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RISK AVERSION, INTERTEMPORAL SUBSTITUTION AND CONSUMPTION: THE CARA-LQ PROBLEM

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Abstract

This paper employs the recursive utility approach, based on quadratic felicity functions and constant absolute risk aversion, to distinguish between risk aversion and intertemporal substitution. Stochastic dynamic programming yields closed-loop linear decision rules for the CARA-LQ problem. Certainty equivalence no longer holds, but instead the decision maker plays a min-max strategy against nature. When applied to a life-cycle consumption problem, one finds a rationale for precautionary saving and a larger sensitivity of changes in consumption to income innovations.

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1 Introduction

Two households with identical preferences over present and future consumption will under certainty save the same, but this does not necessarily imply that these two households will save the same in uncertain environments. This distinction between intertemporal substitution and risk aversion has recently received a great deal of attention (e.g., Kreps and Porteus, 1978, 1979; Epstein and Zin, 1989; Weil, 1989a). Consumers are, in contrast to the timeless von Neuman-Morgenstern utility theory, not indifferent about the timing of the resolution of uncertainty over temporal consumption lotteries, hence the axiom of reduction of compound lotteries must be abandoned in order not to impose such an indifference. When consumers dislike risk more (less) than intertemporal fluctuations, they prefer early (late) resolution of uncertainty. The emphasis is on recursive preferences, because these lead to time-consistent decisions. Most of the attention has been focussed on stochastic interest rates, asset prices and CAPM models with generalised iso-elastic preferences (e.g., Epstein and Zin, 1989; Giovannini and Weil, 1989; Attanasio and Weber, 1989; Weil, 1989b), but relatively little attention has been paid to the implications of risk aversion and stochastic income for the life-cycle hypothesis. The general result is that, if the third derivative of the felicity function is positive, an increase in uncertainty about future income increases saving (e.g., Leland, 1968; Sandmo, 1970). No closed-form analytical solutions are available, but numerical solutions for the case of a felicity function with constant relative aversion show that precautionary saving is an important factor, that there is excess sensitivity of consumption to transitory income and that uncertainty about uninsured medical expenses leads to underspending of the eldery (Zeldes, 1989). However, previous work on the effects of uncertain income on saving have all introduced risk aversion in the felicity function and thus do not distinguish between intertemporal substitution on the one hand and risk aversion on the other hand.

The objective of this paper is to consider risk aversion and intertemporal substitution in the life-cycle consumption problem when labour income is stochastic and not fully diversifiable. Section 2 sets up the problem of consumption and savings. Section 3 solves the general problem with constant absolute risk aversion and quadratic felicity functions. Section 4 applies it to the life-cycle consumption problem and discusses "saving for retirement", "saving for a rainy day" and "making hay while the sun shines". Section 5 concludes the paper. The Appendix considers the continuous-time problem.

2 Risk aversion and intertemporal preferences

The life-cycle consumption problem can be formulated as:

$$\max_{C_t,\ldots,C_{T-1}} \; E[P_t(C_t,\ldots,C_{T-1}) \mid I_t], \; P_t \equiv \sum_{s=t}^{T-1} (1+\theta)^{-(s-t)} F(C_s),$$

$$F' > 0, F'' < 0 \tag{2.1}$$

subject to the budget constraint

$$A_{s+1} = (1+r_s)(A_s + Y_s - C_s), \;\; s = t,\ldots,T-1, \tag{2.2}$$

where θ denotes the rate of time preference, A_s denotes non-human wealth at the beginning of period t, C_s, Y_s and r_s denote consumption, labour income and the interest rate of period s, and I_t denotes the information set at time t. It yields the familiar Euler equation for the "tilt" of the consumption profile (e.g., Hall, 1978; Blanchard and Fischer, 1989, Chapter 6):

$$F'(C_t) = E[(\frac{1+r_t}{1+\theta})F'(C_{t+1}) \mid I_t], \tag{2.3}$$

or the marginal rate of substitution between consumption in two periods must equal the corresponding marginal rate of transformation. When $r_t = \theta$, consumption is a martingale. The problem with this approach is that the felicity function, $F(.)$, does double duty (e.g., Selden, 1978); for example, on the one hand it defines the instantaneous elasticity of intertemporal substitution, $\gamma(C_t) \equiv -F'(C_t)/F''(C_t)C_t$, and on the other hand it defines the elasticity of marginal utility or, in the presence of uncertainty, the coefficient of relative risk aversion, $1/\gamma(C_t)$. Hence, the standard approach does not distinguish between intertemporal substitution and risk aversion. For example, when the felicity function satisfies constant absolute risk aversion, $F(C_t) = -\exp(-\beta C_t)$, $r_t = \theta = 0$, and income obeys a random walk, $Y_t = Y_{t-1} + \epsilon_t, \epsilon_t \sim \mathrm{IN}(o, \sigma^2)$, then it follows that consumption satisfies $C_t = C_{t-1} + \frac{1}{2}\beta\sigma^2 + \epsilon_{t-1}$ or $C_t = (T-t)^{-1}A_t + Y_t - \frac{1}{4}\alpha(T-t-1)\sigma^2$ (see

Caballero, 1987; Kimball and Mankiw, 1989; Blanchard and Fischer, 1989, Chapter 6). This example shows that risk aversion in the face of stochastic shocks induces prudence and leads to precautionary saving. More uncertainty leads to a lower level of consumption, given wealth and income. However, this example is very specific and does not distinguish between two distinct aspects of preferences, viz., risk aversion and intertemporal substitution. In empirical work one finds very low elasticities of intertemporal substitution (e.g, Hall, 1988). This does not contradict prior beliefs about consumer behaviour, but the implied very high coefficients of relative risk aversion do seem unrealistic. Hence, it is important to cut the link between risk aversion and intertemporal substitution.

