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# A "MISTAKEN THEORIES" REFINEMENT by Hideo Suehiro 

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#### Abstract

This paper proposes a new extensive form refinement. The concept is defined for a set of strategy profiles. We conceive a random process by which a player, who experts one of the strategy profiles in this set as a local standard of behavior mode, meets other players, who might hold different expectations from the set. In such a matching environment, we construct an equilibrium criterion called consistency which requires that every player optimally follows his local standard of behavior mode. We show that the criterion provides a nonempty refinement of Nash equilibrium and that the criterion justifies the sequential rationality requirement for perfect information games but not for general games.


## JEL Classification Number: 026

Key Words : refinement, sequential rationality, equilibrium misunderstanding, player matching

[^0]
## 1. INTRODUCTION

Noncooperative game theory has achieved a tremendous success in the refinement for extensive form games since Kreps and Wilson (1982) introduced the notion of sequential equilibrium. The notion can eliminate many unreasonable Nash equilibria by requiring sequential rationality from players. The power of sequential rationality, however, comes from two different parts of the requirement. One is the requirement that a player must hold a belief system about past moves. The other is the equilibrium hypothesis that a player at any information set must expect an equilibrium play in the future. Quite often the second part is crucially responsible for the power of the sequential rationality requirement. A simple example is the game of Figure 1. In the Nash but nonsequential equilibrium (A, $I: d$ ), the threat of $d$ to player I by player II is judged empty because player II at node $r$ must expect player 1 to take $L$ at node $y$, which compels player II to take $a$. But when the Nash equilibrium ( $A, L ; d$ ) is supposed to be common knowledge, why must player II expect the equilibrium play $L$ after the supposition itself is refuted by the fact of deviation $D$ ? The theory of sequential equilibrium can not answer this question since it is a hypothesis exogenously imposed on the theory.

One way to answer this question is by offering a model which describes how a deviation could occur. Fudenberg, Kreps and Levine (1988) provide such a model, in which a deviation occurs because the deviator's payoff turns out to be different from what it was thought to be with high probability. According to them, the Nash equilibrium $(A, L ; d)$ is supported when the game of Figure 1 is viewed as a part of an extended game of the following kind. There are two types ( $A$ and $B$ ) of player I. Nature moves first to randomly choose one of the types and privately inform player I of the realization. Then player I and player II play the game of Figure 1. If player I is type $A$, the payoff assignments to him is exactly as in Figure 1. If player I is type $B$, he receives 0 from $A, 1$ from $D \rightarrow a, 2$ from $D \rightarrow d \rightarrow L$, and 3 from $D \rightarrow d \rightarrow R$. For any small enough positive probability that type $B$ is chosen, a strategy profile in which player I takes $(A, L)$ if he is type $A$ and $(D, R)$ if he is type $\|$ while payer II takes $d$ is a perfect equilibrium ${ }^{2}$ of the extended game. The payoff uncertainty, however, implies that the structure of the conflicts among the players' interests

2 They use a slightly different criterion called near strictness for selecting equilibria in the extended game.
modeled by the payoff assignments is changed from the original game of Figure 1 to the extended game. If, as Rubinstein (1991) argued, noncooperative game theory attempts to develop our understanding of a rational play in a given conflict, we hope that the model of conflict itself is correct. Namely we hope to exclude payoff uncertainty.

Is the equilibrinm hypothesis justified once we exclude payoff uncertainty? Some authors, e.g. Binmore (1987) and Kreps (1989), have argued informally that it might not be justified dne to the possibility that a player holds a different equilibrium understanding, or a "mistaken theory", from other players although all of the understandings are eligible. Some other authors, e.g. McLennan (1985) and Hillas (1987), have even attempted to formalize the notion of an alternative equilibrium play. But none of them has succeeded in providing a theoretical model in which equilibrium misunderstanding induces a deviation, to the extent that Fudenberg. Kreps and Levine (1988) did for the payoff uncertainty argument. The purpose of this paper is to develop a "mistaken theories" refinement ${ }^{3}$ equipped with an explicit model of equilibrium misunderstanding and, in light of the refinement, examine the extent that the equilibrium hypothesis of sequential equilirbium is justified. Section 2 develops the formulation of the refinement called the consistency criterion. Section 3 presents the main results. Section 4 gives some remarks on the approach adopted in this paper.

## 2. CONSISTENT SET OF STRATEGY PROFILES

One might wonder if it is ever possible for rational players to play according to different equilibrium understandings. The game ${ }^{4}$ of Figure 2 illustrates that it is actually possible by the following informal story. Consider a Nash equilibrium $\sigma^{H}=(D ; a, W ; l)$. It is not sequential because player II at node $v$ should take $d$ given the equilibrium hypothesis that player III will take $l$. However imagine MBAs learning game theory at one of two business schools called H-school and S-school. The professors at both schools teach that there are two reasonable ways to play the game; one is $\sigma^{H}$ and the other is another Nash equilibrium $\sigma^{\Sigma}=(A ; a, E ; r)$. But customarily H-school MBAs play $\sigma^{H}$ and S-school MBAs play $\sigma^{S}$. In a recruiting season, each MBA is hired by one of two firms called H-firm and S-firm.

