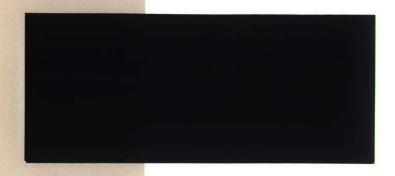


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# Discussion paper







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### DEALER BEHAVIOUR AND PRICE VOLATILITY IN ASSET MARKETS

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## Dealer Behaviour and Price Volatility in Asset Markets

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#### ABSTRACT

We find the equilibrium steady state trading strategy for a dealer, or stock-broker, on an exchange, who gains information from the inflow of orders and must determine how to release the information to the rest of the exchange through time. We show that dealer behaviour is motivated by differences in expected returns and the current order inflow, and information is released at a slower rate than it is received. This model of dealer behaviour generates increased asset price volatility. We also discuss the effects of increased competitiveness in the dealer industry and establish the limiting properties as the industry approaches perfect competition, in particular the increased volatility persists in the limit.

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#### 1. Introduction

Stock-brokers and dealers in financial markets face a flow of buy and sell orders from their clients. If they are dual-capacity traders they can smooth this flow by holding inventories of the stocks they quote and trading on their own account. In this paper we investigate the optimal trading strategy of such a dealer.

It is well known that an optimal purchasing strategy for a dealer in an environment with asymmetric information is based upon a bid-ask spread: the dealer buys from clients at a lower price than it sells to them because of adverse selection; Glosten & Milgrom 1985. This approach does not explain how the dealer determines its optimal inventory by purchasing or selling the stock to the marketplace from period to period. The traditional story assumes that any trade is immediately passed on to an organized exchange and the prices on the exchange come to reflect any information contained in the trades offered to the dealer. Some reflection reveals that this may well not be an optimal strategy for the dealer, because by observing the flow of orders received over the telephone it has gained some information relative to that available in the market place. Thus the dealer may be able to exploit the informational advantage this provides in its future trades and in the level of inventory it holds. In this paper we solve for the optimal intertemporal strategy of a dealer, taking account of this desire to manipulate the flow of information into the market. The model we present can also be interpreted as an extension of Kyle's (1985) model on informed trading, to an environment where new information continues to be acquired by the informed agent.

We show that optimal dealer behaviour will tend to increase the volatility of asset prices above that observed if the dealer passed on all orders, thus the presence of dual capacity dealers will tend to contributed to asset price volatility in excess of that predicted by the underlying flow of information onto a market. We further show that for any large but finite set of dealers in a particular market the increased level of volatility will persist, at the limit the price volatility is not restored to the level of no dealers.

There are two main types of explanation for the excessive asset price volatility noted by Shiller (1981). One class relies on the presence of liquidity, or noise, traders in asset markets to explain variances of prices that are higher than the variance of fundamentals. These explanations rely on the deus ex machina of noise traders to provide all the additional variability in prices. In spite of De Long et al. (1987), which goes some way towards explaining why noise traders exist, such models do not really explain the additional volatility but place it at one further remove. The second type of explanation is based on the belief that there is some feature of the structure or organisation of asset markets which promotes volatility and magnifies the shocks they receive. This work belongs to such a second class. But, we do not rely on bubbles or weakened rationality (e.g. chartists) to derive these results. We show that fully rational dealer behaviour will exaggerate shocks and increase the variance of asset prices.

The idea that dealers can behave in a systematic way to profitably manipulate the flow of information onto an exchange first appears in Kyle (1987). This result is vividly demonstrated by an example in the paper which repays study and so is given below. In the example there is an asset traded which yields a return of zero or unity at the end of play and each of these returns are equally probable. There are two types of traders coming to the dealer; informed, who know the returns, and uninformed who do not. In any period of time the dealer receives one order and there is a probability c that the dealer encounters an informed trader in that period. The informed traders purchase one unit if returns are unity and sell one unit if returns are zero, whilst the uninformed buy and sell with equal probability. We will consider what happens over two periods. In the first period the expected return to the asset, given a sell order, is b = (1-c)/2, and the expected return given a buy order is a = (1+c)/2, so these will respectively describe the sell and buy price of the dealer and the market as a whole. These two prices will differ, because there is adverse selection determining buyers and sellers. In the second period the sell and buy prices will be; 1/2, a<sup>2</sup>/(a<sup>2</sup>+b<sup>2</sup>) respectively, if there was a buy order in the first period and b<sup>2</sup>/(a<sup>2</sup>+b<sup>2</sup>), 1/2 respectively, if there was a sell order in the first period. Kyle points out that a dealer can make a profit by not passing the orders that arrive in the first period onto orders on the exchange. If the dealer observes a buy-sell or a sell-buy sequence of orders and does not pass these trades onto the market, then the market prices will not change between the periods and the dealer is able to make a profit of a-b. This is achieved by simply averaging these trades over time. However, if a buy-buy or a sellsell sequence is observed the dealer can pass these on to the market in the second period at a loss. The loss is made since in the second period the dealer is setting the wrong price relative to its information. It is obliged to charge the same price as the rest of the market in the second period and the market has not observed the first period trades. In spite of these losses on average the dealer is making a profit, hence Kyle concludes that dealers can profitably coexist with an efficient exchange. One can find this example objectionable on a number of fronts. There is no explanation of how the exchange continues if it is on average making a loss. There is the restriction of the dealer to a one unit order flow in each period. There is also no proper determination of the dealer's optimal strategy in this model. These will all be addressed below.

It is worth noting that in the example dealers tend to pass on large swings in trade but not small ones, so the question is; how will dealer behaviour like this affect asset price volatility? In the first period of the example the dealer completely eliminates the price volatility, whilst in the second period there may again be no trade or a high volume of trade as the dealer adjusts its inventories. This second period effect may generate increased price variability particularly if the dealer dumps a sell-sell or a buy-buy sequence on the exchange. (Closer inspection of the example reveals that there is no change in expected volatility even in the second period. Such a conclusion, however, rests on an implicit assumption in our treatment that the market treats two sell orders in the second period as equivalent with a sell order in each period. This is not in general true, one would expect a larger one-off change in demand to have different effect on prices to the effects from the sum of its parts.) So the possibility of dealers smoothing small shocks in the market's information, but passing on large shocks, could conceivably either reduce or exacerbate asset price variability. We will present a model which shows that dealers always tend to increase price volatility.

The example above also indicates something about the manner in which information flows through the dealer onto the market. In each period of the example new information accrues and instead of flowing smoothly in a continual stream onto the exchange, the release of information becomes lumpy and bunched through time. This is in stark contrast to the model of specialist pricing and information release in Kyle (1985), where the information in a single signal is released smoothly through time. In an environment where dealers their are controlling dissemination information, the example predicts that one observes periods where there is a high level of trade and prices contain a great amount of information, and other periods where there is less information contained in prices. This may provide another explanation for the phenomenon of variable patterns of trade during the day as in Admati & Pfleiderer (1986).