An alternative approach is based on:

$$\max_{C_t,\ldots,C_{T-1}} E[U(P_t(C_t,\ldots,C_{T-1})) \mid I_t],$$
$$U' > 0, U'' < 0, U''' > 0 \tag{2.4}$$

subject to (2.2), where the preference function $P_t(.)$ describes the attitude towards intertemporal substitution and the utility function $U(.)$, together with the preference function $P_t(.)$, describes the attitude towards risk. [1] A linear (concave) utility function corresponds to risk neutrality (aversion). Even if the preference function is time separable, the function $U(P_t(.))$ will in general not be time separable and the resulting consumption decisions will be time inconsistent. However, if there is constant absolute risk aversion, $U(P_t) = -\exp(-\beta P_t)$, one obtains the dynamic programming relationship:

$$V_s(A_s) = \max_{C_s} -U(F(C_s))E[V_{s+1}(A_{s+1})^{\frac{1}{1+\theta}} \mid I_s], \; s = T-1, ..., t \tag{2.5}$$

subject to (2.2), where the value function is defined as the expected utility to go, evaluated along the optimal path,

$$V_s(A_s) = -E[\exp\{-\beta[\sum_{s'=s}^{T-1}(1+\theta)^{-(s'-s)}F(C'_s)]\} \mid I_s], \; s = t, ..., T-1, \tag{2.6}$$

[1] For the case that $F(.)$ is quadratic, the utility function alone is sufficient to describe the attitude towards risk.

and $V_T(A_T) = -1$. This corresponds to a special case of recursive utility functions (Epstein and Zin, 1989) and therefore always yields time-consistent consumption plans. When the axiom of reduction of compound lotteries used in von Neumann-Morgenstern utility theory is abandoned to allow for non-difference about the timing of the resolution of uncertainty and an axiom is added to ensure the time consistency of optimal plans, preferences can be represented recursively by

$$V_s(A_s) = \max_{C_s} W_s(C_s, E[V_{s+1}(A_{s+1}) \mid I_s]),\ s = T-1, ..., t \tag{2.7}$$

where $W_t(.)$ denotes the aggregator function (Kreps and Porteus, 1978). Epstein and Zin (1989) analyse various classes of aggregator functions and discuss under what circumstances individuals prefer early or late resolution of uncertainty. Standard von Neumann-Morgenstern theory emerges when the aggregator function is linear in its second argument. Although the preferences based on (2.4)-(2.6) distinguish between risk aversion and intertemporal substitution, they assume indifference about the timing of the resolution of uncertainty.

In general, it is very difficult to obtain closed-form solutions for the problem (e.g., Farmer, 1989). However, when the felicity function is quadratic, interest rates are known and income follows a linear model with normally distributed error terms, linear risk-sensitive rules for consumption can be found. The assumption of a quadratic felicity function is often used, but it is nevertheless a bit awkward, for it implies a finite marginal contribution of consumption to felicity as consumption tends to zero and therefore does not necessarily rule out negative or zero consumption. As an attitude towards risk it is, however, much more problematic, because it implies increasing absolute risk aversion as consumption increases ($-F''/F' = (\alpha - C_s)^{-1}$). Hence, it is probably not too bad an approximation to have a quadratic felicity function and an utility function with constant absolute risk aversion. Section 3 derives some general results, which are then applied to the analysis of consumption and saving in Section 4.

3 The CARA-LQ problem

The state at the beginning of period t is summarised by the vector $\boldsymbol{x}_t$. It summarises all information that occurred before period t as well. The dynamic evolution of the state follows from the linear state equations:

$$\boldsymbol{x}_{s+1} = \boldsymbol{A}\boldsymbol{x}_s + \boldsymbol{B}\boldsymbol{u}_s + \boldsymbol{b}_s + \boldsymbol{\epsilon}_s, \quad \boldsymbol{\epsilon}_s \sim \text{IN}(\boldsymbol{o}, \boldsymbol{V}), \quad s = t, ..., T-1 \qquad (3.1)$$

where $\boldsymbol{u}_s$ denotes the vector of policy instruments at time s and $\boldsymbol{\epsilon}_s$ denotes the vector of normally distributed and serially uncorrelated state disturbances at time s $(\dim(\boldsymbol{\epsilon}_s) = \dim(\boldsymbol{x}_s) = n)$. For example, in the life-cycle consumption $\boldsymbol{x}_s$ includes non-human assets (A_s) and $\boldsymbol{u}_s$ includes consumption (C_s). Intertemporal preferences are described by the preference function:

$$P_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \ldots, \boldsymbol{u}_{T-1}, \boldsymbol{\epsilon}_t, \ldots \boldsymbol{\epsilon}_{T-1}) = \sum_{s=t}^{T-1} (1+\theta)^{-(s-t)} F(\boldsymbol{x}_s, \boldsymbol{u}_s)$$

