.13) $\quad x'Sx < 0, \ \forall x \neq \alpha\iota$, a real scalar (negativity)

Here ι is a vector all elements equal to one. One may add to these properties the one that the rank of the n x n matrix S is n-1 in a way that all principal minors of (n-1) by (n-1) are nonzero.

It follows from (3.10) and from (3.7) that

(3.14) $\quad \iota' u_t = 0$

The covariance matrix $\Omega = E(u_t u_t')$ is then a singular matrix satisfying

$$(3.15) \qquad \iota'\Omega = 0'$$

The Rotterdam specification is attractive because it allows one to express the constraints on the system in terms of the estimated parameters.

As explained in the preceding section a mixed demand system can be obtained from a regular demand system by treating some of the prices as endogenous and the corresponding quantities as exogneous. Let this be the case for the first $n_1 < n$ commodities. For the $n-n_1$ remaining goods the quantities are endogenous and the prices exogenous. (This order is the reverse of that in the preceding section in the interest of convenient exposition). Dropping the time subscript and using the subscript 1 or 2 to indicate the category (3.9) is rewritten as

$$(3.16) \qquad \begin{bmatrix} \Delta \ln y_1 \\ \Delta \ln y_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Delta \ln Q + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \Delta \ln \pi_1 \\ \Delta \ln \pi_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Here $\Delta \ln \pi_1$ and $\Delta \ln y_2$ are endogenous, $\Delta \ln \pi_2$, $\Delta \ln y_1$ and $\Delta \ln Q$ are exogenous.

Since S_{11} is nonsingular one has

$$(3.17) \qquad \Delta \ln \pi_1 = -S_{11}^{-1} b_1 \Delta \ln Q + S_{11}^{-1} \Delta \ln y_1 - S_{11}^{-1} S_{12} \Delta \ln \pi_2 - S_{11}^{-1} u_1$$

This result can be used to eliminate $\Delta \ln \pi_1$ on the right-hand side of the relation for $\Delta \ln y_2$ in (3.16):

$$(3.18) \qquad \begin{aligned} \Delta \ln y_2 = {} & (b_2 - S_{21} S_{11}^{-1} b_1) \Delta \ln Q + S_{21} S_{11}^{-1} \Delta \ln y_1 \\ & + (S_{22} - S_{21} S_{11}^{-1} S_{12}) \Delta \ln \pi_2 + u_2 - S_{21} S_{11}^{-1} u_1 \end{aligned}$$

Subsystems (3.17) and (3.18) together form the mixed demand system. It can be written as

$$(3.19)\quad \begin{bmatrix}\Delta\ln\pi_1\\ \Delta\ln y_2\end{bmatrix} = \begin{bmatrix}-S_{11}^{-1}b_1\\ b_2-S_{21}S_{11}^{-1}b_1\end{bmatrix}\Delta\ln Q$$

$$+ \begin{bmatrix}S_{11}^{-1} & -S_{11}^{-1}S_{12}\\ S_{21}S_{11}^{-1} & S_{22}-S_{21}S_{11}^{-1}S_{12}\end{bmatrix}\begin{bmatrix}\Delta\ln y_1\\ \Delta\ln\pi_2\end{bmatrix} + \begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

where the random components are defined by

$$(3.20)\quad \begin{bmatrix}v_1\\ v_2\end{bmatrix} = \begin{bmatrix}-S_{11}^{-1} & 0\\ -S_{21}S_{11}^{-1} & I\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

An alternative version is

$$(3.21)\quad \begin{bmatrix}\Delta\ln\pi_1\\ \Delta\ln y_2\end{bmatrix} = \begin{bmatrix}c_1\\ c_2\end{bmatrix}\Delta\ln Q + \begin{bmatrix}R_{11} & R_{12}\\ R_{21} & R_{22}\end{bmatrix}\begin{bmatrix}\Delta\ln y_1\\ \Delta\ln\pi_2\end{bmatrix} + \begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

with the following properties on the parameters

$$(3.22)\quad \iota'c_2 = 1,\qquad \iota'R_{21} = -\iota,\qquad \iota'R_{22} = 0'\qquad \text{(adding-up)}$$

$$(3.23\quad R_{12}\iota = \iota,\qquad R_{22}\iota = 0\qquad \text{(homogeneity)}$$

$$(3.24)\quad R_{11} = R_{11}',\qquad R_{22} = R_{22}',\qquad R_{12} = -R_{21}'\qquad \text{(symmetry)}$$

$$(3.25)\quad \begin{matrix} x'R_{11}x < 0 & \forall x \neq 0, \\ x'R_{22}x < 0 & \forall x \neq \alpha\iota \end{matrix}\qquad \text{(negativity)}$$

- see also Bronsard and Salvas-Bronsard (1980). Clearly, the constraints are less easy to impose on the direct estimation of mixed demand system (3.21) than on that of regular demand system (3.9).

