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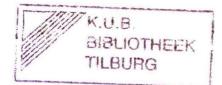# AXIOMATIC CHARACTERIZATIONS OF THE WALRAS CORRESPONDENCE FOR GENERALIZED ECONOMIES

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Axiomatic characterizations of the Walras correspondence for generalized economies[1]

by

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Abstract

A generalized (pure exchange) economy consists of a list of agents, utility functions and initial endowments for each agent, and a net trade vector, that describes the trade between the economy and the outside world. In this paper we extend the definition of Walras equilibria to generalized economies without free disposal of commodities. An existence result is presented and we provide two axiomatic characterizations of Walras allocations based on the axioms of consistency and converse consistency. Also, a characterization of the utility vectors corresponding to strictly positive Walras allocations is given. It is shown that the assumption of free disposal can, for generalized economies, lead to non-Pareto optimal Walras allocations.

1 Introduction

The theory of competitive equilibria in exchange economies was first formulated in *Walras* (1874) and it has been studied extensively since. The existence of Walras equilibria is the subject of investigation in *Arrow* and *Debreu* (1954) (see also *McKenzie* (1954)), who provide an existence proof in the general case, and in *Mas-Colell* (1974), who provided an existence theorem without assuming completeness or transitivity of preferences. For a historical survey of the development of the theory of Walras equilibria the reader is referred to Arrow and Hahn (1971).

The goal of this paper is to axiomatize the Walras correspondence by means of its consistency properties. The consistency principle unifies important developments in diverse areas ranging from abstract game theoretic models to concrete taxation and apportionment problems. Various solution concepts have been characterized on the basis of consistency properties. This line of research was initiated by *Sobolev* (1975), who axiomatized the prenucleolus, and it was followed by (among others) *Peleg* (1985, 1986) (axiomatizations of the core and the prekernel), *Lensberg* (1988) (axiomatization of the Nash bargaining solution), *Hart* and *Mas-Colell* (1989) (axiomatization of the Shapley value), *Peleg* and *Tijs* (1994), *Norde et al.* (1993), and *Peleg et al.* (1994) (axiomatizations of solutions of strategic games), *van Heumen et al.* (1994) (axiomatizations of solutions of Bayesian games), and by *Peters et al.* (1994) (axiomatizations of the Kalai-Smorodinsky and the egalitarian bargaining solutions). A survey on parts of this line of research is provided in *Thomson* (1990).

The existing attempts to characterize the Walras correspondence may be divided into three categories: the social choice approach, the indirect approach, and the direct approach.

In the social choice approach the initial endowments of the traders are fixed and, as a result, an exchange economy is completely determined by the profile of utilities of its members. Hence, the Walras correspondence can be viewed as a social choice correspondence and one can use suitable social choice axioms in order to obtain a characterization. Papers that follow this approach are *Nagahisa* (1991, 1992, 1994), *Nagahisa* and *Suh* (1993), and *Gevers* (1986). In the current paper we do not fix the initial bundles and therefore our approach cannot be placed in this category.

In the indirect approach one attempts to characterize the Walras correspondence by comparing it to other well-known correspondences or by its implementability properties. For example, *Dagan* (1994) characterizes the Walras correspondence as a subcorrespon-

dence of the core which has some additional properties, including consistency and a form of converse consistency. His proof uses the *Debreu* and *Scarf* (1963) limit theorem. For a comprehensive list of papers that follow the indirect approach we refer to *Thomson* (1988, section 5).

The third category consists of papers that deal with the specific properties of the competitive equilibrium. Two examples are the *Thomson* (1988, 1992) papers. However, the first paper deals with competitive equilibria from equal initial bundles, and the second paper is an exploratory study that mainly contains definitions. Hence, our results are not covered by the papers of *Thomson*.

Our approach can be classified as a direct one. In order to deal with a variable number of agents we use the concept of generalized economies due to *Thomson* (1992) (see also *Dagan* (1994)). A generalized (pure exchange) economy is a list of agents, utility functions and initial endowments for each agent, and a net trade vector, that describes the trade between the economy and the outside world. We focus on Walras allocations, allocations of the commodities available that are part of a Walras equilibrium. In order to guarantee the existence of a Walras equilibrium when there is no trade with the outside world (i.e. the net trade vector equals 0) we make the standard assumptions that the initial endowments are strictly positive and that the utility fuctions are continuous, quasi-concave, and satisfy local non-satiation. In order to prove converse consistency of the Walras correspondence we also assume that the utility functions are smooth.

The paper is organized as follows. In section 2 we formally introduce generalized economies. Then, in section 3, we prove that under the assumption that there is no free disposal of commodities the Walras corresponence is the unique solution that satisfies consistency, converse consistency, Pareto optimality for two-agent economies, and necessity of a medium of exchange. The necessity of a medium of exchange axiom is a price oriented exchange axiom. It states that for one-agent economies the agent in the economy maximizes his utility given his budget set with respect to some price vector. Here, the price vector has to be such that the supply of commodities is equal to the demand for commodities. We show that this axiom can be replaced by non-emptiness for one-agent economies if we restrict ourselves to the family of generalized economies that have at least one Walras allocation. Section 4 contains an existence result for Walras allocations. In this section we provide sufficient conditions on generalized economies such that there exists at least one Walras allocation. In section 5 we show that the axioms consistency, Pareto optimality, and non-emptiness suffice to characterize the utility

vectors corresponding to strictly positive Walras allocations. Finally, in section 6 we briefly consider how the analysis changes if there is free disposal of commodities.

2 Generalized economies

In this section we will introduce some notations and definitions.

By $\mathbf{R}_+^\ell$ we denote the non-negative orthant of $\mathbf{R}^\ell$ and the strictly positive orthant of $\mathbf{R}^\ell$ is denoted by $\mathbf{R}_{++}^\ell$. Further, for two vectors $x, y \in \mathbf{R}^\ell$ we denote $x \geq y$ $(x > y)$ if $x - y \in \mathbf{R}_+^\ell$ $(x - y \in \mathbf{R}_{++}^\ell)$. The inner product xy of two vectors x and $y \in \mathbf{R}_+^\ell$ is defined by $xy = \sum_{i=1}^\ell x_i y_i$.

