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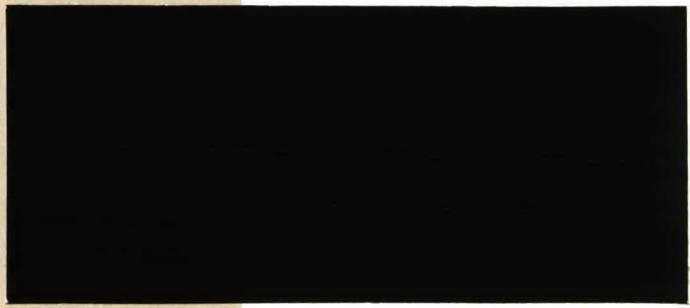
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No. 9214

**DISCRETE CHOICE MODELS OF
FAMILY LABOUR SUPPLY** *R18*

by Arthur van Soest *labour supp*
Choice Theory

April 1992

ISSN 0924-7815

Discrete Choice Models of Family Labour Supply

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April 1992

Abstract

A static neoclassical structural model is presented, explaining labour supply of both spouses in two adults households. Family preferences are described in terms of a direct translog household utility function, with the husband's leisure, the wife's leisure, and family consumer expenditures as its arguments. We assume that the choice set for each family consists of a finite number of points (c, l_m, l_f) , where l_m and l_f are the husband's and wife's hours of leisure per week and c is family income, where account is taken of the main features of the Dutch tax system. In the basic model, error terms are incorporated in the same way as in the multinomial logit model. Compared to models developed earlier in the literature, this model has several advantages. Since tangency conditions and duality theory are not needed, coherency of the model is a priori guaranteed. No parameter restrictions have to be imposed, and regularity of preferences can be tested. Moreover, the model is an adequate framework to build in extensions and a more general error structure.

We discuss several extensions, in which we allow for hours restrictions and random preferences, and account for the fact that wage rates of non-workers are not observed. Estimation of some of the extended models by exact ML is intractable since it involves integrating out the unobserved error terms. Instead, the likelihood function is approximated, replacing the integral by a mean based on random draws. The models are estimated using cross-section data on Dutch families from 1987. Results are compared using estimated confidence bounds of labour supply elasticities for the average family.

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I am grateful to Stephen Jenkins, Menno Pradhan, and others for useful comments at the 'Modelling the Labour market' conference in Florence and a seminar at CentER. The Netherlands Central Bureau of Statistics (CBS) has kindly provided the data. The views expressed in this paper are those of the author and do not necessarily reflect those of the CBS.

1. Introduction

In recent years, the neoclassical static model of individual labour supply has received a lot of attention. See e.g. Moffitt (1990a) for a collection of applications to European and US data sets. Issues such as the impact of non-linear taxation, fixed costs of working, unemployment benefits, measurement errors, unobserved preference variation, endogeneity of wage rates, and institutional or demand side restrictions on hours worked, have been addressed extensively in the individual framework. For households with two adults, the usual approach in empirical work is to analyse male and female labour supply recursively: The husband is assumed to decide first, without taking account of e.g. the wife's wage rate, and the wife decides conditionally upon her husband's labour supply and earnings. This approach for example imposes the *a priori* restriction that the elasticity of the husband's labour supply with respect to the wife's wage rate or, in case of hours restrictions, her working hours, is equal to zero.

During the past decades, labour market participation of females in The Netherlands has strongly increased (both measured in terms of hours worked, and measured in the number of people with a paid job), while average working hours of males have decreased substantially. In an influential recent report, the government is advised to consider several policy measures (tax and benefits reforms, child care improvements, etc.) to stimulate female labour supply and a more equal distribution of paid work among spouses (WRR, 1990).

In order to study the impact of such policy measures, a model in which labour supply of both spouses is modeled simultaneously seems more appropriate than the single individual model. The natural extension of the structural version of the latter, is the model based on joint utility maximisation of a household utility function, with family consumer expenditures, the husband's leisure and the wife's leisure as its arguments. This is the framework which we use in this paper.¹⁾

Compared to the large number of empirical applications of the single individual model, the applications of the two adults model are scarce. Exceptions are Hausman and Ruud (1984), Ransom (1987, 1989), and Kapteyn et al. (1990). Gertler and Newman (1991) estimate a generalisation for the case that more than two family members supply labour. In all these models, hours worked by the two spouses are treated as mixed discrete and continuous random variables. Tangency conditions are used to solve the family's joint utility maximisation problem. Second order conditions must be satisfied in

order to make this a valid approach. If, for example, nonnegativity of the Slutsky matrix is violated, solving the system of tangency conditions no longer corresponds to maximising direct utility. Moreover, the system tangency conditions may have zero or more than one solutions. Thus, as in the single individual model, parameters must satisfy restrictions to guarantee model coherency (cf., e.g., Blundell, 1990, and Van Soest et al., 1992). As a consequence, flexibility of the preference specification can be reduced substantially.

Moreover, the continuous framework implies that utility maximisation becomes intricate in case of non-standard restrictions. Allowing for more general budget sets (non-linear taxation, joint filing, unemployment benefits, fixed costs, hours constraints) leads to an essentially more complicated model. This limits the extent to which policy measures can be analysed. Model tractability arguments have also affected the sophistication of the stochastic specifications used. Random preferences, optimisation errors, and measurement errors due to incomplete wage observations, have not yet been incorporated simultaneously.

In this paper, labour supply is treated as a discrete random variable. The approach is related to the work of e.g. Dickens and Lundberg (1985), Tummers and Woittiez (1991), and Van Soest et al. (1990) on the single individual model. Working hours per week of both spouses are grouped, such that the choice set of each family consists of a finite number of leisure income combinations only. Utility is thus maximised over a finite set. The disadvantage of making the rounding error is, in our view, more than compensated by the simplicity of the resulting utility maximisation problem. We do not rely on tangency conditions and coherency of the model is automatically guaranteed. Specific features of the budget set due to taxation, unemployment benefits, or hours constraints, can be handled without essentially complicating the model. Thus the approach remains fully structural, in the sense that all policy simulations which are possible in the continuous model can also be carried out with this model.

In order to be able to enrich the stochastic specification of the model, we apply approximate ML estimation using simulated frequencies, as discussed by Lerman and Manski (1981) and Gourieroux et al. (1990). This allows for the simultaneous incorporation of several types of errors. Various specifications of the model are estimated and compared, in terms of the extent to which they fit the data, as well as in terms of their implications for own and cross wage and income elasticities of labour supply of the two spouses.

The paper is organised as follows. The basic model is presented in section 2. Data and some details of the incorporated tax and benefits systems, which approximate the actual systems in the Netherlands, are described in section 3. In section 4, we present the estimation results of the basic model. In section 5, we discuss several extensions of the model. We consider more general stochastic structures and incorporation of hours constraints. Section 6 concludes.

2. The Basic Model

We assume that each individual can freely choose among the alternatives in the choice set of leisure income combinations $\{(c_j, lm_j, lf_j); j=1,2,\dots,m\}$. Here $lm_j = TE - hm_j$ and $lf_j = TE - hf_j$, where TE is the (fixed) time endowment, set equal to 80 hours per week, and hm_j and hf_j are working hours per week of husband and wife, respectively. We only consider numbers of working hours which are multiples of some fixed interval length IL , i.e. $hm_j = jmIL$, for some $jm \in \{0, \dots, m_{ind} - 1\}$, and $hf_j = jfIL$, with $jf \in \{0, \dots, m_{ind} - 1\}$. The choice set thus contains $m = m_{ind}^2$ points. In the data, most integer values of actual working hours between 1 and 60 are present, and $IL=1$ would seem a natural choice. In the empirical part of this paper however, we choose $IL=12$ or $IL=10$, in order to limit the computational burden of the estimation procedure. Correspondingly, m_{ind} is set equal to 5 or 6, and the number of choice opportunities is 25 or 36.

c_j denotes the family's after tax income, including husband's and wife's earnings, possible unemployment benefits, unearned family income such as child allowances, etc.²⁾ Details on included taxes and benefits are presented in the next section. For the moment, the before tax wage rate is treated as an observed exogenous variable for all individuals, including non-workers.

We work with the following direct translog specification of the direct utility function:

$$U(v) = v'Av + b'v \quad (2.1)$$

where $v = (\log c, \log lm, \log lf)'$ is the vector of logs of commodity quantities, A is a symmetric 3×3 matrix with entries α_{ij} ($i, j=1,2,3$), and $b = (\beta_1, \beta_2, \beta_3)'$. Preference variation across families through observed characteristics can be incorporated through the parameters:

$$\beta_i = \sum \beta_{ik} x_k, \quad i=1,2,3, \quad \alpha_{ij} = \sum \alpha_{ijk} x_k, \quad i,j=1,2,3. \quad (2.2)$$

The x_k 's reflect family characteristics, such as family composition or the husband's or wife's age. The index indicating the family is suppressed. In the empirical analysis, some of the parameters will be assumed to be constant across families to reduce the computational burden.

For given A and b, it is straightforward to derive the region in (c,lm,lf) space where U is quasi-concave. If U is increasing in c, it is quasi-concave at (c,lm,lf) if and only if HC is positive definite, where HC denotes the matrix of second order derivatives of c with respect to lm and lf, along the indifference surface at (c,lm,lf):

$$HC = -U_c^{-1} \begin{bmatrix} c_{1m} & 1 & 0 \\ c_{1f} & 0 & 1 \end{bmatrix} HU \begin{bmatrix} c_{1m} & 1 & 0 \\ c_{1f} & 0 & 1 \end{bmatrix}' \quad (2.3)$$

Here U_c is the partial derivative of U with respect to c, HU denotes the matrix of second order partial derivatives of U, and $c_{1m} = -U_{1m}/U_c$ and $c_{1f} = -U_{1f}/U_c$ are the marginal rates of substitution of male and female leisure with family consumption. All derivatives are evaluated at (c,lm,lf). They are easily obtained from (2.1).

