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TECHNOLOGICAL CHANGE IN MARKETS WITH NETWORK EXTERNALITIES

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Abstract

This paper examines a duopoly model in which firms have to decide simultaneously on product innovation and compatibility: successor technologies can be designed to be compatible or incompatible with the established industry standard. We show that in markets with homogeneous consumers, there may be a market bias towards a new standard (excess momentum), despite the presence of network externalities and an installed base of users of the old standard. In markets with heterogeneous consumers, sufficient conditions for the coexistence of two incompatible networks are derived. Excess momentum and excess inertia can arise. Finally, equilibria may exhibit too little as well as too much variety, relative to the social optimum.

Keywords: Network Externalities, Compatibility, Product Innovation.
JEL Classification: D62, O33.

1   Introduction

Decisions on innovation often have implications for compatibility; for instance, the decision to develop and introduce the compact-disc technology was at the same time a decision to have a dramatically new product which was incompatible with the existing long-playing records technology. A more recent case concerns the successors of the Compact Cassette (CC) technology. Philips has developed the Digital Compact Cassette technology, compatible with the CC technology in the sense that old tapes can be played on the new equipment. In contrast, Sony decided to pursue an incompatible technology named Mini Disc. It is therefore clear that decisions on technology are simultaneously decisions about compatibility. This paper considers markets with such features, endogenizes firms' decisions concerning technological change, and examines the nature of market incentives for innovation. We are concerned with questions such as the following: Does the existence of an installed base (of an old technology) make innovation more difficult? What role does the principle of differentiation play in determining product/technology variety in markets with network externalities?[1]

We consider a dynamic duopoly market in which the firms have to decide on the nature of the successor technology: an *upgrading* of the existing technology allows a firm to retain compatibility with the established industry network, whereas a major innovation commits it to a *replacement* technology which is incompatible with the extant standard. Firms compete in prices after they make their technological choices.

This economic process is formally modelled as a sequential game, and we study the fulfilled expectations equilibria of this game. Markets with only homogeneous consumers as well as markets with heterogeneous consumers are considered. For markets with homogeneous consumers, we focus, by having consumers coordinating, on equilibria which are efficient for them. For this class of equilibria we prove that innovations may occur in excess of what is socially desirable. The source of this inefficiency is the presence of network

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[1]The principle of differentiation says that firms competing in prices wish to differentiate themselves from each other in the product space, in order to reduce competition (see Tirole (1988), chapter 7).

externalities, since in the absence of network externalities, market outcomes would be socially efficient in our model. For markets with heterogeneous consumers, sufficient conditions for the coexistence of two incompatible networks (an upgraded and a new, incompatible technology), are derived. It is shown that, depending on the parameter values, such market outcomes may exhibit excessive innovation or too little innovation. Furthermore, we show that these equilibria may exhibit too little as well as too much technology/product variety (relative to the social optimum).

Our paper is part of a more general literature on the role of network externalities in markets (see for instance, Katz and Shapiro (1985), (1986a), (1986b), (1992), Farrell and Saloner (1985), (1986a), (1986b)). In particular, Farrell and Saloner (1985, 1986a) and Katz and Shapiro (1992) investigate the impact of network externalities on technological innovation and are more closely related to our paper.

Farrell and Saloner (1985) consider a model where a new technology is exogenously made available and where the presence of network externalities gives rise to a type of coordination game. They show that with common-knowledge about payoffs from a new technology, the socially optimal outcome is always attained; however, if these payoffs are private information then too little innovation, (excess inertia) as well as too much innovation, (excess momentum), may arise.[2] In contrast, in our paper, the nature of new technology is endogenously determined, firms have complete information about relevant variables and we focus on equilibria that rule out inefficiencies due to coordination failures (see assumptions 2.4 and 2.5 below). The results in our paper thus point to different sources of sub-optimal innovation. These sources are: (i) firms do not take consumer benefits into account while they make their technology decisions, and (ii) consumers who make adoption decisions for new technologies do not care about the "stranded base" effect that occurs when the network of users of an old technology ceases to grow.

Farrell and Saloner (1986b) study consumer adoption strategies where there is one existing technology and one entrant (or new) incompatible tech-

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[2] A well-known example of excess inertia is the existence of superior alternatives for the QWERTY-layout of keyboards, see David (1985).

nology. Here again the nature of technological change is, in a sense, exogenously given. Moreover, the incumbent firm which 'sponsors' the existing technology is not allowed any opportunity of technological change.

Katz and Shapiro (1992) provide a very general analysis of the problem of product innovation in markets with network externalities. A basic difference between our paper and Katz and Shapiro is that they allow for technology choices to be independent of compatibility choices, *i.e.*, decisions about technological change do not preclude any decisions about compatibility. This goes against the grain of the example about technological change in the audio market, given above. In terms of the modelling there are two interesting differences between the two papers; first, unlike their paper where only one firm can innovate, we allow for both firms in the market to make decisions concerning innovation, and second, whereas they have homogeneous consumers in the market we also consider markets with heterogeneous consumers. Thus, our paper complements the analysis by Katz and Shapiro.