A *generalized economy* is a list $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle$, where N (sometimes denoted $N(E)$) is a non-empty finite set of agents, $w^i \in \mathbf{R}_{++}^\ell$ is the strictly positive initial endowment of agent $i \in N$, $u^i : \mathbf{R}_+^\ell \to \mathbf{R}$ is the utility function of agent $i \in N$, and $\Theta \in \mathbf{R}^\ell$ is the net trade vector for the economy, satisfying $\sum_{i \in N} w^i + \Theta \geq 0$. The commodity space is $\mathbf{R}_+^\ell$, the non-negative orthant of $\mathbf{R}^\ell$. The net trade vector Θ indicates how much of each good is imported in the economy and, hence, a good that is exported from the economy will have a negative sign in the vector Θ. We will assume throughout the paper that the utility functions of the agents are continuous, quasi-concave and locally non satiated. A utility function $u : \mathbf{R}_+^\ell \to \mathbf{R}$ is called locally non satiated if for every commodity bundle $x \in \mathbf{R}_+^\ell$ and every open neighbourhood of x there exists a $y \in \mathbf{R}_+^\ell$ that is in this neighbourhood and that has a strictly higher utility than x $(u(y) > u(x))$. The class of all generalized economies with continuous, quasi-concave and locally non satiated utility functions is denoted by $\mathcal{E}$.

In this paper we opt for discussing economies in which there can be no free disposal of commodities. In a concluding section we will briefly discuss the influence of the possibility of free disposal of commodities on our results.

An *allocation* in a generalized economy is an assignment of resources to the agents in the economy that is a redistribution of the resources that are available in the economy. Formally, denoting $A(E)$ for the set of allocations in economy E, we have

$$A(E) = \left\{ (x^i)_{i \in N} \mid x^i \in \mathbf{R}_+^\ell \text{ for each } i \in N \text{ and } \sum_{i \in N} x^i = \sum_{i \in N} w^i + \Theta \right\}$$

for each $E \in \mathcal{E}$. An allocation $x = (x^i)_{i \in N} \in A(E)$ is *efficient* or *Pareto optimal* for E if there is no other allocation $(y^i)_{i \in N} \in A(E)$ such that $u^i(y^i) \geq u^i(x^i)$ for all $i \in N$

with strict inequality for at least one $i \in N$. It is easy to prove that efficient allocations exist since the utility functions of the agents are continuous and the set of allocations is compact.

We will explain why a net trade vector Θ appears in our economies on the basis of the concept of a reduced economy, which is important when studying consistency (see section 3). Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle \in \mathcal{E}$ be a generalized economy. Let $S \subseteq N$, $S \neq \emptyset$, and let $x \in A(E)$. The *reduced economy* of E with respect to S and x is

$$E^{S,x} = \langle S; (w^i)_{i \in S}; (u^i)_{i \in S}; \Theta + \sum_{i \in N \setminus S} (w^i - x^i) \rangle.$$

The interpretation of the reduced economy $E^{S,x}$ is the following: Suppose the agents in N agree on the allocation x and the agents in $N \setminus S$ leave the economy with their respective allocated bundles. Then the agents in S can reconsider their situation, only now they are left with a net trade obligation of $\Theta + \sum_{i \in N \setminus S} (w^i - x^i)$ with the outside world. Hence, the agents is S are now part of the reduced economy $E^{S,x}$. Note that even if the original economy is an ordinary economy ($\Theta = 0$), then the reduced economy with respect to some subset of agents and some allocation will in general have a net trade vector $\Theta \neq 0$.

We will show that the reduced economy $E^{S,x}$ is indeed a generalized economy. Since E is a generalized economy and $(x^i)_{i \in N} \in A(E)$, we have $\sum_{i \in N} x^i = \sum_{i \in N} w^i + \Theta$ and, consequently, $\sum_{i \in S} x^i = \sum_{i \in S} w^i + \sum_{i \in N \setminus S} (w^i - x^i) + \Theta$. But $x^i \in \mathbf{R}_+^\ell$ for each $i \in N$, so $\sum_{i \in S} x^i \geq 0$ and therefore

$$\sum_{i \in S} w^i + \left(\Theta + \sum_{i \in N \setminus S} (w^i - x^i) \right) \geq 0.$$

Note that we implicitly proved that $(x^i)_{i \in N} \in A(E)$ implies that $(x^i)_{i \in S} \in A(E^{S,x})$. Further, it is straightforward to check that for $E \in \mathcal{E}$, $(x^i)_{i \in N} \in A(E)$ and $S \subseteq T \subseteq N(E)$, $S \neq \emptyset$, it holds that $E^{S,x} = (E^{T,x})^{S,x^T}$, where x^T denotes $(x^i)_{i \in T}$.

Let $\mathcal{E}_0 \subseteq \mathcal{E}$ be a family of generalized economies. A *solution* on $\mathcal{E}_0$ is a function ϕ which assigns to each economy $E \in \mathcal{E}_0$ a subset $\phi(E)$ of $A(E)$. Examples of solutions on the class $\mathcal{E}$ of generalized economies are the *Pareto optimal solution*, defined by $PO(E) = \{x \in A(E) \mid x \text{ is efficient } \}$ for all $E \in \mathcal{E}$, and the *Walras correspondence*, defined by $W(E) = \{(x^i)_{i \in N} \in A(E) \mid \text{ there exists a price vector } p \in \Delta \text{ such that } x^i \in D^i(p) \text{ for each } i \in N\}$. Here, $\Delta = \{p \in \mathbf{R}_+^\ell \mid \sum_{j=1}^\ell p_j = 1\}$ and $D^i(p)$ is the *demand set of agent i with respect to the price vector p* defined by $D^i(p) := \{x^i \in \mathbf{R}_+^\ell \mid px^i \leq pw^i \text{ and } u^i(x^i) \geq u^i(y) \text{ for all } y \in \mathbf{R}_+^\ell \text{ with } py \leq pw^i\}$.

Our definitions of the Pareto optimal solution and the Walras correspondence for generalized economies are extensions of the usual definitions for ordinary economies ($\Theta = 0$). Using the same line of reasoning as for ordinary economies, it can be proved that $W(E) \subseteq PO(E)$ for all generalized economies $E \in \mathcal{E}$, where the inclusion is in general a strict inclusion. Further, we easily obtain the following result with respect to Walras allocations.