The contents of the paper are organized in the following way, section 2 contains a description of the model where there is one dealer and a definition of an equilibrium, section 3 describes the solution process for this model, section 4 extends this model to the many dealer case and examines the questions of price volatility. In Section 5 we examine how changes in the rules on the pricing of deals may alter the results here. Section 6 contains a conclusion.

#### 2. A Model of Dealer Behaviour

In the model below we will consider one dealer who has a monopoly of a particular flow of orders; this in one sense clearly conflicts with the belief that the dealer industry is competitive. There is, however, a fundamental way in which a dealer does have a monopoly over the flow of orders it observes and so must have information which is distinct from that observed in the rest of the market. It is this lack of competition which we are concerned with here and hence we will begin with a model where there is one dealer who adjusts the flow of orders from clients onto the market. This allows a simpler exposition of the key features of the many dealer case. We will also model the dealer as a player with strictly better information than the rest of the market, because it observes the information contained in its

order flow. There will clearly be information about returns to stocks other than that contained in the flow of information to the dealer, but we will assume this information is commonly available. Hence such information does not alter the dealer's position of being better informed about its own observed trades than the market. Hence for a first approximation, it may be reasonable to abstract from other information flows onto the market and examine how the dealer optimally releases information contained in the trades it makes. We will also attack the problem of what happens if there are a number of different dealers in a market in section 4 to investigate how this will affect the release of information onto a market.

We will assume there are two players; one player is a dealer who observes a flow of orders for an asset from clients in each period. In the light of this order flow the dealer chooses a demand to place on the exchange in each period. The second player can be thought of as a market maker who observes the sequence of total demands from dealers and noise traders and then sets the price equal to the expected discounted returns from the asset. The returns to the asset traded are denoted v and this is determined by the sum of a sequence of independent identically distributed  $N(0,\sigma_0^2)$  random variables  $\theta_t$ . Thus the returns to the asset traded follow a random walk. The realisations of returns are unknown to all agents in the market until after the final period of play. However, the appropriate distributions generating the variables are all common knowledge. The play in the model is assumed to take place during an infinite number of periods t=...,-2, -1, 0, 1, 2, ... This allows us to study the steady state of the model, but it does present a problem of interpretation since it appears that the returns are never obtained by the dealer in finite time. The intuition is clear; trading in assets may be very frequent relative to the annual payment of dividends. One can rationalize our approach observations t= 0, -1, -2, .. as being players' priors, which are also observations on the steady state. This truncates the infinite past. We will also assume that in any period there is a probability x that all information is revealed and returns realized. Thus returns are actually obtained in finite time. The value  $(1-\gamma)$  acts as an additional discount factor in the dealer's objectives, so if &' is the dealer's actual discount factor &=  $\delta'(1-\chi)$  is the factor which accounts for the probability of termination.

The flow of orders to the dealer in period t is informative about the returns to the stock traded, but there is a noise element which limits the value of the dealer's information. The order inflow i, is equal to 9,+u,, where  $u_t$  is an independent identically distributed  $N(0,\sigma_{tt}^{2})$  random variable representing the uninformed noise traders who come to the dealer. The increment in expected returns  $\theta_+$  is also the demand of a body of informed traders, who place their orders with the dealer. The variance  $\sigma_{ij}^{2}$  alters the information in the trades that the dealer observes. The dealer is obliged to meet these demands at the prices that reign on the exchange in the period the order inflow arrives. Within that period the dealer chooses a different demand or order outflow o, to place on the market in the light of the observed order inflow. The dealer's inventory will, of course, be adjusted at the rate  $(o_t - \theta_t - u_t)$  per period. There are many ways in which dual capacity traders are constrained in pricing shares, Roell (1989). In London the dealers must offer the best currently quoted price, in Italy banks offer the next market clearing price, Roell (1989 pl.). The assumption that dealers offer the current market clearing price is a reasonable approximation to this. The informed traders might object to dealers passing on their information and so also benefiting from it. They may also object to dealers (themselves) being net purchasers of the asset when there is a net demand from clients; (we show that dealers always behave in this way below). For these reasons we also consider in Section 5 the possibility that dealers cannot immediately act on information in order inflows. The results here substantiate all our conclusions under the original pricing hypothesis.

The market maker does not observe the private flow of orders to the dealer, but instead observes the sum of orders from all agents in the market (including the dealer) in each period. All other information is common knowledge, so the market maker will interest itself only in the flow of orders from the dealer  $\mathbf{o}_t$  as these will be the only informative data on the returns to the asset. The market maker's observations of the dealer's orders will be disturbed by some independent and identically distributed  $N(0,\sigma_w^2)$  noise trade  $\mathbf{w}_t$ . Thus in each period prices  $\mathbf{p}_t$  are set to satisfy

$$p_{t} = E[v|q_{t}, q_{t-1}, ...], \text{ where } q_{t} := o_{t} + w_{t}.$$
 (1)

This formalisation of market prices improves on the example, because the market correctly accounts for the optimal strategy of the dealer when forming expected returns and hence prices. In particular the market will not be expecting to make losses, because it is allowing for the behaviour of the dealer who sits on the order stream and derives information.

To complete the model we must now describe the dealer's objectives. At the beginning of each time period the dealer chooses a level of ot to maximise the discounted value of its future expected profits, given its own private information on its current and past order flow and the state of the market's information. This is the usual dynamic programming approach and as such has the credibility properties associated with a sequential equilibrium. This model cannot be placed in a proper game-theoretic form as there has been no specification of the market maker's objectives and the game tree is doubly infinite. We will assume that the dealer has a discount factor of  $\delta:=(1-\gamma)\delta'$  and acts to maximise the discounted value of expected profits

$$\max_{\{o_t, o_{t+1}, \dots\}} \mathbb{E} \left[ \sum_{r=t}^{\infty} \delta^{r-t} (\varphi v - p_r) (o_r - i_r) \mid i_t, i_{t-1}, \dots \right]$$
For all t,  $i_t$ ,  $i_{t-1}$ , ... (2)

The factor  $\psi=\chi/(1-\delta)$  discounts returns to the date they are received. Lines (1) and (2) in effect describe the equilibrium condition for this model. In (2) it is clear that the dealer is not constrained to balance its books in the stock in any period, instead we allow the dealer to be able to build up a position or go short in the stock it is trading. This appears to be the correct modelling approach for a number of reasons, first we should not constrain the dealer to balance its books in a stock but instead explain why it is optimal for the dealer to behave in this way. If such a constraint is observed in practise it must surely be a consequence of some optimizing behaviour from the dealer, hence the correct approach is to examine how dealers act if unconstrained. Second, it seems unreasonable to restrict the dealer from building a profitable position in the stock if it is able to do

this. There are obviously a number of ways in which dealers can gain from using their inside information and this is one of the most obvious.