The paper is organized as follows. Section 2 considers the model with homogeneous consumers. Equilibria are derived and compared to welfare maximizing outcomes. Section 3 investigates the model with heterogeneous consumers. We study coexistence of networks of incompatible technologies and provide examples in which inefficiencies occur. Finally, section 4 contains the conclusion.

2 A model with homogeneous consumers

2.1 The model

Consider the following dynamic duopoly. There are three periods, $t = 0, 1, 2$, and two firms, denoted by A and B. At the beginning of the game (period 0), the firms simultaneously decide whether to upgrade (u) or to replace (r) their initial technologies or products. Each firm has a set of actions $\{u, r\}$ at this stage of the game. In period 1, the firms sell their initial products, while in period 2, they sell their upgraded or new technologies. This sequential structure incorporates the idea that developing successor technologies is a

time consuming activity.

The initial (or period-1) technologies, s_A and s_B, are compatible with each other. A firm can choose to go in for an upgraded product, u_i (which is compatible with the existing industry standard) or a completely new product, r_i which is incompatible with the extant industry standard. Furthermore, we assume that u_A and u_B are compatible, while r_A and r_B are incompatible with each other. In period 2 only u_i or r_i are available to consumers. (This is not a restriction, since even if firms had the option of persisting with the old standard they would not do so in equilibrium.)

The marginal cost of firm i when it produces technology x_i is c_{x_i}, where $x_i = s_i, u_i, r_i$. In period 1, the firms have identical marginal costs, denoted by $c \equiv c_{s_A} = c_{s_B}$. However, cost asymmetries arise in the subsequent period. We assume that firm A has a marginal cost advantage with respect to upgrading, $c_{u_A} < c_{u_B}$, while firm B has a cost advantage when it introduces a new technology, $c_{r_B} < c_{r_A}$. Additionally, producing an upgraded version is more costly than producing the initial product, and replacing a technology is in turn more costly than upgrading a product. This can be written as $c_{r_i} > c_{u_j} > c$, $i,j = A,B$.

Consumers are infinitesemal. In period $t = 1,2$, N_t is the exogenously given size of the set of consumers, that is, it has measure N_t. The consumers in period 2 belong to a new generation; they are not in the market in period 1. A consumer living in period t has a completely inelastic demand for one unit of the good in that period, therefore, period-1 consumers cannot postpone their purchase decision until period 2. Adoption decisions in each period are made simultaneously by all consumers in that period.

The following assumption captures the idea that consumers are homogeneous.

Assumption 2.1 *Consumers are homogeneous in the sense that they have the same gross benefit function $b(x,n)$, where x is the technology that is purchased, and n is the size of the set of consumers buying ultimately compatible products (the network size at the end of period 2).*

Consumers maximize their net benefits $b(x,n) - p$ by making a choice

between the available products, where p is the price in the period at hand for technology x. Their utility reservation level is 0.

Network externalities are incorporated in the model by assuming that the benefit function is increasing in the number of consumers purchasing compatible items:

Assumption 2.2 $b(x,n) > b(x,m)$ *whenever* $n > m, \forall x$.

The following assumption concerns symmetry of benefit functions across firms, given equal network sizes.

Assumption 2.3 *Given network size* n, $b(s_A,n) = b(s_B,n)$, $b(u_A,n) = b(u_B,n)$, *and* $b(r_A,n) = b(r_B,n)$.

Since a consumer decides simultaneously with the other consumers in regarding his product, he must have beliefs about what the other consumers will do. The beliefs of a period-t consumer are captured in μ_t, defined as the beliefs on the fraction of consumers in period t purchasing from firm A.

Period-1 consumers can observe the pair of actions chosen by the firms at time 0. The structure of the game is understood by the players (there is complete information) and this is common knowledge.

The firms compete in prices. The price of firm i in period t is denoted by p_{it}, $i = A, B$, $t = 1, 2$. Let $m_i \in \{u, r\}$ denote the action of firm i in period 0, $i = A, B$. In period 1, firm i earns profits equal to $\pi_{i1}(p_{i1}, p_{j1}) = n_{i1}(p_{i1}, p_{j1})(p_{i1} - c)$, $t = 1, 2$, $i \neq j$, when it has sales of n_{i1}. Profits in period 2 are defined in an equivalent manner: $\pi_{i2}(p_{i2}, p_{j2}) = n_{i2}(p_{i2}, p_{j2})(p_{i2} - c_{x_i})$, where $x_i = u_i$ if $m_i = u$, and $x_i = r_i$ if $m_i = r$.

Finally, we implicitly assume that gross benefits are sufficiently large so that all consumers buy. Thus, prices will not be too high relative to gross benefits.