Proposition 2.1 Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle$ be a generalized economy, let x be a Walras allocation for E ($x \in W(E)$), and let $p \in \Delta$ be a price vector such that $x^i \in D^i(p)$ for each $i \in N$. Then it holds that $p\Theta = 0$.

Proof. Because the utility functions of the agents are locally non satiated, it holds that $px^i = pw^i$ for each $i \in N$. Hence, $\sum_{i \in N} px^i = \sum_{i \in N} pw^i$. Further, because $x \in A(E)$, we have $\sum_{i \in N} x^i = \sum_{i \in N} w^i + \Theta$ and, consequently, $p \sum_{i \in N} x^i = p(\sum_{i \in N} w^i + \Theta)$. Now, the proposition follows by substituting $\sum_{i \in N} pw^i$ for $p \sum_{i \in N} x^i$. $\qquad \square$

3 Characterizations of the Walras correspondence

In this section we will provide two axiomatic characterizations of the Walras correspondence based on the axioms of consistency and converse consistency. Both axioms are defined using the reduced economies introduced in the previous section.

A solution ϕ on a family of economies $\mathcal{E}_0$ is *consistent* ($CONS$) if it satisfies the following condition. If $E \in \mathcal{E}_0$, $S \subseteq N(E)$, $S \neq \emptyset$, and $x \in \phi(E)$, then $E^{S,x} \in \mathcal{E}_0$ and $x^S \in \phi(E^{S,x})$. Our notion of consistency is only slightly different from *Dagan's* (1994) notion. The difference is that Dagan puts $E^{S,x} \in \mathcal{E}_0$ as a condition, whereas we put it as a consequence. Therefore, Dagan's notion seems to be weaker at first sight. However, for all families $\mathcal{E}_0 \subseteq \mathcal{E}$ that we consider in this paper it holds that $E^{S,x} \in \mathcal{E}_0$ for all $E \in \mathcal{E}_0$, $S \subseteq N$, $S \neq \emptyset$, and all $x \in A(E)$, so in the context of this paper the two definitions are equivalent. The following lemma was also proved in *Dagan* (1994).

Lemma 3.1 The Walras correspondence on the class $\mathcal{E}$ is consistent.

Proof. Let E be a generalized economy, let $S \subseteq N(E)$, $S \neq \emptyset$, and let $x \in W(E)$. We already saw that $E^{S,x} \in \mathcal{E}$. Further, there exists a price vector $p \in \Delta$ such that $x^i \in D^i(p)$ for each $i \in N$. Since $x \in A(E)$, we know that $x^S \in A(E^{S,x})$. Hence, using the fact that $x^i \in D^i(p)$ for each $i \in S$, it follows that $x^S \in W(E^{S,x})$. $\qquad \square$

Remark 3.2 Proposition 2.1 implies that the Walras correspondence will often be empty. However, it follows by lemma 3.1 and existence of Walras allocations for ordinary economies ($\Theta = 0$) that Walras allocations also exist for all reduced economies of ordinary economies with respect to Walras allocations.

Now, let us define converse consistency. A solution ϕ on a family of generalized economies $\mathcal{E}_0$ is *converse consistent* ($COCONS$) if for every $E \in \mathcal{E}_0$ with at least two agents ($|N(E)| \geq 2$) and for every $x \in A(E)$ that is efficient the following condition is satisfied. If for every $S \subseteq N(E)$, $S \notin \{\emptyset, N(E)\}$, it holds that $E^{S,x} \in \mathcal{E}_0$ and $x^S \in \phi(E^{S,x})$, then $x \in \phi(E)$. Actually, $COCONS$ is very similar to the converse consistency for the strong Nash equilibrium used in *Peleg* and *Tijs* (1994).

The Walras correspondence satisfies converse consistency if we put a smoothness condition on the utility functions.

A utility function $u : \mathbf{R}_+^\ell \to \mathbf{R}$ is *smooth* if the following conditions are satisfied:
(i) If $x \in \mathbf{R}_{++}^\ell$, $y \in \mathbf{R}_+^\ell$, and $u(x) = u(y)$, then $y \in \mathbf{R}_{++}^\ell$.
(ii) If $x \in \mathbf{R}_{++}^\ell$ then there exists a unique price vector $p \in \Delta$ such that $py \geq px$ for all $y \in G(x)$, where $G(x) = \{y \in \mathbf{R}_+^\ell \mid u(y) \geq u(x)\}$.
Note that conditions 1 and 2 imply that $u(x) > u(y)$ for all $x \in \mathbf{R}_{++}^\ell$ and $y \in \mathbf{R}_+^\ell$. Smoothness also implies weak monotonicity, i.e. if a utility function $u : \mathbf{R}_+^\ell \to \mathbf{R}$ is smooth, then $u(y) \geq u(x)$ for all $x \in \mathbf{R}_+^\ell$ and $y \in \mathbf{R}_{++}^\ell$ with $y \geq x$. Further, smoothness implies that for each generalized economy $E \in \mathcal{E}$ and each Walras allocation $x \in W(E)$ there exists a unique price vector $p \in \Delta$ such that $x^i \in D^i(p)$ for each $i \in N(E)$.

Lemma 3.3 Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle$ be such that u^i is smooth for each $i \in N$ and let x be a Walras allocation for E. Then there exists a unique price vector $p \in \Delta$ such that $x^i \in D^i(p)$ for each $i \in N$.

Proof. Obviously, there exists at least one price vector $p \in \Delta$ such that $x^i \in D^i(p)$ for each $i \in N$, by the definition of Walras allocations. To prove uniqueness, let $p \in \Delta$ be a price vector such that $x^i \in D^i(p)$ for each $i \in N$ and fix $j \in N$ be fixed. Then, because u^j is locally non satiated $px^j = pw^j$. Also, if $y \in \mathbf{R}_+^\ell$ and $u^j(y) > u^j(x^j)$, then $py > pw^j = px^j$. Thus, using continuity of u^j, $py \geq px^j$ for all $y \in \mathbf{R}_+^\ell$ such that $u^j(y) \geq u^j(x^j)$. Further, $u^j(x^j) \geq u^j(w^j)$ and $w^j \in \mathbf{R}_{++}^\ell$. Hence, we know that $x^j \in \mathbf{R}_{++}^\ell$. Now, it follows from condition (ii) of smoothness that p is unique. $\square$

Let $\mathcal{F}$ be the family of generalized economies where the agents have smooth utility functions. Note that within the family $\mathcal{F}$ we have enough assumptions to guarantee the

validity of the two welfare theorems for ordinary economies. Hence, if $E \in \mathcal{F}$ has a net trade vector $\Theta = 0$, then the two welfare theorems are valid for this economy. Further, it is easily seen that for each economy $E \in \mathcal{F}$, each $S \subseteq N(E)$, $S \neq \emptyset$, and $x \in A(E)$, the reduced economy of E with respect to S and x is a member of the family $\mathcal{F}$. It follows that the Walras correspondence is consistent on $\mathcal{F}$.