In the discrete choice model at hand, convexity of preferences plays no role. Utility maximisation takes place over a finite set, and neither first nor second order conditions are required. On the other hand, the economic interpretation of the model would be lost if the monotonicity condition that, in some 'relevant region' of (c,lm,lf) space, (including, e.g., all sample observations), U is increasing in c, is not fulfilled. The reason is that in defining the choice set, interior points of the budget set are already excluded. U is increasing in c at (c,lm,lf) if and only if

$$2(\alpha_{11} \log c + \alpha_{21} \log lm + \alpha_{31} \log lf) + \beta_1 > 0 \quad (2.4)$$

Since coherency of the model is guaranteed whether (2.4) is satisfied or not, it is not necessary to impose (2.4) *a priori*. Thus, neither (2.3) nor (2.4) have to be imposed. It can be checked *ex post* whether the estimates imply that these restrictions are satisfied or not. If (2.3) is not satisfied, there is nothing to worry about, since our economic model accounts for that. If (2.4) is violated, we have to reconsider our economic model.

Imposing (2.3) or (2.4) involves imposing a number of inequality restrictions on the parameters of the model. This limits the flexibility of the locally second order flexible translog specification. Moreover, if (2.3) is used, these restrictions will be data dependent. The use of tangency conditions requires imposition of this type of restrictions to guarantee model coherency. The approach followed in this paper is an attractive way to avoid this. Flexibility is retained, computational tractability is retained, and the economic constraints (2.3) and (2.4) can be tested.

Random disturbances are added to the utilities of all choice opportunities in the same way as in the well-known multinomial logit model (cf., e.g., Maddala, 1983):

$$U_j = U(c_j, l_{mj}, lf_j) + \epsilon_j, \quad (j=1, \dots, m) \quad (2.5)$$

$$\epsilon_j \sim EV(I) \quad (j=1, \dots, m), \quad \epsilon_1, \dots, \epsilon_m \text{ independent,}$$

where $EV(I)$ denotes the type I extreme value distribution with cumulative probability function $\Pr[\epsilon_j < \epsilon] = \exp(-\exp(-\epsilon))$ ($\epsilon \in \mathbb{R}$). We assume that the family chooses j for which U_j is largest. The probability that j is chosen is then given by

$$\Pr[U_j > U_k \text{ for all } k \neq j] = \exp(U(c_j, l_{mj}, lf_j)) / \sum_{k=1}^m \exp(U(c_k, l_{mk}, lf_k)) \quad (2.6)$$

Properties of this $EV(I)$ distribution imply $E\{\epsilon_j\} = 0$ and $V\{\epsilon_j\} = \pi^2/6$. The assumption that all variances are equal obviously limits the flexibility of the error structure of the model. However, it is necessary to obtain the simple expressions for the probabilities in (2.6). The choice of the magnitude of the common variance can be interpreted as a normalisation. Equivalently, one of the parameters of the utility function could be normalised, and $V\{\epsilon_j\}$ could be estimated as a parameter. The chosen normalisation has the advantage that the sign of the normalised parameter is known *a priori*.

The error structure of the model can be compared with the one in the more traditional kinked budget constraint continuous models, which usually include either random preferences or optimisation errors or both.³⁾ In the case of family labour supply, both types of errors would typically be bivariate, with a univariate term for each spouse.

The assumption of independence of irrelevant alternatives (IIA), is an implicit assumption for the multinomial logit model. Because of the IIA assumption, the ϵ_j 's strictly cannot be interpreted as reflecting random preferences, which could for example be due to unobserved family characteristics.⁴⁾ A natural way to interpret the ϵ_j 's is the assumption that they represent unobserved alternative specific utility components. In this case, the U_j 's represent the actual utilities, and there are no optimisation errors. Alternatively, they could be interpreted as optimisation errors, implying that the actual utilities are given by $U(c_j, lm_j, lf_j)$ and do not contain a random component. Because $P[U_i > U_j | U(c_i, lm_i, lf_i), U(c_j, lm_j, lf_j)]$ depends on $U(c_i, lm_i, lf_i) - U(c_j, lm_j, lf_j)$ and not on, for example, lm_i and lm_j , the errors cannot be interpreted as measurement errors on observed working hours.

Still, the large number of errors incorporated, one for each alternative, suggests that in estimating the model these errors might very well pick up part of the random preferences effects, and it may be hard to distinguish random preferences from optimisation errors in practical estimation. This problem sometimes also arises in the single individual kinked budget constraint model. Explicit incorporation of random preferences by adding error terms to β_2 and β_3 will be discussed in section 5.

3. Data and Budget Sets

The data we use stem from the Socio Economic Panel (SEP) wave drawn in October 1987 and were collected by the Netherlands Central Bureau of Statistics. We only use observations concerning families with at least husband and wife, with both partners between 16 and 65 years of age. After eliminating a few observations with missing values of explanatory variables, 2859 families remained. In 13.0% of these families, neither spouse has a paid job. In 3.1% of all cases, only the wife works. In 49.7% of all cases, only the husband works, and in the remaining 34.1% of all cases, both spouses have a paid job.

Working hours include regular overtime if it is paid, as well as hours worked in secondary jobs. They refer to the usual working week and thus do not correct for the number of holidays, etc. For 20 males and 17 females, it is known that he or she has a paid job, but the number of working hours could not be retrieved. The 33 families of these individuals are retained in the sample. Likelihood contributions of these families can easily be adjusted to take account of the missing information.

After tax earnings include allowances for shift work, paid overtime, tips, etc. Before tax wage rates are computed from after tax earnings and hours worked, using an approximation of (the inverse of) the Dutch taxation system. For approximately 8% of working males and 6% of working females, no wage rate could be computed. Most of these people did not answer the earnings questions, not being salaried employees.

We distinguish two types of (after tax) family income other than the husband's and wife's earnings: child benefits and other income, mainly capital income. Income of other household members is not included. It is thus assumed that, for example, earnings of children do not affect the husband's and wife's labour supply decisions. The husband's and wife's unemployment benefits are also excluded from the other income measure. Some sample statistics on participation rates, working hours, wage rates, other income, and a number of individual and family characteristics are mentioned in table 1. There appears to be a significantly negative correlation between other family income (excluding child benefits) and male and female working hours and participation.

In order to compute the complete budget sets, we assume that before tax wage rates do not depend on hours worked, implying that, due to the tax system, average after tax wage rates are decreasing. This assumption can in principle be generalised along the lines of Moffitt (1984) and Tummers and Woittiez (1991), who find that after tax earnings are an S-shaped function of hours worked, implying lower before tax wage rates for part-time jobs. This extension is beyond the scope of this paper.

The Dutch 1987 tax system for individuals basically consists of eleven tax brackets, with marginal tax rates gradually increasing from 0 to 70%. Some simplifying assumptions are necessary for our purposes, since the data do not contain all necessary information on deductables, health insurance premiums, etc. Spouses file separately, but the width of the husband's tax-free bracket is increased if his wife does not work, or if her earnings are extremely low (and vice versa). This implies that, if the husband's earnings are given and are not too small, the first part of the wife's budget set is virtually horizontal. On average, this concerns approximately her first four hours of work per week. In the current policy debate, this non-convexity in the female's budget set is seen as one of the reasons for the relatively low rate of female labour market participation in The Netherlands (cf. WRR, 1990).⁵⁾

The data contain information on various types of unemployment benefits for those who are actually unemployed. On the other hand, there is no

information on what someone would receive if he or she were to become unemployed. Since the level of unemployment benefits strongly depends on someone's labour market history, we feel that it may very well be correlated with time persistent unobserved individual characteristics. To avoid these problems, we have only taken into account the unemployment assistance a family receives when family income, excluding child benefits, is below the official poverty line for a two adults household without children, which is about 50% of average after tax earnings of working males in the sample. Child benefits then make up for the differences between the poverty lines for families with and without children. Unemployment insurance benefits, which generally have limited duration, are thus ignored.⁶⁾ This stylised benefits system implies that the first part of someone's budget set is horizontal only if partner's earnings and other family income are low or zero.

Table 1. Sample statistics

Variable (Description)	Mean	St. dev.	Number
NCH (number of children)	1.09	1.12	2859
DCH<6 (dummy children younger than 6)	0.28	0.45	2859
AGEM (age husband)	41.13	11.05	2859
AGEF (age wife)	38.63	11.02	2859
EDLM (education level, husband)	2.72	1.08	2859
EDLF (education level, wife)	2.34	0.97	2859
CHB (child benefits)	33.55	47.65	2859
OFI (other family income)	39.47	147.90	2859
WBM (before tax wage rate, husband)	26.71	17.35	2176
WBF (before tax wage rate, wife)	18.46	8.83	989
HM (working hours per week, husband*)	35.42	17.88	2839
HF (working hours per week, wife*)	9.68	14.66	2842
DEM (dummy employed, husband)	0.84	0.36	2859
DEF (dummy employed, wife)	0.40	0.49	2859

*: including non-workers

Pearson Correlation Coefficients

	HF	WBM	WBF	DEM	DEF	OFI
HM	0.14	-0.12	0.03	0.86	0.14	-0.28
HF		-0.04	0.03	0.15	0.82	-0.08
WBM			0.23		-0.02	0.10
WBF				0.03		0.06
DEM					0.16	-0.34
DEF						-0.10

In order to obtain wage rate predictions for non-workers and for workers whose wage rate is not observed, we have estimated wage equations for males

and females. Selection bias was taken into account in the usual way, by adding a reduced form participation equation and allowing for non-zero correlation between the two equations (cf. Heckman, 1979). The two equations model was then estimated by maximum likelihood, for males and females separately. The endogenous wage variable we used was the log of before tax hourly earnings. Explanatory variables include dummies for the education levels, age variables, the minimum wage, and the regional unemployment rate. Estimation results are mentioned in table 2.

We find a significantly negative correlation coefficient for males and an insignificant one for females. Estimated slope coefficients are generally in line with common findings. The wage rate increases with the education level and with age and, according to the product terms, the increase with age is strongest for the highest education levels. The impact of the regional unemployment rate is insignificant. The impact of the minimum wage rate on the wage rate seems very high, particularly for females. However, the minimum wage by law only varies with age, and thus this regressor may simply correct for the imperfect fit of the log quadratic age pattern.