2.2 Equilibrium concept

The equilibrium notion used is fulfilled expectations equilibrium (Katz and Shapiro (1985) use a similar equilibrium notion). An equilibrium consists of

technology choices (m_A^*, m_B^*), prices (p_{At}^*, p_{Bt}^*), consumer beliefs μ_t^*, and sizes of consumer groups purchasing from each firm (n_{At}^*, n_{Bt}^*), $t = 1, 2$, such that

(t=2) Given (m_i, m_j), (p_{i1}, p_{j1}), (n_{i1}, n_{j1}), and given p_{j2}^*, each firm $i \neq j$ maximizes its period-2 profits π_{i2} by selecting $p_{i2} = p_{i2}^*$ (as a function of the history of the game). Period-2 consumers make their purchase decisions in order to maximize their net benefits, and their beliefs are fulfilled: $\mu_2^* N_2 = n_{A2}^*$ and $(1 - \mu_2^*)N_2 = n_{B2}^*$. Notice that there is a coordination problem when $(m_i, m_j) = (u, r)$, because in this case consumers have to choose between incompatible technologies (u_i and r_j).

(t=1) Given (m_i, m_j), and given p_{j1}^*, each firm $i \neq j$ maximizes its total profits $\pi_{i1} + \pi_{i2}$ by selecting $p_{i1} = p_{i1}^*$ (when consumers and both firms act optimally afterwards). Period-1 consumers make their purchase decisions in order to maximize their net benefits, and their beliefs are fulfilled. However, there is no coordination problem because the products for sale (s_A and s_B) are compatible with each other.

(t=0) Given m_j^*, each firm $i \neq j$ maximizes its total profits $\pi_{i1} + \pi_{i2}$ by selecting $m_i = m_i^*$ (when consumers and both firms act optimally afterwards).

We require an equilibrium to be subgame perfect. Therefore, it is ensured that no firm, taking the other firm's equilibrium actions as given, wishes to change its own actions at each stage of the game. Furthermore, given the prices in period t, consumers maximize their net benefits by making their adoption decisions.

In this paper, we are concerned with the welfare properties of equilibrium. In particular, we will argue that, in the presence of network externalities, market incentives are inappropriate. In view of these results, we concentrate our attention on the 'good' equilibrium. Informally speaking, an equilibrium is good if it is not Pareto-dominated by some other equilibrium. We look at equilibria in which period-2 consumers do not choose both upgraded and replacement technologies; this is motivated by the idea that an equilibrium with such a property will be Pareto-inefficient. Formally,

Assumption 2.4 *In an equilibrium, all period-2 consumers choose the same technology.*

This assumption is best motivated through an argument from the finite consumer case. As Katz and Shapiro (1986b) argue, in case of finitely many homogeneous consumers, coexistence of two networks (of upgraded and replaced products) implies that the payoff from the two networks is equal. However, in that case a single consumer can always gain from switching, thus setting into movement a wave of switches by other consumers, which destroys the equilibrium with coexisting networks.

The second assumption concerns multiple equilibria with single networks. To avoid any coordination problem in this context, we require that,

Assumption 2.5 *When there exist two equilibria with single networks, consumers in period 2 coordinate on the network that maximizes their net surplus. If two networks yield the same surplus for period 2 consumers they choose the network that yields the (over periods 1 and 2) maximal net surplus.*

We now make precise the notions of the inefficiencies that can occur. The social optimum is defined as the outcome that maximizes welfare. Welfare is defined as the sum of the total profits and the net consumers' surpluses. A social planner (who wishes to implement the social optimum) may not only have to restrict the innovation decisions of the firms, but also the adoption decisions of period-2 consumers.

We say that an equilibrium exhibits *excess inertia* when period-2 consumers adopt the upgraded technology although the adoption of the new technology would yield a higher welfare level. Notice that when both firms upgrade their product, consumers in period 2 are not able to purchase a new technology; therefore, in some cases the inefficiency may be attributed to the firms. *Excess momentum* arises when consumers in period 2 purchase the new technology (possibly again because they have no other choice, due to the technology choices of the firms), while (the introduction and) the adoption of an upgraded product would be preferred by a social planner.

2.3 Analysis

In this subsection equilibria will be calculated and compared to socially optimal outcomes. We focus on the technological choices of firms in period 0, i.e., (m_A, m_B).

Given the compatibility structure, an installed base of size N_t is formed for the initial industry standard. Indeed, all the period-1 consumers purchase the good from one of the two firms, irrespective of (m_A, m_B). The reason is that, since both s_A and s_B are compatible with an upgraded product of any of the two firms (and incompatible with any new technology), consumers are indifferent between the two brands. Therefore, in an equilibrium the firms price at marginal cost in period 1 ($p_{A1}^* = p_{B1}^* = c$) and make zero profits. However, notice that if at least one firm upgrades its product, then period-1 consumers' benefits depend on the *adoption decisions of consumers* in period 2.

To verify what happens in period 2, we distinguish four cases, corresponding to the pairs of actions that can be chosen in period 0.

(i) $(m_A, m_B) = (u, u)$. Because u_A and u_B are compatible with one another and with the earlier "vintages", the relevant network size is $N_1 + N_2$. Consumers compare $b(u_A, N_1 + N_2) - p_{A2}$ and $b(u_B, N_1 + N_2) - p_{B2}$. Firm A is able to capture the entire market because is has a marginal cost advantage. Therefore, firm B sets the lowest price at which it would not make a loss were it to make sales, $p_{B2}^* = c_{u_B}$, while firm A sets p_{A2}^* "just below" c_{u_B}. Sales volumes are $n_{A2}^* = N_2$ and $n_{B2}^* = 0$ (and $\mu_2^* = 1$).