$$+ (1+\theta)^{-(T-t)} F_T(\boldsymbol{x}_T) \qquad (3.2)$$

where $T, \theta, F(.)$ and $F_T(.)$ denote the horizon, the pure rate of time preference, the felicity function and the final asset value function, respectively. Clearly, it has been assumed that preferences are time separable and that the felicity function describes intertemporal preferences. The attitude towards risk is described by a separate utility function $U(P_t), U' > 0, U'' < 0$ for risk aversion, $U'' > 0$ for risk loving, and $U''' > 0$. The optimal policy instruments for period t and the planned policy instruments planned during period t for period s, say $\boldsymbol{u}_s = \boldsymbol{u}_{s/t}, s > t$, follow from:

$$\max_{\boldsymbol{u}_t, \ldots, \boldsymbol{u}_{T-1}} \boldsymbol{E}[U(P_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \ldots, \boldsymbol{u}_{T-1}, \boldsymbol{\epsilon}_t, \ldots, \boldsymbol{\epsilon}_{T-1})) \mid I_t] \qquad (3.3)$$

where the relevant information set includes $\boldsymbol{x}_t$. In the presence of shocks planned policy instruments may be revised as time proceeds. A more natural approach is based on stochastic dynamic programming. This yields the following recurrence relationship:

$$V_s(x_s) = \max_{u_s} \exp[-\beta F(x_s, u_s)] E[V_{s+1}(x_{s+1})^{\frac{1}{1+\theta}} \mid I_s], \; s = T-1, ..., t,$$
$$V_T(x_T) = -\exp[-\beta F_T(x_T)] \quad (3.4)$$

subject to (3.1), where $V_s(x_s)$ denotes the value function for period s and gives the maximum value of expected utility from period s onwards (as given by (3.3)). Two crucial assumptions were required to obtain this relationship: (i) time separability of the preference function; and (ii) a constant Arrow-Pratt coefficient of absolute risk aversion. Of course, the function $U(P(.))$ is not time separable. Nevertheless, equation (3.4) corresponds, in contrast to the approach of Selden (1978; 1979) [2], to a special case of recursive utility discussed in Kreps and Porteus (1978, 1979) and in Epstein and Zin (1989) and therefore optimal policy actions derived from (3.4) are time consistent. Since $-\frac{1}{\beta}\log(E[-U(P)]) \cong E(P) - \frac{1}{2}\beta\text{var}(P)$, constant absolute risk aversion corresponds locally to the mean-variance approach.

In general it is very difficult to obtain a closed-form solution to the above problem. However, if a quadratic preference function,

$$F(x_s, u_s) = q'x_s - \frac{1}{2}x_s'Qx_s + r'u_s - \frac{1}{2}u_s'Ru_s, \; s = t, ..., T-1,$$
$$F_T(x_T) = z_T'x_T - \frac{1}{2}x_T'Z_Tx_T, \quad (3.2')$$

where Q is a symmetric semi-positive definite matrix and R and Z_T are symmetric positive definite matrices, is used, a closed-loop solution with linear policy feedback rules can be found (Whittle, 1982).

Theorem: For the problem (3.4) subject to (3.1), (3.2) and (3.2') the value functions are given by

[2] Ordinal certainty-equivalent preferences have been developed for the two-period problem. Consumers first use a utility function, which describes their attitude towards risk, to convert future consumption into its certainty-equivalent level and then use this in a preference function over current and future consumption. The extension to multi-period problems suffers from time inconsistency (Johnsen and Donaldson, 1985).

$$V_s(x_s) = -\exp[-\frac{1}{2}\beta(x_s - \bar{x}_s)' Z_s (x_s - \bar{x}_s)],\ s = t, ..., T-1, \qquad (3.5)$$

and the optimal policy rules are linear and given by

$$u_s = K_s x_s + k_s,\ s = t, ..., T-1, \qquad (3.6)$$

where the matrices of feedback coefficients are given by

$$K_s = -(R + B'\tilde{Z}_{s+1}B)^{-1}B'\tilde{Z}_{s+1}A$$

$$= -R^{-1}B'[(1+\theta)Z_{s+1}^{-1} - \beta V + BR^{-1}B']^{-1}A,\ s = t, ..., T-1, \qquad (3.7)$$

the vectors of policy constants are given by

$$k_s = -(R + B'\tilde{Z}_{s+1}B)^{-1}[B'\tilde{Z}_{s+1}(b_s - \bar{x}_{s+1}) - r],\ s = t, ..., T-1, \qquad (3.8)$$

the positive definite and symmetric matrices Z_s obey the Riccatti recursions

$$Z_s = Q + A'[(1+\theta)Z_{s+1}^{-1} - \beta V + BR^{-1}B']^{-1}A,\ s = T-1, ..., t, \qquad (3.9)$$

the vectors $\bar{x}_s$ obey the recursions

$$\begin{aligned} Z_s\bar{x}_s &= q + A'[(1+\theta)Z_{s+1}^{-1} - \beta V + BR^{-1}B']^{-1} \\ &\quad [\bar{x}_{s+1} - b_s - BR^{-1}r],\ s = T-1, ..., t, \end{aligned} \qquad (3.10)$$

and the matrices $\tilde{Z}_s \equiv [(1+\theta)Z_s^{-1} - \beta V]^{-1}$, $s = t+1, ..., T$, must be positive definite for a well defined solution.