The following lemma implies that the Walras correspondence is converse consistent on $\mathcal{F}$.

Lemma 3.4 Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle \in \mathcal{F}$ and let $x \in A(E)$. Then x is a Walras allocation for E if and only if x is efficient and $x^i \in W(E^{\{i\},x})$ for all $i \in N$.

Proof. Clearly, if $x \in W(E)$ then x is efficient. Further, by consistency of the Walras correspondence we know that $x^i \in W(E^{\{i\},x})$ for all $i \in N$. Assume now that x is efficient and that $x^i \in W(E^{\{i\},x})$ for all $i \in N$. Then, for each $i \in N$ there exists a price vector p^i such that $x^i \in D^i(p^i)$. Following the proof of lemma 3.3, we see that p^i is the unique price vector such that $x^i \in D^i(p)$.

Now, by our assumption, x is Pareto optimal in E. Clearly, this implies that x is Pareto optimal in the ordinary economy $\langle N; (x^i)_{i \in N}; (u^i)_{i \in N}; 0 \rangle$ (this property will reappear in section 6 under the name self-efficiency and we refer to this section for an explanation of this property). Now, we can apply the second welfare theorem to the ordinary economy $\langle N; (x^i)_{i \in N}; (u^i)_{i \in N}; 0 \rangle$ and we conclude that there exists a $p \in \Delta$ that supports x^i for each $i \in N$. Therefore, by uniqueness of the prices p^i such that $x^i \in D^i(p^i)$ for each $i \in N$, we may conclude that $p^i = p$ for all $i \in N$. Thus, x is a Walras allocation for E. $\square$

Corollary 3.5 The Walras correspondence is converse consistent on the family $\mathcal{F}$ of generalized economies.

Proof. Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle \in \mathcal{F}$ with $|N| \geq 2$ and let $x \in A(E)$ be efficient and such that for every $S \subseteq N$, $S \notin \{\emptyset, N\}$, it holds that $x^S \in W(E^{S,x})$. Then, obviously, $x^i \in W(E^{\{i\},x})$ for all $i \in N$. Now, by lemma 3.4 it follows that $x \in W(E)$. $\square$

Smoothness of the utility functions is a necessary condition in corollary 3.5 in the sense that converse consistency may be violated if the utility functions are not smooth.

Dagan (1994) also proved that the Walras correspondence satisfies converse consistency. However, his notion of converse consistency is different from ours. The most important difference between the two notions is that the notion of *Dagan* (1994) can be applied only to generalized economies with more than two agents, whereas ours is also applicable for two-agent economies because we only consider efficient allocations.

In order to characterize the Walras correspondence we need two more axioms. A solution ϕ on a family of generalized economies $\mathcal{E}_0$ satisfies *Pareto optimality for two-agent generalized economies* $(PO(2))$ if for every two-agent generalized economy $E \in \mathcal{E}_0$ all $x \in \phi(E)$ are efficient. Further, ϕ satisfies *necessity of a medium of exchange for one-agent generalized economies* $(NMEX(1))$ if $\phi(E) = W(E)$ for every one-agent generalized economy $E \in \mathcal{E}_0$. The necessity of a medium of exchange axiom can be interpreted as a price oriented exchange axiom that states that for one-agent economies the agent in the economy maximizes his utility given his budget set with respect to some price vector such that the supply of commodities is equal to the demand for commodities.

Using lemma 3.4 and corollary 3.5 we obtain the following result.

Theorem 3.6 The Walras correspondence is the unique solution on $\mathcal{F}$ that satisfies $NMEX(1)$, $PO(2)$, $CONS$ and $COCONS$.

Proof. Clearly, the Walras correspondence satisfies $NMEX(1)$ and $PO(2)$. Further, in lemma 3.1 and corollary 3.5 we proved that the Walras correspondence satisfies $CONS$ and $COCONS$. Assume that ϕ is a solution on $\mathcal{F}$ that also satisfies the four foregoing axioms and let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle$ be in $\mathcal{F}$.

If $|N| = 1$, then $\phi(E) = W(E)$ by $NMEX(1)$.

If $|N| = 2$, let first $x \in W(E)$. Then x is efficient and, by $CONS$ of the Walras correspondence, $x^i \in W(E^{\{i\},x}) = \phi(E^{\{i\},x})$ for all $i \in N$. So, by $COCONS$ of ϕ we know that $x \in \phi(E)$. Now, let $x \in \phi(E)$. By $PO(2)$, x is efficient and $x^i \in \phi(E^{\{i\},x}) = W(E^{\{i\},x})$ for all $i \in N$ by $CONS$ of ϕ. Hence, by lemma 3.4, $x \in W(E)$.

We proceed by induction on the number of agents in an economy. Suppose that we proved that the Walras correspondence and ϕ coincide for all economies with at most k agents ($k \geq 2$). Now, suppose $|N| = k + 1$.

If $x \in W(E)$, then x is efficient and, by the induction hypothesis, $x^S \in W(E^{S,x}) = \phi(E^{S,x})$ for every $S \subseteq N$, $S \notin \{\emptyset, N\}$ and by $COCONS$ of ϕ it holds that $x \in \phi(E)$.

If $x \in \phi(E)$, then $x^S \in \phi(E^{S,x}) = W(E^{S,x})$ for every $S \subseteq N$, $S \neq N$, by $CONS$ of ϕ. Note that we cannot apply $COCONS$ of the Walras correspondence, because we do not know whether x is efficient. Therefore, we proceed in the following way. For each $i \in N$ it holds that x^i is part of a Walras allocation in some reduced economy and, therefore, there is a price vector $p^i \in \Delta$ such that $x^i \in D^i(p^i)$. It follows from the proof of lemma 3.3 that each p^i is unique. We claim that $p^i = p^j$ for any two agents i and $j \in N$, for if $p^i \neq p^j$, consider the reduced economy $E^{S,x}$ where $S = \{i, j\}$. Since $x^S \in W(E^{S,x})$, there is some price vector p such that both $x^i \in D^i(p)$ and $x^j \in D^j(p)$.