#### 3. The Optimal Form of Dealer Behaviour

The solution process for this model proceeds in two steps, we first posit a form for the dealer's optimal strategy and deduce the way in which the market will extract information from knowledge of its structure. Then, given the market will behave in this way, determine the optimal response of the dealer and show that this optimal response is indeed of the form first suggested. Finally one can solve for the coefficients of the optimal strategy: this solution technique owes a great deal to that found in Cukierman & Meltzer (1986) or Kyle (1985). The solution technique does have the deficiency that we have restricted our search for equilibria to those which are of the form first postulated. There may be other types of equilibria which are not discovered by this process. It is important to note that in employing this solution technique we have not restricted the dealer's actions in any way; the dealer's optimal response to the belief formation by the market is completely unrestricted. It simply turns out that this response is of the form first postulated. Second, the linear quadratic Gaussian nature of the model leads one to have strong faith in the essential linearity of the solution we find, moreover, we can exclude the possibility of a multiplicity of solutions due to beliefs off the equilibrium path since every information set is on the equilibrium path. This must also imply that no inadmissible strategies are used by players in equilibrium.

The first step is to guess a form for the optimal strategy for the dealer, this consists of describing the way  $\mathbf{o}_t$  is chosen at every point in time for every state, we will assume

$$o_{t} = \alpha i_{t} + \beta E_{t} v + \gamma p_{t-1}. \tag{3}$$

Here  $\alpha, \beta$  and  $\gamma$  are constants yet to be determined. The first term in this expression represents the new information observed in period t,  $E_t v$  is the dealer's expected value for returns in period t and the variable  $P_{t-1}$ 

represents the market's expected returns in period t-1. These last two variables describe all the relevant information on the state of players' information in period t. Thus the strategy postulated does respond to the level of both players' information about the returns at each period of time.

The market observes the past sequence of variables  $q_t = o_t + w_t$ , from these observations it will infer something about the values  $\theta_t$  and hence v. Given the linear form of the dealer's strategy postulated in (3) this process of inference will be the exponential weighting of observations given below (this follows from the standard least squares projection result and is given in detail in the appendix)

$$E[v | q_t, q_{t-1}, \dots] = \frac{(1-\mu)}{\beta} \sum_{j=0}^{\infty} \mu^{j} (q_{t-j} - \gamma p_{t-j-1}),$$
 (4)

Where

$$\rho = \frac{\sigma_{\vartheta}^{2}}{\sigma_{\vartheta}^{2} + \sigma_{u}^{2}}, \quad \omega = \frac{\sigma_{w}^{2}}{\sigma_{\vartheta}^{2}}, \quad B = 2 + \left[ \left( \frac{\alpha}{\beta \rho} \right)^{2} + \frac{\rho \omega}{\left( \rho \beta \right)^{2}} + \frac{\alpha}{\beta \rho} \right]^{-1},$$

$$\mu = (B/2) - \sqrt{\left( B/2 \right)^{2} - 1}.$$

(Henceforward we will use the notation  $E_t$  to denote expectations taken relative to the dealer's available information at time t, so this notation is equivalent to the notation  $E[.|i_t,i_{t-1},i_{t-2},...]$  used above.) The equivalent expected returns for the dealer  $E_t v$  can also be solved applying the well known conditional expectation properties of the normal distribution  $E_t v = \rho \sum_{j=0}^{\infty} i_{t-j}$ . Using the information given in these statements we can now complete the description of the determination of prices. Prices have been determined as a function of past values of  $q_t$ , by employing (1) and (4) we have

$$p_t = \frac{(1-\mu)}{\beta} \sum_{j=0}^{\infty} \mu^{j} (q_{t-j} - \gamma p_{t-j-1}).$$

Leading once, multiplying the above by  $\mu$  and differencing gives the following difference equation in  $p_t$ ;  $p_t^{-\mu p}_{t-1} = \frac{(1-\mu)}{\beta}(q_t - \gamma p_{t-1})$ . Solving for  $p_{t-1}$  gives

$$p_{t} = \frac{(1-\mu)}{\beta} \sum_{j=0}^{\infty} g^{j} q_{t-j} \qquad \emptyset := \mu - \frac{(1-\mu)}{\beta} \gamma.$$
 (5)

We have now determined prices in this model and explained how the dealer forms its beliefs on returns, it remains to determine the dealer's optimal behaviour. The first step is to take (5) and substitute it into the dealer's objectives (2)

$$\mathbf{E}_{\mathbf{t}}[\sum_{\mathbf{r}=\mathbf{t}}^{\infty} \, \boldsymbol{\delta}^{\mathbf{r}-\mathbf{t}} ( \boldsymbol{\psi} \boldsymbol{v} - \frac{(1-\boldsymbol{\mu})}{\beta} \, \sum_{j=0}^{\infty} \, \boldsymbol{\phi}^{j} \boldsymbol{q}_{\mathbf{r}-j}) \, (\boldsymbol{o}_{\mathbf{r}}^{-1}\boldsymbol{i}_{\mathbf{r}}) \, ].$$

Now optimize, by calculating the value for  $\mathbf{o}_{\mathsf{t}}$  which maximizes the above. This can be found from the first order condition, or so called Euler equation, for the model

$$0 = E_{t}[\psi v - p_{t}(o_{t} - i_{t}) \frac{(1 - \mu)}{\beta} - \frac{(1 - \mu)}{\beta} \sum_{r=t+1}^{\infty} (\delta \emptyset)^{r-t} o_{r}].$$

Rearranging this and substitution from (5) for  $p_t$  gives the following relation describing the dealer's optimal strategy

$$2o_{t} = i_{t} + \frac{\beta \psi}{(1-\mu)} E_{t}v - \omega \sum_{j=0}^{\infty} \omega^{j} q_{t-1-j} - E_{t} \sum_{r=t+1}^{\infty} (\delta \omega)^{r-t} o_{r}.$$
 (6)

Now lead this once take expectations relative to  $\mathbf{E}_{\mathsf{t}}$  of the resulting equation and then multiply through by  $\emptyset\delta$  and difference, this gives:

$$(2-\emptyset \delta) \circ_{\mathsf{t}} - \emptyset \delta \mathsf{E}_{\mathsf{t}} \circ_{\mathsf{t}+1} \; = \; \mathsf{i}_{\mathsf{t}} \; + \; (1-\emptyset \delta) \frac{\beta \forall}{(1-\mu)} \; \mathsf{E}_{\mathsf{t}} \mathsf{v} - \emptyset (1-\delta \emptyset^2) \sum_{\mathsf{j}=0}^{\infty} \emptyset^{\mathsf{j}} \mathsf{q}_{\mathsf{t}-1-\mathsf{j}}.$$