Proof: Assume that the functional form of the value function (3.5) is correct. When $\bar{x}_T \equiv Z_T^{-1}z_T$, this is up to a constant true for $s = T$. Substitution of (3.1), (3.2') and (3.5) into (3.4) and taking expectations yields

$$\begin{aligned} V_s(\mathbf{x}_s) &= \max_{\mathbf{u}_s} \exp[-\beta F(\boldsymbol{x}_s, \boldsymbol{u}_s)]\Omega \int V_{s+1}(G(\boldsymbol{x}_s, \boldsymbol{u}_s, \epsilon_s))^{\frac{1}{1+\theta}} \\ &\quad \exp(-\frac{1}{2}\epsilon_s' \mathbf{V}^{-1}\epsilon_s) d\epsilon_s \\ &= \max_{\mathbf{u}_s} - \int \exp[-\beta S(\mathbf{u}_s, \epsilon_s; \mathbf{x}_s, s) + constant] d\epsilon_s, \\ &\quad s = T-1, ..., t, \end{aligned} \qquad (3.11)$$

where $\Omega \equiv ((2\pi)^n \mid V \mid)^{-\frac{1}{2}}$ and $S(.)$ is a positive definite quadratic form in $\mathbf{u}_s$ and ϵ_s:

$$\begin{aligned} S(\mathbf{u}_s, \epsilon_s; \mathbf{x}_s, s) &= q'\mathbf{x}_s - \frac{1}{2}\mathbf{x}_s'\mathbf{Q}\mathbf{x}_s + \mathbf{r}'\mathbf{u}_s - \frac{1}{2}\mathbf{u}_s'\mathbf{R}\mathbf{u}_s \\ &- \frac{1}{2}(1+\theta)^{-1}(\mathbf{A}\mathbf{x}_s + \mathbf{B}\mathbf{u}_s + \mathbf{b}_s + \epsilon_s - \overline{\boldsymbol{x}}_{s+1})' \\ &\quad Z_{s+1}(\mathbf{A}\mathbf{x}_s + \mathbf{B}\mathbf{u}_s + \mathbf{b}_s + \epsilon_s - \overline{\boldsymbol{x}}_{s+1}) \\ &+ \frac{1}{2}\epsilon_s'(\beta\mathbf{V})^{-1}\epsilon_s, \ s = T-1, ..., t. \end{aligned} \qquad (3.12)$$

A standard result on the maximisation and integration of Gaussian densities says that (3.11) is equal to

$$V_s(\boldsymbol{x}_s) = -\exp[-\beta S(\mathbf{u}_s^*, \epsilon_s^*; \boldsymbol{x}_s, s)], \ s = T-1, ..., t, \qquad (3.13)$$

where $\mathbf{u}_s^*$ and ϵ_s^* are the values of $\mathbf{u}_s$ for which $S(\mathbf{u}_s, \epsilon_s; \boldsymbol{x}_s, s)$ attains its maximum and the values of ϵ_s for which $S(\mathbf{u}_s, \epsilon_s; \boldsymbol{x}_s, s)$ attains its minimum (maximum) for $\beta > 0$ $(\beta < 0)$. This yields, upon use of the Householder matrix identity, the optimal policy rules for period s, viz. (3.6)-(3.8), provided that the matrices $\tilde{Z}_{s+1}$ are non-singular. Upon substitution of ϵ_s^* and $\boldsymbol{u}_s^*$ into (3.12) and (3.13), one obtains the recursions (3.9)-(3.10). □

It is clear from the proof of the above theorem that the optimal policy instruments for period t, $\mathbf{u}_t$, and the instruments planned during period t for period s, $\mathbf{u}_{s/t}, s = t+1, ..., T-1$, can alternatively be found as the outcome of a deterministic, difference game against nature (cf., Whittle, 1982):

$$\max_{\mathbf{u}_t, ..., \mathbf{u}_{T-1}} \operatorname{ext}_{\epsilon_t ... \epsilon_{T-1}} \sum_{s=t}^{T-1} (1+\theta)^{-(s-t)} \left[F(\mathbf{x}_s, \mathbf{u}_s) + \frac{1}{2}\epsilon_s'(\beta \mathbf{V})^{-1}\epsilon_s \right]$$

$$+(1+\theta)^{(T-t)} F_T(\mathbf{x}_T) \qquad (3.14)$$

subject to (3.1) and (3.2'), where 'ext' stands for 'min' when $\beta > 0$ and for 'max' when $\beta < 0$. This sequential open-loop procedure leads to the same outcomes as the closed-loop procedure of the above theorem. A risk-averse ($\beta > 0$) decision maker is pessimistic and treats nature as a non-cooperative, belligerent player. A max-min strategy is therefore used to hedge against shocks that may work to his disadvantage. Alternatively, Z_s is replaced by $\tilde{Z}_s$ in order to increase the effective shadow penalty on uncertain state variables as the decision maker is worried that shocks will frustrate the achievement of targets. On the other hand, a risk-loving decision maker is optimistic and assumes that shocks work out to his advantage. A cooperative max-max strategy is therefore used.

When risk aversion increases, the intensity of the feedback rule (measured by the norm of K_s) increases, the norm of the closed-loop transition matrix $A + BK_s$ reduces and there is a possibility that $\tilde{Z}_s$ may become negative definite and there is an infinite expected loss of utility. Hence, such a neurotic policy can lead to a "nervous break-down". On the other hand, when risk loving tends to infinity, the decision maker becomes "complacent" and considers discretionary policy inappropriate (the norm of K_s tends to zero as $\beta \to -\infty$).