4. Estimation Results Basic Model

The model introduced in section 2 has been estimated by maximum likelihood, using the full sample of 2859 families. Unobserved wage rates were replaced by wage rate predictions based upon the estimates in table 2, without taking account of the error term in the wage equation. Included family characteristics in (2.2) are the number of children (NCH), a dummy for children younger than six (DCH<6), the male's log age (LAGE) and log age squared (L2AGE) (in β_2), and the female's log age and log age squared (in β_3 and α_{23}). β_1 and the α_{ij} 's other than α_{23} are assumed not to depend on family characteristics. Estimation results are mentioned in table 3, both for $IL=12$ and $m_{ind}=5$ (25 choice opportunities), and $IL=10$ and $m_{ind}=6$ (36 choice opportunities).

Most of the parameter estimates for the two cases correspond rather well to each other, in particular the slope parameters including family characteristics. There are some relatively large and significant differences between the estimated α 's. In general, estimated standard errors are remarkably small, and many parameters are significantly different from zero. Since parameter estimates by themselves in this model are not very informative, some more computations are necessary to interpret the results.

Table 2: Wage rates and participation model

	Males		Females	
	parameter	t-value	parameter	t-value
<u>Participation Equation</u>				
Constant	-73.82	-11.53	-34.81	-6.39
DED2	0.36	3.78	0.21	2.89
DED3	-0.01	-0.01	1.12	1.41
DED4	-2.67	-2.13	0.69	0.54
DED5	-2.48	-1.99	0.83	0.62
LAGE (= log age)	45.66	11.42	21.15	6.24
L2AGE (= LAGE squared)	-6.48	-11.93	-3.16	-6.77
DED3*LAGE	0.19	0.76	-0.17	-0.76
(DED4+DED5)*LAGE	0.92	2.79	0.05	0.14
DWEST	0.11	1.38	0.13	2.17
UNEMPR	-0.84	0.51	-1.31	-0.90
WMIN	-2.12	-2.89	-0.02	-0.03
NCH	0.08	1.89	-0.26	-8.32
DCH<6	-0.29	-2.68	-0.77	-10.51
<u>Log Wage rate Equation</u>				
Constant	-10.56	-6.82	-1.17	-0.59
DED2	0.07	2.74	0.05	1.45
DED3	-0.51	-2.00	-0.42	-1.23
DED4	-2.03	-6.17	-1.33	-2.85
DED5	-1.86	-5.57	-1.21	-2.57
LAGE	6.70	7.02	1.02	0.81
L2AGE	-0.92	-6.94	-0.14	-0.82
DED3*LAGE	0.20	2.83	0.15	1.62
(DED4+DED5)*LAGE	0.68	7.68	0.47	3.65
DWEST	0.08	4.85	0.09	3.67
UNEMPR	-0.73	-1.66	0.34	0.51
WMIN	0.57	3.67	0.84	3.21
$\sigma(\eta)$	0.35	95.83	0.33	61.56
ρ	-0.54	-9.78	0.11	0.94

Explanation:

The education variable ED ranges from 1 (lowest level) to 5 (highest level). The variables DED2, DED3, DED4 and DED5 are dummies for the corresponding levels (DED3=1 if ED=3 and DE3=0 otherwise, etc.)

DWEST is a dummy variable indicating whether the family lives in the western part of The Netherlands (with largest population and industrial density).

UNEMPR is the unemployment rate (males and females jointly) in the region; 11 regions (provinces) are distinguished.

WMIN is the log of the before tax minimum wage (in DFL) fixed by law.

$\sigma(\eta)$ is the standard deviation of the error term in the wage equation.

ρ denotes the correlation coefficient between the error terms in the participation equation and the wage equation.

Table 3: Estimation results basic model

	$m_{ind}=5$		$m_{ind}=6$	
	param.	t-value	param.	t-value
α_{11} ($\log^2 c$)	-0.850	-3.20	-1.084	-3.82
α_{22} ($\log^2 lm$)	-3.030	-8.25	-1.808	-5.12
α_{33} ($\log^2 lf$)	0.125	0.31	1.593	4.42
α_{12} ($\log c \times \log lm$)	0.145	0.61	-0.307	-1.25
α_{13} ($\log c \times \log lf$)	-2.226	-9.54	-2.318	-10.16
α_{230} ($\log lm \times \log lf$)	5.319	2.47	5.019	2.53
α_{231} ($\log lm \times \log lf \times lage$)	-2.585	-2.14	-2.582	-2.32
α_{232} ($\log lm \times \log lf \times l2age$)	0.361	2.14	0.364	2.33
α_{233} ($\log lm \times \log lf \times nch$)	-0.500	-2.46	-0.438	-2.30
α_{234} ($\log lm \times \log lf \times dch<6$)	-1.655	-3.56	-1.765	-4.22
β_1 ($\log c$)	33.472	5.11	41.158	6.02
β_{20} ($\log lm$)	158.440	6.87	146.341	6.96
β_{21} ($\log lm \times lage$)	-80.688	-6.72	-74.904	-6.81
β_{22} ($\log lm \times l2age$)	11.509	6.98	10.693	7.07
β_{23} ($\log lm \times nch$)	3.681	2.12	3.209	1.97
β_{24} ($\log lm \times dch<6$)	14.850	3.76	15.809	4.46
β_{30} ($\log lf$)	106.523	4.04	82.866	3.40
β_{31} ($\log lf \times lage$)	-52.182	-3.60	-42.818	-3.19
β_{32} ($\log lf \times l2age$)	8.434	4.14	7.046	3.74
β_{33} ($\log lf \times nch$)	5.201	3.42	4.589	3.27
β_{34} ($\log lf \times nch6$)	15.854	4.51	16.364	5.24

It is easy to check that (2.4) is satisfied for all sample observations, i.e. utility increases with consumption. Economic theory is thus supported by the estimates. (2.3) is satisfied for most sample observations: According to the estimates based on $m_{ind}=5$ and $m_{ind}=6$, the utility function is not quasi-concave at 0.8% and 6.3% of all sample points, respectively. These observations are high income families in which the wife has a full-time job. Note that in the traditional model, this finding would not have been possible, since quasi-concavity of U would have been imposed *a priori* to

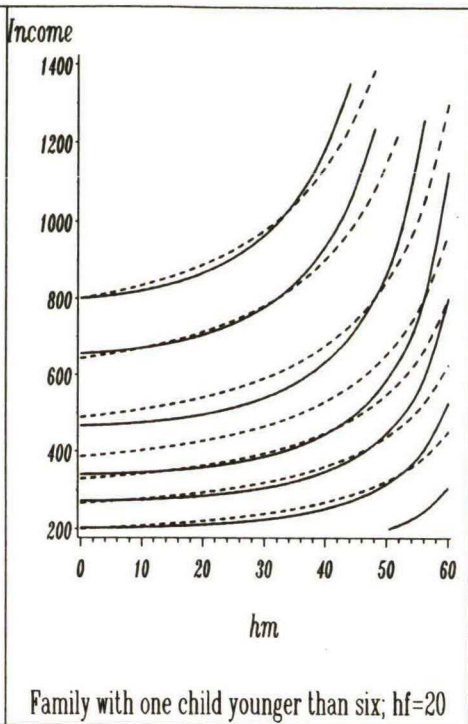
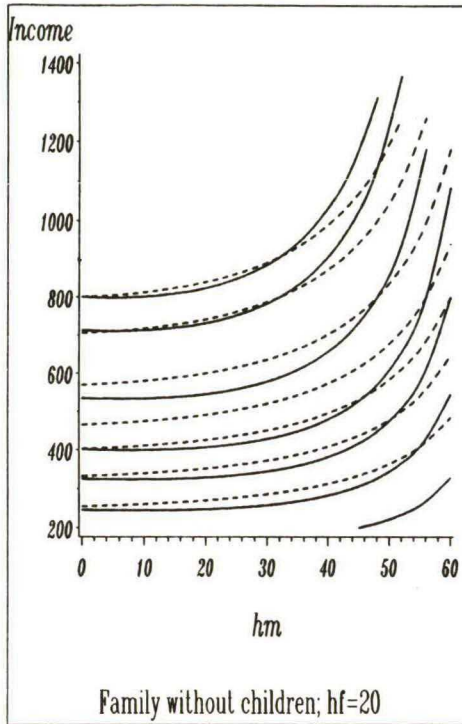
guarantee coherency, i.e. a unique solution of the system of tangency conditions and inequality constraints.

It can be shown that the estimated effects of family size, children younger than six and age on preferences for leisure all correspond to intuition. The coefficients of NCH and DCH₆ imply a negative impact on female labour supply. *Ceteris paribus*, labour supply decreases with age for most males and females. In figure 1, some indifference curves are drawn, for fixed working hours of one of the spouses. Solid lines refer to $m_{ind}=5$ and dashed lines refer to $m_{ind}=6$. Comparison of the curves gives some idea about the differences between the two sets of point estimates. Differences between curves for families of different composition confirm the stylised fact that the impact of family composition is much larger for females than for males. Moreover, the difference between the shapes of the curves for males and females suggest that the elasticity of substitution is much larger for females than for males, implying that females have larger own wage rate elasticities than males. Finally, the concave parts of some of the dashed curves are in line with the fact that according to the $m_{ind}=6$ estimates, U is not quasi-concave at all sample points.

