The Political Economy of a Changing Population

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ABSTRACT

In the context of an overlapping generations model with intragenerational inequality and majority voting, I study how the taxation of the old and retired generation is affected when the population growth rate changes. A fall in the birth rate leads to two opposite effects. On the one hand, the old generation acquires more political power because their relative size in the voting population increases. This exerts downward pressure on the taxation of the old. On the other hand, the tax burden on the young (used to repay the public debt held by the old) increases, so that their support for a low tax rate on assets held by the old decreases. In general, the number of equilibria is either zero or two, one of which involves zero taxation while the other involves partial taxation of the assets held by the old.

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1. Introduction

Birth rates have fallen dramatically in Western countries over the past few decades. Coupled with the fact that life expectancy is increasing, the prediction is that the share of retired people in Western populations will substantially grow in the future. Therefore, in this paper, I extend the real debt repudiation model of Calvo (1988, Section I) to an overlapping generations model with intergenerational inequality in order to investigate the politico-economic consequences of a fall in the birth rate. On the one hand, the political power of the old and retired generation increases because their relative number increases, which leads to pressure for a lower tax rate on their asset holdings. On the other hand, a larger share of old people in the population also raises the tax burden on the young, who prefer a shift towards more taxation of the old. In general, the net effect on the taxation of the old is ambiguous and depends on the distribution of debt holdings.

The remainder of this paper is organised as follows. Section 2 extends Calvo's (1988, Section I) model of real debt repudiation to an overlapping generations context with intragenerational inequality. Section 3 derives, for a given pre-tax rate of return on debt, the voting outcome (by majority rule) for the tax rate on public debt held by the old. In particular, I analyse how the tax rate is affected by a fall in the population growth rate. In Section 4, I close the model and study the existence and characteristic features of politico-economic equilibria. In general, either no or two equilibria exist, one of which involves zero and the other partial taxation of debt held by the old. Section 5 concludes the paper.

2. The model

A convenient framework for analysing the politico-economic consequences of a fall in the birth rate is obtained through the extensions of Calvo's (1988, Section I) model of real debt repudiation to an overlapping generations context with intragenerational inequality and voting by majority rule. I assume that the economy has an infinite horizon, so that individuals born at different points in time face the same optimisation problem.

The society is assumed to consist of families which are connected over time through altruism. In each period a new generation of young agents is born which lives for two periods. Therefore, in any period t, two generations are alive, a young and an old. An old agent gives birth to $\eta>1$ young agents. The size of the old generation in period t is normalised to unity, so that the size of the young generation in period t is η. Hence, the population growth rate is $\eta-1>0$.

Agents are risk-neutral and only consume when they are old.[2] Consider some family denoted by " ˜ ". The utility of the parent and his children are given by, respectively,

$$\tilde{U}_{t-1,t} = \tilde{c}_{t-1,t} \,, \tag{2.1}$$

$$\tilde{U}_{t,t} = \xi \tilde{c}_{t-1,t} + \beta \tilde{c}_{t,t+1}{}^{e}, \tag{2.2}$$

where $\tilde{U}_{t-1,t}$ and $\tilde{c}_{t-1,t}$ are the period t utility and consumption, respectively, of an agent born in period t-1 and β is the subjective discount rate. Expectations are denoted by superscript " ^{e} ". The old care about their own consumption only, while the young are assumed to be altruistic towards the old.[3] The parameter ξ measures the degree of altruism. A young person becomes less selfish as ξ increases.

When born, a descendant of family " ˜ " receives a positive income $\tilde{y}$. The income differs across members of different families, but is the same for descendants of the same family.[4] Out of this income, the newly-born pays an amount of taxes, x_t, which will be used to repay the public debt held by the old. Only distortionary taxation is available. As in Calvo (1988), the "deadweight" cost of taxation is captured by the loss function z(x), which is twice continuously differentiable with $z(0)=0$, $z'(0)=0$ and $z''(.)>0$. After-tax income, $y-x_t-z(x_t)$, can either be invested in physical capital with an exogenous gross real rate of return R,[5] or in newly issued public debt. Each period a new issue of public debt is used to finance a public investment (e.g., infrastructure) of per new young capita size b^A, which is assumed to be exogenous (cf. Calvo, 1988). Therefore, I assume, for convenience, that it does not enter the utility function of agents.