(ii) $(m_A, m_B) = (u, r)$. Consumers compare $b(u_A, N_1 + \mu_2 N_2) - p_{A2}$ and $b(r_B, (1 - \mu_2) N_2) - p_{B2}$. Define $\alpha = b(u_A, N_1 + N_2) - c_{u_A}$ and $\beta = b(r_B, N_2) - c_{r_B}$, the maximum benefits that firm A and firm B can offer, respectively.

- If $\alpha \geq \beta$ then firm A wins all the sales; $p_{A2}^* = c_{u_A} + (\alpha - \beta) = b(u_A, N_1 + N_2) - b(r_B, N_2) + c_{r_B}$ and $p_{B2}^* = c_{r_B}$, while $(n_{A2}^*, n_{B2}^*) = (N_2, 0)$ (the case where $\alpha = \beta$ follows from assumption 2.5).

- If $\alpha < \beta$ then firm B captures the market; $p_{A2}^* = c_{u_A}$ and $p_{B2}^* = c_{r_B} + (\beta - \alpha) = b(r_B, N_2) - b(u_A, N_1 + N_2) + c_{u_A}$, while $(n_{A2}^*, n_{B2}^*) = (0, N_2)$.

(iii) $(m_A, m_B) = (r, u)$. As in the previous case, define $\alpha' = b(r_A, N_2) - c_{r_A}$ and $\beta' = b(u_B, N_1 + N_2) - c_{u_B}$,

   - If $\alpha' > \beta'$ then firm A attracts all the consumers; $p_{A2}^* = b(r_A, N_2) - b(u_B, N_1 + N_2) + c_{u_B}$ and $p_{B2}^* = c_{u_B}$, while $(n_{A2}^*, n_{B2}^*) = (N_2, 0)$.
   - If $\alpha' \le \beta'$ then firm B captures the market; $p_{A2}^* = c_{r_A}$ and $p_{B2}^* = b(u_B, N_1 + N_2) - b(r_A, N_2) + c_{r_A}$, while $(n_{A2}^*, n_{B2}^*) = (0, N_2)$ (the case where $\alpha' = \beta'$ follows again from assumption 2.5).

(iv) $(m_A, m_B) = (r, r)$. Firm B obtains the entire market demand because of its marginal cost advantage; $p_{A2}^* = p_{B2}^* = c_{r_A}$ and $(n_{A2}^*, n_{B2}^*) = (0, N_2)$.

Now that the outcome is known for each pair of actions (m_A, m_B), the game can be "folded back" and considered as a one-shot game where the firms choose their actions simultaneously. A Nash equilibrium in this one-shot game is an equilibrium pair (m_A^*, m_B^*) in the overall game. Below, four parameter ranges are distinguished in which the strategic form is given (with firm A on the horizontal axis, and firm B on the vertical axis). The payoffs in these matrices are $(\pi_{A1} + \pi_{A2}, \pi_{B1} + \pi_{B2})$, the total profits of firm A and firm B, respectively.

The equilibria of the game are compared to the outcomes that would be selected by a social planner who wishes to maximize welfare. Because of the marginal cost asymmetries, if the social optimum is attained then either u_A or r_B is produced and adopted in period 2. In the first case, the welfare level is equal to

$$W(u_A) \equiv N_1(b(s_i, N_1 + N_2) - c) + N_2(b(u_A, N_1 + N_2) - c_{u_A}), \qquad (1)$$

while in the latter case, it equals

$$W(r_B) \equiv N_1(b(s_i, N_1) - c) + N_2(b(r_B, N_2) - c_{r_B}). \qquad (2)$$

A social planner will enforce introduction and adoption of r_B if and only if $W(u_A) < W(r_B)$, or

$$N_1(b(s_i, N_1 + N_2) - b(s_i, N_1)) < N_2(\beta - \alpha). \tag{3}$$

We will use inequality (??) to verify if market outcomes are socially efficient.

A. $\alpha \geq \beta$ and $\alpha' > \beta'$. When α is strictly larger than β, the equilibria are (u, u) and (u, r). The identification of (u, r) as a Nash equilibrium is straightforward. To see that (u, u) is an equilibrium, note that $c_{u_B} - c_{u_A} \geq \alpha' - \beta'$ if and only if $b(u_B, N_1 + N_2) - c_{u_A} \geq b(r_A, N_2) - c_{r_A}$. The latter inequality holds strictly because $\alpha \geq \beta$, and $\beta > b(r_A, N_2) - c_{r_A}$. When α equals β, there is a third equilibrium: (r, r). (See figure 1.)

	u	r
u	$N_2(c_{u_B} - c_{u_A})$, 0	$N_2(\alpha - \beta)$, 0
r	$N_2(\alpha' - \beta')$, 0	0 , $N_2(c_{r_A} - c_{r_B})$

Figure 1: The game in case A.

Because $\alpha \geq \beta$ and $N_1(b(s_i, N_1 + N_2) - b(s_i, N_1)) > 0$, inequality (??) does not hold. Therefore, welfare maximizing outcomes are (u, u) and (u, r), since they lead to the adoption of u_A in period 2. Equilibrium outcome (r, r) exhibits excess momentum.