The optimal decision rules presented in the above Theorem incorparate the risk-neutral decision rules developed in the literature on optimal economic planning (e.g, Chow, 1975; Kendrick, 1981; Preston and Pagan, 1982) as special cases and thus relax multi-period certainty equivalence. Section 4 discusses an application to the life-cycle consumption problem and van der Ploeg (1984) develops the theory in continuous time and presents applications to the supply of a monopolist and the depletion of exhaustible resources. Extensions to allow for imperfect observations of the state vector and Kalman filters can be found in Bensousson and van Schuppen (1983) and Whittle (1982). Just as the linear decision rules depend on the covariance matrices (V), the Kalman filter now depends on the penalty matrices of the preference function (Q). A risk-averse decision maker increases the effective variance of the incoming observations, since he is afraid to incorporate possibly faulty information that may lead to large losses in utility. This leads to the use of biassed estimates of the state vector.

4 Precautionary saving and consumption

The method discussed in the previous section can be applied to the life-cycle consumption problem discussed in Section 2. A constant interest rate, $r_s = r$, $s = t, ..., T-1$,, a quadratic felicity function with α denoting the bliss level of consumption, a constant coefficient of absolute risk aversion, β, and an AR(1) process for shocks to income are assumed. More general stochastic processes for income are considered at the end of this section. The information set in period t includes A_t and Y_{t-1} so that current income is assumed to be unknown when current consumption is decided upon [3]. Hence, the problem for the consumer in period t is:

$$\max_{C_t,...,C_{T-1}} E\left[-\exp\{-\beta\left[\sum_{s=t}^{T-1}(1+\theta)^{-(s-t)}(\alpha C_s - \frac{1}{2}C_s^2)\right]\} \mid A_t, Y_{t-1}\right] \quad (4.1)$$

subject to the consolidated budget constraint,

$$\sum_{s=t}^{T-1}(1+r)^{-(s-t)}C_s = A_t + \sum_{s=t}^{T-1}(1+r)^{-(s-t)}Y_s, \quad (4.2)$$

and the stochastic process for income,

$$Y_s - \bar{Y}_s = \rho(Y_{s-1} - \bar{Y}_{s-1}) + \epsilon_s, \ \epsilon_s \sim \text{IN}(0, \sigma^2), \ s = t, ..., T-1 \quad (4.3)$$

where $\bar{Y}_s$, $s \geq t$, are constants. Transitory shocks to income correspond to $\rho = 0$ and permanent shocks to $\rho = 1$. It is assumed that $\rho < 1 + r$ is

[3]If current income is assumed to be known when consumption is decided upon, the qualitative results are unaffected

satisfied, so that human wealth is finite as $T \to \infty$. From (3.14) and (4.3) it is clear that current consumption, C_t, and consumption planned during period t for period s, $C_{s/t}, s = t+1, ..., T-1$, can be found from:

$$\max_{C_t,...,C_{T-1}} \text{ext}_{\epsilon_t,...,\epsilon_{T-1}} \sum_{s=t}^{T-1} \left[(1+\theta)^{-(s-t)}(\alpha C_s - \frac{1}{2}C_s^2) + \frac{1}{2}(\epsilon_s^2/\beta\sigma^2) \right] \quad (4.4)$$

subject to the expected consolidated budget constraint,

$$\sum_{s=t}^{T-1}(1+r)^{-(s-t)}C_s = A_t + H_t +$$

$$\sum_{s=t}^{T-1}(1+r)^{-(s-t)}[\rho^{(s-t-1)}(Y_{t-1} - \bar{Y}_{t-1}) + \sum_{s'=t}^{s} \rho^{s-s'}\epsilon_{s'}] \quad (4.5)$$

where the deterministic component of human wealth is defined as

$$H_t \equiv \sum_{s=t}^{T-1}(1+r)^{-(s-t)}\bar{Y}_s. \quad (4.6)$$

This yields

$$C_{s/t} = \alpha - \left(\frac{1+\theta}{1+r}\right)^{s-t} \lambda_t < \alpha, \quad s = t, ..., T-1 \quad (4.7)$$

and

$$\begin{aligned} \epsilon_{s/t} &= -\beta\sigma^2 \left[\sum_{s'=s}^{T-1} (1+r)^{-(s'-t)}\rho^{s'-s} \right] \lambda_t \\ &= -\beta\sigma^2 \left[\frac{(1+r)^{-(s-1-t)} - (1+r)^{-(T-t-1)}\rho^{T-s}}{1+r-\rho} \right] \lambda_t, \; s \geq t \quad (4.8) \end{aligned}$$

where $C_t = C_{t/t}$ and $-(\beta\lambda_t V_t) > 0$ denotes the marginal utility of non-human wealth at the beginning of period t. Equation (4.7) shows the familiar "tilt" of the planned consumption stream; if the interest rate exceeds (is below) the rate of time preference, planned consumption increases (decreases) over time and the consumer initially saves (dissaves). Also, consumption decreases as the marginal utility of non-human wealth increases. Equation (4.8) shows that a risk-averse (risk-loving) consumer is pessimistic (optimistic) in the sense that he takes into account that shocks to income may turn out to be negative (positive). When there are no stochastic shocks or when the consumer is risk neutral ($\beta\sigma^2 = 0$), the perceived shocks to future income are zero ($\epsilon_{s/t} = 0, \quad s \geq t$). In general ($\beta\sigma^2 \neq 0$), however, the present approach departs from certainty equivalence.