Denote the gross real rate of return on debt by R_{bt}. For a risk-neutral agent to be indifferent between investing in physical capital and in public debt, the expected after-tax return on public debt should be equal to the return on physical capital:

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[2] The assumption that consumption only takes place in the second period of an agent's life is only made to simplify the algebra. Given that utility derived from second-period consumption is linear, none of the results are affected by abstracting from consumption in the first period.

[3] The main results all go through if the old also care about the consumption of the young. What is crucial (both here and in the literature on social security) is that the young care about the old. Otherwise, no majority voting equilibrium could exist in which the debt holdings of the old are not completely taxed away.

[4] The income earned by a young person can be thought of as income generated through the exploitation of a family property.

[5] This is the same as assuming the existence of a linear storage technology or the possibility of investing abroad against a fixed gross rate of return R. The key assumption is that there is an "outside" investment opportunity which is untaxed.

$$(1-\theta_{t+1}^{e})\ R_{bt} = R,\ 0\leq\theta_{t+1}^{e}\leq 1, \tag{2.3}$$

where θ_{t+1}^{e} is the tax rate (or "repudiation rate" in Calvo’s, 1988, words) expected for the next period on public debt issued in period t. Once debt has been issued, its gross rate of return is fixed, but not its net rate of return. The investor recognises this and demands an interest rate R_{bt} which compensates him for the expected ex post incentives to tax debt. For future convenience, I assume that $\beta R=1$.

The per young capita tax payment follows from the government budget constraint,

$$x_t = [(1-\theta_t)R_{b,t-1}b^A + \alpha\theta_t R_{b,t-1}b^A]/\eta,\ 0\leq\theta_t\leq 1, \tag{2.4}$$

where $R_{b,t-1}$ is the gross real rate of return on public debt held by the old (determined in the previous period), b^A is the average amount of public debt held by the old, θ_t is the tax rate on debt repayment to the old and α ($0\leq\alpha<1$) are the distortionary losses per unit of repayment reduction.[6] Because $\alpha<1$, a higher value of θ_t lowers the per capita taxes paid by the young. Note that each young person only pays taxes for a number of $1/\eta<1$ old individuals.

Given that agents are indifferent between the two investment opportunities (eq.(2.4)), I follow Calvo (1988) in assuming that they all invest an equal amount in physical capital, such that the market for public debt exactly clears. Hence, apart from a shift by some amount k, the income distribution is preserved in the distribution of debt holdings, which I denote by G(.). The distribution of debt holdings is continuous and its density function is assumed to be positive only on a connected interval on $\mathbb{R}^{+}$. Hence, for every agent debt holdings are positive, with a uniquely defined median. The (expected) consumption of an old, respectively young, person with debt holdings $\tilde{b}$, becomes:[7]

$$\tilde{c}_{t-1,t} = (1-\theta_t)R_{b,t-1}\tilde{b} + (k-x_{t-1}-z(x_{t-1}))R, \tag{2.5}$$

$$\tilde{c}_{t,t+1}^{e} = (1-\theta_{t+1}^{e})R_{bt}\tilde{b} + (k-x_t-z(x_t))R. \tag{2.6}$$

6 α can be thought of as transaction costs associated with debt repudiation (cf. Calvo, 1988).

7 I abstract from the possibility of leaving bequests or gifts, as in Tabellini (1991). Altruism manifests itself only in the voting behaviour of agents. In fact, as long as agents care more about their own consumption than about the consumption of their family members, they would choose not to leave any bequests nor to make any gifts, because the marginal utility from their own consumption is higher than the marginal utility of the consumption of their family members.