Before turning to the issue of estimation first some attention should be paid to the random components. As is readily verified from (3.20), the counterpart (3.14) for the mixed demand system is

$$(3.26) \qquad \iota' v_2 = 0 \quad \text{or} \quad j'v = 0$$

with $j' = (0', \iota')$. The adding-up condition applies only to the second, the quantity endogenous part of the mixed system. Let Σ be $E(v_t v_t')$. Then (3.26) means

$$(3.27) \qquad j'\Sigma = 0'$$

implying singularity of Σ.

The relation between v and u is given by (3.20). Let

$$(3.28) \qquad C = \begin{bmatrix} -S_{11}^{-1} & 0 \\ -S_{21}S_{11}^{-1} & I \end{bmatrix}$$

Note $j'C = \iota'$. The reverse of (3.20) is

$$(3.29) \qquad u = C^{-1}v$$

with

$$(3.30) \qquad C^{-1} = \begin{bmatrix} -S_{11} & 0 \\ -S_{21} & I \end{bmatrix}$$

Clearly,

$$(3.31) \qquad \Sigma = C\,\Omega\,C', \qquad \Omega = C^{-1}\,\Sigma\,C'^{-1}$$

express the relations between the two covariance matrices Σ and Ω.

Mixed demand systems (3.19) or (3.21) give the basic decision rules with the exogenous variables on the right-hand side. The disturbance v_1 and v_2 are then distributed independently of $\Delta\ln Q$, $\Delta\ln y_1$ and $\Delta\ln\pi_2$, the exogenous variables in question. It is clear that both u_1 and u_2 involve v_1, the random component of $\Delta\ln\pi_1$. This set of variables appears on the right-hand side of (3.16). Estimating that mode of the system without taking into account the endogenous nature of $\Delta\ln\pi_1$ results in inconsistencies. Still (3.16) is more attractive to estimate than (3.21) because of the simple nature of the constraints. One can estimate (3.16) using all relevant constraints by some instrumental variables method as was done by Meyermans (s.a.). An alternative is to use a Maximum Likelihood procedure to estimate (3.16) with properly accounting for the endogenous nature of $\Delta\ln\pi_2$. That is the topic of the next section.

4. Maximizing the Likelihood

Mixed demand system (3.21) is the natural starting point for the formulation of the likelihood function. It is assumed that the vector of disturbance terms, v_t, is normally distributed with the zero vector as mean and Σ as covariance matrix. We also assume that $E(v_s v_t') = 0$ for $s \neq t$ and that, in keeping with the exogenous nature of $\Delta \ln Q_t$, $\Delta \ln y_{1t}$ and $\Delta \ln \pi_{2t}$, the v_t are distributed independently of the explanatory variables in (3.21).

Because of (3.27) the covariance matrix Σ is singular and the joint density of v_t is not defined. Delete one of the equations for endogenous quantities, i.e. an equation of the second part of the system. The reduced disturbance vector $\tilde{v}_t$ will now have a covariance matrix $\tilde{\Sigma}$ of full rank n-1. Let the deleted equation be the last. The joint density for a sample of T independent realization of the endogenous variables can then be written as

$$(4.1) \qquad \ln L_n = -\left[T(n-1)\ln 2\pi + T\ln|\tilde{\Sigma}| + \Sigma_t \tilde{v}_t' \tilde{\Sigma}^{-1} \tilde{v}_t\right]/2$$

which is <u>likelihood function</u> when $\tilde{v}_t$ is expressed in observations and unknown parameters. Following Barten (1969), <u>mutatis mutandis</u>, one can express (4.1) also as

$$(4.2) \qquad -\left[T(n-1)\ln 2\pi - T\ln n_2 + T\ln\left|\Sigma + \frac{1}{n_2} jj'\right| + \Sigma_t v_t'\left[\Sigma + \frac{1}{n_2} jj'\right]^{-1} v_t\right]$$

which is the <u>quasi-likelihood function</u>. It is independent of the identity of the deleted equation, as long as this is one of the second subset. Maximizing (4.1) with respect to the coefficients of the first n-1 equations results in the same value as maximizing (4.2) with respect to the coefficients of the full system. This means that the identity of the deleted equation does not matter for mixed demand systems in the same sense as it does not matter in the case of regular (or inverse) systems - see Barten (1969). Adding-up conditions (3.22), (3.26) and (3.27) will enable one to reconstruct the required information for the deleted equation.