Lead this once and multiply the above by  $\emptyset$  taking expectations relative to  $\mathsf{E}_\mathsf{t}$  and differencing again gives

$$g\delta E_{t}^{o}_{t+2} - 2E_{t}^{o}_{t+1} + go_{t} = gi_{t} - \frac{\beta y}{1-\mu} (1-g)(1-g\delta)E_{t}^{v},$$
 (7)

and for r>0,

$$g\delta E_{t^{o}_{t+r+2}} - 2E_{t^{o}_{t+r+1}} + gE_{t^{o}_{t+r}} = -\frac{\beta \psi}{1-\mu} (1-g) (1-g\delta) E_{t^{v}}.$$
 (8)

The characteristic equation of the difference equations in (7) and (8) have the roots

$$\frac{1 \pm \sqrt{1 - \omega^2 \xi}}{\delta \omega}, \text{ let } \xi = \frac{1 - \sqrt{1 - \omega^2 \xi}}{\delta \omega}.$$

If -1 < g < 1, one of these roots is stable and the other is unstable. It is unlikely that the dealer will expect its sales in the market to become unboundedly large in the future. Also any such behaviour will conflict with the transversality condition attached to the optimization above. Hence we will define  $\xi$  to be the stable root and treat it as the only solution to the difference equations. As a result the solution to (8) is

$$E_{t_{t+r}} = \xi^* + (E_{t_{t+1}} - \xi^*) \xi^{r-1} \qquad r \ge 1$$
 (9)

$$\xi^* = \frac{(1-\emptyset)(1-\emptyset\delta)}{2-\emptyset-\emptyset\delta} \frac{\beta \psi}{1-\mu} E_t v.$$

This solution together with (7) can be employed to determine  $E_t^o_{t+1}$ . Substitute from (9) for  $E_t^o_{t+2}$  into (7), then rearrange to solve for  $E_t^o_{t+1}$  gives

$$E_{t}o_{t+1} = \frac{\emptyset}{2 - \xi \xi \emptyset} (o_{t}i_{t}) + \frac{\beta \psi}{1 - \mu} E_{t}v \frac{(1 - \emptyset)(1 - \emptyset \xi)(2 - \emptyset - \xi \emptyset \xi)}{(2 - \emptyset \xi \xi)(2 - \emptyset - \emptyset \xi)}.$$
(10)

Employing (10) and (9) we can now calculate the sum at the end of (6) and hence determine the dealer's optimal strategy. To do this we must now let the time when the value of the asset is realised tend to infinity, otherwise the value of the last term on the right of (6) will be dependent on the time period. The idea behind this device is to model the notion that there are a large number of periods between today's trade and the point in time when

returns are received. One means of achieving this end would be to recast the model as a discrete approximation to continuous time with a finite horizon T, but an infinite set of periods between zero and T. To do this the variances must be multiplied by a term ( $\Delta t$ ), which denotes the period length, and discount factors are  $\delta^{\Delta t}$ ; the limits are taken as  $\Delta t$ +0. This seems unnecessarily cumbersome, hence we will simply let horizons be unboundedly large. The solution to (6) after calculation of the sum  $E_t \Sigma (g \delta)^T o_{t+r}$  becomes

Division by the factor on the left and substitution for  $p_{t-1}$  reveals that (11) does have the form first postulated for the optimal strategy;  $o_t = \alpha i_t^+ \beta E_t^- v + \gamma p_{t-1}^-$ . All that remains is to solve for the coefficients  $\alpha, \beta, \gamma$  by equating these with the appropriate expressions in (11). First take  $\gamma$  and equate this with the expression above, substitute for  $\xi$  from its definition, then substitute for  $\gamma$  in the definition of  $\emptyset$  (5). This can be used to establish

$$\varnothing = \frac{2\mu}{1+6\mu^2}.\tag{12}$$

Now equate  $\beta$  with the appropriate factor of (11), substitute for  $\emptyset$ ,  $\xi$  and re-arrange, this gives

$$\delta u^2 - 2\tau u + 1 = 0$$

where

$$\tau = \frac{\delta(1-\gamma)+1}{2-\gamma}.$$

Taking the stable solution

$$\mu = \frac{1}{6} \left( \tau - \sqrt{\tau^2 - \delta} \right). \tag{13}$$

These facts can then be used to completely characterise the coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$  of the dealer's supply. We have now succeeded in characterising the dealer's optimal strategy in this model and it is indeed a linear function of today's new information  $i_t$  and the two state variables  $p_{t-1}$ ,  $E_t v$ . These respectively describe the state of the market's information on returns and the state of the dealer's own private information on returns.

The sign of  $\gamma$  is particularly significant, and it appears that the dealer acts in a way which is negatively related to the market's expected returns from the asset traded. Hence it will prefer to supply the asset when high returns are expected by the market and when the level of returns the dealer expects are relatively low. When the converse is true it will prefer to purchase the asset from the market place.

If the value  $(\alpha+\rho\beta-1)$  is calculated we can work out the extent to which the dealer's inventory changes in response to order inflows (since  $o_t^{-i}_t^{=}(\alpha-1)i_t^{+}\beta E_t^{v+}\gamma p_{t-1}^{-}$ ). Some algebra gives  $\alpha=\mu$ . Then to determine  $\beta$  we must solve  $\mu^2-\beta\mu+1=0$ . This gives

$$(\alpha-1+\rho\beta)>0.$$

Order inflows in isolation from their information content lead to a reduction in inventory levels, but this ignores the informational effects which in fact lead to offsetting increases in inventories. The magnitude of the parameter  $\beta$  implies that inventories actually increase in response to a net demand from clients. Thus dealers act to supply their clients and also purchase the asset on their own account. One may, therefore, say that dealers trade against their clients, Roell 1989. The informed clients do not necessarily loose because of this. If the variance of noise trade is relatively large it is quite possible that dealers are net purchasers of the asset, whilst the clients are net sellers.

The relative sizes of the parameters  $\beta$  and  $\alpha$  determines the informational content of the market maker's new observations. The rate at which the market learns, or acquires new information, from the dealer is determined by the dealer's discount factor  $\delta$  (12). Dealers with higher discount factors select strategies with a higher weight  $\mu$  in the signal extraction problem. Thus as dealers become less myopic they choose a strategy which reduces the informational content of current actions. This implies that the market maker places more weight on past information when solving its signal extraction problem.