Then, either $p \neq p^i$ or $p \neq p^j$, which means that either x^i or x^j is supported by two different price vectors, contradicting unicity of the supporting price vectors. Now, we may conclude that there exists a price vector p such that $x^i \in D^i(p)$ for each $i \in N$. But this means that $x \in W(E)$. □

Dagan (1994) also provides a characterization of the Walras correspondence using consistency and converse consistency. Besides the fact that his notion of converse consistency is different from ours, there is a more important difference between the two characterizations. *Dagan* (1994) needs a domain of economies that includes all economies with a finite number of agents (variable number of agents), whereas it follows from the proof of theorem 3.6 that we only need a domain that satisfies the condition that for every economy in the domain all its reduced economies are also in the domain.

In our characterization we can replace the 'necessity of a medium of exchange' axiom by a 'non-emptiness' axiom if we restrict the family of economies.

Let $\mathcal{G}$ be the family of generalized economies where agents have smooth utility functions and that allow for a Walras allocation, i.e. $\mathcal{G} = \{E \in \mathcal{F} \mid W(E) \neq \emptyset\}$. A solution ϕ on $\mathcal{G}$ satisfies *non-emptiness for one-agent generalized economies* $(NEM(1))$ if $\phi(E) \neq \emptyset$ for every one-agent generalized economy $E \in \mathcal{G}$.

Theorem 3.7 The Walras correspondence is the unique solution on $\mathcal{G}$ that satisfies $NEM(1)$, $PO(2)$, $CONS$, and $COCONS$.

Proof. Clearly, the Walras correspondence satisfies $NEM(1)$, $PO(2)$, and $COCONS$ on $\mathcal{G}$. Further, it follows from $CONS$ of the Walras correspondence on $\mathcal{F}$ that for each economy $E \in \mathcal{G}$, each $S \subseteq N(E)$, $S \neq \emptyset$, and $x \in W(E)$, the reduced economy of E with respect to S and x is a member of the family $\mathcal{G}$. It follows that the Walras correspondence is consistent on $\mathcal{G}$.

Now, let ϕ be a solution on $\mathcal{G}$ that also satisfies the four axioms. If $E \in \mathcal{G}$ with $|N(E)| = 1$, then $|A(E)| = 1$ and, because both ϕ and the Walras correspondence satisfy $NEM(1)$, $\phi(E) = W(E) = A(E)$. This proves that ϕ satisfies $NMEX(1)$ on $\mathcal{G}$. Now we can follow the proof of theorem 3.6 and show that $\phi(E) = W(E)$ for all $E \in \mathcal{G}$. □

Remark 3.8 If ϕ is a solution on $\mathcal{G}$ that satisfies $PO(2)$ and $CONS$, then $\phi(E) \subseteq W(E)$ for all $E \in \mathcal{G}$.

This remark can easily be proved using the ideas in the proofs of theorems 3.6 and 3.7.

Remark 3.9 The axiom $COCONS$ is not redundant in the characterization in theorem 3.7. This can be seen as follows. Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle \in \mathcal{G}$. Then

i, $j \in N$ are called *symmetric* if $w^i = w^j$ and $u^i = u^j$. An allocation $x \in A(E)$ satisfies *equal treatment* if $x^i = x^j$ whenever i and j are symmetric. Consider the equal treatment Walras correspondence $W_e(E) = \{x \in W(E) \mid x$ satisfies equal treatment$\}$. This correspondence satisfies $NEM(1)$, $PO(2)$, and $CONS$ on $\mathcal{G}$. Further, $W_e(E) \subseteq W(E)$, for each $E \in \mathcal{G}$, where the inclusion is in general a strict inclusion.

4 An existence result

In this section we will provide a sufficient condition on generalized economies such that there exists at least one Walras allocation.

Let E be a generalized economy where the net trade vector Θ is such that it has at least one coordinate that is strictly positive and at least one coordinate that is stricly negative. This seems to be a reasonable assumption because the case $\Theta = 0$ is known and for $\Theta \neq 0$ that are strictly positive or strictly negative we have non-existence when there is only one agent in the economy. We will compactify the consumption sets of the agents and provide conditions on the compactified economy that are sufficient for the existence of Walras allocations. Existence of Walras allocations for the generalized economy then follows by weak monotonicity of the utility functions.

Let $\rho \in \mathbf{R}_+$ and let E_ρ be the economy which is obtained from E when the consumption sets of the agents are replaced by the cube $Q_\rho = \{x \in \mathbf{R}_+^\ell \mid x_j \leq \rho$ for all $j \in \{1, \ldots, \ell\}\}$. Assume that ρ is large enough, so that $\sum_{i \in N} w^i$ and $\sum_{i \in N} w^i + \Theta$ are interior points of Q_ρ. For $p \in \Delta$ and $i \in N$ denote by $D_\rho^i(p)$ the demand set of agent i with respect to the price vector p, i.e. $D_\rho^i(p) := \{x^i \in Q_\rho \mid px^i \leq pw^i$ and $u^i(x^i) \geq u^i(y)$ for all $y \in Q_\rho$ with $py \leq pw^i\}$. For $p \in \Delta$ denote by $z(p) = \sum_{i \in N} D_\rho^i(p) - \sum_{i \in N} w^i - \Theta$, the excess demand correspondence of E_ρ at p. Also denote $L^+ = \{j \mid \Theta_j > 0\}$ and $L^- = \{j \mid \Theta_j < 0\}$. Note that both L^+ and L^- are non-empty. Further, suppose that the following condition is satisfied: for all $p \in \Delta$ there exists a $\zeta \in z(p)$ such that $\zeta\Theta = 0$ and

$$\frac{\zeta_i}{\Theta_i} \leq \frac{\zeta_j}{\Theta_j}$$

for all $i \in L^-$ and all $j \in L^+$. This condition is rather strong, but this is not surprising, because it is rarely satisfied that there exist Walras allocations if the net vector Θ is not equal to 0.

Theorem 4.1 If all the conditions above are satisfied, then E_ρ has a Walras allocation.