Upon substitution of (4.7) and (4.8) into the expected consolidated budget constraint and letting $T \to \infty$, one obtains a relatively simple expression for λ_t:

$$\lambda_t = \left[\frac{\alpha - \left(\frac{r}{1+r}\right)(A_t + H_t) - \left(\frac{\rho r}{1+r-\rho}\right)(Y_{t-1} - \bar{Y}_{t-1})}{\left(\frac{r(1+r)}{[r(2+r)-\theta]}\right) - \bar{\beta}} \right] \tag{4.9}$$

where

$$\bar{\beta} \equiv \beta\sigma^2 \left[\frac{(1+r)^2 [r(2+r) - \rho(1+r-\rho)]}{(1+r-\rho)^3(2+r)} \right]. \tag{4.10}$$

A higher degree of absolute risk aversion and a higher variance of shocks raise $\bar{\beta}$ and thus increase the marginal utility of wealth and reduce planned consumption. Hence, risk aversion leads to precautionary saving and the building up of financial assets for fear of a rainy day. Since it can be shown that $\bar{\beta}$ is larger for $\rho = 1$ than for $\rho = 0$, persistence in income shocks reinforces this rationale for precautionary saving. The sign of $\partial\bar{\beta}/\partial\rho$ corresponds to the sign of $[2(1+r)^2 + \rho^2 - 3]$, so that as ρ increases from 0 to 1 $\bar{\beta}$ first falls (when r is not too high) and then increases. Hence, for intermediate degrees of autocorrelation in income the opposite of precautionary saving occurs. Textbook theory assumes risk neutrality and leads to the certainty-equivalent level of consumption (associated with $\bar{\beta} = 0$), which is unaffected by the variance of income shocks (when the felicity function is quadratic).

In order to gain a better understanding of our bivariate model of consumption and income, it is from now on assumed that the interest rate equals the rate of time preference ($r = \theta$). This leads to a flat planned consumption profile (cf., Flavin (1981) for $\bar{\beta} = 0$):

$$C_{s/t} = \left[\frac{\left(\frac{\theta}{1+\theta}\right)\left[A_t + \sum_{s=t}^{\infty}(1+\theta)^{-(s-t)}E(Y_s \mid I_{t-1})\right] - \bar{\beta}\alpha}{1-\bar{\beta}} \right]$$

$$= \left[\frac{\left(\frac{\theta}{1+\theta}\right)(A_t + H_t) + \left(\frac{\rho\theta}{1+\theta-\rho}\right)(Y_{t-1} - \bar{Y}_{t-1}) - \bar{\beta}\alpha}{1-\bar{\beta}} \right], \ s \geq t. \qquad (4.11)$$

It follows from $C_t = C_{t/t}$ that the change in actual consumption satisfies

$$C_t - C_{t-1} = \left(\frac{\theta}{1-\bar{\beta}}\right)\left\{\sum_{i=0}^{\infty}(1+\theta)^{-i}\left[E(Y_{t+i} \mid I_t) - E(Y_{t+i} \mid I_{t-1})\right]\right\} + \left(\frac{\bar{\beta}\theta}{1-\bar{\beta}}\right)(\alpha - C_{t-1}), \qquad (4.12)$$

so that marginal felicity obeys

$$F'(C_t) = \alpha - C_t = \left[1 - \left(\frac{\theta\bar{\beta}}{1-\bar{\beta}}\right)\right]F'(C_{t-1}) - \left[\frac{\theta(1+\theta)}{(1-\bar{\beta})(1+\theta-\rho)}\right]\epsilon_t. \qquad (4.13)$$

As usual "news" does not matter, so that the change in consumption does not depend on the past history or on previously anticipated changes in income (Hall, 1978). The marginal propensity to consume out of income shocks equals $[\theta(1+\theta)/(1-\bar{\beta})(1+\theta-\rho)]$, so that the marginal propensity to consume out of unanticipated transitory shocks is $\theta/(1-\bar{\beta})$ and out of permanent shocks is $(1+\theta)/(1-\bar{\beta})$. Hence, consumption reacts as usual much stronger to permanent than to transitory innovations in labour income. It should be noted that risk aversion raises the sensitivity of changes in consumption to unanticipated changes in income. This excess sensitivity could be referred to as "making hay while the sun shines". This excess sensitivity does not occur with risk-neutral consumers and a felicity function with constant absolute risk aversion (Kimball and Mankiw, 1989), but it does occur with risk-neutral consumers and felicity functions with constant relative risk aversion (Barsky et al., 1986). Empirical research indicates that the observed sensitivity of consumption to current income is greater than is warranted by the permanent-income, life-cycle hypothesis, even when the role of current income in signaling changes in permanent income is taken into account (Flavin, 1981).

Risk aversion also introduces a degree of persistence in the stochastic process for consumption. This can be seen from solving (4.12):

$$C_s = \alpha + \left[1 - \left(\frac{\theta\bar{\beta}}{1-\bar{\beta}}\right)\right]^{s-t} (C_t - \alpha)$$

$$+ \left[\frac{\theta(1+\theta)}{(1-\bar{\beta})(1+\theta-\rho)}\right] \sum_{s'=t+1}^{s} \left[1 - \left(\frac{\theta\beta}{1-\bar{\beta}}\right)\right]^{s-s'} \epsilon_{s'}, \; s > t, \tag{4.14}$$

where C_t is less than the certainty-equivalent or risk-neutral level of consumption, C_t^{CE}. Figure 1 illustrates what happens. Given a known variable future income stream, the risk-neutral level of consumption is completely smoothed in a way that the consolidated budget constraint is satisfied.