#### 4. Many Dealers and Price Volatility

In this section we show how the model solved above can be extended to solve for the case with a fixed finite number of dealers. This is then used to examine the price variability of a market with optimal dealer behaviour. We also address the question of how changes in the degree of competitiveness among dealers changes the properties of the equilibrium we have discovered and ask whether as the number of dealers becomes large we approach a fully competitive market.

In the one dealer model a flow of orders presented themselves to the single dealer in every period. We now assume there are n identical dealers with identical discount factors operating in the market, with objectives equivalent to those described in (2). Each dealer trades at the market price and we assume there is no significant non-price competition between dealers, hence from the point of view of the customer one dealer is identical to another. Thus we let the noise traders  $\mathbf{u}_t$  be randomly allocated to dealers, so dealers have equal levels of noise trade. Once each dealer has an equal proportion of noise trade the informed clients are also indifferent between dealers, Roell (1989). Thus in each period the order inflow of any dealer is  $\frac{1}{n}$  t. We will assume that it is common knowledge that customers are so-allocated.

Now postulate a form for the symmetric equilibrium strategy of dealer i

$$o_{t}^{i} = \frac{\alpha'}{n} i_{t} + \frac{\beta'}{n} E_{t} v + \frac{\gamma'}{n} p_{t-1}.$$
 (14)

Given dealers adopt such a strategy total demand facing the market maker in any period satisfies  $q_t=\alpha'i_t+\beta'E_tv+\gamma'p_{t-1}+w_t$  as in section 3. Thus, the signal extraction problem faced by the market maker has an identical solution to that described in the appendix. Hence mutatis mutandis define  $\mu'$  to be the appropriate weight for the signal extraction problem and dealer i's optimization problem becomes

$$\text{Max E}_{t_{j=0}}^{\sum_{j=0}^{\infty}} \delta^{j} (\gamma^{v} - \frac{(1-\mu')}{\beta'}) \sum_{j=0}^{\infty} q_{t-j}) (o_{t-j}^{i} - \frac{1}{n} i_{t-j}). \tag{15}$$

The first order condition for this problem, the analogue of equation (6), will be

$$0 = \mathbb{E}_{\mathbf{t}} \left[ \frac{\beta' \psi}{1 - \mu'} \ \mathbf{v} \ - \ \sum_{\mathbf{k} = 0}^{\infty} \ \mathbf{g}^{\mathbf{k}} \mathbf{q}_{\mathbf{t} - \mathbf{k}} \ - \ \mathbf{o}^{\mathbf{i}}_{\mathbf{t}} \ + \ \frac{1}{n} \mathbf{i}_{\mathbf{t}} \ - \ \sum_{\mathbf{j} = 1}^{\infty} \ \delta^{\mathbf{j}} \mathbf{g}^{\mathbf{j}} \mathbf{o}^{\mathbf{i}}_{\mathbf{t} + \mathbf{j}} \right],$$

where

$$\varnothing := \mu' - (1 - \mu') \frac{\chi'}{\beta'}$$

Summing these over all dealers i gives an equation for total supply

$$(n+1)E_{t}q_{t}=i_{t}+n\frac{\beta' \forall}{1-\mu'}E_{t}v-n \bigotimes_{k=0}^{\infty}\bigotimes_{k=0}^{k}q_{t-1-k}-E_{t}\sum_{j=1}^{\infty}\delta^{j} \bigotimes_{j}q_{t+j}.$$
 (16)

This relation can then be transformed in the manner described by (6) through (8). To give a difference equation for  $E_tq_{t+s}$ 

$${^{n}\varnothing} \delta {^{E}}_{t}{^{q}}_{t+2} - {^{(n-1)}} {^{E}}_{t}{^{q}}_{t+1} + {^{\varnothing}} {^{E}}_{t}{^{q}}_{t} = {^{\varnothing}} {^{i}}_{t} - {^{n}\beta \psi \over 1-\mu} \; (1-\varnothing) \, (1-\varnothing\delta) {^{E}}_{t}{^{v}}.$$

Taking the stable root and substituting into (15) gives the form of the equilibrium strategy as in (11). Employing equivalent operations to those described in section 3 we can show that

$$\emptyset = \frac{(n+1)\mu}{n\delta\mu^{2}+1},$$

$$\mu' = \frac{1}{2\delta n} \left[ (n+1)\tau' - \sqrt{(\tau')^{2}(n+1)^{2}-\delta n} \right],$$

$$\frac{\chi'}{\beta'} = \frac{n\mu'}{1-\mu'} \frac{\delta(\mu')^{2}-1}{n\delta(\mu')^{2}+1},$$

$$\alpha' = \frac{1+n\delta(\mu')^{2}}{n+1} = \mu'$$

$$\tau' = \frac{1+\delta n(1-\gamma)}{1+n(1-\gamma)}.$$
(17)

The ratio of  $\gamma'/\beta'$  is, of course, determined from the definition of  $\mu$  in (4). This completes the description of all the aggregate variables in this equilibrium. We will not verify that given aggregate behaviour is of this form then individual behaviour of each dealer must also be. This is based largely upon the first order conditions (16).

Let us first examine how increases in the number of dealers and hence increased competitiveness affects the behaviour of market prices. First consider the rate at which new information is reflected in prices, that is the determination of the parameter u' in the signal extraction problem. As n increases the value " decreases, tending to zero as n tends to infinity. As the dealers become closer to a competitive industry their equilibrium strategy results in current actions being very informative, relative to past information. There is thus little difference between the information of dealers and the information of the market. The reason for this can be seen if we note that as n tends to infinity so does α' tend to zero. The dealer's strategy consequently places less weight on the new information and gets to be responsive only to their expectations of returns. This is precisely the information the rest of the market is seeking to learn. Also, as n becomes large so does the ratio  $\gamma'/\beta'$  tend to minus one. Hence, as the dealer industry becomes competitive it is simply the difference between the beliefs of the dealers and the beliefs reigning in the market place which motivates dealers to trade. This is the sole remaining motive for trade once their market power and ability to influence the market's information acquisition has vanished. The extent to which this motivates dealers in a fully competitive industry can be seen if we solve for  $\beta'$  from (4) as  $\mu$  tends to zero. This demonstrates that  $\beta'$  becomes unboundedly large, and any discrepancy between market's and dealers' information about returns to the asset will result in very large swings in dealers positions to restore the balance between expectations. In summary, as n becomes large the behaviour does approach the perfectly competitive outcome. Any difference in information would result in large swings of trade, and so dealers and market have the same levels of information. We will show below that the level of price volatility does not converge to the competitive level but stays at a higher level.