[^1]H-firm hires II-school MBAs mainly and S-firm hires S-school MBAs mainly. However there is a small chance that by some mistake in the recruiting process an MBA inadvertently goes to the wrong firm. Suppose that there are the same large number of H -school MBAs and S -school MBAs, and that the chance of an H -school MBA's going to S -firm and the chance of an S-school MBA's going to H -firm are equal and independent. After the recruiting season, a group of three MBAs is formed by random matching in a firm and play the game of Figure 2. Assume that it is private information whether a particular player is an H school MBA or an S-school MBA, and that it is never known to MBAs whether they are in II-firm or S-firm. If all the above is common knowledge, it is rational for every MBA to follow his own school custom. Examine the nonsequential action $a$ after $A$ prescribed by $\sigma^{H}$ for an H-school MBA acting as player II. In the play environment described above, he would not interpret the deviation $A$ from the H -school custom as a trembling hand mistake by payer I who is an H-school MBA, but rather would believe that player I must be an $S$-school MBA, since $\sigma^{S}$ does prescribe $A$ for player I. Given this information, he attempts to reason whether player III will be an H -school MBA or an S-school MBA. There are two possibilities. One is that he is in H-firm but an S-school MBA has been mistakenly placed there and acted as player I. In this case, player III will also be an H -school MBA with high probability. The other possibility is that player II himself has been mistakenly placed at S-firm and he is playing against player I who is an S-school MBA and has visited the right firm. In this case, player III will likely be an S-school MBA. Due to the symmetry of the play environment, player II assesses the probability $\frac{1}{2}$ that player III will be an H -school MBA who will play $l$ and the probability $\frac{1}{2}$ that player III will be an S-school MBA who will play $r$. Player II prefers $a$ to the lottery of $\frac{1}{2} l+\frac{1}{2} r$ implied by $d$, as $\sigma^{H}$ prescribes for him. Thus it is rational for an H -school player II seeing $A$ to act according to his custom $\sigma^{H}$. The reader may easily verify that this is always the case for all MBAs from both schools. Thus it is possible for rational players to play according to different equilibrium understandings, including even a nonsequential equilibrium.

The above story is predicated on a specific matching device of MBAs with different equilibrium understandings. The essence of the story, however, is that the final distribution over the set of players' identity configurations simultaneously generates player's beliefs about the opponents' past moves and about the opponents' future moves. We shall formalize
this essence to define the following "mistaken theories" refinement. We shall apply the embedding technique of Fudenberg, Kreps and Levine (1988) to the incomplete information with respect to payers' equilibrium understandings. We consider any finite extensive form game $G$ with perfect recall. Let I be the set of players. Due to the Kuhn's (1953) theorem, the strategic opportunity for each player $i \in I$ is represented by the set $\Sigma_{i}$ of behaviorally mixed strategies. Consider any finite set $C \subset \prod_{i \in \mathbf{I}} \Sigma_{i}$ of strategy profiles. Then consider any fully mixed probability distribution $q \in I n t \Delta\left(C^{\mathbf{I}}\right)$ over the set of possible configurations of payers' identities. A configuration $\left(\sigma^{j}\right)_{j \in \mathbf{I}} \in C^{\mathbf{I}}$ assigns to each player $j$ a strategy profile $\sigma^{j}$ from the set $\left({ }^{\circ}{ }^{5}\right.$ We define an extensive form game $G(C, q)$ associated with the original game $G$ via the pair $(C, q)$. The tree of $G(C, q)$ consists of $\left|C^{\mathbf{I}}\right|$ replicas of the tree of $G$. A replica is denoted as $T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ corresponding to the element $\left(\sigma^{j}\right)_{j \in \mathbf{I}} \in C^{\mathbf{I}}$. Let $\gamma: U_{(\sigma), \in \mathbf{I} \in \mathbb{C}} T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right) \rightarrow T$ be the mapping which maps node $x$ in a replica tree $T\left(\left(\sigma^{\lambda}\right)_{j \in \mathbf{I}}\right)$ to the corresponding node $\gamma(x)$ in the tree $T$ of the original game $G$. The player set of $C(C, q)$ is $\mathbf{I} \times C$. Player $(i, \sigma) \in \mathbf{I} \times C$ owns node $x \in T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ if and only if node $\gamma(x)$ is owned by player $i$ in the original game $G$ and the replica tree $T\left(\left(\sigma^{j}\right)_{j \in \mathrm{I}}\right)$ is such that $\sigma^{i}=\sigma$. If a player owns no node in a replica tree $T\left(\left(\sigma^{j}\right)_{j \in \mathfrak{I}}\right)$, then he is a dummy player in the tree $T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$. If the player $(i, \sigma) \in \mathbf{I} \times C$ is active in a replica tree $T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$, his payoff at terminal node $z \in T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ is identical with the payoff to player $i$ at lerminal node $\gamma(z)$ in the original game $G$. A dummy player gets zero payoff. An active player $(i, \sigma) \in \mathbf{I} \times C$ in a replica tree $T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ has a set of possible actions at his node $x \in T\left(\left(\sigma^{j}\right)_{j \in I}\right)$ which is identical with the set of possible actions at node $\gamma(x)$ in the original game $G$. At the node $x$, however, the information set $h(x)$ containing node $x$ in the game $G(C, q)$ is replicated with respect to his opponent's identity profile
 initial node $w \in T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ is chosen by Nature with probability $q\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right) \rho(\gamma(w))$ where $\rho(\gamma(w))$ is the probability that initial node $\gamma(w)$ is chosen in the original game $G .^{6}$ This means that the replica tree $T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ is played with probability $q\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$. This completes

[^2]the definition of $G(C, q)$. With the definition of actions and information sets for the game $G(C, q)$, the space of behavior strategies $s_{i, \sigma}$ for player $(i, \sigma) \in \mathbf{I} \times C$ in the game $G(C, q)$ is $\Sigma_{i}$ itself. Call a strategy profile $s=\left(s_{i, \sigma}\right)_{(i, a) \in \mathbf{I} \times C} \in \prod_{i \in \mathbf{I}} \prod_{\sigma \in C} \Sigma_{i}$ an implementation of $C$ ' if $s_{i, \sigma}=\sigma_{i}$ for any $(i, \sigma) \in \mathbf{I} \times C$. Now we define the criterion that the set $C$ is a collection of simultaneously reasonable ways to play the original game $G$.