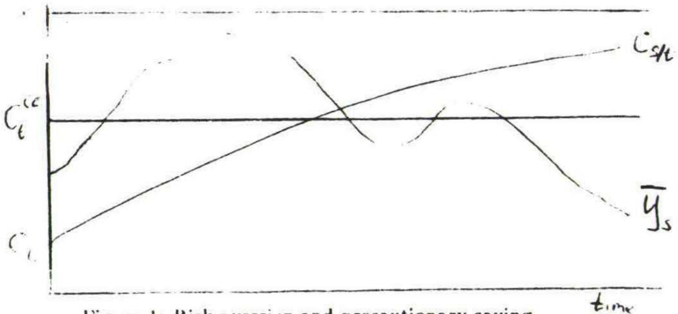

Figure 1: Risk aversion and precautionary saving

However, when there is risk aversion, consumption is initially below its certainty-equivalent level as the consumer is "saving for a rainy day". In the absence of shocks, consumption subsequently rises to its bliss level because each period the consumer is pleasantly surprised to find out that bad shocks did not materialise after all and that consequently his total (human and non-human) wealth has risen. In this sense risk aversion has a similar effect as depressing the rate of time preference or increasing the return on assets, because this would also lead to an upward-sloping consumption profile. It is useful to derive an expression for saving, S_t:

$$\begin{aligned} S_t &\equiv \left(\frac{\theta}{1+\theta}\right) A_t + Y_t - C_t = \left(\frac{\bar{\beta}}{1-\bar{\beta}}\right) \left[\alpha - \left(\frac{\theta}{1+\theta}\right) A_t - Y_t\right] \\ &+ \left(\frac{1}{1-\bar{\beta}}\right) \left[\epsilon_t - \sum_{i=1}^{\infty}(1+\theta)^{-i} \Delta \boldsymbol{E}(Y_{t+i} \mid I_{t-1})\right], \; s \geq t. \end{aligned} \tag{4.15}$$

With risk neutrality ($\bar{\beta} = 0$), saving corresponds to the present discounted value of expected future declines in labour income. This is what Campbell (1987) calls "saving for a rainy day". Since future rain fall is uncertain, this seems a misnomer given that Campbell's expression applies to a deterministic or certainty-equivalent environment. It may be better to refer to "saving for retirement", because the only thing one can be sure of in life is that one becomes older and has to retire and suffer a loss in income. It is easy to show that saving under risk aversion exceeds the certainty-equivalent level of saving and it seems more appropriate to reserve the expression "saving for a rainy day" for this type of prudent behaviour. If future labour income is fully predictable or the consumer is risk neutral, "saving for a rainy day" would not and "saving for retirement" would occur. Atkinson and Stiglitz (1980, p.73) distinguish between the **life-cycle** motive for saving, giving rise to a transfer of purchasing power from one period to another period when the time profiles of income and desired consumption do not coincide, and the **precautionary** motive, relevant when individuals want to save in order to provide insurance against future periods in which their incomes are low or their needs (e.g., uninsured medical expenses) are high. The former motive corresponds to "saving for retirment" (or for the finance of education or of the purchase of home) and the latter to "saving for a rainy day".

Finally, it is worthwhile to consider more stochastic processes for shocks to income such as the AR(p) model:

$$\psi(L)(Y_t - \bar{Y}_t) = \epsilon_t, \;\; \psi(L) \equiv 1 + \sum_{i=1}^{p} \psi_i L^i, \;\; \epsilon_t \sim \mathrm{IN}(0, \sigma^2). \tag{4.16}$$

Equation (4.3) corresponds to $p = 1$ and $\psi_1 = -\rho$. It follows that (4.8) becomes $\epsilon_{s/t} = -\beta\sigma^2\gamma_{s-t}\lambda_t, \;\; s \geq t$, where

$$\Gamma(L^{-1}) \equiv \left(\frac{(1+r)L}{[(1+r)L-1]\,\psi(L)}\right) = \sum_{i=-\infty}^{\infty} \gamma_i L^{-i}. \tag{4.17}$$

Upon substitution together with (4.7) into the expected consolidated budget constraint, one can solve for λ_t and C_t. In particular, when changes in income shocks follow an AR(1) process, as is popular in the literature (e.g., Campbell and Deaton, 1989), one has $p = 2, \psi_1 = -(1+\psi), \psi_2 = \psi, \gamma_i = \frac{(1+r)^{2-i}}{(1+r-\psi)r}, i \geq 0$, so that the first equation of (4.11), (4.12), and (4.15) hold with

$$\bar{\beta} = \beta\sigma^2 \left[\frac{1+r}{(1+r-\psi)(1-\psi)}\right]$$

$$\left[\left(\frac{1+r}{(2+r)r^2}\right) - \left(\frac{\psi^2}{[(1+r)^2-\psi]\,[(1+r)\psi-1]}\right)\right]. \tag{4.18}$$

Note that with $\psi = 0$ and $\rho = 1$, expressions (4.10) and (4.15) coincide. Table 1 shows that the extent of "saving for a rainy day" depends crucially on the characteristics of the stochastic process for income. The first point to note is that, when income shocks follow an AR(1) process with $\rho = 0.5$, $\bar{\beta} < 0$ and thus consumption is initially above its risk-neutral level and no precautionary saving occurs. In fact, the opposite of precautionary saving occurs. Since the term in the small square brackets of the right-hand side of (4.10) attains a minimum value of $\frac{3}{4}(r - 0.1547)(r + 2.1547)$ when $\rho = \frac{1}{2}(1+r)$, $\bar{\beta}$ is negative unless $r > 0.1547$. The second point to note is that permanent shocks to income ($\rho = 1, \psi = 0$) lead to much more precautionary saving than transitory shocks to income ($\rho = 0$). The final point to note is that persistence in the shocks to changes in income lead to even greater values of $\bar{\beta}$ and thus to even more precautionary saving.