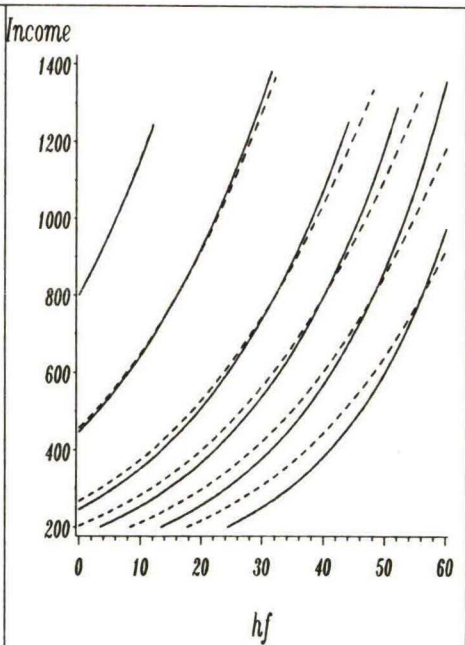
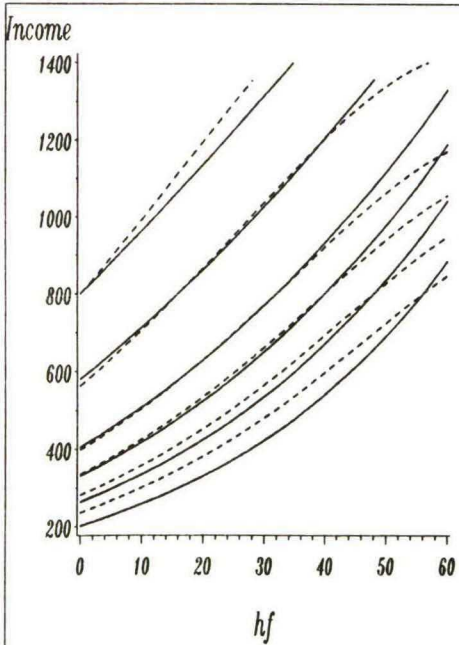
Elasticities for the average family, i.e. the family with average characteristics, wage rates and other income, are presented in table 4. We present 12 elasticities: of the husband's and the wife's hours worked and of the husband's and wife's participation probability, with respect to the before tax wage rates of husband and wife, and with respect to other family income. The tax and benefits system described in section 3 is fully taken into account.

In order to be able to judge the preciseness of the estimates, i.e. to take account of the standard errors of the parameter estimates, the elasticities are calculated 100 times, for 100 independent draws of the parameter values from the estimated asymptotic distribution of the estimator of the parameter vector. For each case, we present the median elasticity, and the first and ninth decile. The last two columns can thus be interpreted as the bounds of a two sided 80% confidence interval. The elasticity estimates appear to be quite accurate, corresponding to the small standard errors of most parameter estimates. Moreover, in all 12 cases the confidence intervals corresponding to the two sets of estimates overlap, which suggests that the error made by grouping working hours into a limited number of categories does not affect the results too much.

Fig. 1: Indifference Curves Basic Model



$m_{ind}=5$: —; $m_{ind}=6$: - - -



Family without children; $hm=40$

Family with one child younger than six; $hm=40$

All own wage elasticities are significantly positive on the 10% level. Corresponding to earlier findings for The Netherlands, the own wage sensitivity of females is larger than for males. The effect of an increase in the own wage largely takes place through an increasing participation probability. Cross wage elasticities of hours worked are significantly negative, both for males and females, suggesting that male and female leisure are substitutes. Cross wage sensitivity appears to be much smaller than own wage sensitivity. Surprisingly, the elasticity of the participation probability of the husband with respect to the female wage rate is significantly positive, and the estimate seems extremely accurate. The elasticity of male labour supply with respect to other family income is small but significantly negative. Surprisingly, the female's other income elasticity is even smaller and not significantly different from zero.

Table 4: Elasticities for the average family; Basic Model

$m_{ind}=5:$									
	wage rate male			wage rate female			other family income		
	median	Q10	Q90	median	Q10	Q90	median	Q10	Q90
hm	0.131	0.113	0.151	-0.044	-0.051	-0.037	-0.038	-0.041	-0.035
hf	-0.096	-0.157	-0.037	0.937	0.880	0.985	-0.001	-0.011	0.009
pm	0.090	0.083	0.098	0.014	0.013	0.016	-0.002	-0.003	-0.001
pf	-0.088	-0.137	-0.045	0.704	0.669	0.739	0.001	-0.008	0.008
$m_{ind}=6:$									
	wage rate male			wage rate female			other family income		
	median	Q10	Q90	median	Q10	Q90	median	Q10	Q90
hm	0.153	0.136	0.172	-0.036	-0.042	-0.030	-0.034	-0.037	-0.030
hf	-0.171	-0.237	-0.116	1.027	0.961	1.078	-0.009	-0.018	0.001
pm	0.098	0.090	0.107	0.013	0.012	0.015	-0.002	-0.003	-0.001
pf	-0.158	-0.208	-0.111	0.776	0.738	0.813	-0.006	-0.014	0.001

Explanation:

hm: working hours males; hf: working hours females
 pm: participation rate males; pf: participation rate females
 Q10: first decile; Q90: ninth decile

The results obtained so far seem quite satisfactory. Economic theory is supported and most elasticity estimates correspond to our expectations. Still, table 5 shows that the model hardly fits the data at all. In this table, we present actual and simulated bivariate cell frequencies of male and female labour supply, and marginal cell frequencies for males and females separately.⁷⁾ The table shows that both models strongly

mispredict most cell frequencies. This problem already arises at the univariate level, for males as well as females.

Table 5: Observed and Predicted Cell Frequencies Basic Model

hm	hf	actual	prediction		h	actual	prediction	
			m _{ind} =5	m _{ind} =6			m _{ind} =5	m _{ind} =6
0	0	12.845	4.899	5.406	<u>males:</u>			
0	10	0.814	0.989	0.938	0	15.959	6.194	6.605
0	20	1.062	0.208	0.176	10	0.637	6.827	6.119
0	30	0.460	0.057	0.046	20	2.052	11.450	8.922
0	40	0.602	0.027	0.023	30	2.654	23.157	18.294
0	50	0.177	0.014	0.016	40	58.033	31.027	29.205
					50	20.665	21.345	30.854
10	0	0.354	4.636	4.396	<u>females:</u>			
10	10	0.071	1.174	0.958	0	61.783	50.239	53.700
10	20	0.000	0.496	0.365	10	8.988	21.872	20.723
10	30	0.035	0.280	0.199	20	11.253	12.239	10.744
10	40	0.177	0.155	0.117	30	5.909	7.143	6.081
10	50	0.000	0.086	0.084	40	10.510	4.710	4.269
					50	1.557	3.797	4.482
20	0	1.062	5.368	4.535				
20	10	0.142	2.601	1.935				
20	20	0.425	1.652	1.139				
20	30	0.071	0.950	0.632				
20	40	0.283	0.545	0.381				
20	50	0.071	0.334	0.300				
30	0	1.415	10.204	8.660				
30	10	0.177	5.558	4.253				
30	20	0.354	3.378	2.404				
30	30	0.425	1.958	1.341				
30	40	0.212	1.207	0.864				
30	50	0.071	0.851	0.772				
40	0	33.652	14.481	14.517				
40	10	6.299	7.044	6.393				
40	20	6.582	4.071	3.460				
40	30	3.963	2.417	1.978				
40	40	6.900	1.645	1.397				
40	50	0.637	1.370	1.459				
50	0	12.456	10.652	16.186				
50	10	1.486	4.505	6.245				
50	20	2.831	2.433	3.200				
50	30	0.955	1.481	1.885				
50	40	2.335	1.132	1.487				
50	50	0.602	1.141	1.850				

Explanation:

hm, hf: hours categories males and females:

0: 0-5, 10: 6-15, 20: 16-25, 30: 26-35, 40: 36-45, 50: >45.

actual: sample fraction (in %)

predicted: predicted fraction (in %), using the estimates in table 3.

males, females: marginal distributions.

For both sexes, the rate of non-participation is underpredicted, as well as the number of people working 31-42 hours a week, the interval which includes the common full-time working week in the Netherlands. The number of part-time workers is strongly overpredicted, as well as the number of people working more than 42 hours per week.

The misfit of the model is easily affirmed formally, using a chi-squared diagnostic test (cf. Andrews, 1988): The explained sum of squares of a regression of a vector $(1, \dots, 1)' \in \mathbb{R}^n$, where n is the number of observations, on the vectors of differences between observed and predicted cell frequencies and the score vectors, follows, under the null hypothesis of no misspecification, a chi-squared distribution with m degrees of freedom.⁸⁾

The finding that the standard model cannot explain the peaks in the hours distribution is not new and has been used to motivate models incorporating explicit hours restrictions in the individual neoclassical labour supply model (cf., e.g., Dickens and Lundberg, 1985, and Tummers and Woittiez, 1991), the results of which suggest that there are too few part-time jobs. Apparently, our approach of grouping working hours into rather broad intervals of 10 or 12 hours per week does not solve this problem.

One way to extend the model without leaving the framework sketched in section 3, is to allow more parameters to depend on individual characteristics. Lagrange multiplier tests suggest that in particular α_{22} and α_{33} depend on individual characteristics. Reestimating the model with flexible α_{22} and α_{33} indeed yields a statistically better fit (in terms of a log likelihood ratio test and several t-test. Estimated elasticities for the average family differ somewhat from those in table 3. For example, the 80% confidence intervals ($IL=10$, $m_{ind}=6$) for the own wage elasticities of hours worked are $[0.078, 0.108]$ for males and $[1.104, 1.176]$ for females. Qualitative conclusions remain unchanged. The number of observations at which utility is not convex increases to 12.4% of the sample. Monotonicity of U in c remains satisfied for all observations. Extending the model in this direction however does not change the conclusions from table 5: The misfit between actual and simulated cell frequencies remains as apparent as before.⁹⁾

5. Extensions

In this section we discuss three extensions of the basic model introduced above. The extensions should meet three major shortcomings of the basic model. First, it does not fit the data, in the sense that the number of part-time jobs is strongly underpredicted. Secondly, it treats the problem of unobserved wage rates in an unsatisfactory way. In the third place, it does not allow for random preferences. We shall first describe the three extensions, and then discuss the empirical results of models incorporating one or more of the extensions.