## Definition

A nonempty finite set $C$ of behavior strategy profiles in $G$ is said to be consistent if and only if there exists a sequence $\left\{q^{m}\right\}_{m=1}^{\infty}$ in Int $\Delta\left(C^{\mathrm{I}}\right)$ such that

1) the sequence $\left\{q^{m}\right\}_{m=1}^{\infty}$ is convergent in $\Delta\left(C^{\mathbf{I}}\right)$ and the limit is $\lim _{m \rightarrow \infty} q^{m}\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)>$ 0 if and only if $\sigma^{j}$ s are identical across $j \in \mathbf{I}$, and
2) for each $m$ fixed, the implementation of $C$ is a perfect equilibrium of the game $G^{\prime}\left(C, q^{m}\right)$.

The interpretation of the criterion is straightforward. For each equilibrium understanding in $C$ to be simultaneously eligible, you must be able to find a near-by situation $G\left(C, q^{m}\right)$ in which 1) equilibrium misunderstanding in the form of heterogeneous identity configuration $\left(\sigma^{j}\right)_{j \in \mathbf{I}}$ such that $\sigma^{i} \neq \sigma^{i^{\prime}}$ for some $i, i^{\prime} \in \mathbf{I}$ is almost probability zero and 2) it is rational for every player $i$ to implement $\sigma_{i}$ if he believes the equilibrium $\sigma$. Note that we do not require any specific form of matching probability except the condition that a theory mistake is very rare. The reader may easily verify that the informal MBA matching story given above for the game of Figure 2 can be formally stated as saying that the set $C=\left\{\sigma^{H}, \sigma^{S}\right\}$ is consistent hy taking the sequence $\left\{q^{m}\right\}_{m=1}^{\infty}$ as $q^{m}\left(\sigma^{\mathrm{I}}, \sigma^{\mathrm{II}}, \sigma^{\mathrm{III}}\right)=\frac{1-6_{\epsilon} \mathrm{m}}{2}$ if $\sigma^{I}=\sigma^{I I}=\sigma^{\text {III }}=\sigma^{H I}$ or $\sigma^{S}$ and $q^{m}\left(\sigma^{1}, \sigma^{I I}, \sigma^{I I I}\right)=\epsilon^{m}$ otherwise where $\epsilon^{m}$ is a small positive number converging to zero. The "mistaken theories" refinement selects an equilibrium $\sigma$ if there exists a consistent set $C$ such that $\sigma \in C$.

## 3. MAIN RESULTS

We now present our main results about what kind of refinement the consistency criterion provides us. First note that, for any perfect equilibrium $\sigma$, the singleton set $C=\{\sigma\}$ is automatically consistent, since any probability $q \in \operatorname{Int} \Delta\left(C^{\mathbf{1}}\right)$ is degenerate to pick the replica tree $T\left(\left(\sigma^{j}\right)_{j \in \mathbf{I}}\right)$ with $\sigma^{j}=\sigma(j \in \mathbf{I})$ for sure. Since any finite extensive form game
with perfect recall has at least one perfect equilibrium, the following existence result is immediate.

## Proposition 1

For any finite extensive form game with perfect recall, there exists at least one consistent set of strategy profiles. Specifically a singleton set of perfect equilibrium is consistent.

Second, the consistency criterion allows any kinds of behavior modes to be matched. Once a player $(i, \sigma)$ sees a deviation from his equilibrium understanding $\sigma$, the original expectation that they are playing $\sigma$ with almost probablity one is upset. In this occasion there is no necessity that he seeks the deviator's equilibrium understanding $\sigma^{\prime}$ from the set of Nash equilibria of the original game. It might be possible that variety of behavior modes beyond Nash equilibria support each other in a player matching environment. The following result, however, says that it is not possible. (See Appendix for the proof.)

## Proposition 2

For any finite extensive form game $G$ with perfect recall, if a nonempty finite set $C$ of behavior strategy profiles in $G$ is consistent, then any strategy profile $\sigma \in C$ is a Nash equilibrium of the original game $G$.

With Proposition 1 and Proposition 2 together, the consistency criterion provides a nonempty refinement between Nash equilibrium and perfect equilibrium. The notion of sequential equilibrium also provides a refinement between Nash equilibrium and perfect equilibrium. Then does the consistency criterion induce the equilibrium hypothesis of sequential equilibrium and justify the sequential rationality requirement? The answer is affirmative for the following class of games. (See Appendix for the proof.)

## Proposition 3

For any generic perfect information game, which has a unique perfect equilibrium, there exists a unique consistent set of strategy profiles. The set is a singleton set of the perfect equilibrium.

Now the requirement that a player expects an equilibrium play in the future at any information set is not an exogenously imposed hypothesis but an indogenously induced implication
of the "mistaken theories" refinement for a generic perfect information game. For the game of Figure 1, for example, we can claim that the Nash equilibrium $(A, L ; d)$ is unreasonable, without resorting to the equilibrium hypothesis in the theory of sequential equilibrium. This result is in a sharp contrast with the result of Fudenberg, Kreps and Levine (1988), which was demonstrated in Section 1, and comes from the fact that we exclude payoff uncertainty. Proposition 3, however, does not generalize beyond perfect information games. An immediate counter example is the game of Figure 2. As we showed, the Nash equilibrium $\sigma^{H}=(D ; a, W ; l)$ is supported by the "mistaken theories" refinement even though it is not sequential. The equilibrium hypothesis apparently collapses for an H -school player II after $A$ in the MBA matching story since he does not expect the equilibrium action $l$ prescribed for player III by $\sigma^{H}$ but rather expects a lottery $\frac{1}{2} l+\frac{1}{2} r$ from player III.