B. $\alpha \geq \beta$ and $\alpha' \leq \beta'$. One can easily verify in figure 2 that (u, u) and (u, r) are equilibrium pairs when α is strictly larger than β. When α is equal to β, there is a third equilibrium: (r, r). To see this, note that $c_{r_A} - c_{r_B} \geq \beta' - \alpha'$ if and only if $b(r_A, N_2) - c_{r_B} \geq b(u_B, N_1 + N_2) - c_{u_B}$. The latter inequality holds strictly because $\alpha = \beta$ and $\alpha > b(u_B, N_1 + N_2) - c_{u_B}$.

It is easy to verify that the social optimum is effected in (u, u) and (u, r), just as in the previous case. Equilibrium outcome (r, r) exhibits excess momentum.

	u	r
u	$N_2(c_{u_B} - c_{u_A})$, 0	$N_2(\alpha - \beta)$, 0
r	0 , $N_2(\beta' - \alpha')$	0 , $N_2(c_{r_A} - c_{r_B})$

Figure 2: The game in case B.

	u	r
u	$N_2(c_{u_B} - c_{u_A})$, 0	0 , $N_2(\beta - \alpha)$
r	$N_2(\alpha' - \beta')$, 0	0 , $N_2(c_{r_A} - c_{r_B})$

Figure 3: The game in case C.

C. $\alpha < \beta$ and $\alpha' > \beta'$. Figure 3 shows that equilibrium pairs are (u, r) and (r, r).

In both equilibria technology r_B is adopted in period 2. Therefore, welfare is maximized by the market forces if and only if inequality (??) holds. The interpretation of inequality (??) is that in order to achieve coincidence of private and social incentives, the increase in the surplus of period-1 consumers when the size of their network grows with N_2 has to be less than the advantage of r_B over u_A for period 2 consumers. The market outcomes are inefficient (in the sense that they exhibit excess momentum) if and only if inequality (??) does not hold.

D. $\alpha < \beta$ and $\alpha' \leq \beta'$. As can be seen in figure 4, equilibrium pairs are (u, r) and (r, r).

	u	r
u	$N_2(c_{u_B} - c_{u_A})$, 0	0 , $N_2(\beta - \alpha)$
r	0 , $N_2(\beta' - \alpha')$	0 , $N_2(c_{r_A} - c_{r_B})$

Figure 4: The game in case D.

Just as in case C, we find that the market outcomes (u, r) and (r, r)

are optimal from a welfare point of view if and only if inequality (??) holds.

The results of the analysis are summarized in the following proposition.

Proposition 2.1 *Suppose assumptions 2.1-2.5 are satisfied. Then, if $\alpha > \beta$, (u, u) and (u, r) are equilibrium pairs and socially optimal outcomes; if $\alpha = \beta$, equilibrium pairs are (u, u), (u, r) and (r, r), socially optimal outcomes are (u, u) and (u, r); if $\alpha < \beta$, equilibrium pairs are (u, r) and (r, r), these pairs are socially optimal if and only if (??) holds.*

An important insight of the analysis so far is that excess innovation is possible whereas too little innovation is impossible, when consumers coordinate efficiently across equilibria. To be precise, excess momentum occurs in the case where $\alpha = \beta$ and $(m_A^*, m_B^*) = (r, r)$, and in the case where $\alpha < \beta$ and (??) does not hold. In these cases, the network for the initial standard ceases to grow after period 1. A part of the explanation for this result is that period-2 consumers ignore the effects of their adoption decisions on period-1 consumers. In our model there is also another explanation, namely that the firms do not consider the effects on consumers when they decide on product innovation and compatibility. This deviation from social optima arises due to the presence of network externalities, by means of the 'stranded base' phenomenon (Farrell and Saloner (1986a)). In the absence of such externalities, there will be no connection between the two periods, and in period 2 the firm with the product which yields the largest social surplus will simply capture the market, via standard (differentiated cost) price competition.

In this context the precise structure of the model becomes important; we have assumed that an upgraded technology is compatible with the initial industry standard in general, not only with the previous version of the same firm's technology. This structural assumption precludes the possibility of an upgrading firm making implicit transfers across generations of consumers, via charging high prices in period 1 and low prices in period 2.

We conclude this section with an observation on the role of the relationship between assumption 2.5 and impossibility of excess inertia. Suppose that $\alpha < \beta$, and that (??) holds. When assumption 2.5 is not

satisfied, it is possible that consumers in period 2 coordinate on u_A instead of r_B. When firms expect this outcome, they set prices $(p^*_{A2}, p^*_{B2}) = (b(u_A, N_1 + N_2) - b(r_B, 0) + c_{rB}, c_{rB})$. Note that (u, r) is an equilibrium pair of actions for the firms. The corresponding welfare level is $W(u_A) < W(r_B)$. This example demonstrates that when consumers are not able to coordinate on the outcome that maximizes their surplus, excess inertia can occur.