Proof. For $p \in \Delta$ let $z^*(p)$ be the set of all $\zeta \in z(p)$ that satisfy the conditions mentioned above, i.e. $z^*(p) = \{\zeta \in z(p) \mid \zeta\Theta = 0 \text{ and } \frac{\zeta_i}{\Theta_i} \leq \frac{\zeta_j}{\Theta_j} \text{ for all } i \in L^- \text{ and } j \in L^+\}$. Then the multifunction z^* has non-empty and convex values and it is upper semi-continuous. Now, let $A \subseteq \mathbf{R}^\ell$ be such that $\bigcup_{p \in \Delta} z^*(p) \subseteq A$. We may choose A to be convex and compact. For $\zeta \in A$ we define

$$f(\zeta) = \{q \in \Delta_\Theta \mid q\zeta \geq \overline{q}\zeta \text{ for all } \overline{q} \in \Delta_\Theta\},$$

where $\Delta_\Theta = \{q \in \Delta \mid q\Theta = 0\}$. Then the multifunction f has non-empty convex values and it is upper semi-continuous.

Since both z^* and f are non-empty and convex valued upper semi-continuous multifunctions, the multifunction h defined by $h(p, \zeta) = f(\zeta) \times z^*(p)$ for $(p, \zeta) \in \Delta \times A$, has a fixed point $(\overline{p}, \overline{\zeta}) \in f(\overline{\zeta}) \times z^*(\overline{p})$. We will show that $\overline{\zeta} = 0$, which implies that E_ρ has a Walras allocation.

Because $\overline{p} \in f(\overline{\zeta}) \subseteq \Delta_\Theta$, we know that $\overline{p}\Theta = 0$ and $\overline{p}\overline{\zeta} \geq q\overline{\zeta}$ for all $q \in \Delta_\Theta$. Further, since $\overline{\zeta} \in z^*(\overline{p})$, we know by local non satiation that $\overline{p}\overline{\zeta} = \overline{p}\Theta$. Hence, $0 \geq q\overline{\zeta}$ for all $q \in \Delta_\Theta$. Defining $r = -\frac{1}{\Theta_i} + \frac{1}{\Theta_j}$, it is easily seen that $\frac{1}{r}\left(-\frac{1}{\Theta_i}e^i + \frac{1}{\Theta_j}e^j\right) \in \Delta_\Theta$, for all $i \in L^-$ and $j \in L^+$. Hence, $q\overline{\zeta} \leq 0$ for all $q \in \Delta_\Theta$ implies in particular that $0 \geq -\frac{\overline{\zeta}_i}{\Theta_i} + \frac{\overline{\zeta}_j}{\Theta_j}$ for all $i \in L^-$ and $j \in L^+$. However, $\overline{\zeta} \in z^*(\overline{p})$ implies $\frac{\overline{\zeta}_j}{\Theta_j} \geq \frac{\overline{\zeta}_i}{\Theta_i}$ for $i \in L^-$ and $j \in L^+$. We conclude that $\frac{\overline{\zeta}_i}{\Theta_i} = \frac{\overline{\zeta}_j}{\Theta_j}$ for all $i \in L^-$ and $j \in L^+$. Hence, $\overline{\zeta} = k\Theta$ for some $k \in \mathbf{R}$. Because $\overline{\zeta} \in z^*(\overline{p})$, it holds that $\overline{\zeta}\Theta = 0$ and, hence, we conclude that $k = 0$ and, consequently, $\overline{\zeta} = 0$. $\square$

Now, let $E \in \mathcal{F}$ be a generalized economy where the agents have smooth utility functions and let $\rho \in \mathbf{R}_+$ be such that the conditions in theorem 4.1 are satisfied. Then it follows by weak monotonicity of the utility functions that $W(E_\rho) \subseteq W(E)$ and, hence, using theorem 4.1 we see that the set of Walras allocations for E is non-empty.

5 A characterization of the Walras utilities

In this section we will provide a characterization of the vectors of utilities that correspond to strictly positive Walras equilibria, based on consistency and non-emptiness.

Since we only want to consider strictly positive Walras allocations and their utility vectors, we introduce the notation

$$W_{++}(E) = W(E) \cap \mathbf{R}^\ell_{++}$$

for all $E \in \mathcal{E}$. Further, we no longer need condition (i) of smoothness, so we define $\mathcal{F}_{++}$ to be the family of all generalized economies in $\mathcal{E}$ where the agents have utility functions that satisfy condition (ii) of smoothness. Further, let $\mathcal{G}_{++}$ be the family of generalized economies in $\mathcal{F}_{++}$ that allow for a strictly positive Walras allocation.

Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle \in \mathcal{E}$. We define a function $U_E : A(E) \to \mathbf{R}^{N(E)}$ by $U_E\left((x^i)_{i \in N(E)}\right) = \left((u^i(x^i))_{i \in N(E)}\right)$ for every $x \in A(E)$. We denote

$$U(E) = \{U_E(x) \mid x \in A(E)\}.$$

A u-*solution* on $\mathcal{E}$ is a function ψ on $\mathcal{E}$ that assigns to each $E \in \mathcal{E}$ a subset $\psi(E)$ of $U(E)$. The u-*Walras correspondence* is defined by

$$uW(E) = \{(u^i(x^i))_{i \in N(E)} \mid (x^i)_{i \in N(E)} \in W_{++}(E)\}$$

for all $E \in \mathcal{E}$.

In the next lemma we show that we can enlighten utility vectors corresponding to strictly positive Walras allocations. Lemma 5.1 is a modified 'ancestors property' for the u-Walras correspondence (see *Peleg et al.* (1994)).

Lemma 5.1 Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle \in \mathcal{G}_{++}$, $x \in W_{++}(E)$ and $p \in \Delta$ such that $x^i \in D^i(p)$ for each $i \in N$. Then there exists a generalized economy $\overline{E} = \langle N \cup \{0\}; (w^i)_{i \in N \cup \{0\}}; (u^i)_{i \in N \cup \{0\}}; \Theta \rangle \in \mathcal{G}_{++}$ with the property that $uW(\overline{E}) = \{v\}$, where $v^0 = pw^0$ and $v^i = u^i(x^i)$ for all $i \in N$.