The last terms in (2.5) and (2.6) are the physical capital investments multiplied by R.[8]

3. Voting on the taxation of the old

In each period, the tax policy, the combination of x and θ, is decided upon by majority rule.[9] To find the voting outcome in period t, we need to combine the two generations currently alive into one voting population. Because x_{t-1} is given, the utility of the old is monotonically decreasing in θ_t. To find the optimal tax rate $\tilde{\theta}_t$ preferred by a young agent with debt holdings $\tilde{b}$, combine (2.2) and (2.4)-(2.6), to obtain utility as a function of θ_t:

$$\tilde{U}_{t,t} = \xi[(1-\theta_t)R_{b,t-1}\tilde{b} + (k-x_{t-1}-z(x_{t-1}))R] +$$
$$\beta\{(1-\theta_{t+1}{}^{e})R_{bt}\tilde{b} + [k- ((1-\theta_t)R_{b,t-1}b^A + \alpha\theta_t R_{b,t-1}b^A)/\eta - z(((1-\theta_t)R_{b,t-1}b^A + \alpha\theta_t R_{b,t-1}b^A)/\eta)]R\}. \quad (3.1)$$

Differentiate (3.1) with respect to θ_t and define (remember that $\beta R=1$),

$$b_{Lt} \equiv (1/\xi\eta)(1-\alpha)b^A\ [1+z'(\alpha R_{b,t-1}b^A/\eta)], \quad (3.2)$$
$$b_{Ut} \equiv (1/\xi\eta)(1-\alpha)b^A\ [1+z'(R_{b,t-1}b^A/\eta)], \quad (3.3)$$

If $\tilde{b}\leq b_{Lt}$, the utility of the young increases monotonically for $0\leq\theta_t\leq 1$. Hence, a poor young person (i.e., also a young person with a poor parent) prefers complete taxation of the debt held by the old. The resulting reduction in his own tax payments outweighs the fall in his parent's consumption. In contrast, if $\tilde{b}\geq b_{Ut}$, indirect utility decreases for $0\leq\theta_t\leq 1$. A sufficiently rich young person thus prefer zero taxation of the old. Finally, if $b_{Lt}<\tilde{b}<b_{Ut}$, indirect utility increases on the interval $[0,\tilde{\theta}_t)$, decreases on the interval $(\tilde{\theta}_t,1]$, and reaches a maximum at $\theta_t=\tilde{\theta}_t$, which is defined by the first-order optimisation condition,

$$\tilde{b} = (1/\xi\eta)(1-\alpha)b^A\ \{1+z'[((1-\tilde{\theta}_t)R_{b,t-1}b^A+\alpha\tilde{\theta}_t R_{b,t-1}b^A)/\eta]\}. \quad (3.4)$$

If $\tilde{b}$ increases, the preferred tax rate $\tilde{\theta}_t$ falls. The increase in the parent's consumption, as a result of the fall in $\tilde{\theta}_t$, more than outweighs the fall in future consumption by the young as a result of the higher income tax x_t to be paid.

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[8] I assume that k is large enough for consumption to be positive under any admissible policy outcome.

[9] Note that the current voting outcome does *not* depend on previous voting outcomes, as we will see below. Hence, there are no intertemporal linkages in voting behaviour.

To determine the chosen tax policy, one needs to combine the young and the old generation into one voting population. As shown above, preferences are single-peaked in θ_t. Hence, the tax policy chosen by majority rule is the median of the distribution of most preferred policies by the total voting population (also, see Tabellini, 1991). One can distinguish three cases:

(i) A majority of all voters is in favour of zero debt taxation. This is the case if $1 \cdot 1 + \eta \cdot (1-G(b_{Ut})) \geq (1+\eta)/2$. The first term on the left hand side is the total number of old multiplied by the share (unity) of the old generation in favour of zero debt taxation. Similarly, the second term is the total number of young in favour of zero debt taxation. The condition reduces to,

$$G(b_{Ut}) \leq (1+\eta)/2\eta. \tag{3.5}$$

(ii) A majority of all voters is in favour of complete debt taxation. This is the case if $G(b_{Lt}) \geq (1+\eta)/2\eta$. Because none of the old favours complete debt taxation, the number of young in favour of $\theta_t=1$ must be at least half of the total voting population.