Because no lack of information is involved we will work with (4.1). As said earlier we will not estimate the coefficients of the mixed demand system directly, but rather those of the regular system. Likelihood function (4.1) has then to be expressed in terms of $\tilde{\Omega}$ and $\tilde{u}_t$, where $\tilde{\Omega}$ is Ω with the last row and column deleted and $\tilde{u}_t$ is the u_t vector without the last element. It follows from (3.20) that

$$(4.3) \qquad \tilde{v}_t = C_* \tilde{u}_t$$

C_* is the matrix C, defined by (3.28), with the last row and column deleted. Consequently,

$$(4.4) \qquad \tilde{\Sigma} = C_* \tilde{\Omega} C_*'$$

Then

$$(4.5) \qquad \tilde{v}_t' \tilde{\Sigma}^{-1} \tilde{v}_t = \tilde{u}_t' C_*' C_*'^{-1} \tilde{\Omega}^{-1} C_*^{-1} C_* \tilde{u}_t = \tilde{u}_t' \tilde{\Omega}^{-1} \tilde{u}_t$$

and

$$(4.6) \qquad \ln|\tilde{\Sigma}| = \ln|\tilde{\Omega}| + 2\ln|C_*|$$

We can then rewrite (4.1) as

$$(4.7) \qquad \ln L_n = -\left[T(n-1)\ln 2\pi + T\ln|\tilde{\Omega}| + 2T\ln|C_*| + \Sigma_t \tilde{u}_t' \tilde{\Omega}^{-1} \tilde{u}_t\right]/2$$

which differs from the likelihood function of a system with all prices exogenous by the presence of $T\ln|C_*|$.

It is useful to look somewhat closer into $|C_*|$. The matrix C_* is a lower block-triangular matrix. Its determinant is then given by $|-S_{11}^{-1}|$. Since S_{11} is a negative definite matrix $-S_{11}^{-1}$ is a positive definite matrix. One clearly has $\ln|C_*| = -\ln|-S_{11}|$. Let S_* be the matrix S of (3.9) with last row and column deleted. Barten and Geyskens (1975) use the Cholesky decomposition

(4.8) $$S_* = -BHB'$$

where B is a lower triangular matrix with ones on the diagonal and H is a (n-1) x (n-1) diagonal matrix with the Cholesky values $h_1, \ldots, h_{n-1}$ as diagonal elements. From (4.8) it follows that

(4.9) $$S_{11} = -B_1H_1B_1'$$

where B_1 is the $n_1 \times n_1$ leading block of B and H_1 is the $n_1 \times n_1$ diagonal matrix of the first n_1 Cholesky values. It follows from the nature of B_1 that $|B_1| = 1$. Consequently

(4.10) $$\ln|-S_{11}| = 2\ln|B_1| + \ln|H_1| = \sum_{i=1}^{n_1} \ln h_i$$

We now rewrite (4.7) as

(4.8) $$\ln L_n = -\left[T(n-1)\ln 2\pi + T\ln|\tilde{\Omega}| - 2T\sum_{i=1}^{n_1}\ln h_i + \sum_t \hat{u}_t'\tilde{\Omega}^{-1}\hat{u}_t\right]/2$$

One can next follow the same path as outlined in Barten and Geyskens except that the first- and second-order derivatives of $\ln L_n$ with respect to the h_i have to be adjusted. It turns out that these adjustments involve only a minor change in the computer program package DEMMOD which was originally designed for regular demand systems.

In most of the earlier experiments the Cholesky values h_i were less than one in absolute value. Their logarithm is then negative. For fixed h_i the likelihood of a mixed demand system will be less than that of a regular demand system.

One can expect that for the mixed demand system the estimates of the h_i, $i = 1, \ldots, n$, will be somewhat higher, i.e. closer to one, than for the regular system in order to reduce the maximum in the least way. This means that also the estimates of Slutsky matrix S will tend to be higher for the mixed case than for the regular case.