Changes in dealers' discount factors produce similar effects to those in the one-agent case. We can also assess the impact of changes in the informativeness of dealers' signals using the parameter  $\rho.$  As this shrinks so does the proportion of noise in the dealers' order inflow increase. By (16) the parameters  $\mu$  and  $\alpha$  will not be affected by a change in  $\rho,$  however,  $\beta$  and  $\gamma$  must both respond to this change. A decrease in  $\rho$  generates a higher value of  $\beta$  and  $\gamma.$  Hence, as current signals become less informative on returns dealers' equilibrium strategies become more responsive to their beliefs on returns. This increases the relative information on returns in their period-to-period behaviour and thus preserves the optimal rate of learning by the players in the market.

The rate of learning by the market is the key feature of the optimal dealer strategy, it is this which determines how much profits dealers are able to generate. In the first example we saw that it was a dealer's ability to slow or stop the market's information acquisition, which generates a dealer's profit opportunities. Similarly, in this more complex model profit opportunities come from delaying the learning of the market and trading this off against the potential risks of future adverse information. Thus, it is the discount factor and the number of competitors which determines  $\mu$ , and hence all other features of the solution.

#### 4.1 Price Volatility

We now try to compare the price volatility in these models of dealer behaviour with the price volatility in the absence of dealers. The benchmark against which we measure volatility will be determined by the equilibrium that reigns when the market maker observes the sum of the dealers' order inflow and noise trade;  $\mathbf{i_t}^+ \mathbf{w_t}$ . We will show that dealer behaviour always increases the variability of prices over this benchmark. We consider two different ways of measuring price variability. First, the variance of prices is used. This is not the usual way of measuring price volatility. Shiller (1981) employs a measure  $\mathrm{Var}[\mathbf{p_t}^-\mathbf{p_{t-1}}|\mathbf{p_{t-1}}]$  so we also compare this measure.

A natural measure of price volatility would be the unconditional variance of prices, however, given the assumed infinite past in the model this variance is unboundedly large. To avoid this problem we treat periods; -1,-2,-3,..., as past history, which can be conditioned upon in the calculation of the variances. Hence we compare the price variances in period t conditional upon a particular past history for a given regime. Obviously as t becomes large the conditioning becomes increasingly less important, moreover, the particular events during the past have no effect on the values of these variances. We will also interest ourselves in the rate at which these variances would grow as t tends to infinity.

Suppose there are no dealers, so the market maker observes  $q_t^* = i_t^* + w_t$  in each period. The expected returns determines prices under this regime.

$$p_{t}^{*} = \frac{\sigma_{\theta}^{2}}{\sigma_{\theta}^{2} + \sigma_{u}^{2} + \sigma_{w}^{2}} \sum_{j=0}^{\infty} (i_{t-j}^{*} w_{t-j}^{*}),$$

$$V[p_{t}^{*}|I_{-1}] = \frac{(\sigma_{\theta}^{2})^{2}}{\sigma_{\theta}^{2} + \sigma_{u}^{2} + \sigma_{u}^{2}} (t+1), \qquad t \ge 0.$$

Here  $\mathbf{I}_{-1}$  represents the information set of the market makers before time zero. To calculate the conditional variance in period t in the presence of dealers we use the fact that

$$\mathbf{p}_{\mathsf{t}} = \left(\frac{1-\mu}{\beta}\right) \sum_{\mathsf{j}=0}^{\infty} \mu^{\mathsf{j}} (\alpha^{\mathsf{j}}_{\mathsf{t}-\mathsf{j}} + \beta^{\mathsf{E}}_{\mathsf{t}-\mathsf{j}}^{\mathsf{v}} + w_{\mathsf{t}-\mathsf{j}}).$$

Using a similar approach to that taken in the appendix and correctly evaluating all the finite geometric progressions, one can calculate the variance of this to be

$$\begin{split} \mathbb{V} \big[ \mathbb{P}_{\mathsf{t}} \big| \mathbb{I}_{-1} \big] &= \frac{\sigma_{\mathsf{w}}^2}{\varrho} \; \frac{ (1 - \mu)}{1 + \mu} \; (1 - \mu^{\mathsf{t} + 2}) \; + \frac{ (1 - \mu)^2}{\beta^2} \; (\sigma_{\theta}^2 + \; \sigma_{\mathsf{u}}^2) \; \bigg[ \; \frac{\beta^2 \varrho^2}{(1 - \mu^2)} \; (\mathsf{t} + 1) \\ &+ \; \frac{2\beta \varrho}{1 - \mu} \; \frac{1 - \mu^{\mathsf{t} + 2}}{1 - \mu} \; X \; + \; X^2 \; \frac{1 - \mu^{2\mathsf{t} + 4}}{1 - \mu^2} \bigg] \; , \bigg[ \; X \; = \; \alpha \; - \; \frac{\beta \varrho \mu}{1 - \mu} \bigg] \; . \end{split}$$

To compare the two expressions it is simply a matter of comparing terms with the factor (t+1) this gives us

$$V[p_t^*|I_{-1}] < V[p_t|I_{-1}].$$

Therefore we can conclude that the activity of dealers will always tend to increase price variances. Furthermore, for any given past history price variances with the presence of dealers grows at a faster rate than in their absence. These properties are also robust to changes in period length. As the time period  $\Delta t$  between trading periods shrinks  $\delta$  tends to unity and the variances  $\sigma_{\theta}^2$ ,  $\sigma_{u}^2$ ,  $\sigma_{w}^2$  are multiplied by a factor  $\Delta t$ . The inequality between these two variances is preserved.

If one measures volatility using the variance of the difference of prices one must calculate

$$\begin{split} \mathbf{p}_{t}^{*} - \mathbf{p}_{t-1}^{*} &= \frac{\sigma_{\theta}^{2}}{\sigma_{\theta}^{2} + \sigma_{u}^{2} + \sigma_{w}^{2}} \left( \mathbf{i}_{t} + \mathbf{w}_{t} \right) \\ \mathbf{p}_{t} - \mathbf{p}_{t-1} &= \left( \frac{1 - \mu}{\beta} \right) \left[ \alpha \mathbf{i}_{t} + \beta \mathbf{E}_{t} \mathbf{v} + \mathbf{w}_{t} - (1 - \mu) \mathbf{p}_{t-1} \right] \\ &= \left( \frac{1 - \mu}{\beta} \right) \left[ (\alpha + \beta \rho) \mathbf{i}_{t} + \beta \mathbf{E}_{t-1} \mathbf{v} + \mathbf{w}_{t} - (1 - \mu) \mathbf{p}_{t-1} \right] \end{split}$$

Now calculating the conditional variances

$$\begin{aligned} \text{Var}[\textbf{p}_{t}^{*} - \textbf{p}_{t-1}^{*} | \textbf{p}_{t-1}] &= \frac{(\sigma_{\theta}^{2})^{2}}{\sigma_{\theta}^{2} + \sigma_{u}^{2} + \sigma_{w}^{2}} \\ \text{Var}[\textbf{p}_{t} - \textbf{p}_{t-1} | \textbf{p}_{t-1}] &= (1 - \mu)^{2} (\frac{\alpha}{\beta} + \rho)^{2} \text{Var}[\textbf{i}_{t}] \\ &+ (1 - \mu)^{2} \text{Var}[\textbf{E}_{t-1} \textbf{v} | \textbf{p}_{t-1}] + \frac{(1 - \mu)^{2}}{\beta^{2}} \sigma_{w}^{2} \end{aligned}$$