Beyond perfect information games, although the exact form of the equilbrium hypothesis in the theory of sequential equilibrium is not justified by the consistency criterion, is it still possible to justify some implication of the hypothesis in the form of backward induction? The power of the equilibrium hypothesis is that we can solve a rational play backward since a player chooses his optimal action at any information set by assuming that the rest of the play will follow the pattern already calculated backward up to that information set. Then, as Kohlberg and Mertens (1986) argued, ${ }^{7}$ one might hope that, for any game $G$ and any proper subgame $g$ of $G$, if an equilibrium $\sigma$ in $G$ is selected by the consistency criterion, then the restriction $\left.\sigma\right|_{g}$ of the equilibrium to the subgame $g$ is also selected as an equilibrium for the game $g$ by the consistency criterion so that we can solve $\left.\sigma\right|_{g}$ first and then solve $\sigma$ given the obtained $\left.\sigma\right|_{g}$. By Proposition 3, of course, the hope is fulfilled for a generic perfect information game. But, again, it does not generalize beyond perfect information games. The game of Figure 3 is a generic counter example. The strategy profile $(A, w)$ is not a Nash equilibrium of game $g$. By Proposition 2, therefore, the profile is never supported for the game $g$ by the consistency criterion. For the backward induction property to hold, the prescription $(A, w)$ should not be given for the proper subgame $g$ by any strategy profile $\sigma$ of the entire game of Figure 3 which is supported by the consistency criterion. But a set of three strategy profiles $\sigma=(A ; l, w), \sigma^{\prime}=(S ; r, w), \sigma^{\prime \prime}=(W ; r, s)$ is consistent by taking the sequence $\left\{q^{m}\right\}_{m=1}^{\infty}$ as $q^{m}\left(\sigma^{1}, \sigma^{I I}\right)=\frac{1-6 c^{m}}{3}$ if $\sigma^{1}=\sigma^{11}=\sigma, \sigma^{\prime}$
${ }^{7}$ This property is their property (B11).
or $\sigma^{\prime \prime}$ and $q^{m}\left(\sigma^{1}, \sigma^{I I}\right)=\epsilon^{m}$ otherwise where $\epsilon^{m}$ is a small positive number converging to zero. ${ }^{8}$ Thus the consistency criterion does not satisfy the backward induction property for general games.

## 4. CONCLUDING REMARKS

This paper has proposed a "mistakeu theories" refinement, which models equilibrium misunderstandings among rational players, and examined the extent that the equilibrium hypothesis of sequential equilibrium is endogenously justified. We conclude this paper with three remarks on the embedding approach adopted by this paper to accomplish these tasks. First, one of the advantages of our approach is to avoid a counterfactual argument which many refinements after Kreps and Wilson (1982), e.g. Cho and Kreps (1987), had to rely on in invoking so called forward induction. Forward induction assumes that a deviation from a presupposed equilibrium can convey information about the deviator's motivation of his deviation. But if the presupposed equilibrium is really presumed, there should not exist any motivation to deviate. In our approach, a deviation can convey information about the deviator's behavior mode endogenously via a presumed matching model of different equilibrium understandings without invoking such a counterfactual argument as forward induction.

Second, in order to develop a satisfactory "mistaken theories" refinement, it is not enough to allow a positive possibility that some of your opponents has an equilibrium understanding different from your own. Some seminal works, e.g. Binmore (1987) and Kreps (1989). do examine such a possibility. But what would happen to that opponent's rationality if he also foresees the positive possibility that he must play with a player like you who has an equilibrium understanding different from his own? The argument proposed without an explicit matching model can lead to an unsatisfactory conclusion. An example is the Selten's (1975) horse game of Figure 4. The Nash but nonsequential equilibrium $\sigma=(D, a, R)$ can be supported by allowing a positive possibility that player I and III

[^3]jointly see a different way $\sigma^{\prime}=(A, a, L)$ to play the game. ${ }^{9}$ If player II who initially believed $\sigma$ sees $\Lambda$ from player I, he updates his guess about his opponents' equilibrium understanding from $\sigma$ to $\sigma^{\prime}$. Given the updated guess, player II's choice of $a$ is rational. But if we allow a positive possibility that player III who believes $\sigma^{\prime}$ play against player II who believes $\sigma$, it is natural to allow also a positive possibility that player III who believes $\sigma^{\prime}$ play against player I who believes $\sigma$. Once we allow the latter possibility, it is not rational for player III who believes $\sigma^{\prime}$ to follow the prescription $L$. If player III is called upon to move, he should conclude that he is not at node $y$ but at node $x$ since there is no player's identity configuration, consisting of $\sigma, \sigma^{\prime}$, which justifies $A \rightarrow d$ while $\sigma$ does preseribe $I$ ) for player I. Thus the set $\left\{\sigma, \sigma^{\prime}\right\}$ is not consistent. ${ }^{10}$

Third, some may still hesitate to use an embedding technique of modeling possible deviations for the reason that the embedding twists the game in question from the original game to the embedded one. Which is the true model that you want to analyse? For the kind of embedding proposed by Fudenberg, Kreps and Levine (1988), we would conclude that the answer is a matter of reality or a matter of players' perceptions of reality. ${ }^{11}$ However, it is possible to interpret our embedding not as twisting the nature of conflict described by the original game but as a model of a way that people cope with that given conflict. Then a set of strategy profiles being consistent carries the implication that there exists a way to cope with the given conflict which admits all strategy profiles in the set. For example, let us interpret the game of Figure 3 as an augmented "battle of the sexes" game. In the subgame $g$, the players face the "battle of the sexes" game extended by a safe option $A$ for player I. Plaver II has the right to decide if they play the subgame $g$. One natural way that people cope with the coordination problem like the "battle of the sexes" game is to agree by preplay communication. Interestingly enough, we can interpret the matching device $\left\{q^{m}\right\}_{m=1}^{\times}$given to support the set $\left\{\sigma, \sigma^{\prime}, \sigma^{\prime \prime}\right\}$ in Section 3 as a model of preplay