5. The Vegetable Market

Vegetables come to the market fresh or preserved. Storage costs for fresh vegetables are rather high while it takes some time to grow additional supplies. One can expect that the price is set to absorb the given supply of fresh vegetables. The possibility of destroying part of the supply to maintain a minimum price exists but is rarely used. Imports of fresh vegetables are relatively unimportant. Canned or otherwise preserved vegetables are easy to store without losing their quality. The difference between demand and supply can be bridged by changes in inventories rather than by price adjustments. Of course, canned and fresh vegetables are mutual substitutes. The price formation of fresh vegetables takes into account the prices set for canned ones. The seasonal variations in the supply of fresh vegetables will be partly compensated by opposite variations in the demand for canned vegetables. The market for vegetables appears to be well suited for description by a mixed demand system.

The models of the preceding sections express individual consumer rationality. We will assume that they also valid in the aggregate, for the whole market.

These models also apply to the full consumer allocation problem. To what extent can they be used for vegetables only? Under weak separability of the preferences in vegetables and various other commodity groups (meat, clothing, etcetera) the demand for the group of vegetables as a whole can be described as a function of total available means and the price indexes of the groups. The demand for vegetables as a group or rather its log-change $\Delta\ln Q_t$ acts as the explanatory variable of the subsystem for a particular market. For this market only relative prices matter, not the general price index of the group. If all prices go up by the same factor also m, total expenditure for the group goes up by that factor and the $\pi_i = p_i/m$ remain unchanged. The endogeneity of some of the relative prices in the subsystem is not in contradiction with the exogenous nature of $\Delta\ln Q_t$. The models presented earlier can be meaningfully applied to the market for vegetables.

The data to which the mixed demand system is applied are quarterly data collected by the Agricultural Econòmic Institute of the Belgian Ministry of Agriculture. This Institute observes the purchasing behaviour for foodstuffs of a shifting panel of about 300 families and publishes quarterly average prices and quantities. Our data stand for the first quarter of 1975 and ends for the last quarter of 1984. The time series cannot be easily extended after this last observation because the format of the published data changed.

From the available data some 12 types of vegetables were selected: cauliflower, lettuce, spinach, tomatoes, carrots, Belgian endives, Brussels sprouts, beans, frozen spinach, canned tomatoes, canned peas and carrots and frozen beans. The first 8 form the category of fresh vegetables, the last 4 are of the preserved kind.

There is in principle no major difficulty in handling a system of 12 types of vegetables. For the purpose of a numerical illustration, however, a system of lower dimension suffices. The 12 kinds of vegetables have therefore been aggregated to a set of 8 composed as follows:

1. CFSS (.12): cauliflower, Brussels sprouts
2. LTSP (.14): lettuce, spinach
3. CTBN (.13): carrots, beans
4. TOMA (.26): tomatoes
5. BEND (.26): Belgian endives
6. PCBC (.04): canned peas and carrots, frozen beans
7. SPIF (.02): frozen spinach
8. TOMC (.03): canned tomatoes

The numbers in between brackets are the shares of expenditure on the type of vegetables in the total budget for vegetables averaged over the sample period. Obviously the fresh vegetables dominate the preserved ones.

The data for the fresh vegetables display considerable seasonal variability. Tomatoes e.g. are low in quantity in the first quarter and high in the third one. Their prices show an opposite pattern. The compensating price variation is not enough to eliminate seasonal effects from the expenditure shares, which range from 7 percent of the first quarter of 1976 to 50 percent in the third quarter of 1980. To allow for the possibility that the seasonal variability in supply of fresh vegetables is not completely absorbed by price changes seasonal dummies have been added to the equations of the system. As the system is in first differences of the variables also first differences of the seasonal dummies have been taken. This means that, for example, the dummy for the first quarter has a one in quarter one and minus one in quarter two and zero in quarter three and four. Four of such quarter dummies are fully collinear. The one for the second quarter has therefore been deleted. The coefficients of the remaining season dummies measure the difference with respect to the second quarter.

The data cover 40 quarters. Taking first differences leaves one with 39 usable observations. These have been employed to estimate the coefficients of a regular demand system with endogenous prices for the fresh vegetables. The results for the b_i and the s_{ij} are given in Table 1.