3 A model with heterogeneous consumers

3.1 The model

The results of the analysis in the previous section depend heavily on the assumption that consumers are homogeneous. We saw that, under general assumptions, excess momentum is easygoing. Excess inertia does not arise under the standard assumptions, but is a possibility if period-2 consumers focus on the "wrong" technology (assumption 2.5 is violated). However, preferences for different technologies may put more weight in some consumers' product choice decisions than network sizes. This observation leads us to the idea that heterogeneity among consumers may result in the coexistence of networks of incompatible technologies, and possibly in excess inertia without having consumers coordinating on a "wrong" equilibrium.

We model the idea of consumer heterogeneity in terms of greater or lesser inclination for new products. When a consumer has to make a choice between an upgraded product and a new one, her decision will *inter alia* depend on her attachment to the old product (or standard). In the model a period-2 consumer does not have the old product, but she may be more or less familiar with it. A more conservative consumer is inclined to buy the upgraded product, whereas a more innovative consumer is willing to adopt the new technology.

This heterogeneity is introduced in period 2, where consumers may have a choice between upgraded and replacement technology; in period 1, consumers can only adopt the initial industry standard. We examine heterogeneity in terms of types of period 2 consumers. We denote the type of a consumer

by θ, where $\theta \in [0,1]$, and the differences across consumers are modelled in terms of benefit functions, $b_\theta(x,n)$. We formally define heterogeneity as follows:

Assumption 3.1 *Consumers in period 2 are heterogeneous with respect to their benefit function. They are uniformly distributed with respect to their type θ. Their benefit function satisfies:*
(i) $b_\theta(u_i, n)$ is (weakly) decreasing in $\theta, \forall n, i = A, B$,
(ii) $b_\theta(r_i, n)$ is (weakly) increasing in $\theta, \forall n, i = A, B$.

This assumption replaces assumption 2.1 above. It models the idea that given a fixed network size, a more conservative (lower θ type) consumer derives more gross benefits from an upgraded product than an innovative (high θ type) consumer.

The assumption on network externalities is maintained:

Assumption 3.2 *For period 1 consumers, $b(x,n) > b(x,m)$ whenever $n > m$, $\forall x$; and for period 2 consumers $b_\theta(x,n) > b_\theta(x,m)$ whenever $n > m$, $\forall x, \theta$.*

The rest of the set-up of the model remains unchanged.

3.2 Equilibrium concept

The same equilibrium concept as in the previous section is used. Assumption 2.4 is dropped; if heterogeneous preferences are strong relative to network advantages, then consumers in period 2 may find it in their interest to adopt incompatible technologies.

The notions of excess inertia and excess inertia as explained in section 2.2 are extended in the following way. If the number of period-2 consumers that purchase an upgraded product in an equilibrium is too high to attain the maximum welfare level, then that equilibrium exhibits excess inertia. Likewise, if too many consumers (more than the number that would be socially optimal) adopt a new, incompatible technology in period 2 in an equilibrium, then excess momentum occurs.

A related issue concerns the degree of variety of products available in the market. In the context of markets with heterogeneous consumers, it allows an explicit examination of the interaction between the forces of differentiation and compatibility, in a dynamic environment. We say that an equilibrium exhibits too much (too little) variety if there are more (less) active networks in period 2 than what is socially optimal.

3.3 Analysis

We explore the implications of heterogeneity in consumers tastes for the co-existence of incompatible networks, and the welfare properties of the equilibrium more generally: Under what conditions will two incompatible networks coexist? Does the market generate excessive or too little incentives for technological change? Is there too little or too much variety in the market?

We first derive sufficient conditions for coexistence of two incompatible networks in equilibrium. The following lemma is useful in deriving these conditions.

Lemma 3.1 *If* $\pi_{A2}(u,r) > 0$ *and* $\pi_{B2}(u,r) > 0$, *then* (u,r) *or* (r,u) *is the equilibrium pair of actions for the firms.*

Proof: Recall that $c_{u_A} < c_{u_B}$ and $c_{r_A} > c_{r_B}$. Therefore $\pi_{A2}(u,u) > 0$, $\pi_{B2}(u,u) = 0$, $\pi_{A2}(r,r) = 0$, and $\pi_{B2}(r,r) > 0$. Since profits in period 1 are equal to zero, the game in simplified strategic form is given by figure 5. The

	u	r
u	$\pi_{A2}(u,u)$, 0	$\pi_{A2}(u,r)$, $\pi_{B2}(u,r)$
r	$\pi_{A2}(r,u)$, $\pi_{B2}(r,u)$	0 , $\pi_{B2}(r,r)$

Figure 5: The game in simplified strategic form.

proof follows immediate from figure 5. □

We next consider conditions on the primitives of the model that are sufficient to meet the requirements derived in the above lemma. A first step in

this direction is the definition of *strong preferences*. We say that consumers have strong preferences for a good if they are willing to overcome network disadvantages suffered by that good. More formally, consumers have strong preferences if for $i, j = A, B$, and for some $\epsilon > 0$, $\theta_u > 0$ and $\theta_r < 1$ the following inequalities are satisfied,

$$\forall \theta \in [0, \theta_u) : b_\theta(u_i, N_1) - c_{u_i} \geq b_\theta(r_j, N_2) - c_{r_j} + \epsilon$$
$$\forall \theta \in (\theta_r, 1] : b_\theta(r_j, 0) - c_{r_j} \geq b_\theta(u_i, N_1 + N_2) - c_{u_i} + \epsilon. \qquad (4)$$

Inequalities (??) state that extreme types, *i.e.*, types near 0 and near 1, have strong preferences. When products are valued at marginal costs, the net benefits of u_i for a consumer of a type near 0, when she is the only person buying that product, are larger than the net benefits she could obtain by joining the network of r_j buyers. On the other hand, a consumer type near 1 prefers r_j irrelevant of its network size when the products are priced at marginal cost. Proposition 3.1 shows that if inequalities (??) are satisfied then in an equilibrium both firms can earn positive profits under (u, r).