[^4]communication process. Through discrete time $m=1,2, \cdots$, players are trying to reach a consensus about which equilibrium among $\sigma, \sigma^{\prime}, \sigma^{\prime \prime}$ should be played. The conversation at each moment can stochastically change each player's current understanding about how to play but no player can see the inside of his opponents' minds for sure. If the stochastic nature of the conversation is described by the above sequence $\left\{q^{m}\right\}_{m=1}^{\infty}$, the players can agree to go for a play at any point of time by convincing themselves that everybody is satisfied with their own current understanding and, as the players conduct the conversation long enough to let $m$ go to infinity, the possibility of real consensus in the sense that all the players do share the same understanding goes to one. This is what we expect from preplay communication as a method to solve a coordination problem. Furthermore the way that this specific communication process solves the coordination problem is in agreement with our intuition. In this specific model, for example, it is very possible that player II's mind sticks to $\sigma$ throughout the conversation until a play. Then player II should take the quit option $I$ since he expects that if he goes to the subgame $g$ by $r$, player I of identity $\sigma$ will be upset and take the safe option $A$. And there does exist a good reason on the side of player I to take the safe option $A$ if he expected $\sigma$. before the play. The player I's understanding in this case is that they failed to agree on either the coordination to $(S, w)$ or the coordination to $(W, s)$ through the conversation until the play. So, even if he is forced to play the subgame $g$ by player II, he does not see any clear intention of player II either to play $(S, w)$ or to play $(W, s){ }^{12}$ Thus the prescription $(A, w)$ for $g$ by $\sigma$ makes sense even though the backward induction property breaks down here. One would say that, for the game of Figure 3, the embedding of the "mistaken theories" refinement is offering a model of a way that people cope with the coordination problem by failing to satisfy the backward induction property.

## APPENDIX

Proof of Proposition 2 : Suppose that a finite set $C$ of strategy profiles in a finite extensive form game ( $;$ with perfect recall is consistent. Let $\left\{q^{m}\right\}_{m=1}^{\infty}$ be the sequence in Int $\Delta\left(C^{\mathbf{I}}\right)$ which supports the consistency of $C$. Fix any $m \in \mathbf{N}$. Then the implementation

[^5]$s$ of $C$ is a perfect equilibrium of game $G\left(C, q^{m}\right)$. Therefore the implementation $s$ of $C$ is a Nash equilibrium of game $G\left(C, q^{m}\right)$. This means that for any $i \in I$, any $\sigma \in C$ and any $s_{i, a}^{\prime} \in \Sigma_{i}$, we have
\[

$$
\begin{equation*}
U_{i, \sigma}^{G\left(C, q^{m}\right)}\left(s_{i, \sigma}, s_{-(i, \sigma)}\right) \geq U_{i, \sigma}^{G\left(C, q^{m}\right)}\left(s_{i, \sigma}^{\prime}, s_{-(i, \sigma)}\right) \tag{1}
\end{equation*}
$$

\]

where, for any $s^{\prime} \in \prod_{i \in \mathrm{I}} \prod_{\sigma \in C} \Sigma_{i}$. the expression $U_{i, \sigma}^{G\left(C . q^{m}\right)}\left(s^{\prime}\right)$ denotes the expected utility which player $(i, \sigma)$ receives in game $G\left(C, q^{m}\right)$ when a play is conducted according to the strategy profile $s^{\prime}$. For any $\sigma^{\prime} \in \prod_{i \in I} \Sigma_{i}$, let $U_{i}\left(\sigma^{\prime}\right)$ denote the expected utility which player $i$ receives in the original game $G$ when a play is conducted according to the strategy profile $\sigma^{\prime}$. Then, by utilizing the definition $s_{j, \sigma}=\sigma_{j}^{j}\left(j \in \mathbf{I}, \sigma^{j} \in C\right)$ of the implementation $s$ of C. we can write

$$
V_{i, \sigma}^{G\left(C, q^{m}\right)}\left(s_{i, \sigma}^{\prime}, s_{-(i, \sigma)}\right)=\sum_{\left(\sigma^{j}\right)_{i \neq i} \in C^{\mathbf{n}},} q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right) U_{i}\left(s_{i, \sigma}^{\prime},\left(\sigma_{j}^{j}\right)_{j \neq i}\right) .
$$

Hence, by dividing both sides of (1) by $\sum_{\left(\sigma^{\prime}\right)_{j \neq i} \in C^{I},}, q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)>0$, we have

$$
\begin{align*}
& \quad \sum_{\left(\sigma^{j}\right)_{j \neq i} \in C^{\text {I }}:}\left(\frac{q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)}{\left(\sigma^{1}\right)_{j^{*} \in i} \in C^{\text {II }}, q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)}\right) U_{i}\left(\sigma_{i},\left(\sigma_{j}^{j}\right)_{j \neq i}\right) \\
& \geq \sum_{\left(\sigma^{j}\right)_{j \neq i} \in C^{\text {I }}:}\left(\frac{q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)}{\left.\sum_{\left(\sigma^{\prime}\right)_{j \neq i} \in C^{\mathbf{I}}, q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)}\right) U_{i}\left(s_{i, \sigma}^{\prime},\left(\sigma_{j}^{j}\right)_{j \neq i}\right) .}\right. \tag{2}
\end{align*}
$$

The inequality (2) must hold for any $m \in \mathbf{N}$. Let $m$ go to infinity. Since the consistency requires that $\lim _{m \rightarrow \infty} q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)>0$ if and only if $\sigma^{j}=\sigma$ for any $j \neq i$, we have

$$
\lim _{m \rightarrow \infty}\left(\frac{q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)}{\sum_{\left.\left(\sigma^{\prime}\right)_{\neq 1} \in C^{\prime}\right)} q^{m}\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)}\right)= \begin{cases}1 & \text { if } \sigma^{j}=\sigma \text { for any } j \neq i \\ 0 & \text { otherwise } .\end{cases}
$$