Hours restrictions

The fact that the basic model cannot capture the data has become apparent from table 5. A possible explanation is that it does not account for the lack of available part-time jobs. In the basic model, no distinction has been made between full-time and part-time jobs. Several explanations for the lack of part-time jobs can be given. Because of fixed costs of hiring workers, or, equivalently, increasing returns to scale of the worker's production, employers may be reluctant to hire part-time workers. This may be an incentive to offer lower wages to part-time workers. Results of Tummers and Woittiez (1991) suggest that this is indeed the case to some extent, but that this is not enough to explain the lack of part-time jobs. The reason may be the fact that, at least in the Netherlands, most wages are largely determined by collective bargaining between unions and employer organisations in sectors, and resulting agreements generally do not allow to discriminate between full-time and part-time workers. In this paper, we have assumed that before tax wage rates do not depend on hours worked and thus do not allow for wage discrimination of part-time workers.

Employers may also simply not offer part-time jobs or refuse to hire part-time workers. As a consequence, part-time jobs will be scarce and average search costs for a part-time job will be relatively high. Dickens and Lundberg (1985) have incorporated this idea explicitly into a model of labour supply in a framework with a limited number of job offers, in which most people are restricted in the choice of their working hours. It has been shown (Tummers and Woittiez, 1991, Van Soest et al., 1990) that such a model fits the data much better than the standard model. Still, with no information used on the actual number of job offers or the restrictions

individuals actually face, estimation of the job offer mechanism is based upon indirect information (on actual working hours) only. The question arises whether explaining the phenomena in the data require this whole extra (reduced form) branch of the model.

The multinomial logit framework we use in this paper allows for a much simpler approach: we include alternative specific constant terms for the alternatives in which either the male or the female works part-time. These constants reflect monetary or non-monetary drawbacks of working part-time, e.g. search costs of part-time jobs (which, in our static framework, obviously cannot be incorporated explicitly), or unattractive job characteristics. If $m_{ind}=6$ and $IL=10$, this implies that six extra parameters are included. Equation (2.5) is replaced by

$$U_j = U(c_j, lm_j, lf_j) + \gamma_m(lm_j) + \gamma_f(lf_j) + \epsilon_j \quad (j=1, \dots, 36), \quad (4.1)$$

where, for $s=m, f$,

$$\begin{aligned} \gamma_s(ls) &= \gamma_{sk} && \text{if } hs=80-ls=10k, k=1,2,3, \\ \gamma_s(ls) &= 0 && \text{otherwise} \end{aligned} \quad (4.2)$$

The γ_{sk} 's ($s=m, f, k=1,2,3$) are expected to be negative; $-\gamma_{sk}$ reflects the 'disutility' of a part-time job.

As in the Dickens and Lundberg model, the assumptions that the extra parameters do not depend on characteristics such as wage rates, education level, family composition, etc. is *ad hoc*, and implicitly reflects the assumption that hours restrictions are homogeneous across the labour market. It implies that the relative lack of part-time jobs is uncorrelated with these characteristics.

A drawback of introducing the alternative specific parameters is that the parameterisation depends on the chosen way in which working hours are categorised. For different values of m_{ind} and IL , a different number of parameters must be used, and results for various values of m_{ind} can no longer be compared. The same drawback is implicitly present in the Dickens and Lundberg model. If m_{ind} is large, it may be worthwhile to circumvent this problem by further parameterising the γ_{sk} 's. This is put into practice in one of the empirical models discussed below.

Errors in wage rate predicitions

One of the problems with the type of labour supply models discussed above is that before tax wage rates of many individuals, including all non-workers, are not observed. The usual approach is to replace wage rates of non-workers by wage rate predictions, whereas for workers, actual wage rates are used (cf., e.g., most papers in Moffitt, 1990a). The estimates discussed in section 3 are based upon this approach. The approach in principle does not lead to consistent estimates, since it assumes that wage rates of non-workers are predicted without error (cf., e.g., MaCurdy et al., 1990).

Another *ad hoc* alternative is to use predicted wage rates for workers as well as non-workers. Because of the non-linearities in the model and the chosen distribution of the error terms, this would only yield consistent estimates if all families based their decision on (our) predictions instead of on their actual wage rates. This seems an implausible assumption, in particular since there will be variables known to the individual and helpful for predicting, but not present in our data.

In this section, we explicitly take account of the fact that unobserved wage rates are predicted with error. The estimation results in table 2 were already used to construct wage rate predictions. Now, we also use the estimated standard deviations of the error terms in these wage equations to take account of prediction errors. The labour supply model itself essentially remains unchanged. Moreover, we retain the assumption that the error terms in the wage equations and the error terms in (2.5) are independent, and thus do not allow for the possible endogeneity of before tax wage rates. We come back to this in section 6.

In order to describe the way in which the prediction errors are incorporated, we rewrite the model in rather general terms. The labour supply model yields probabilities of working hours combinations of husband and wife as a function of before tax wage rates of husband and wife (W_{bm} and W_{bf}) and several family characteristics (X), including other family income:

$$\Pr[(h_m, h_f) = (h_{mj}, h_{fj})] = F_j(W_{bm}, W_{bf}, X) \quad (j=1, \dots, m), \quad (4.3)$$

where F_j is given by (2.6). The index indicating the family is suppressed. The likelihood contribution in case of observed W_{bm}^0 and W_{bf}^0 and choice $(h_{m_{job}}, h_{f_{job}})$ is given by

$$L = F_{\text{job}}(Wbm^O, Wbf^O, X) \quad (4.4)$$

Measurement errors in Wbm^O and Wbf^O are thus ignored. The estimated wage equations for males and females are given by

$$\log Wbs = Z'_s \pi_s + \eta_s \quad (s=m, f) \quad (4.5)$$

where Z_m and Z_f are vectors of individual characteristics (cf. table 2) and η_m and η_f are unobserved error terms. In the previous section, we replaced Wbm^O and Wbf^O in (4.4), if they were not observed, by $Wbm^P = \exp(Z'_m \pi_m)$ and $Wbf^P = \exp(Z'_f \pi_f)$, thus ignoring η_m and/or η_f . However, for given probability density p of (Wbm, Wbf) , conditional on Z_m and Z_f and determined by π_m and π_f and the density of (η_m, η_f) , the correct likelihood contribution in case none of the wage rates are observed is given by

$$L = \int_0^\infty \int_0^\infty F_{\text{job}}(Wbm, Wbf, X) p(Wbm, Wbf) dWbm dWbf \quad (4.6)$$

Similar expressions, involving a single integral, can be given if either Wbm or Wbf is not observed.

In general, (4.6) cannot be written as a sum of normal probabilities, and computation of L requires numerical integration. There are various ways in which this can be avoided. The first is to derive a simulated moment estimator, generalising McFadden's (1989) estimator for the multinomial probit model. This implies that in the first order conditions corresponding to maximizing the likelihood, scores must be replaced by fixed instruments, and that probabilities or partial derivatives of probabilities are replaced by smooth unbiased simulators. McFadden shows that the resulting estimator is consistent, irrespective of the number of replications per individual on which the simulators are based.¹⁰⁾

An easier alternative, also based on replacing expectations by simulated means, is to approximate the integral in (4.6) by

$$L_R = \frac{1}{R} \sum_{r=1}^R F_{\text{job}}(Wbm_r, Wbf_r, X) \quad (4.7)$$

where $(Wbm_1, Wbf_1), \dots, (Wbm_R, Wbf_R)$ are R independent draws from the distribution of (Wbm, Wbf) (conditional on Z_m and Z_f). Similarly, if only the husband's wage rate is unobserved, L is replaced by

$$L_R = \frac{1}{R} \sum_{r=1}^R F_{\text{job}}(Wb_{m_r}, Wbf^O, X) \quad (4.8)$$

and an analogous expression can be given if only Wbf is observed. The approximate likelihood function is maximised, in which, for non-workers, L is replaced by L_R (see Lerman and Manski, 1981, and Gourieroux and Monfort, 1990). The resulting estimator is inconsistent for fixed R but will be consistent if R tends to infinity with the number of observations. If R tends to infinity at a large enough rate (i.e., to be precise, if $n/R^{\frac{1}{2}}$ tends to 0), the asymptotic distributions of approximate ML estimator and exact ML estimator coincide. In fact, for a fixed number of observations, the estimator will converge to the ML-estimator if R tends to infinity.

Random preferences

Because of the Independence of Irrelevant Alternatives assumption implicitly present in the multinomial logit model, the ϵ_j 's in (2.5) cannot be interpreted as random preferences due to unobserved family characteristics. Random preferences can be incorporated explicitly by adding an error term to some of the parameters, for example β_2 and β_3 , corresponding to the linear terms of male and female leisure in the direct utility function. We thus replace the expressions for β_2 and β_3 in (2.2) by

$$\beta_i = \sum \beta_{ik} x_k + \xi_i, \quad i=2,3 \quad (4.9)$$

where we assume that ξ_2 and ξ_3 are mutually independent, independent of other errors in the model, homoskedastic, and normally distributed with mean 0. Conditional on ξ_2 and ξ_3 , we retain the same labour supply model as before, including the IIA assumption. The probabilities unconditional on ξ_2 and ξ_3 (but for given wage rates) however are given by

$$\begin{aligned} \Pr[(hm, hf) = (hm_j, hf_j)] = \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pr[(hm, hf) = (hm_j, hf_j) | (\xi_2, \xi_3)] p_{\xi}(\xi_2, \xi_3) d\xi_2 d\xi_3 \end{aligned} \quad (4.10)$$

where p_{ξ} denotes the probability density function of (ξ_2, ξ_3) .