Proposition 3.1 *Suppose assumptions 3.1 and 3.2 hold. If there exist $\theta_u > 0$ and $\theta_r < 1$ such that inequalities (??) hold, then in an equilibrium $\pi_{A2}(u, r) > 0$ and $\pi_{B2}(u, r) > 0$.*

Proof: Suppose that $(m_A, m_B) = (u, r)$, and that there exist $\theta_u > 0$ and $\theta_r < 1$ such that inequalities (??) hold.

First, suppose $p_{B2}^* \geq c_{r_B}$; then given that inequalities (??) are satisfied, for some $\hat{\theta} < \theta_u$, there is an $\epsilon > 0$ such that,

$$b_{\hat{\theta}}(u_A, N_1) - c_{u_A} \geq b_{\hat{\theta}}(r_B, N_2) - p_{B2}^* + \epsilon$$

Given assumptions 3.1 and 3.2, this implies that for any $p_{u_A}^*$ such that $c_{u_A} \leq p_{A2}^* \leq c_{u_A} + \epsilon$, for all $\theta \in [0, \hat{\theta}]$, optimal choice is brand A, and hence $n_{A2} > 0$. Since p_{A2}^* is optimal, the equilibrium profits must be strictly positive too, *i.e.*, $\pi^*(u, r) > 0$. It is easy to show that in an equilibrium $p_{B2}^* < c_{r_B}$ generates a contradiction. This completes the proof for $\pi_{A2}^*(u, r) > 0$.

Second, suppose that $p_{A2}^* \geq c_{u_A}$ is given. Analogous arguments as in the previous case hold for firm B. This completes the proof. $\square$

The following corollary is an immediate consequence of proposition 3.1 and lemma 3.1, and gives sufficient conditions for coexistence of two incompatible networks in equilibrium.

Corollary 3.1 *Suppose that assumptions 3.1 and 3.2 hold. If there exist an $\epsilon > 0$ and $\theta_u > 0$ and $\theta_r < 1$ such that inequalities (??) hold, then in any equilibrium $n^*_{A2} > 0$ and $n^*_{B2} > 0$.*

These conditions can be weakened if one allows for conditions on beliefs of agents; here the focus has been on the primitives of the model, however. The sufficient conditions are fairly intuitive: If there is some measure of agents whose choice of a brand is relatively insensitive to the size of the networks then incompatible networks will coexist. In contrast to section 2, assuming that period-2 consumers adopt one technology cannot be justified by Pareto-optimality because there may be two groups of consumers with strong preferences for incompatible products.

The next question that arises naturally concerns the welfare properties of coexistence equilibria. To examine them we construct examples which suggest that equilibria with coexisting networks can, depending on parameter values, exhibit excess inertia as well as excess momentum. The examples suggest that, in markets with heterogeneous consumers, and with network effects, market incentives do not reflect social benefits accurately.

Consider the following benefit functions. In period 1, the benefit function is

$$b(x, n) = a + k_1(n), x = s_A, s_B.$$

In period 2, a consumer of type θ has benefit function

$$b_\theta(x, n) = \begin{cases} b + k_2(n) & \text{if } x = u_A, u_B \text{ and } 0 \leq \theta \leq \frac{1}{2} \\ (b - z) + k_2(n) & \text{if } x = u_A, u_B \text{ and } \frac{1}{2} < \theta \leq 1 \\ (d - y) + k_3(n) & \text{if } x = r_A, r_B \text{ and } 0 \leq \theta \leq \frac{1}{2} \\ d + k_3(n) & \text{if } x = r_A, r_B \text{ and } \frac{1}{2} < \theta \leq 1. \end{cases}$$

The parameters satisfy $a < b < d$, *i.e.*, there is technological progress. For simplicity, we assume that $N_1 = N_2 \equiv N$ and $c = 0$.

Example 1: Excess momentum. Suppose that $N = 1/2$; $k_1(n) = k_2(n) = k_3(n) = n$, $\forall n$; $b = 2$, $z = 1.75$, $d = 3$, $y = 2.625$; $c_{u_A} = 0.05$, $c_{r_A} = 3.5$, $c_{u_B} = 2.75$ and $c_{r_B} = 2.75$.