Adding-up conditions (3.2) are met automatically. The homogeneity and symmetry conditions are imposed. Negativity condition (3.5) is satisfied freely. The estimated coefficients characterize equilibrium relationships between prices and quantities. They do not represent a pure impulse-response effect. One may observe that TOMA and BEND have b_i values larger than their average expenditure shares. Specifically endives have a strong b value. This vegetable is commonly considered a luxury. The other (than TOMA and BEND) vegetables, fresh or not, have all rather low marginal propensities to consumers. That for SPIF, frozen spinach, is even negative, but not significantly so.

Table 1. Estimates of b and S with endogenous prices for fresh vegetables, Belgium 1975 - 1984 [a]

i. type	b_i	s_{ij}							
		CFSS	LTSP	CTBN	TOMA	BEND	PCBC	SPIF	TOMC
1. CFSS	.016 *.056*	-.412 *.039*	.216 *.034*	-.053 *.019*	.030 *.042*	.177 *.044*	.037 *.011*	-.009 *.006*	.014 *.007*
2. LTSP	.060 *.059*	.216 *.034*	-.275 *.037*	.063 *.017*	.054 *.040*	-.063 *.043*	-.006 *.008*	.018 *.005*	-.007 *.005*
3. CTBN	.013 *.029*	-.053 *.019*	.063 *.017*	-.167 *.013*	.028 *.023*	.121 *.025*	.006 *.005*	-.006 *.005*	.009 *.003*
4. TOMA	.298 *.073*	.030 *.043*	.054 *.040*	.028 *.023*	-.511 *.058*	.321 *.055*	.029 *.011*	.029 *.006*	.019 *.007*
5. BEND	.587 *.090*	.177 *.044*	-.063 *.043*	.121 *.025*	.321 *.055*	-.591 *.077*	.036 *.011*	.002 *.005*	-.003 *.006*
6. PCBC	.017 *.011*	.037 *.011*	-.006 *.008*	.006 *.005*	.029 *.011*	.036 *.011*	-.073 *.011*	-.023 *.006*	-.007 *.007*
7. SPIF	-.001 *.005*	-.009 *.006*	.018 *.005*	-.006 *.003*	.029 *.006*	.002 *.005*	-.023 *.006*	-.026 *.007*	.014 *.005*
8. TOMC	.009 *.009*	.014 *.007*	-.007 *.005*	.009 *.003*	.019 *.007*	-.003 *.006*	-.007 *.007*	.014 *.005*	-.040 *.007*

[a] Asymptotic standard errors are given in italics

The Slutsky coefficients s_{ij} are in absolute value somewhat larger than one usually finds in a system of this size. Of the 28 independently estimated ones 19 are twice their asymptotic standard errors, in absolute value, which is also better than usual given that there are 39 observations. Of the 21 pairs of different goods 7 s_{ij} have the negative sign of Hicksian complementarity. Of these 2 are significantly negative, namely that for CTBN (carrots and beans) and CFSS (cauliflower and Brussels sprouts) and for SPIF (frozen spinach) and PCBC (peas, carrots and beans, preserved). Domination of substitution is plausible, not only because of the mathematical properties of the matrix S, but also because of the nutritional properties of these vegetables.

Given the point estimates of Table 1 one can calculate the coefficients of (3.19) or (3.21), the mixed demand system. The results are given in Table 2. Under the assumptions made these coefficients correspond with impulse-response effects. They are "reduced form" coefficients. No asymptotic standard errors have been calculated. The results satisfy properties (3.22) through (3.25).

Note that in Table 2 the first five equations have the log-change in (normalized) prices as dependent variables, while the last three the log-change in quantities (multiplied by the w_i). After $\Delta\ln Q$, the first five exogenous variables are the log-change in quantities (multiplied by the w_i) and the last three the log-change in prices. Only the coefficients of c_1 and R_{12} are elasticities. All the others would have to be divided and/or multiplied by the relevant expenditure shares to turn them into elasticities.