If we let the number of dealers tend to infinity we have

$$\text{Var}[\textbf{p}_{t}^{-\textbf{p}_{t-1}}|\textbf{p}_{t-1}] \rightarrow \frac{(\sigma_{\vartheta}^{2})^{2}}{\sigma_{\vartheta}^{2} + \sigma_{u}^{2}} + \text{lim Var}[\textbf{E}_{t-1}\textbf{v}|\textbf{p}_{t-1}]$$

This clearly exceeds the variance when dealers are not present. Using both measures of price volatility we have now shown that the presence of dealers exacerbates price volatility and that this result happens even as competition approaches perfect competition.

This result appears odd given the arguments given above that as the dealer industry becomes more competitive so information transmission from dealers to the industry becomes more transparent. However, if one views these results in the light of those on information transmission in Kyle 1989 it should be clear that there may continue to be incentives to effect the informational context of prices even as agents become insignificant. Similarly, it is quite possible for dealer activity to affect aggregate variability even as dealers become individually small.

#### 5. A Different Pricing Rule for Dealers

We have shown that dealers will always trade against the order stream, so they are net sellers when their clients are also selling the asset and net buyers when their clients buy. In this section we consider what happens when dealers do not trade against their clients in this way. We suppose that the clients of a dealer; informed or uninformed; are able to insist that dealers do not trade on their current information in the market. Instead dealers must wait one period before they can act on the information they have acquired from their order inflow. This implies that the information relevant to the price in period t is that observed by the dealer in period t-1. In effect the dealers must pay the price in period t-1 to the order inflow in period t. Below we solve for the steady state strategy of the dealer when it is forced to pay a price  $p_{t-1}$  to today's order inflow.

Each dealer's optimization problem now becomes

$$\max E_{t} \sum_{r=t}^{\infty} \delta^{r-t} \{ \psi v(o_{r}^{-i}_{r}) - p_{r}^{o_{r}} + p_{r-1}^{i}_{r} \}.$$

Since dealers cannot react to i<sub>t</sub> in the period that it is observed, the first term in the dealer's optimal strategy vanishes and we will postulate the following form for each dealer's actions

$$o_t = \frac{1}{n} \beta E_t v + \frac{1}{n} \gamma p_{t-1}$$

We use the notation  $\beta$ ,  $\gamma$  for the dealer's strategy but these denote different values to those used previously. The signal extraction problem faced by the market is identical to the one solved in the appendix with  $\alpha$ =0.

$$\mu = (B/2) - \sqrt{(B/2)^2 - 1}$$

$$B = 2 + \frac{\rho \beta^2}{\omega}$$

Now as above substitute for  $\mathbf{p_r}$  and  $\mathbf{p_{r-1}}$  in the dealers' objective functions and solve for the Euler conditions

$$(n+1)q_{t} = \frac{n_{\beta}}{1_{\mu}} v^{E}_{t}v - n_{\emptyset} \sum_{i=0}^{\infty} g^{i}q_{t-1-j} - E_{t} \sum_{i=1}^{\infty} g^{i}\delta^{i}q_{t+j}.$$

Comparison with the equivalent condition (16) shows that the only difference here is the absence of the first term on the right in  $i_t$ . The absence of this term considerably simplifies the solution of  $E_t q_{t+j}$ , but the same procedure is followed to give the optimal strategy

$$\begin{split} \mathbf{q}_t &= \left[\frac{1-\xi \emptyset \delta}{n+1-n\xi \emptyset \delta}\right] \left[1 \; - \; \frac{\emptyset \delta \left(1-\xi\right) \left(1-\emptyset\right)}{\left(1-\xi \emptyset \delta\right) \left(n+1-\emptyset-n\emptyset \delta\right)}\right] \frac{\beta n \gamma}{1-\mu} \; \mathbf{E}_t \mathbf{v} \\ &- \left[\frac{1-\xi \emptyset \delta}{n+1-n\xi \emptyset \delta}\right] n \emptyset \; \sum_{j=0}^{\infty} \; \emptyset^j \mathbf{q}_{t-j} \,. \end{split}$$

We can solve for the coefficients  $\beta$ ,  $\gamma$ ,  $\mu$  in the manner described above, to give

$$\emptyset = \frac{(n+1)\mu}{1+n\delta\mu^2}$$

$$0 = n\delta\mu^2 - (n+1)\tau'\mu + 1$$

$$\mathcal{L} = \frac{n\mu}{1-\mu} \frac{\delta\mu^2 - 1}{n\delta\mu^2}.$$

Therefore the coefficients  $(\gamma,\beta)$  of the optimal strategy when the dealer is constrained to offer last period's price are <u>identical</u> to those found in Section 4.

The strategy of the dealers in this section is identical to that in the previous section apart from the constraint that  $\alpha$ =0 here. Thus, all the analysis of the form of the dealers' optimal strategy also applies to the case where dealers are constrained to offer last periods price to thier clients. The results on increased competition in this case are precisely equivalent, because we showed that  $\alpha'$  tends to zero as n tends to infinity. This also ensures that the results on price volatility of Section 4.1 also hold true in this section.

To conclude, if we allow for the fact that the clients of dealers are reluctant to allow dealers to trade against them, we can again use the techniques developed here to discover the form of the steady state strategy.

Its form is identical to the strategy in Section 4 with  $\alpha$ =0. All the results on excess volatility are still true in this case.

#### 6. Conclusions

We have found an equilibrium for a model where dealers gain information from their private inflow of orders. They then release this information to the market in a manner which optimally provides them with opportunities to profit from trade. It is the dealers' ability to slow the markets rate of learning which creates the opportunity for the dealers to make profits; just as is the case in our initial example. This has a number of effects. It increases the volatility of the asset market price and it slows the rate at which the information is released onto the market. We have also established that dealer behaviour converges to a perfectly competitive limit, where there is still increased price volatility. Finally, it seems possible to view these results as support for the results of Hart & Kreps (1986) that speculation may increase price volatility by furnishing an example where it invariably has this effect.