(iii) The median of preferred tax rates, θ_{ct}^M, lies between zero and one. In fact, this is tax rate preferred by a young person with debt holdings of,

$$b_{ct}^M = (1/\xi\eta)(1-\alpha)b^A \ \{1+z'[((1-\theta_{ct}^M)R_{b,t-1}b^A+\alpha\theta_{ct}^M R_{b,t-1}b^A)/\eta]\}. \tag{3.6}$$

Note that $b_{Lt}<b_{ct}^M<b_{Ut}$. The median θ_{ct}^M is defined by the requirement that at most half of the population prefer a lower tax rate θ_t (all the of old and all of the young richer than b_{ct}^M) and that at most half the population prefer a higher tax rate (all young poorer than b_{ct}^M). Hence, θ_{ct}^M is implicitly defined by,

$$G(b_{ct}^M) = (1+\eta)/2\eta. \tag{3.7}$$

Both a higher level of public debt, b^A, and a higher interest rate $R_{b,t-1}$ raise the tax burden on the young, who prefer a shift towards increased taxation of the old. Both b_{Lt} and b_{Ut} fall, making $\theta_t=1$ more likely and $\theta_t=0$ less likely. Starting from a situation of partial taxation of the old, the outcome for θ_t rises if b^A or $R_{b,t-1}$ increases. An increase in ξ makes the young more altruistic towards the parents and lowers θ_t. Similarly, an increase in the marginal cost α leads to a decrease in the taxation of the old. Finally, a fall in the birth rate, η (maintaining $\eta>1$), results in *two opposite* effects. On the one hand, it raises the political power of the old through their weight in the voting population. This effect manifests itself on the right hand side of (3.7), which increases if η falls, and would require a lower tax rate θ_{ct}^{M}, ceteris paribus. On the other hand, a fall in η also raises the tax burden on the young, who prefer a shift towards increased taxation of the old. This effect appears on the left hand side of (3.7). The net effect of a fall in the birth rate is ambiguous and depends on G(.).[10]

4. Equilibrium

So far, I have been concerned with the chosen tax policy when the interest rate is given on debt due for repayment. In this section, I study the existence and some characteristic features of the equilibria of the model. These are found if we impose the arbitrage condition (2.3) and the requirement that expectations be rational (i.e., $\theta_t^e=\theta_t$).

Corresponding to the voting outcomes, we have three candidate equilibria in each period t. The candidate equilibrium with complete taxation of the old ($\theta_t=1$) can be excluded immediately. If agents expect their investments to be taxed away completely, they will not be prepared to invest in the first place. The second candidate is an equilibrium with $\theta_t=0$. Use (2.3), lagged by one period, and use the rationality of expectations. Hence, substitute (3.3) into (3.5) and impose $R_{b,t-1}=R/(1-0)=R$. An equilibrium with zero taxation of the old exists if and only if,

$$G\{(1/\xi\eta)(1-\alpha)b^A[1+z'(Rb^A/\eta)]\} \leq (1+\eta)/2\eta, \tag{4.1}$$

Finally, there is a possibility for an equilibrium with partial taxation of the old. Combine the equilibrium condition $R_{b,t-1}=R/(1-\theta_{ct}^{M})$ with (3.6) and (3.7). Hence, an equilibrium with partial taxation exists if and only if there exists a $\theta_t=\theta_{ct}^{M}$ such that,

$$G\{\ (1/\xi\eta)(1-\alpha)b^A\{1+z'[Rb^A(1+\alpha\theta_{ct}^{M}/(1-\theta_{ct}^{M}))/\eta]\}\ \} = (1+\eta)/2\eta,\ 0<\theta_{ct}^{M}<1. \tag{4.2}$$

The left hand side of (4.2) is increasing in θ_{ct}^{M} for $0\leq\theta_{ct}^{M}<1$ and is equal to the left hand side of (4.1) if $\theta_{ct}^{M}=0$. The following proposition is immediate:

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[10] If only the young had voting power, the effect of a decrease in η would be unambiguous. Starting from a situation of partial taxation of the old, a fall in η leads to an unambiguous increase in θ.