Therefore the inequality (2) in the limit is

$$
\begin{equation*}
U_{i}\left(\sigma_{i},\left(\sigma_{j}\right)_{j \neq i}\right) \geq U_{i}\left(s_{i, \sigma}^{\prime},\left(\sigma_{j}\right)_{j \neq i}\right) . \tag{3}
\end{equation*}
$$

Since the inequality (3) holds for any $i \in I$ and any $s_{i, \sigma}^{\prime} \in \Sigma_{i}$, the strategy profile $\sigma$ is a Nash equilibrium of the original game $G$. \|

Proof of Proposition 3 : Let $G$ be a perfect information game satisfying the following two conditions. First there exists a unique perfect equilibrium $\sigma^{*}$ in pure strategies. Second,
at any decision node $x$, the action $a^{*}(x)$ prescribed by $\sigma^{*}$ is a unique best action among the set $A(x)$ of available actions at node $x$ for player $i(x)$, who owns the node $x$, given the successive play according to $\sigma^{*}$. The second condition can be formally stated as follows. Let $Z$ be the set of terminal nodes of $G$ and, for any node $y$, let $Z(y)$ be the subset of terminal nodes which are reached via node $y$. For any $i \in I$ and any $z \in Z$, the payoff to player $i$ at node $z$ is denoted by $u_{i}(z)$. For any noninitial node $y$, let $p(y)$ denote the immediate predecessor of node $y$ and let $\alpha(y)$ denote the action which leads node $p(y)$ to node $y$. For any $l \in\{0\} \cup N$. let $p^{t}(y)$ denote the $l$-th predecessor of node $y$ as long as such a predecessor exists. For any node $y$ and any terminal node $z \in Z(y)$, let $l(y, z)$ denote the number satisfying $y=p^{(y, z)}(z)$. Then the second condition ${ }^{13}$ is that

$$
\begin{aligned}
& \sum_{: \in Z\left(\alpha^{-1}\left(\alpha^{-}(s)\right)\right)} \prod_{l=0}^{l\left(\alpha^{-1}\left(\alpha^{-}(s)\right),=z\right)-1} \sigma_{i\left(p^{\prime+1}(z)\right)}\left(\alpha\left(p^{l}(z)\right)\right) u_{i(x)}(z) \\
& >\sum_{=\in Z\left(\alpha^{-1}(\alpha)\right)} \prod_{i=0} \prod_{\left.\alpha^{-1}(\alpha)=z\right)-1} \sigma_{i\left(p^{\prime+1}(z)\right)}^{*}\left(\alpha\left(p^{l}(z)\right)\right) u_{i(x)}(z) \quad \text { for any } a \in A(x) \backslash a^{*}(x) . \text { (4) }
\end{aligned}
$$

It is well known that a perfect information game satisfying these two conditions is generic. Take such a perfect information game $G$ fixed. We shall show that the uinique consistent set of stratregy profiles in $G^{\prime}$ is $C^{\prime}=\left\{\sigma^{*}\right\}$.

Suppose that a finite set $C$ of strategy profiles in $G$ is consistent. Let $\left\{q^{m}\right\}_{m=1}^{\infty}$ be the sequence in $\operatorname{Int} \Delta\left(C^{\mathbf{I}}\right)$ which supports the consistency of $C$. Fix any $m \in \mathbf{N}$. Then the implementation $s$ of $C$ is a perfect equilibrium of game $G\left(C, q^{m}\right)$. Therefore the implementation $s$ of $C$ is a sequential equilibrium strategy profile of game $G\left(C, q^{m}\right)$. Let $\mu^{m}: \gamma^{-1}(X) \rightarrow[0,1]$ be the Kreps and Wilson (1982) belief system of game $G\left(C, q^{m}\right)$ constituting the sequential equilibrium ( $s, \mu^{m}$ ) where $X$ is the set of decision nodes of $G$. Consider any player $(i, \sigma) \in \mathbf{I} \times C$ and any information set $h$ of player $(i, \sigma)$ in $G\left(C, q^{m}\right)$. For any node $y \in h$, since $G$ is a perfect information game, there exists a node $x$ of $G$ such that $\gamma(h)=x$ and there exists an opponent's identity profile $\left(\sigma^{j}\right)_{j \neq i} \in C^{\prime \backslash i}$ such that $y=\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)$. Therefore we can write $\mu^{m}(y)=\mu^{m}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)\right)$.

[^6]Then the sequential rationality of strategy $s_{i, \sigma}$ for player $(i, \sigma)$ at information set $h$ implies

$$
\begin{aligned}
& \sum_{\left(\sigma^{j}\right), \neq i \in C^{T} \mid \cdot}\left[\mu^{m}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i}\right)\right)\right. \\
& \left.\left\{\sum_{a^{\prime} \in A(x)} \sigma_{i}\left(a^{\prime}\right)\left(\sum_{: \in Z\left(\alpha^{-1}\left(a^{\prime}\right)\right)} \prod_{l=0}^{l\left(\alpha^{-1}\left(a^{\prime}\right), z\right)-1} \sigma_{i\left(p^{i+1}(z)\right)}^{i\left(p^{l+1}(z)\right)}\left(\alpha\left(p^{l}(z)\right)\right) u_{i}(z)\right)\right\}\right]
\end{aligned}
$$

$$
\begin{align*}
& \left.\left\{\sum_{z \in Z\left(\alpha^{-1}(a)\right)} \prod_{t=0}^{4 o^{-1}(a), z-1} \sigma_{i\left(p^{\prime+1}(z)\right)}^{i\left(p^{t+1}(z)\right)}\left(\alpha\left(p^{l}(z)\right)\right) u_{i}(z)\right\}\right] \quad \text { for any } a \in A(x) \text {. } \tag{5}
\end{align*}
$$