#### Appendix

Begin by noting that  $\mathrm{E}[\mathbf{v}|\mathbf{q}_t,\dots]$  is equivalent to the conditional expectation of  $\mathbf{v}$  given the variables  $\mathbf{d}_t := \mathbf{q}_t - \gamma \mathbf{p}_{t-1}$  since at the time expectations are taken the data  $\gamma \mathbf{p}_{t-1}$  are common knowledge. Given the linearity of the structure we can employ the least squares projection result to deduce that the conditional expectation is linear in the data, hence

$$E[v|d_t,..] = \sum_{j=0}^{\infty} \lambda_j d_{t-j}.$$

And the coefficients  $\lambda_i$  minimise the expected squared prediction error

$$E[\{v - \sum_{j=0}^{\infty} \lambda_{j} d_{t-j}\}^{2}] = E[\{v - \sum_{j=0}^{\infty} \lambda_{j} (\alpha i_{t-j} + \beta E_{t-j} v + w_{t-j})\}^{2}]$$

$$\text{E}[\{(\sum\limits_{i=0}^{\infty}\vartheta_{t-j})-\sum\limits_{j=0}^{\infty}\lambda_{j}(\alpha^{i}_{t-j}+\beta\rho\sum\limits_{k=0}^{\infty}{}^{i}_{t-j-k})^{+w}_{t-j})\}^{2}].$$

Noting that  $i_t = \theta_t + u_t$  and rearranging gives

$$\begin{split} & \mathbb{E}[\{\mathbf{v} - \sum_{\mathbf{j}=0}^{t} \mathbf{d}_{\mathbf{t}-\mathbf{j}}\}^{2}] = \mathbb{E}[\{\theta_{\mathbf{t}}^{2}\{1 - \lambda_{0}(\alpha + \beta \rho)\}^{2} + \theta_{\mathbf{t}-1}^{2}\{1 - \lambda_{1}(\alpha + \beta \rho) - \lambda_{0}\beta \rho\}^{2} + \theta_{\mathbf{t}-2}^{2}\{1 - \lambda_{2}(\alpha + \beta \rho) - \lambda_{1}\beta \rho - \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots \\ & + \mathbf{u}_{\mathbf{t}}^{2}\{\lambda_{0}(\alpha + \rho \beta)\}^{2} + \mathbf{u}_{\mathbf{t}-1}^{2}\{\lambda_{1}(\alpha + \rho \beta) + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{0}\rho \beta\}^{2} + \theta_{\mathbf{t}-3}^{2} \cdots + \theta_{\mathbf{t}-2}^{2}\{\lambda_{2}^{2}(\alpha + \rho \beta) + \lambda_{1}\rho \beta + \lambda_{1$$

Now differentiate this with respect to  $\lambda_j$  and lead the resulting equation once and difference with the original, this gives

$$0 = \sigma_{9}^{2} \{ -(\alpha + \beta \rho) [1 - \lambda_{j} (\alpha + \beta \rho) - \lambda_{j-1} \rho \beta - \dots] - \rho [1 - \lambda_{j+1} (\alpha + \beta \rho) - \dots] \} +$$

$$+ \sigma_{u}^{2} \{ (\alpha + \rho \beta) [\lambda_{j} (\alpha + \rho \beta) + \lambda_{j-1} \dots] = \rho [\lambda_{j+1} (\alpha + \rho \beta) + \lambda_{j} \dots] \} +$$

$$+ (\lambda_{j} - \lambda_{j+1}) \sigma_{w}^{2}.$$
(A.1)

Lead once and difference again gives

$$\begin{split} 0 &= \sigma_{\bf j}^{\ 2} \{ -(\alpha + \beta \rho) [\lambda_{\bf j+1} (\alpha + \beta \rho) - \lambda_{\bf j} \alpha] + \alpha [\lambda_{\bf j+2} (\alpha + \beta \rho) - \lambda_{\bf j+1} \alpha] \} + \\ &+ \sigma_{\bf u}^{\ 2} \{ (\alpha + \rho \beta) [-\lambda_{\bf j+1} (\alpha + \rho \beta) + \alpha \lambda_{\bf j}] - \alpha [-\lambda_{\bf j+2} (\alpha + \rho \beta) + \alpha \lambda_{\bf j+1}] \} + \\ &+ (\lambda_{\bf j}^{\ -2} \lambda_{\bf j+1} + \lambda_{\bf j+2}) \sigma_{\bf w}^{\ 2}. \end{split}$$

Rearranging this equation we can write it as a second order homogeneous difference equation

$$0 = \lambda_{j+2}^{-B} \lambda_{j+1}^{+\lambda_{j}}$$

Where

$$B = 2 + \left[ \left( \frac{\alpha}{\rho \beta} \right)^2 + \frac{\omega}{\rho \beta^2} + \frac{\alpha}{\rho \beta} \right]^{-1}.$$

The only stable solution to this difference equation is of the form  $\lambda_j = C_\mu^j$  where  $\mu$  is as defined in (4). To find the value for the constant C take (A.1) when j=0 and substitute from  $\lambda_j = C_\mu^j$ .

$$\rho\sigma_{9}^{2} = C\{\left[1+(\beta\rho/\alpha)\right]^{2} - (\beta\rho/\alpha) - \lambda(1+(\beta\rho/\alpha))\right] + (1-\lambda)\beta\rho\omega/(\alpha)^{2}\}$$

Now use the fact that  $\mu^2$ -B $\mu$ +1=0; substitute for B and rearrange to solve for  $\rho\omega$ , then substitute this into the above and solve for C. This gives C =  $(1-\mu)/\beta$ .

#### References

Admati, A.R. & Pfleiderer, P. (1986) "A theory of intraday trading patterns" Research Paper No. 927, Graduate School of Business, Stanford University.

Cukierman, A. & Meltzer, A.H. (1986) "A theory of ambiguity, credibility, and inflation under discretion and asymmetric information" Econometrica 54. 1099-1128.

De Long, B.; Shleifer, A.; Summers, L. & Waldmann, R. "The economic consequences of noise traders" Discussion Paper No. 1348, Harvard Institute of Economic Research.

Glosten, L.R. & Milgrom, P.R. (1985) "Bid, ask and transaction prices in a specialist market with heterogenously informed traders" <u>Journal of Financial</u> Economics 14, 71-100.

Hart, O.D. & Kreps, D.M. (1986) "Price destabilizing speculation" <u>Journal of</u> Political Economy 94, 927-952.

Kyle, A.S. (1985) "Continuous auctions and insider trading" <u>Econometrica</u> 53, 1315-1335.

Kyle, A.S. (1987) "Dealer competition against an organized exchange" Unpublished.

Kyle, A.S. (1989) "Informed speculation with imperfect competition" Review of Economic Studies 56, 317-356.

Roell, A. (1989) "Dual-capacity trading in the presence of asymmetric information" Discussion Paper No. 8943, CentER for Economic Research, Tilburg University.

Shiller, R.J. (1981) "Do stock prices move too much to be justified by subsequent changes in dividends? American Economic Review 71, 421-436.

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