Unobserved random preferences thus complicate ML estimation in a similar way as unobserved wage rate components. Expressions for the likelihood involve a complicated bivariate integral if both wage rates are observed. If, for example, neither of the two wage rates are observed, the combination of the previous extension with random preferences leads to the following expression for the likelihood contribution:

$$L = \int_0^{\infty} \int_0^{\infty} \int_{-\infty}^{\infty} F_{\text{job}}(W_{bm}, W_{bf}, X | \zeta_2, \zeta_3) p_{\zeta}(\zeta_2, \zeta_3) p(W_{bm}, W_{bf}) d\zeta_2 d\zeta_3 dW_{bm} dW_{bf} \quad (4.11)$$

where $F_{\text{job}}(W_{bm}, W_{bf}, X | \zeta_2, \zeta_3)$ is defined as before, but now conditional on (ζ_2, ζ_3) . The integral in (4.11) can, as before, be approximated by

$$L_R = \frac{1}{R} \sum_{r=1}^R F_{\text{job}}(W_{bm_r}, W_{bf_r}, X | \zeta_{2r}, \zeta_{3r}) \quad (4.12)$$

Here $(W_{bm_1}, W_{bf_1}, \zeta_{2r}, \zeta_{3r}), \dots, (W_{bm_R}, W_{bf_R}, \zeta_{2r}, \zeta_{3r})$ are R independent draws from the distribution of $(W_{bm}, W_{bf}, \zeta_2, \zeta_3)$ (conditional on Z_m and Z_f). Similar expressions can easily be obtained for the case that one of the two wage rates is observed.

Estimation Results

Estimation results of various extended versions of the model are presented in table 6. All results are based upon $m_{\text{ind}}=6$ and $IL=10$, i.e. the family's choice set consists of 36 alternatives. The first column refers to the basic model with extra parameters to reflect hours restrictions (cf. equations 4.1-4.2). The error structure is thus the same as in the basic model and exact ML estimation is used. The estimates for β_1 and for the α_{ij} 's which are not allowed to depend upon family characteristics, strongly differ from those in table 3, and so do some of their significance levels.

The parameters related to family characteristics however are largely in accordance with the earlier findings. The estimates of the α 's and β 's imply that utility (the γ_{sk} 's, $s=m, f$, $k=1, 2, 3$, not taken into account) is an increasing function of family income for all observations. The estimated utility function is increasing in family consumption and quasi-concave for 99.9% of all observations in the sample.

Table 6: Estimation results extended models

	I		IIa (R=5)		IIb (R=10)		III (R=10)	
	par.	t-val.	par.	t-val.	par.	t-val.	par.	t-val.
α_{11}	0.189	0.64	-1.342	-3.97	-1.380	-3.99	-1.415	-4.08
α_{22}	-4.003	-11.05	-5.038	-13.25	-4.977	-13.03	-5.118	-13.17
α_{33}	-6.618	-10.00	-6.970	-10.53	-6.957	-10.50	-7.030	-10.55
α_{12}	1.248	4.80	0.087	0.32	0.094	0.34	0.016	0.06
α_{13}	-0.733	-3.03	-1.558	-6.40	-1.601	-6.46	-1.668	-6.74
α_{230}	5.126	2.47	4.187	2.02	4.170	2.00	4.143	1.97
α_{231}	-2.148	-1.86	-2.038	-1.75	-2.044	-1.75	-2.070	-1.75
α_{232}	0.286	1.76	0.274	1.68	0.275	1.68	0.278	1.68
α_{233}	-0.497	-2.61	-0.481	-2.50	-0.471	-2.45	-0.476	-2.47
α_{234}	-1.574	-3.76	-1.459	-3.47	-1.468	-3.49	-1.443	-3.43
β_1	-3.605	-0.50	32.021	4.09	32.931	4.11	34.468	4.29
β_{20}	125.474	5.78	148.631	6.67	150.631	6.68	152.496	6.71
β_{21}	-67.945	-5.96	-64.698	-5.59	-65.938	-5.66	-65.546	-5.57
β_{22}	9.716	6.20	9.237	5.81	9.411	5.87	9.366	5.79
β_{23}	3.666	2.25	3.523	2.14	3.438	2.09	3.486	2.12
β_{24}	13.945	3.93	12.953	3.64	13.038	3.66	12.822	3.60
β_{30}	131.329	5.17	152.627	5.97	151.613	5.94	153.957	5.98
β_{31}	-49.787	-3.64	-50.880	-3.71	-49.912	-3.64	-50.127	-3.63
β_{32}	8.066	4.21	8.195	4.26	8.061	4.19	8.099	4.17
β_{33}	5.066	3.60	4.932	3.47	4.856	3.42	4.899	3.45
β_{34}	14.430	4.61	13.640	4.35	13.704	4.36	13.539	4.31
γ_{m1}	-3.742	-14.81	-3.740	-15.02	-3.734	-14.99	-3.738	-15.01
γ_{m2}	-3.143	-22.67	-3.133	-22.52	-3.131	-22.45	-3.130	-22.48
γ_{m3}	-3.256	-26.26	-3.238	-26.05	-3.237	-26.03	-3.235	-25.99
γ_{f1}	-1.805	-20.14	-1.801	-20.21	-1.800	-20.19	-1.798	-20.09
γ_{f2}	-1.365	-11.74	-1.358	-11.73	-1.358	-11.74	-1.354	-11.67
γ_{f3}	-1.522	-11.90	-1.521	-11.92	-1.521	-11.93	-1.519	-11.90
$\sqrt{V\{\gamma_m\}}$							0.254	1.18
$\sqrt{V\{\gamma_f\}}$							0.064	0.22

Explanation:

- I: Model with parameters reflecting hours restrictions ((4.1)-(4.2)); imputed wage rates for non-workers
- II: Model with parameters reflecting hours restrictions; wage rate prediction errors taken into account; approximate ML-estimates using (4.7)-(4.8)
- III: II extended with random preferences; approximate ML-estimates using (4.12)

All estimates for the γ_{sk} 's ($s=m,f, k=1,2,3$) are significantly negative. This confirms the interpretation that they reflect hours restrictions and strongly suggests that the model including these parameters is an improvement with respect to the basic model. This conclusion is easily confirmed using either one of the familiar tests for the null that all γ_{sk} 's

are equal to zero (Lagrange Multiplier, Wald or Likelihood Ratio test; the null is strongly rejected by either of these).

Table 7 can be compared to table 5. It contains actual and simulated bivariate and univariate cell frequencies of hours worked. The results for the model at hand (model I) suggest that we have indeed to a large extent achieved what this extension of the model was designed for: This model fits the data much better than the basic model, in the sense that actual and predicted cell frequencies are quite similar. It should however be admitted, that the model specification is still rejected by chi-squared diagnostic tests similar to those discussed in section 3.

Table 8 presents 80%-confidence bounds for the elasticities of the average family, computed in the same way as in table 4. The hours restrictions parameters are fully taken into account. Comparing the upper panel of the table with the outcomes in table 4 shows that incorporating the hours restrictions parameters indeed significantly changes the conclusions about some of the elasticities. In particular, the female's own wage rate elasticity is much smaller than before (between 0.47 and 0.60 instead of between 0.88 and 0.98). Cross wage elasticities also decrease in absolute value, the effect of the husband's wage rate on the wife's hours no longer being significant. Elasticities with respect to other family income remain very small. Surprisingly, the impact of other income on the wife's working hours is now significantly positive.¹¹⁾

The Model II estimates in table 6 refer to the model in which not only hours restrictions parameters are included, but also prediction errors are taken into account in wage rates of individuals whose wage rate is not observed. The likelihood function is approximated using (4.7) and (4.8), with $R=5$ (Model IIa) and $R=10$ (Model IIb). We used the estimation results for the wage equations mentioned in table 2, and assumed that the error terms in husband's and wife's wage equation are independent.¹²⁾ The fact that parameters in the wage equation are estimated is not taken into account in computing the standard errors of the labour supply model parameter estimates. These standard errors might therefore be slightly underestimated.¹³⁾

Parameter estimates for both $R=5$ and $R=10$ are mentioned in table 6. The two sets of parameter estimates are very similar, suggesting that $R=5$ already yields reasonable accuracy, even though, according to theory, consistency of the approximate ML-estimator requires R to tend to infinity with the number of observations. We have not obtained estimates based upon larger values of R .¹⁴⁾ On the other hand, differences with model I seem

quite substantial and significant, at least for the α_{ij} 's and for β_1 , suggesting that taking wage rate prediction errors into account does make a difference. The estimates for the slope coefficients with respect to family characteristics again are very well in line with the results obtained earlier. Moreover, the hours restrictions parameter estimates are virtually identical to those for model I. The model IIa estimates imply that the direct utility function would be decreasing in family consumption at 3.3% of all sample points. For these observations, the micro-economic foundation of the model is lost. At 0.4% of sample points, indifference surfaces are not convex.

Predicted cell frequencies for model IIa are presented in table 7. Those for model IIb are virtually identical to those for model IIa and therefore not presented. The extent to which models IIa and IIb fit the data appears to be quite similar to the model I case. Chi-squared diagnostic test results are also similar, and suggest that the model is still misspecified.

Comparing the confidence bounds of elasticities for the R=5 and R=10 case, presented in table 8, again shows that models IIa and IIb yield virtually identical results. Surprisingly, the confidence intervals are not very different from those corresponding to model I, even though some of the parameter estimates do differ. The husband's labour supply elasticity with respect to other family income is significantly negative but small, whereas the wife's is now again insignificant.¹⁵⁾

Results for the model with explicit random preferences ξ_2 and ξ_3 are presented in the righthand column of table 6 (model III). We have assumed that ξ_2 and ξ_3 are independent. Approximate ML was used, based upon (4.12) and similar likelihood contributions in case of observed wage rates. Hours restrictions were also allowed for. Again, results obtained with R=5 and R=10 appeared to be virtually identical. We present results for R=10 only. Differences with the model II results are minor. The estimates for the standard deviations of the random preference terms seem rather inaccurate, and the importance of incorporating these errors is not confirmed.¹⁶⁾

Correspondingly, confidence bounds for elasticities are also quite similar to those obtained for the previous specification (cf. table 8). Apparently, the large number of error terms already in the model (one for each of the 36 alternatives) makes it hard to distinguish separate random preference terms, even though the specification of the other errors is restrictive and implies IIA. Obviously, allowing for more flexibility with respect to the way in which random preferences are incorporated, might change this result. It remains for instance to be checked whether ξ_m and ξ_f are independent of

each other and of the η_m and η_f , which might reflect similar unobserved individual or family characteristics.