One may note that the effect of the exogenous quantities as represented by R_{11} and R_{21} is uniformly negative while that of $\Delta\ln Q_t$ is positive, even strongly positive for the five price formation equations. Part of $\Delta\ln Q$ is due to $\Delta\ln y_1$ - see (3.7). One can separate that part out from $\Delta\ln Q$ and attribute it to $\Delta\ln y_1$. Let

$$\Delta\ln Q_t = \bar{W}_{1t}\Delta\ln Q_{1t} + \bar{W}_{2t}\Delta\ln Q_{2t}$$

with

Table 2. Estimates of c and R with endogenous prices for fresh vegetables, Belgium 1975 - 1984

i. type	c_i	r_{ij}							
		CFSS	LTSP	CTBN	TOMA	BEND	PCBC	SPIF	TOMC
1. CFSS	5.214	-8.297	-7.335	-4.759	-4.870	-5.318	.627	.185	.187
2. LTSP	5.351	-7.335	-10.63	-6.170	-5.100	-5.092	.579	.251	.169
3. CTBN	5.964	-4.759	-6.170	-12.25	-5.568	-6.298	.602	.177	.221
4. TOMA	5.769	-4.870	-5.100	-5.568	-6.654	-5.667	.583	.221	.196
5. BEND	6.338	-5.318	-5.092	-6.298	-5.667	-7.109	.627	.188	.185
6. PCBC	.612	-.627	-.579	-.602	-.583	-.627	-.010	-.003	.013
7. SPIF	.196	-.185	-.251	-.177	-.221	-.188	-.003	-.017	.020
8. TOMC	.192	-.187	-.169	-.221	-.196	-.185	.013	.020	-.033

$$\Delta\ln Q_{1t} = \Sigma_{i=1}^{5}(\bar{w}_{it}/\bar{W}_{1t})\Delta\ln q_{it} = (1/\bar{W}_{1t})\Sigma_{i=1}^{5}\Delta\ln y_{it}$$

$$\Delta\ln Q_{2t} = \Sigma_{i=6}^{8}(\bar{w}_{it}/\bar{W}_{2t})\Delta\ln q_{it} = (1/\bar{W}_{2t})\Sigma_{i=6}^{8}\Delta\ln y_{it}$$

$$\bar{W}_{1t} = \Sigma_{i=1}^{5}\bar{w}_{it},\ \bar{W}_{2t} = \Sigma_{i=6}^{8}\bar{w}_{it}$$

Here, $\Delta\ln Q_{1t}$ is the average log-change for the goods of the quantity exogenous groups. A similar definition holds for $\Delta\ln Q_{2t}$. The $\Delta\ln Q_{1t}$ part of $\Delta\ln Q_t$ is already exogenous in its own right, because the relevant $\Delta\ln y_{it}$ are exogenous. The exogenous nature of $\Delta\ln Q_t$ implies then the additional exogeneity of $\Delta\ln Q_{2t}$. One can therefore replace $c\Delta\ln Q_t$ by

$$c\ \bar{W}_{2t}\Delta\ln Q_{2t} + c\ \Sigma_{i=1}^{5}\Delta\ln y_{it}$$

For our sample $\bar{W}_{2t}$ is in the mean .09, which scales down the c vector considerably. The r_{ij} with $j = 1, \ldots, 5$ have to be increased by c_i, which reduces their absolute value also substantially. Note that the diagonal elements of $R_{11} + c_1\iota'$ still remain negative. An increase of the supply of a good will depress its price but it might increase the price or quantity of another good. This becomes clear from Table 3 which states the total effects of exogenous quantity changes.

It is tempting to associate positive signs with complementarity and negative ones with substitution. Conventionally, measures of such interactions refer to situations where utility is kept constant. The entries of Table 4 do not correspond with constant utility. The Allais characterization of substitution and interaction could be fruitfully used here - see Barten (1989). We will not go further in this issue in the present paper.

A few words about the seasonal dummies. The point estimates of their coefficients together with their asymptotic standard errors are given in the part of Table 4 headed by "Regular". All entries have been multiplied by 100 for easier presentation. As already explained the coefficients refer to differences with respect to the second quarter. Note that the three columns add up to zero. The seasonal effects appear to be very strong for Belgian endives for which supply goes up in the fourth quarter to reach a peak in the

Table 3. Total effects of exogenous quantities

i. type	$r_{ij} + c_i$				
	CFSS	LTSP	CTBN	TOMA	BEND
1. CFSS	-3.083	-2.121	.455	.344	-.104
2. LTSP	-1.984	-5.279	-.819	.251	.259
3. CTBN	1.205	-.206	-6.286	.396	-.334
4. TOMA	.899	.669	.201	-.885	.102
5. BEND	1.020	1.246	.040	.671	-.771
6. PCBC	-.015	.033	.010	.029	-.015
7. SPIF	.011	-.055	.019	-.025	.008
8. TOMC	.005	.023	-.029	-.004	.007