Proof. Pick an agent $0 \notin N$ and endow this agent with a utility function u^0 such that $u^0(y) = py$ for all $y \in \mathbf{R}^\ell$. Further, the initial endowment of agent 0 is some $w^0 \in \mathbf{R}^\ell_{++}$. We enlarge the economy E by adding agent 0, so that we obtain the economy

$$\overline{E} = \langle N \cup \{0\}; (w^i)_{i \in N \cup \{0\}}; (u^i)_{i \in N \cup \{0\}}; \Theta \rangle.$$

Note that $(y^i)_{i \in N \cup \{0\}} := (w^0, x^1, \dots, x^n)$ is a Walras allocation for $\overline{E}$, because $y^i \in D^i(p)$ for each $i \in N \cup \{0\}$. This implies that $(pw^0, (u^i(x^i))_{i \in N}) \in uW(\overline{E})$.

To prove the other inclusion $(uW(\overline{E}) \subseteq \{(pw^0, (u^i(x^i))_{i \in N})\})$, we will first show that a price vector $p' \neq p$ does not support a strictly positive Walras allocation for economy $\overline{E}$. Notice that agent 0 has a linear utility function. This implies that for a price vector $p' \neq p$, an allocation $y \in D^0(p')$ has the property that $y_k = 0$ for at least one $k \in \{1, \dots, l\}$. Hence, for each price vector $p' \neq p$ it holds that $D^0(p') \cap \mathbf{R}^\ell_{++} = \emptyset$. Now, the proof is concluded by noting that $U_{\overline{E}}(y) = U_{\overline{E}}(z)$ for all Walras allocations y and z for economy $\overline{E}$ that are supported by price vector p. $\quad\square$

In the proof of lemma 5.1 we use a linear utility function. Clearly, this utility function does not satisfy condition (i) of smoothness. It is for this reason that we have to restrict to strictly positive Walras allocations.

In order to characterize the u-Walras correspondence, we need the following three axioms.

A u-solution ψ on a family $\mathcal{E}_0 \subseteq \mathcal{E}$ satisfies *non-emptiness* (NEM) if $\psi(E) \neq \emptyset$ for every $E \in \mathcal{E}_0$.

A u-solution ψ on a family $\mathcal{E}_0 \subseteq \mathcal{E}$ is *Pareto optimal* (PO) if for every $E \in \mathcal{E}_0$ and every $v \in \psi(E)$ all $x \in A(E)$ with $U_E(x) = v$ are efficient allocations. Note that this is equivalent to saying that for every $E \in \mathcal{E}_0$ and every $v \in \psi(E)$ it holds that there is no allocation $y \in A(E)$ such that $u^i(y) \geq v^i$ for all $i \in N(E)$ with strict inequality for at least one $i \in N(E)$.

A u-solution ψ on a family $\mathcal{E}_0 \subseteq \mathcal{E}$ is *consistent* ($CONS$) if for all $E \in \mathcal{E}_0$ and all $v \in \psi(E)$ the following condition is satisfied. If $x \in A(E)$ is such that $U_E(x) = v$, then $E^{S,x} \in \mathcal{E}_0$ and $(v^i)_{i \in S} \in \psi(E^{S,x})$ for all $S \subseteq N(E)$, $S \neq \emptyset$.

Theorem 5.2 The u-Walras correspondence is the unique u-solution on $\mathcal{G}_{++}$ that satisfies NEM, PO, and $CONS$.

Proof. It is easily verified that the u-Walras correspondence satisfies NEM and PO on $\mathcal{G}_{++}$. Consistency of the u-Walras correspondence on $\mathcal{G}_{++}$ follows from the observation that if $x \in W(E)$ and $y \in A(E)$ such that $U_E(y) = U_E(x)$, then $y \in W(E)$ (this is a well-known property of the Walras allocations, see e.g. the non-discrimination axiom in *Nagahisa* (1991)).

Now, let ψ be a u-solution on $\mathcal{G}_{++}$ that satisfies the three axioms and let $E \in \mathcal{G}_{++}$. If $|N(E)| = 1$, then $|A(E)| = |U(E)| = 1$ and by NEM $uW(E) = \psi(E) = U(E)$. If $|N(E)| \geq 2$, let $v \in \psi(E)$ and $x \in A(E)$ such that $U_E(x) = v$. Then, by PO of ψ, x is an efficient allocation. Also, $x^i \in A(E^{\{i\},x})$ and, by $CONS$ of ψ, $U_{E^{\{i\},x}}(x^i) = v^i \in \psi(E^{\{i\},x})$ for all $i \in N(E)$. Because $E^{\{i\},x}$ is a one-agent economy, this implies that $v^i \in uW(E^{\{i\},x})$ and, hence, $x^i \in W_{++}(E^{\{i\},x})$ for all $i \in N(E)$. Now, lemma 3.4 implies that $x \in W_{++}(E)$. Hence, $v = U_E(x)$ is in $uW(E)$. Thus, $\psi(E) \subseteq uW(E)$ for all $E \in \mathcal{G}_{++}$.

We shall use lemma 5.1 to prove that $uW(E) \subseteq \psi(E)$ for all $E \in \mathcal{G}_{++}$. Let $E \in \mathcal{G}_{++}$ and let $v \in uW(E)$. Choose $x \in W_{++}(E)$ such that $v = U_E(x)$. Further, let $p \in \Delta$ be a price vector such that $x^i \in D^i(p)$ for each $i \in N(E)$. Consider the economy $\overline{E}$ that we defined in lemma 5.1. For this economy we have $uW(\overline{E}) = \{\overline{v}\}$, where $\overline{v}^0 = pw^0$

and $\bar{v}^i = u^i(x^i)$ for all $i \in N(E)$. Since $\psi(\overline{E}) \subseteq uW(\overline{E})$, it follows by NEM of ψ that $\psi(\overline{E}) = \{\bar{v}\}$. Hence, by $CONS$ of ψ, we know that v is an element of ψ of the reduced economy of $\overline{E}$ with respect to $N(E)$ and $(w^0, (x^i)_{i \in N(E)})$. This reduced economy is the economy E. Hence, we conclude that $v \in \psi(E)$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

6   Economies with free disposal of commodities

If we assume that commodities are freely disposable, then we have to consider for a generalized economy $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle$ the set of allocations

$$A_f(E) = \{(x^i)_{i \in N} \mid x^i \in \mathbf{R}_+^\ell \text{ for all } i \in N, \sum_{i \in N} x^i \le \sum_{i \in N} w^i + \Theta\}$$

and, correspondingly, a *solution* on a family $\mathcal{E}_0 \subseteq \mathcal{E}$ is a function ϕ that assigns to each economy $E \in \mathcal{E}_0$ a subset $\phi(E)$ of $A_f(E)$.