It is possible to show that the following configuration can be sustained in an equilibrium: $m_A^* = u, m_B^* = r$; $p_{A1}^* = p_{B1}^* = 0$, $p_{A2}^* = 2.75, p_{B2}^* = 3.25$; $n_{A2}^* = n_{B2}^* = 1/4$. It is easy to check that the inequalities (??) are not satisfied; they are, thus, not necessary for coexistence of two networks. Denote the social welfare levels attained in case of a single network with only upgraded product, with only replacement product, and coexisting networks by $W(u)$, $W(r)$ and $W(u,r)$, respectively. It is easy to compute these expressions for this example. They are as follows,

$$W(u) \approx .5a + 1.54,$$

$$W(r) = .5a - .031,$$

$$W(u,r) = .5a + 1.175.$$

Thus, the equilibrium with coexistence exhibits excessive momentum relative to the socially optimal outcome. In the absence of network effects, however, such a coexistence equilibrium is in the present example socially optimal. The intuition behind this inefficiency is as follows: Consumers have strong preferences for some type of product and firms exploit this feature to create 'local' monopoly type effects; but in the presence of network externalities this generates too high an incentive to introduce a new standard, and leads to excessive momentum.

Example 2: Excess inertia. Suppose that $N = 1$; $k_1(n) = k_2(n)$, $\forall n$, $k_1(n) = n$ for $n \leq N$ and $k_1(n) = N + 1/16(n - N)$ for $n > N$, $k_3(n) = n \forall n$; $b = 2$, $z = 1.15$, $d = 3$, $y = 1.15$; $c_{u_A} = 2$, $c_{r_A} = 3.5$, $c_{u_B} = 2.2$ and $c_{r_B} = 2.25$.

It is possible to show that the following configuration can be sustained in an equilibrium: $m_A^* = u$, $m_B^* = r$; $p_{A1}^* = p_{B1}^* = 0$, $p_{A2}^* \approx 3.03$, $p_{B2}^* = 3.5$; $n_{A2}^* = n_{B2}^* = 1/2$. Denote the social welfare levels attained, as in example 1, above. For this example, they are computed as follows

$$W(u) = a + 1.55$$

$$W(r) = a + 2.175$$

$$W(u,r) = a + 2.172$$

The social optimum is not attained in the coexistence equilibrium. In particular, the outcome where all period-2 consumers adopt the new standard yields a higher welfare level. We observe excess inertia. This is not true in the absence of network externalities; then there exists an equilibrium with coexisting networks and it is socially optimal. This social sub-optimality emerges due to the desire of firms to exploit 'local' monoploy effects of product differentiation, which generates too high (low) a market share for u (r) technology, leading to 'too little' innovation, relative to the social optima.

The two examples above also demonstrate that market equilibrium may exhibit excessive variety. The following example shows that an equilibrium may exhibit too little variety (as in Farrell and Saloner (1986b)). In view of the general intuition underlying the principle of maximal differentiation, this is an interesting issue.

Example 3: Too little variety. Suppose that $N = 1$; $k_1(n) = 2n$, for $n \leq 3/2, k_1(n) = 1.5 + n$ for $n > 3/2$; $k_2(n) = n$, $k_3(n) = 1.5n$ for $n \leq 1$, $k_3(n) = .5 + n$ for $n > 1$; $b = 1, z = .75$; $d = 4, y = 2$; $c_{u_A} = 2, c_{r_A} = 3.5$, $c_{u_B} = 2.20, c_{r_B} = 2.25$.

It is possible to show that the following can be sustained in an equilibrium: $m_A^* = u, m_B^* = r$; $p_{A1}^* = p_{B1}^* = 0, p_{A2}^* = 2, p_{B2}^* = 3.5$; $n_{A2}^* = 0, n_{B2}^* = 1$. The social welfare levels can be computed as follows:

$$W(u) = a + 4.125$$

$$W(r) = a + 4.250$$

$$W(u,r) = a + 5.125$$

Thus, the socially optimal outcome involves coexistence of networks, whereas there is an equilibrium in which one network covers the entire market. This difference between social optima and market equilibrium is due to two reasons: first, the network effects that period 2 consumers generate for period

1 consumers, and second, the coordination problem faced by the consumers preferring an upgraded product in case they would like to switch to the other network.

The examples demonstrate that a market equilbrium may exhibit excess momentum (example 1) or excess inertia (example 2). Moreover, it may exhibit too much variety (examples 1 and 2) or too little variety (example 3).

4   Conclusion

This paper has been concerned with the question: In the presence of network externalities, what is the nature of market incentives for innovation? We consider a dynamic duopoly model in which the firms have to decide on the nature of the successor technology: an *upgrading* of the old product allows a firm to retain compatibility with the established industry network, whereas a major innovation commits it to a *replacement* technology which is incompatible with the extant standard.

We show that in markets with homogeneous consumers, major innovations may occur in excess of what is socially desirable (excess momentum), and that too little innovation (excess inertia) is not possible if consumers can coordinate on networks which yield their generation maximum payoffs. For markets with heterogeneous consumers, sufficient conditions for the co-existence of two incompatible networks (of an upgraded and a replacement technology) are derived and it is shown that, depending on the parameter values, such market outcomes can exhibit too little or excessive innovation. Finally, we show that equilbrium may exhibit excessive as well too little product/technology variety, relative to the social optimum.

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