Table 4. Seasonal effects for the Vegetables Market Belgium 1975 - 1984

type	Regular			Mixed		
	Q1	Q3	Q4	Q_1	Q_3	Q_4
1. CFSS	-5.37 *1.71*	-9.20 *5.42*	-9.76 *2.75*	-18.2	-38.3	-33.3
2. LTSP	-8.07 *1.78*	-1.65 *5.38*	-4.75 *2.81*	-56.7	-44.1	-66.6
3. CTBN	-4.41 *.920*	.211 *2.94*	-5.90 *1.45*	-.136	10.2	-10.8
4. TOMA	-15.1 *2.15*	-17.1 *7.06*	-32.0 *3.61*	-.359	-24.7	-33.6
5. BEND	33.8 *2.68*	24.9 *8.65*	50.0 *4.33*	57.7	24.1	61.5
6. PCBC	-.362 *.420*	.105 *1.33*	.968 *.678*	1.37	-.851	1.29
7. SPIF	-.478 *.211*	1.72 *.687*	.717 *.350*	-1.26	.539	-.994
8. TOMC	-.068 *.232*	1.01 *.757*	.626 *.388*	-.112	.312	-.297

first quarter. The second quarter sees a decline and the bottom is reached in the third quarter. The prices move inversely. Apparently in the fourth and first quarter the price decrease is not enough and in the third quarter the price increase is too strong as follows from the positive coefficients of the quarter dummies. Non-price factors appear to pick up the extra supply.

This interpretation is confirmed by the transformation of the dummy coefficients by matrix C, defined by (3.28). This transformation gives the values for the dummy coefficients in the mixed mode of the system. They are given in the last three columns of Table 4. The first five rows show the seasonal effects on the price formation. Positive signs mean that the prices are not low enough in comparison to the situation of the second quarter. Negative signs indicate the opposite.

The seasonal effects are quite important in terms of explanatory power of the model. Still they are somewhat puzzling. Their attribution to shifts in preferences is disputable. The data refer to household demand and are net of the effects of market interventions or import/export fluctuations. The seasonal effects reflect an undeniable seasonal pattern in the part of consumer behaviour not explained by exogenous quantity and price changes.

It is of some interest to compare the results given in Table 1 with those obtained under the assumption that all quantities are endogenous and all prices (and $\Delta \ln Q$) are exogenous. Table 5 presents the point estimates. The b_i display roughly the same pattern as in Table 1. The s_{ij} are in absolute value usually smaller. This corresponds with the higher Cholesky values obtained for the mixed case as was to be expected. Table 6 gives the two sets of Cholesky values together with their standard errors, calculated as if the model in question were the correct one. The first five Cholesky values correspond with the S_{11}-part of matrix S. They are all substantially higher in the price endogenous case than for the price exogenous situation.

Table 5. Estimates of b and S with endogenous prices for fresh vegetables, Belgium 1975 - 1984 [a]

i. type	b_i	s_{ij}							
		CFSS	LTSP	CTBN	TOMA	BEND	PCBC	SPIF	TOMC
1. CFSS	.039	-.130	.005	-.029	.040	.017	.024	-.000	.015
	.019	*.024*	*.013*	*.010*	*.020*	*.019*	*.009*	*.005*	*.006*
2. LTSP	.030	.005	-.034	-.002	-.020	.026	.014	.007	.003
	.015	*.013*	*.011*	*.007*	*.014*	*.014*	*.006*	*.003*	*.003*
3. CTBN	.027	-.029	-.002	-.115	.019	.061	.002	-.001	.006
	.018	*.010*	*.007*	*.009*	*.014*	*.015*	*.004*	*.002*	*.003*
4. TOMA	.282	.040	-.020	.019	-.136	.072	.017	.009	-.001
	.035	*.020*	*.014*	*.014*	*.038*	*.033*	*.009*	*.005*	*.005*
5. BEND	.613	.017	.026	.061	.072	-.181	.008	.007	-.008
	.046	*.019*	*.014*	*.015*	*.033*	*.044*	*.008*	*.004*	*.005*
6. PCBC	.006	.024	.014	.002	.017	.008	-.051	-.016	.002
	.008	*.009*	*.006*	*.004*	*.009*	*.008*	*.012*	*.007*	*.007*
7. SPIF	-.000	-.000	.007	-.001	.009	.007	-.016	-.024	.017
	.004	*.005*	*.003*	*.002*	*.005*	*.004*	*.007*	*.007*	*.005*
8. TOMC	.004	.015	.003	.006	-.001	-.008	.002	.017	-.035
	.005	*.006*	*.003*	*.003*	*.005*	*.005*	*.007*	*.005*	*.007*