Table 7: Observed and Predicted Cell Frequencies; Extended Models

hm	hf	actual	prediction					
			model I	model IIa				
males:								
			hm	actual	model I	model IIa		
0	0	12.845	12.243	12.154				
0	10	0.814	1.352	1.263				
0	20	1.062	1.133	1.043	0	15.959 15.842 15.612		
0	30	0.460	0.419	0.428	10	0.637 0.635 0.632		
0	40	0.602	0.628	0.653	20	2.052 2.048 2.056		
0	50	0.177	0.067	0.072	30	2.654 2.645 2.662		
					40	58.033 58.063 58.273		
					50	20.665 20.768 20.766		
10	0	0.354	0.411	0.406				
10	10	0.071	0.053	0.052				
10	20	0.000	0.067	0.068	females:			
10	30	0.035	0.037	0.038	hf	actual	model I	model IIa
10	40	0.177	0.059	0.061	0	61.783	61.494	61.336
10	50	0.000	0.006	0.007	10	8.988	9.113	9.121
					20	11.253	11.368	11.394
20	0	1.062	1.124	1.134	30	5.909	5.935	5.965
20	10	0.142	0.194	0.194	40	10.510	10.527	10.611
20	20	0.425	0.288	0.286	50	1.557	1.563	1.572
20	30	0.071	0.158	0.156				
20	40	0.283	0.254	0.255				
20	50	0.071	0.029	0.031				
30	0	1.415	1.457	1.463				
30	10	0.177	0.256	0.259				
30	20	0.354	0.364	0.365				
30	30	0.425	0.197	0.197				
30	40	0.212	0.328	0.334				
30	50	0.071	0.042	0.044				
40	0	33.652	33.504	33.442				
40	10	6.299	5.424	5.493				
40	20	6.582	7.265	7.340				
40	30	3.963	3.919	3.937				
40	40	6.900	6.937	7.024				
40	50	0.637	1.014	1.037				
50	0	12.456	12.754	12.737				
50	10	1.486	1.834	1.860				
50	20	2.831	2.251	2.293				
50	30	0.955	1.204	1.210				
50	40	2.335	2.321	2.284				
50	50	0.602	0.404	0.382				

Explanation:

hm, hf: hours categories males and females:

0: 0-5, 10: 6-15, 20: 16-25, 30: 26-35, 40: 36-45, 50: >45.

actual: sample fraction (in %)

predicted: predicted fraction (in %), using the estimates in table 6.

males, females: marginal distributions.

Table 8: Elasticities for the average family; Extended Models

Model I:

	wage rate male			wage rate female			other family income		
	median	Q10	Q90	median	Q10	Q90	median	Q10	Q90
hm	0.104	0.078	0.122	-0.015	-0.023	-0.010	-0.026	-0.031	-0.022
hf	0.051	-0.049	0.122	0.524	0.470	0.599	0.016	0.004	0.029
pm	0.125	0.105	0.143	-0.004	-0.009	0.001	-0.018	-0.022	-0.014
pf	0.033	-0.044	0.085	0.383	0.330	0.426	0.009	-0.001	0.018

Model IIa:

	wage rate male			wage rate female			other family income		
	median	Q10	Q90	median	Q10	Q90	median	Q10	Q90
hm	0.073	0.050	0.091	-0.017	-0.025	-0.012	-0.030	-0.035	-0.026
hf	0.011	-0.087	0.077	0.453	0.399	0.521	0.009	-0.004	0.020
pm	0.093	0.076	0.111	-0.008	-0.014	-0.004	-0.022	-0.026	-0.018
pf	-0.012	-0.083	0.034	0.319	0.269	0.365	0.002	-0.008	0.010

Model IIb:

	wage rate male			wage rate female			other family income		
	median	Q10	Q90	median	Q10	Q90	median	Q10	Q90
hm	0.076	0.053	0.093	-0.017	-0.025	-0.012	-0.030	-0.035	-0.026
hf	0.005	-0.093	0.072	0.472	0.417	0.543	0.008	-0.005	0.019
pm	0.097	0.080	0.114	-0.008	-0.014	-0.003	-0.022	-0.027	-0.018
pf	-0.017	-0.090	0.029	0.335	0.284	0.382	0.001	-0.009	0.009

Model III:

	wage rate male			wage rate female			other family income		
	median	Q10	Q90	median	Q10	Q90	median	Q10	Q90
hm	0.064	0.046	0.086	-0.016	-0.022	-0.012	-0.030	-0.035	-0.025
hf	0.037	-0.028	0.126	0.453	0.392	0.522	0.013	0.002	0.026
pm	0.085	0.070	0.105	-0.008	-0.013	-0.004	-0.022	-0.026	-0.019
pf	0.007	-0.040	0.073	0.313	0.257	0.370	0.004	-0.004	0.014

Explanation:

hm: working hours males; hf: working hours females
 pm: participation rate males; pf: participation rate females
 Q10: first decile; Q90: ninth decile

6. Conclusions

In this paper, we have compared several types of models explaining family labour supply behaviour. In section 2, we have introduced an alternative for the common neoclassical approach, based upon grouping working hours into categories, and utility maximisation over a finite set. This approach has several advantages in comparison to the traditional model: It does not require any *a priori* assumptions for model coherency, it is computationally more tractable, and it allows for extensions of the standard

model, such as incorporating a kinked tax system, constraints on working hours, fixed costs of working, etc. Moreover, if the number of categories increases, the model can be interpreted as an approximation of the more traditional continuous model.

Estimation results for the basic discrete choice model are discussed in section 4. From an economic point of view, these results seem quite satisfactory: The utility function is quasi-concave and increasing with family consumption, and labour supply elasticity estimates seem rather accurate and quite reasonable. Still, a simple comparison of observed and predicted hours distribution reveals that the model does not fit the data.

In section 5 we discuss several extensions. To remove the misfit, we introduce a few extra parameters which can be interpreted as hours constraints. Incorporation of these parameters has a substantial impact on elasticity estimates, and in particular significantly reduces the estimated female's own wage elasticity.

It is shown that extensions with more random terms can be handled by using approximate ML, based upon simulated frequencies, instead of exact ML. Thus it becomes possible to treat the problem of unobserved wage rates in a satisfactory way. Moreover, we can explicitly allow for random preferences. Compared to the first extension however, incorporating these two features does not substantially affect the elasticity estimates.

Obviously, numerous directions remain in which the model can be further extended. Many of these can easily be implemented in the discrete choice framework which we use. The relatively large data set available makes such extensions worthwhile and necessary, if we want the model to be accepted by simple diagnostic tests. Score tests can be used to investigate which directions of extension are most promising.

A few examples illustrate this point. In model I, the assumption that the three hours restrictions parameters for females do not depend upon family characteristics (LAGEF, L2AGEF, NCH, DCH<6) is strongly rejected. (the test statistic is 77.0, which exceeds $\chi^2_{12;0.01}=26.2$). In particular, age seems to play a role. A similar test for males leads to rejection at the 5% level but acceptance at the 1% level (the test statistic being 23.9). In the same model, the hypothesis that the coefficients in the utility function corresponding to the squared quantities α_{11} , α_{22} and α_{33} do not depend upon family characteristics is also rejected (the test statistic being 66.4, exceeding $\chi^2_{12;0.01}$). The scores needed for these tests are easily computed, multiplying scores with respect to the parameter assumed constant with

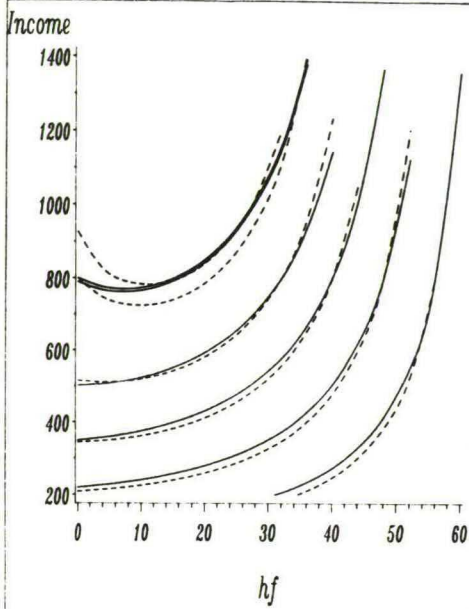
family characteristics, and do not require new evaluations of the likelihood function.

In model III, endogeneity of wage rates can be accounted for by allowing for correlation between the error terms in the wage equation and the random preference terms, i.e. between η_m and ζ_m and between η_f and ζ_f . This essentially boils down to including the wage rate residual in the expressions for β_2 and β_3 . The hypothesis of no correlation is strongly rejected (the test statistic is 19.0 and exceeds $\chi^2_{2;0.01}$). The obvious problem with these tests is that each test is only feasible if the type of misspecification tested for is the only one present. Thus, if many of these tests lead to rejection of the null, it is still hard to know which alternative should be chosen.

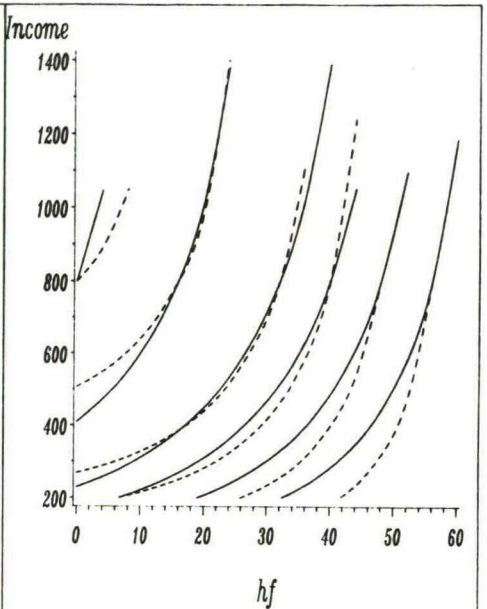
Because of the misspecification which is still present, the practical value of calculated elasticities is not clear. The sensitivities of labour supply with respect to the own and the partner's wage rates are rather small. Moreover, they tend to become smaller the more general and realistic the model on which their calculation is based becomes. This suggests that the true elasticities which we are trying to estimate may also be rather small. This conjecture is confirmed by the results of estimating one more extension: Model IIa (in section 5), in which γ_{fk} ($k=1,2,3$), i.e. the hours restrictions parameters for females, are allowed to depend on the female's characteristics (LAGE, L2AGE, NCH, DCH6). The extension fits the data significantly better than the original Model IIa (cf. the LM test result mentioned above). Estimated indifference curves for the average family (as in section 3) are drawn in figure 2, both for the original model IIa and its extension. Differences between the curves do not seem to be too large, although some of the curves of the extended model have a larger curvature, implying lower own wage elasticities. This is confirmed by the 80%-confidence bounds for the male's and female's own wage elasticities of hours worked, which are given by $[-0.005, 0.048]$ and $[0.269, 0.362]$, respectively. These elasticities are smaller than the Model IIa estimates in table 8. Income elasticities slightly increase (in absolute value), but remain very small.