Table 1:
Effects of stochastic process for income on "saving for a rainy day"

$(\beta/\beta\sigma^2)$	$\rho = 0$	$\rho = 0.5$	$\rho = 1$ *or* $\psi = 0$	$\psi = 0.5$
$r = 0.02$	0.01961	-0.80440	1313.3762	4956.103
$r = 0.04$	0.03846	-0.63436	344.6275	1230.755

5 Conclusions

The life-cycle consumption problem with known interest rates and a stochastic process for income has been considered. Intertemporal substitution and constant absolute risk aversion are distinguished in order to get a better grasp of precautionary saving. When the felicity function is quadratic and income follows a general autoregressive process, analytical decision rules for consumption and saving can be found. In general, consumption does not follow a random walk. Two motives for precautionary saving can be distinguished: (i) "saving for retirement" in view of anticipated future declines in labour income, (ii) "saving for a rainy day" when consumers are risk averse and hedge against unanticipated future declines in labour income. In addition, risk aversion increases the sensitivity of changes in consumption to unanticipated changes in income which can be referred to as "making hay while the sun shines". This is consistent with the evidence reported in Flavin (1981). However, Campbell and Deaton (1989) argue that there is excess sensitivity of consumption to anticipated changes in income and too little sensitivity of consumption to unanticipated changes in income and suggest that this is one of the reasons why consumption is so smooth. The presence of "making hay while the sun shines" therefore makes it more difficult to explain the puzzle of why consumption is so smooth. Future work will consider the insurance aspects of future taxes and its effect on precautionary saving and the break-down of Ricardian equivalence (cf. Barsky et al., 1986; Kimball and Mankiw, 1989).

The solution of the general CARA-LQ problem has also been given and can be applied to other problems, such as the problem of investment under uncertainty or the problem of inventory management, as well.

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Appendix

For completeness a brief discussion of the continuous-time problem is given. The consumer's preference function is now given by

$$P_t \equiv \int_t^\infty \exp[-\theta(s-t)]F(C(s))ds$$

and the budget constraint is given by

$$dA(s) = [r(s)A(s) + \bar{Y}(s) - C(s)]dt + \sigma dW(s), \quad s \geq t$$

where $Y(s)$ denotes the deterministic component of labour income and $W(s)$ denotes a standard Wiener process. Effectively, it has been assumed that income at time s is serially uncorrelated and normally distributed with mean $\bar{Y}_s$ and variance σ^2. Hence, non-human wealth is driven by Brownian motion with drift. The results given in van der Ploeg (1984) show that this problem (for $\beta > 0$) is equivalent to a deterministic, differential, max-min game against nature (cf., Section 4):

$$\max_C \min_\epsilon \int_t^\infty \{\exp[-\theta(s-t)]F(C(s)) + \frac{1}{2}(\epsilon(s)^2/\beta\sigma^2)\}ds$$

subject to

$$\dot{A}(s) = rA(s) + \bar{Y}(s) + \epsilon(s) - C(s), \quad s \geq t.$$

This yields the following solutions:

$$C(s,t) = \alpha - \exp[-(r-\theta)(s-t)]\lambda(t), \quad s \geq t$$

$$\epsilon(s,t) = -\beta\sigma^2 \exp[-r(s-t)]\lambda(t), \quad s \geq t$$

where $C(s,t)$ and $\epsilon(s,t)$ denote the planned or (perceived) level of consumption and income shock at time s given information at time t and $\lambda(t)$ denotes the marginal value of wealth at time t. Hence, a risk-averse consumer ensures prudence by having a negative bias in the expectation on income shocks. This bias diminishes for more distant income shocks. Substitution into the intertemporal budget constraint gives

$$\lambda(t) = \left(\frac{\alpha - r[A(t) + H(t)]}{\left(\frac{r}{2r-\theta}\right) - \frac{1}{2}\beta\sigma^2} \right)$$

where the deterministic component of human wealth is defined by

$$H(t) \equiv \int_t^{\infty} \exp[-r(s-t)]\bar{Y}(s)ds.$$

When the market rate of interest equals the pure rate of time preference $(r = \theta)$, one has

$$C(s,t) = \left(\frac{\theta[A(t) + H(t)] - \frac{1}{2}\alpha\beta\sigma^2}{1 - \frac{1}{2}\beta\sigma^2} \right)$$

$$< C^{CE}(t) = \theta[A(t) + H(t)], \quad s \geq t$$

where $C^{CE}(t)$ denotes the certainty-equivalent (or risk-neutral) level of consumption. Hence, risk aversion leads consumers to engage in precautionary saving and as a result the expected profile of consumption over the life cycle is upward-sloping and there is excess sensitivity of consumption to unanticipated changes in income.

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