Proposition 1: Existence and number of equilibria. Let $\alpha>0$:

(a) If (4.1) fails to hold, no politico-economic equilibrium exists.

(b) If (4.1) holds with equality, a unique equilibrium with zero debt taxation exists.

(c) If (4.1) holds with inequality, two equilibria exist, one with zero and the other with partial debt taxation.

Proposition 1 preserves the key properties of Calvo's (1988, Section I) model as summarised in his Proposition I, which says that, in general, his model has either zero or two equilibria. Existence of a unique equilibrium is a special case. Here, the intuition for a possible multiplicity is the same as in his model. If agents expect (partial) taxation of their investments, (before-tax) repayment obligations of the government are relatively large because these agents require a correspondingly higher interest rate. But large repayment obligations create an ex post incentive to tax them away once they are fixed (see Section 3). In contrast, if no taxation is expected, the bond interest rate will be so low that any ex post incentive for taxation is absent. The equilibrium with zero debt taxation Pareto dominates the one with partial taxation. The difference in welfare results from the fact that in the latter case the tax on income and the implied distortionary losses z(.) are higher, only because of the losses associated with the taxation of debt repayments.

Contrary to Calvo (1988), a zero marginal cost of debt taxation ($\alpha=0$) does *not* necessarily lead to complete debt taxation. Therefore, an equilibrium can exist even if $\alpha=0$, although its existence becomes less likely (for given values of the other parameters).[11] In Calvo's model, all agents hold the same amount of government debt. Hence, if $\alpha=0$, the agent's income tax payments are reduced one for one per unit of debt taxed away. A benevolent government minimises total distortionary losses through complete taxation of any outstanding government debt. Because there is no distributional conflict in his model, ex post every agent would be in favour of complete debt taxation. Therefore, if $\alpha=0$, no equilibrium can exist in the standard Calvo model. If debt were unequally distributed in his model, the richer agents might be opposed to debt taxation, because the reduced value of their debt repayments is not compensated by the reduction in their income tax payments. This would allow for the potential existence of an equilibrium even if $\alpha=0$, because the tax rate on debt depends on the relative wealth of the agent represented by the government. As with a fall in α, an increase in b^A raises the left hand sides of (4.1) and (4.2), which renders the existence of equilibria less likely. The reason is that for any given value of $R_{b,t-1}>0$ (hence, also for potential equilibrium values) complete taxation of debt is more likely if gross debt service costs are higher. Finally, an decrease in the population growth rate, $\eta>1$ raises the left hand sides of (4.1)

[11] In fact, if $\alpha=0$, either no equilibrium or a unique equilibrium with zero debt taxation exists.

and (4.2), which reflects a decrease in the number of young in favour of low debt taxation. The right hand sides of (4.1) and (4.2) also rise, which reflects the increased political power of the old. In general, the combined effect of a fall in η on the number of equilibria is ambiguous, although for η large enough, there always exist two equilibria. The reason is that, while the tax burden on the young becomes negligible, their degree of altruism towards the old remains unaffected. Hence, the minor increase in their own expected consumption from an increase in the tax rate on debt is easily outweighed by the effect on their own utility of a fall in their parent’s consumption.

5. Conclusions

In this paper I have investigated the politico-economic effects of a slowdown in the birth rate and hence an increase in the number of old and retired people in the voting population. In general, this affects the taxation of the old in two opposite directions. On the one hand, the political power of the old increases, which exerts a downward pressure on the taxation of their assets. On the other hand, the tax burden on the young, needed to maintain the living standards of the old, increases. This raises their support for a shift towards more taxation of the old.

A potentially interesting extension of the present paper would be to allow for capital accumulation and capital taxation (which particularly hurts the old), in order to study the politico-economic consequences of a fall in the population growth rate for long-run economic growth. The intuition would be that the expected choice of the tax rate on capital affects the willingness to save for retirement and thereby the growth rate of the economy.

References

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