Given this fact, we proceed with the following induction. For any nonterminal node $x \in X$ in the original game $G$, let $L(x)$ be the hight of the subgame starting from node $x$ in $G$, defined by $L(x)=\max _{z \in Z(x)} l(x, z)$. Let $X$ be partitioned into $X=\sum_{L=1}^{L} \chi_{L}$ by defining $\backslash L \equiv\{x \in X \mid L(x)=L\}$ and $\bar{L}=\max _{x \in X} L(x)$. The induction will be with respect to $L$. Consider $I=1$. Take any $x \in \chi_{1}$. Then, since node $x$ is adjacent to terminal nodes in $Z(x)$, the inequality (4) reduces to

$$
\begin{equation*}
u_{i(x)}\left(\alpha^{-1}\left(a^{*}(x)\right)\right)>u_{i(x)}\left(\alpha^{-1}(a)\right) \quad \text { for any } a \in A(x) \backslash a^{*}(x) \tag{6}
\end{equation*}
$$

Consider any $\sigma \in C$ and the corresponding player $(i(x), \sigma)$ in game $G\left(C, q^{m}\right)$. Then the inequality (5) for his sequential rationality also reduces to

$$
\begin{aligned}
& \sum_{\left.\left(\sigma^{\prime}\right), \neq u s\right) \in C \mid i(x)}\left[\mu^{m}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i(x)}\right)\right) \sum_{a^{\prime} \in A(x)} \sigma_{i(x)}\left(a^{\prime}\right) u_{i(x)}\left(\alpha^{-1}\left(a^{\prime}\right)\right)\right]
\end{aligned}
$$

which further simplifies to

$$
\begin{aligned}
& \left(\sum_{\left(\sigma^{\prime}\right), \neq(x) \in C^{1} \mid(x)} \mu^{m \prime \prime}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i(x)}\right)\right)\right)\left[\sum_{a^{\prime} \in A(x)} \sigma_{i(x)}\left(a^{\prime}\right) u_{i(x)}\left(\alpha^{-1}\left(a^{\prime}\right)\right)\right] \\
& \geq\left(\sum_{\left(\sigma^{\prime}\right), \neq i_{i(x)} \in C^{\prime} \mid,(x)} \mu^{m}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i(x)}\right)\right)\right) u_{i(x)}\left(\alpha^{-1}(a)\right)
\end{aligned}
$$

and. since $\left.\sum_{\left(\sigma^{j}\right), \neq(x) \in C^{\boldsymbol{I}},(x)} \mu^{m}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i(x)}\right)\right)\right)=1$, further to

$$
\begin{equation*}
\sum_{a^{\prime} \in A(x)} \sigma_{i(x)}\left(a^{\prime}\right) u_{i(x)}\left(\alpha^{-1}\left(a^{\prime}\right)\right) \geq u_{i(x)}\left(\alpha^{-1}(a)\right) \quad \text { for any } a \in A(x) \tag{7}
\end{equation*}
$$

By the inequality (6), the inequality (7) implies that $\sigma_{i(x)}\left(a^{*}(x)\right)=1$, that is, $\sigma$ is identical with $\sigma^{*}$ over the set $A(x)$. Since this holds for any $x \in \chi_{1}$ and any $\sigma \in C$, we know that any $\sigma \in\left('\right.$ is identical with $\sigma^{*}$ over the set $U_{x \in x} A(x)$.

Now consider $L=2$. Take any $x \in \chi_{2}$. Then for any $z \in Z(x)$, the number $l(x, z)$ is either 1 or 2 . Since we have proved that any $\sigma \in C$ is identical with $\sigma^{*}$ over the set $\cup_{x \in \chi_{1}} A(x)$, this implies that the expression $\sigma_{i\left(p^{l+1}(z)\right)}^{i\left\{\left(p^{\prime+1}\right)\right.}\left(\alpha\left(p^{l}(z)\right)\right)$ in both sides of the inequality (5) can be replaced by the expression $\sigma_{i\left(p^{t+1}(z)\right)}^{*}\left(\alpha\left(p^{t}(z)\right)\right)$. Then for any $\sigma \in$ $C$ and any corresponding player $(i(x), \sigma)$ in game $G\left(C, q^{m}\right)$, the inequality (5) for his sequential rationality reduces to

$$
\begin{aligned}
& \left(\sum_{\left(\sigma^{\prime}\right), \sum_{\neq(x)} \in C^{\text {I }} \boldsymbol{i ( x )}} \mu^{m}\left(\gamma^{-1}(x) \cap T\left(\sigma,\left(\sigma^{j}\right)_{j \neq i(x)}\right)\right)\right) \\
& \left.\left[\sum_{a^{\prime} \in A(x)} \sigma_{i}\left(a^{\prime}\right)\left(\sum_{z \in Z\left(\alpha^{-1}\left(a^{\prime}\right)\right)} \prod_{l=0}^{l\left(\alpha^{-1}\left(a^{\prime}\right), z\right)-1} \sigma_{i\left(p^{\prime}+1\right.}(z)\right)\left(\alpha\left(p^{l}(z)\right)\right) u_{i(x)}(z)\right)\right] \\
& \left.\geq\left(\sum_{\left(\pi^{i}\right),\left(x i(s) \in(\cdot \boldsymbol{T})\left(\sigma_{0}\right)\right.} \mu^{m}( \urcorner^{-1}(x) \cap T\left(\sigma_{,}\left(\sigma^{j}\right)_{j \neq i(x)}\right)\right)\right) \\
& {\left[\sum_{z \in Z\left(\alpha^{-1}(a)\right)} \prod_{i=0}^{\left.\| \alpha^{-1}(a),=\right)-1} \sigma_{i\left(p^{l+1}(z)\right)}\left(\alpha\left(p^{l}(z)\right)\right) u_{i(x)}(z)\right]}
\end{aligned}
$$