Pareto optimal allocations and Walras allocations are defined with respect to the set of allocations $A_f(E)$ and, correspondingly, we denote these solutions by $PO_f(E)$ and $W_f(E)$ respectively. The definitions of the Pareto optimal solution and the Walras correspondence are straightforward generalizations of the definitions we provided in section 2 and therefore we will not repeat them.

A striking difference between the approach without free disposal and the approach with free disposal is that Walras allocations need no longer be Pareto optimal if there is free disposal. This is shown in the following example.

Example 6.1 Let E be a two-agent economy with two goods. The initial endowment of agent 1 is $w^1 = (1,1)$, i.e. agent 1 is endowed with 1 unit of the first good and one unit of the second good, and the initial endowment of agent 2 is $w^2 = (2,1)$. The utility functions of agents 1 and 2 are given by $u^1(x_1, x_2) = 2x_1 + x_2$ and $u^2(x_1, x_2) = x_1 + 2x_2$, respectively. The net trade vector for the economy is $\Theta = (-1, 2)$. A Walras allocation of this economy is given by $\hat{x}^1 = (2,0)$ and $\hat{x}^2 = (0,3)$. The corresponding price vector is $p = (\frac{1}{2}, \frac{1}{2})$. The allocation $y \in A(E)$ defined by $y^1 = (2,0)$ and $y^2 = (0,4)$ satisfies $u^1(y^1) = u^1(\hat{x}^1)$ and $u^2(y^2) > u^2(\hat{x}^2)$. Hence, the Walras allocation $\hat{x}$ is not efficient.

Clearly, the non-efficiency of the Walras allocation in example 6.1 is caused by the fact that in this allocation supply is not equal to demand (it is in the set $A_f(E) \backslash A(E)$), because it follows from our previous results that a Walras allocation in which supply equals demand, is efficient. It would be interesting to find out for which economies there

exists a Walras allocation in which supply equals demand. A partial answer to this question is given in the next proposition.

Proposition 6.2 Let $E \in \mathcal{F}$ be a generalized economy, $x \in A_f(E)$ a Walras allocation, and let $p \in \Delta$ be a price vector such that $x^i \in D^i(p)$ for each $i \in N$. Then the following two assertions hold:

(i) $p\Theta \geq 0$.

(ii) If $p\Theta = 0$, then x is Pareto optimal in E and, moreover, E has a Walras allocation in which supply equals demand.

Proof. Let $\varepsilon = \sum_{i \in N} w^i + \Theta - \sum_{i \in N} x^i$. Then $\varepsilon \in \mathbf{R}_+^\ell$ and, consequently, $p\varepsilon \geq 0$. Further, by local non satiation $pw^i = px^i$ for all $i \in N$, and, consequently, $p\varepsilon = p\Theta$. This proves part (i). To prove part (ii), suppose $p\Theta = 0$ and let $j \in N$. Then $x^j, x^j + \varepsilon \in \mathbf{R}_{++}^\ell$ and, because of weak monotonicity, $u^j(x^j + \varepsilon) \geq u^j(x^j)$. However, because $p(x^j + \varepsilon) = px^j$, it holds that $u^j(x^j) \geq u^j(x^j + \varepsilon)$. Thus, $u^j(x^j + \varepsilon) = u^j(x^j)$ and, hence, $x^j + \varepsilon \in D^j(p)$. So, we found a Walras allocation, $x_\varepsilon = (x^j + \varepsilon, (x^i)_{i \in N \setminus \{j\}})$, in which supply equals demand. Now, Pareto optimality of x follows by noting that $u^i(x_\varepsilon^i) = u^i(x^i)$ for each $i \in N$ and that x_ε is Pareto optimal. □

Note that for an ordinary economy ($\Theta = 0$) and all its reduced economies with respect to some Walras allocation the condition in proposition 6.2 (ii) is satisfied and, hence, these economies have a Walras allocation in which supply equals demand. Of course, we do not need proposition 6.2 to see this, because it follows using consistency of the Walras correspondence.

Since Walras allocations need not be Pareto optimal, we cannot use the axiom $PO(2)$ in order to characterize the Walras allocations. We have to adapt the axiom $PO(2)$ and we do it in the following way.

Let $E = \langle N; (w^i)_{i \in N}; (u^i)_{i \in N}; \Theta \rangle$ be a generalized economy and let $x \in A_f(E)$. Then x is *self-efficient* if x is efficient in the economy $\langle N; (x^i)_{i \in N}; (u^i)_{i \in N}; 0 \rangle$. Hence, an allocation x is self-efficient if the total amount of commodities that is allocated to the agents $(\sum_{i \in N} x^i)$ cannot be re-allocated in such a way that all agents are at least as well off and one agent is strictly better off. Clearly, Walras allocations are self-efficient.

Let $\mathcal{E}_0$ be a family of generalized economies. A solution ϕ on $\mathcal{E}_0$ satisfies *self-efficiency for two-agent generalized economies* $(SE(2))$ if for every two-agent generalized economy $E \in \mathcal{E}_0$ all $x \in \phi(E)$ are self-efficient.

Now we modify the definition of converse consistency to fit the situation in which commodities are freely disposable. Let $\mathcal{E}_0$ be a family of generalized economies. A solution ϕ on $\mathcal{E}_0$ satisfies *converse consistency* $(COCONS)$ if for every $E \in \mathcal{E}_0$ with at least two agents $(|N(E)| \geq 2)$ and for every $x \in A_f(E)$ that is self-efficient the following condition is satisfied. If for every $S \subseteq N(E)$, $S \notin \{\emptyset, N(E)\}$, it holds that $E^{S,x} \in \mathcal{E}_0$ and $x^S \in \phi(E^{S,x})$, then $x \in \phi(E)$.

The following result is easily obtained by making slight adaptations in the proof of theorem 3.6.

Theorem 6.3 The Walras correspondence is the unique solution on $\mathcal{F}$ that satisfies $NMEX(1)$, $SE(2)$, $CONS$ and $COCONS$.

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