The temptation thus exists to conclude that the empirical results imply that the sensitivity of male and female labour supply with respect to wages, tax rates, and other income is quite small, at least in the static neoclassical framework we consider.

Fig. 2: Indifference Curves Extended Models

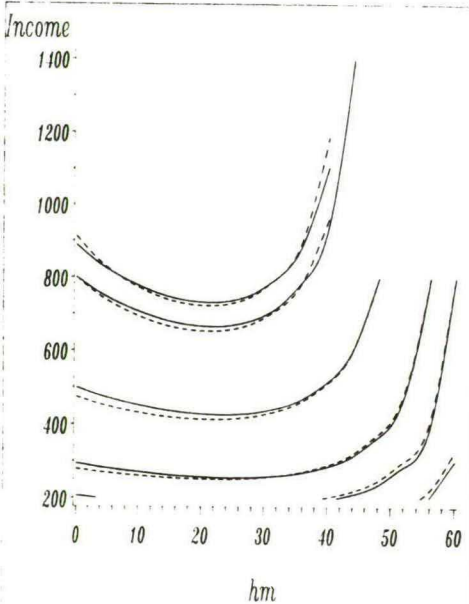


Family without children; $hm=40$

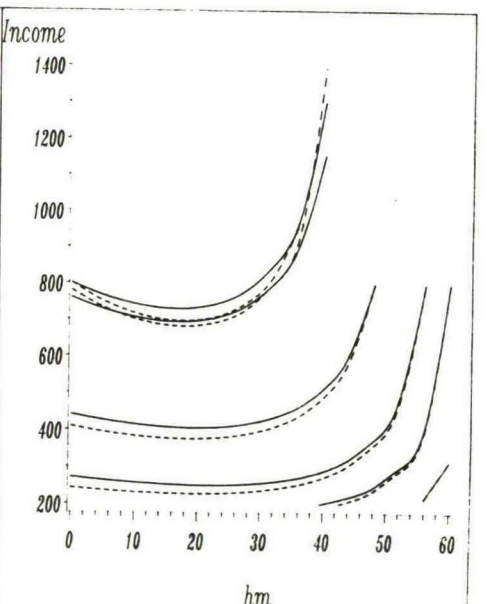


Family with one child younger than six; $hm=40$

Model IIa: —; Model IIa, generalised: - - - -



Family without children; $hf=20$



Family with one child younger than six; $hf=20$

References

- Andrews, D. (1988), 'Chi-square diagnostic tests for econometric models', Journal of Econometrics 37, 135-156.
- Ben-Akiva, M. and S. Lerman (1985), 'Discrete Choice Analysis', MIT Press, Cambridge Mass.
- Blundell, R. (1990), 'Evaluating structural microeconomic models of labour supply', working paper, University College London.
- Burtless, G. and J. Hausman (1978), 'The effect of taxation on labor supply: Evaluating the Gary income maintenance experiment', Journal of Political Economy 86, 1103-1130.
- Chiappori, P.-A. (1991), 'Collective labour supply and welfare', Journal of Political Economy, forthcoming.
- Dickens, W. and S. Lundberg (1985), 'Hours restrictions and labor supply', NBER working paper No. 1638.
- Gertler, P. and J. Newman (1991), 'Family productivity, labour supply and welfare in a low income country', working paper, World Bank, Washington DC.
- Gourieroux, C. and A. Monfort (1990), 'Simulation based econometric methods', working paper, ENSAE-CREST, Paris.
- Hajivassiliou, V. (1991), 'Simulation estimation methods for limited dependent variable models', mimeo, Yale University, to appear as a chapter in G.S. Maddala et al. (eds), Handbook of Statistics, Vol 11.
- Hausman, J. and P. Ruud (1984), 'Family labor supply with taxes', American Economic Review 74, 242-248.
- Heckman, J. (1979), 'Sample selection bias as a specification error', Econometrica 47, 153-161.
- Kapteyn, A., P. Kooreman and A. van Soest (1990), 'Quantity rationing and concavity in a flexible household labor supply Model', The Review of Economics and Statistics 70, 55-62.
- Lerman, S. and C. Manski (1981), 'On the use of simulated frequencies to approximate choice probabilities', in C. Manski and D. McFadden eds, Structural analysis of discrete data with econometric applications, MIT Press, Cambridge Mass., 305-319.
- Maddala, G. (1983), 'Limited dependent and qualitative variables in econometrics', Cambridge University Press, Cambridge, Mass.
- MaCurdy, T., D. Green and H. Paarsch (1990), 'Assessing empirical approaches for analyzing taxes and labor supply', Journal of Human Resources 25, 415-490.
- McFadden, D. (1989), 'A method of simulated moments for estimation of multinomial probits without numerical integration', Econometrica 57, 995-1026.

- Moffitt, R. (1984), 'The estimation of a joint wage hours labour supply model', Journal of Labor Economics 2, 550-566.
- Moffitt, R. (1986), 'A survey and exposition of the maximum likelihood method', Journal of Business and Economic Statistics 4, 317-328.
- Moffitt, R. (1990a), 'Taxation and Labor Supply in Industrial Countries', Journal of Human Resources 25, no. 3.
- Moffitt, R. (1990b), 'The econometrics of kinked budget constraints', Journal of Economic Perspectives 4, 119-139.
- Ransom, M. (1987), 'An empirical model of discrete and continuous choice in family labor supply', Review of Economics and Statistics 59, 465-472.
- Ransom, M. (1989), 'The added worker effect in a microeconomic model of the family with market rationing', working paper, Bigham Young Univ.
- Tummers, M. and I. Woittiez (1991), 'A simultaneous wage and labour supply model with hours restrictions', Journal of Human Resources 26, 393-423.
- Van Soest, A., I. Woittiez and A. Kapteyn (1990), 'Labour supply, income taxes and hours restrictions in the Netherlands', Journal of Human Resources 25, 517-558.
- Van Soest, A. and P. Kooreman (1991), 'On the optimal degree of sophistication of a labour supply model', working paper, Tilburg University.
- Van Soest, A., P. Kooreman and A. Kapteyn (1992), 'Coherency and regularity of demand systems with equality and inequality constraints', Journal of Econometrics, forthcoming.
- WRR (1990), 'Een werkend perspectief; arbeidsparticipatie in de jaren '90', Wetenschappelijke Raad voor het Regeringsbeleid, 's-Gravenhage.

Notes

- 1) Generalisations in which spouses have separate utility functions and are, for example, assumed to reach some Pareto-efficient leisure-consumption allocation, are beyond the scope of this paper. See, e.g., Chiappori (1991) for a discussion of identification and the practical value of such models.
- 2) Due to lack of appropriate data, family expenditures excluding savings etc. could not be computed. As a consequence, the model is purely static and is not consistent with two stage budgetting in a life cycle framework (cf., e.g., Blundell, 1990).
- 3) This is discussed in many papers, starting with Burtless and Hausman (1978). See also e.g. Moffitt (1986, 1990b) for overviews.
- 4) See, for example, Ben-Akiva and Lerman (1985) for a discussion of this issue.
- 5) The income tax reform in 1990 has reduced the number of tax brackets to three for each individual. The non-convexity for married females however has been retained.
- 6) Atkinson and Micklewright (1991) stress the importance of distinguishing between unemployment insurance and unemployment assistance for the decision whether or not to accept a job offer.
- 7) The 33 families with male or female workers whose working hours are not observed (cf. section 3) are not included.
- 8) An extra cell has to be included to allow for unobserved positive working hours. Andrews (1988) also discusses generalisations in which cells are disaggregated using the explanatory variables.
- 9) Detailed estimation results and the analogons of tables 4 and 5 and figure 1 are available upon request. Preliminary estimation results suggest that it is not worthwhile to allow the other parameters, corresponding to log c or a product with log c, to depend on family characteristics.
- 10) See also Hajivassiliou (1991) for a recent survey of simulation estimation methods for a more general class of models.
- 11) As in section 3, this model has also been estimated with α_{22} and α_{33} allowed to depend on family characteristics. Resulting elasticity estimates are not very different, though still somewhat smaller (in absolute value). The other income elasticity for the female's hours of work remains significantly positive.
- 12) In principle, it is also possible to estimate the wage rate equations and the labour supply model simultaneously. This requires a small adjustment of the likelihood function only. However, the number of parameters to be estimated substantially increases. Earlier results with the single individual model (cf. Van Soest and Kooreman, 1991) suggest that coefficients in the wage rate equations will hardly change.

- 13) Again, results in Van Soest and Kooreman (1991) suggest that this problem is of minor importance. In the simultaneous model for a single individual, the correlation matrix between the estimator for the parameters of the wage equation and the estimator for the labour supply parameters, appears to be very small.
- 14) This is due to insufficient memory space for storing large matrices in Fortran, and not to increasing computing time requirements.
- 15) If α_{22} and α_{33} are further parameterised, basically the same happens as what is described in footnote 10 for model I: elasticities slightly fall; the width of the confidence intervals remain similar.
- 16) Standard tests (LM, LR, Wald) are in principle invalid due to the one-sided nature of the alternative. Still, any bivariate confidence region for $(\sqrt{V\{\xi_m\}}, \sqrt{V\{\xi_p\}})$ with a reasonable size contains (0,0).

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