and, thell to

$$
\begin{align*}
& \sum_{a^{\prime} \in A(x)} \sigma_{i}\left(a^{\prime}\right)\left(\sum_{z \in Z\left(\alpha^{-1}\left(a^{\prime}\right)\right)} \prod_{l=0}^{4\left(\alpha^{-1}\left(a^{\prime}\right), z\right)-1} \sigma_{i\left(p^{\prime+1}(z)\right)}^{*}\left(\alpha\left(p^{l}(z)\right)\right) u_{i(x)}(z)\right) \\
& \geq \sum_{z \in Z\left(\alpha^{-1}(a)\right)} \prod_{l=0}^{4\left(\alpha^{-1}(a), z\right)-1} \sigma_{i\left(p^{l+1}(z)\right)}^{*}\left(\alpha\left(p^{\prime}(z)\right)\right) u_{i(x)}(z) \quad \text { for any } a \in A(x) . \tag{8}
\end{align*}
$$

By the inequality (4), the inequality (8) implies that $\sigma_{i(x)}\left(a^{*}(x)\right)=1$, that is, $\sigma$ is identical with $\sigma^{*}$ over the set $A(x)$. Thus we know that any $\sigma \in C$ is identical with $\sigma^{*}$ over the set $\cup_{L=1}^{2} \cup_{x \in \lambda L} A(x)$. The procedure for $L=2$ can be repeated to $L=\bar{L}$ and we conclude that any $\sigma \in C$ is identical with $\sigma^{*}$ over the set $\cup_{L=1}^{L} \cup_{x \in \chi_{L}} A(x)=U_{x \in X} A(x)$. This means that any $\sigma \in\left(\right.$ is actually $\sigma^{*}$ itself. \||

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I

Figure 1


Figure 2


Figure ${ }^{3}$


Figure 4

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[^0]:    ' This paper is based on a part of my dissertation submitted to Stanford Business School and further revised while I have been a research fellow at CentER of Tilburg University. I am sincerely grateful to David Kreps. I am also indebted to Eric van Damme. The supports by the above institutions are acknowledged.

[^1]:    3 The name is by Kreps ( $19 \times 9$ ).
    ${ }^{4}$ This example was developed in conversation with D. Kreps.

[^2]:    ${ }^{5}$ We use superseript $j$ to denote that $\sigma^{j}$ is the equilibrium understanding held by player j. Each $a^{\prime}$ is of the form $a^{\prime}\left(\sigma_{1}^{\prime}\right)_{\mathrm{t}}$ i where $a_{a}^{\prime}$ denotess the strategy for player $i$ preseribed by the strategy profile $\sigma^{J}$ that player $j$ believes.
    ${ }^{6}$ Note that the assignment of initial probabilities is well defined due to the assumption that $q \in \operatorname{Int} \Delta\left(C^{\mathbf{I}}\right)$ is fully mixed.

[^3]:    ${ }^{8}$ The point of the proof is that in game $G\left(C, q^{m}\right)$ for each $m$ fixed, as the trembling hand perturbation goes to zero, player I of identity $\sigma$ seeing $r$ attributes the deviation mostly either to player II of identity $\sigma^{\prime}$ or to player II of identity $\sigma^{\prime \prime}$ with almost equal weights, which induces player I's expectation close to $\frac{1}{2} w+\frac{1}{2} s$ in subgame $g$.

[^4]:    9 The argument here is intended to be similar to the eductive argument which Binmore (1987) proposed to defend $\sigma$.

    10 Although the argument to justify $\sigma$ by $\sigma^{\prime}$ is not satisfactory at all, the outcome of $\sigma$ still can be supported by another consistent set of strategy profiles. Consider $\sigma^{\prime \prime}=\left(D, \frac{2}{3} a+\right.$ $\left.\frac{1}{3} d, R\right)$. The set $\left\{\sigma^{\prime}, \sigma^{\prime \prime}\right\}$ is consistent by taking the sequence $\left\{q^{m}\right\}_{m=1}^{\infty}$ as $q^{m}\left(\sigma^{\prime}, \sigma^{\prime}, \sigma^{\prime}\right)=$ $q^{m}\left(\sigma^{\prime \prime}, \sigma^{\prime \prime}, \sigma^{\prime \prime}\right)=\frac{1}{2}-12 \epsilon^{m}, q^{m}\left(\sigma^{\prime}, \sigma^{\prime \prime}, \sigma^{\prime}\right)=15 \epsilon^{m}, q^{m}\left(\sigma^{\prime}, \sigma^{\prime \prime}, \sigma^{\prime \prime}\right)=5 \epsilon^{m}$, and $q^{m}\left(\sigma^{1}, \sigma^{1 I}, \sigma^{I I I}\right)=$ $\epsilon^{m}$ otherwise where $\epsilon^{m}$ is a small positive number converging to zero.
    11 The conclusion coincides with the claim of Rubinstein (1991).

[^5]:    ${ }^{12}$ This corresponds to the argument in footnote 8.

[^6]:    ${ }^{13}$ Here we use a notational convention that if $l\left(\alpha^{-1}\left(a^{*}(x)\right), z\right)=0$, the expression $\prod_{i=0}^{\mu\left(a^{-1}\left(n^{*}(\tau)\right), z\right)-1} \sigma_{i\left(p^{t+1}(z)\right)}^{*}\left(\alpha\left(p^{l}(z)\right)\right)$ reduces to 1 .