[a] Asymptotic standard errors are given in italics

Table 6. Cholesky values of S with and without endogenous prices [a]

	h_i						
	1	2	3	4	5	6	7
with five endogenous prices	.412 *.039*	.162 *.016*	.153 *.010*	.468 *.049*	.141 *.012*	.010 *.008*	.016 *.009*
with all prices exogenous	.130 *.024*	.033 *.011*	.108 *.012*	.107 *.044*	.062 *.012*	.010 *.010*	.018 *.009*

[a] Asymptotic standard errors are given in italics

The asymptotic standard errors are usually smaller in the exogenous case. Because of the lower absolute values of the s_{ij} also here 19 of the 28 independent coefficients are in absolute value more than twice their standard error.

One can consider $\ln|\tilde{\Omega}|$ as a measure of the generalized variance of the regular demand system and $\ln|\tilde{\Sigma}|$ as its counterpart for the mixed demand system. The former is minimized when all prices are exogenous and obtains the value of -65.670. Its value, given the b and S of Table 1 and, of course, the dummy coefficients, is -60.215, which is larger, as is to be expected. The conversion of $\ln|\tilde{\Omega}|$ to $\ln|\tilde{\Sigma}|$ can be achieved by subtracting $2\ \Sigma_i \ln h_i$. For the price endogenous case one obtains for $\ln|\tilde{\Sigma}|$ the value of -28.067, while the price exogenous variant yields a value of -22.999, clearly larger. As a very rough goodness of fit test this comparison fails to reject. Each variant produces the least variance for the case for which it is appropriate. It is beyond the scope of the present paper to develop a more refined test procedure which could sort out for which goods the prices are exogenous and for which the quantities are given.

6. Concluding remarks

Mixed demand systems are in between the polar cases of regular demand systems with all prices exogenous and inverse demand systems with all quantities exogenous. They are realistic when for some commodities the inventory costs are substantial and prices adjust to available supply while for other goods one can let the quantities demanded adjust to the prices and absorb eventual differences between demand and supply by rather cheap inventory changes.

All these modes of demand systems reflect the basic consumer equilibrium consisting of the budget identity and the Second Law of Gossen. This means that one can start from any mode and solve it for the appropriate set of endogenous variables.

In this paper the regular demand system with the Rotterdam parametrization has served as a starting point for the formulation of a mixed demand system. One of the attractions of the Rotterdam specification is the ease by which one can take into account theoretical constraints on the coefficients. This property is to a certain extent lost in the transition to a mixed demand system.

One way to have your cake and eat it is to estimate the system in its regular mode while taking into account the endogenous nature of some of the prices. One can easily incorporate the various constraints while avoiding inconsistencies of estimation. Following a maximum likelihood approach this turns out to require only a minor adjustment of the estimation procedure for a regular system with all prices exogenous.

The market for vegetables in Belgium provided quarterly data for the period 1975 - 1984. Eight (groups of) vegetables were selected, five of them fresh, the remaining three frozen, canned or otherwise preserved. The prices of the fresh vegetables were taken to be endogenous, those of the others as exogenous.

Seasonal dummies were added to absorb the obvious seasonal pattern in the residuals. Given the fact that the seasonal variation in the supply of fresh vegetables should have fully been reflected in their prices the presence of an unexplained season is puzzling.

The results show that taking into account the endogenous nature of some of the prices tends to increase the absolute value of the price effects in the equilibrium relations. The results are as a whole reasonable but intuition is lacking to serve as the touchstone of plausibility.

One further step could be the use of Allais coefficients to obtain an idea of the pattern of complementarity and substitution implied by the estimates.

One would also like to obtain standard errors of some nature for the various derived coefficients. A Monte Carlo procedure could be a possibility.

Another line of further research is the setting-up of tests for the selection of the commodities for which prices are endogenous and quantities exogenous and for which the reverse holds.

What is perhaps needed is a model which endogenizes this choice on the basis of inventory and/or adjustment costs. A further extension is to model the planting decisions in response to the relative prices obtained in the recent past. Such a dynamic general equilibrium model then explains both prices and quantities in their development over time.

Empirical work usually answers some questions but raises at the same time a host of other ones left unanswered. The present paper is no exception.

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