An Empirical Analysis of the Hedging Effectiveness of Currency Futures

by

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Abstract

Existing research on the hedging effectiveness of currency futures assumes that futures positions are continuously adjusted. This is an unrealistic assumption in practice. In this paper we study the hedging effectiveness for futures positions which are not adjusted during the hedge period. For this purpose an out-of-sample approach is used. Three models are used to determine hedge ratios and hedging effectiveness. These are the minimum-variance model of Ederington (1979), the α-t model of Fishburn (1977), which is a model in which the disutility of a loss is minimized, and the Sharpe-ratio model of Howard and D'Antonio (1984, 1987). For the minimum-variance model and the α-t model it is found that the naively hedged positions yield a higher effectiveness than the unadjusted model-based hedged positions. For the Sharpe-ratio model it is found that both naively and model-based hedged positions lead to a lower hedging effectiveness than unhedged positions.

1. Introduction

Since their introduction in the early 1970s, currency futures have become increasingly popular. For example, Dubofsky (1992, page 314) shows that from October 1989 to September 1990 only, the Chicago Mercantile Exchange has traded more than 7 million futures contracts on German Marks and even more than 9 million futures contracts on Japanese Yen. These contracts are mainly used to hedge currency risk. A naive hedging technique suggests to take a futures position that has an equal magnitude and an opposite sign compared to the spot position. Unfortunately, the price changes of futures and currencies are not perfectly correlated. Naive hedging with currency futures changes currency risk into basis risk.[2] A naive hedge only reduces risk optimally in a situation of zero change of the basis. Therefore much research has been carried out to find the optimal proportion of futures to hedge with, i.e. the hedge ratio. In these studies expressions for the optimal hedge ratio have been derived under the assumption that futures positions are continuously adjusted. In this study we refer to these hedges as model-based or hedge ratio hedges.

In order to test to what extend hedging with this model-based hedge ratio or with the naive hedge ratio improves the utility of a hedger, the hedging effectiveness can be measured. In this study three models are applied to compare the hedging effectiveness of naive and model-based hedges. These are the minimum-variance model of Ederington (1979), the α-t model of Fishburn (1977) and the Sharpe-ratio model of Howard and D'Antonio (1984, 1987).

The purpose of this paper is twofold. The first purpose is to compare the hedging effectiveness of futures for the different *utility functions* which are assumed in determining the hedge ratio and the hedging effectiveness. In the minimum-variance model and in the α-t model hedging effectiveness is perceived as minimizing the risk of a portfolio. In the Sharpe-ratio model the trade-off between risk and return is optimized.

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[2] The change of the basis is the change of the difference between the cash price of a currency and the futures price of the same currency.

As effectiveness refers to attaining an objective and models of hedging effectiveness have various assumptions with regard to the perceived objective of the hedge, the specification of this objective is essential. The second purpose of this paper is to investigate the hedging effectiveness for *futures positions which are not adjusted* during the hedge period. In both the theoretical and the empirical literature it is assumed that futures positions are adjusted continuously. However, this assumption will be highly unrealistic in practice. Treasurers of for example industrial companies will not continuously adjust these futures positions. Therefore, in this paper it is assumed that a hedger first uses a specific period to calculate the model-based hedge ratio. After this period the amount of futures contracts as calculated by this model-based hedge ratio is used to hedge a spot position. In other words, in this paper we use an out-of-sample approach, whereas previous empirical studies have used an in-sample approach. The tests in this paper have been carried out for the three models mentioned above, using three different futures contracts, i.e. US Dollar futures on British Pounds, German Marks and Japanese Yen. These are all traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME). Daily data are used for the period from December 1976 to October 1993. The results of this study indicate that hedges are only effective for the minimum-variance model and the α-t model. Naive hedges are more effective than model-based hedges. For the Sharpe-ratio model, adding futures contracts decreases the utility of a hedger, as the risk-return trade-off is worsened. For this model, both model-based and naive hedges are less effective than the unhedged spot position.

The remainder of this paper is organized as follows. In section 2 previous theoretical and empirical research is shortly discussed. In section 3 the methodology and data description are discussed. Section 4 presents and comments on the most important results. Finally, section 5 gives a summary and some conclusions.

2. Related research

The traditional method of determining the number of futures in a hedge is simply to measure the position in the underlying asset and to take an equal but opposite position in futures contracts. This method is nowadays being referred to as the naive approach. Ederington (1979) was the first to suggest an alternative approach and to define a measure for the effectiveness of a hedge. Other approaches were suggested by Johnson and Walther (1984), who applied the α-t model of Fishburn (1977), and Howard and D'Antonio (1984, 1987). We will shortly discuss these three approaches. The hedge ratio, which is the ratio of the size of the futures position to the size of the spot position, is denoted by λ. In this study three hedge ratios are applied: $\lambda=1$ for the naive hedge, $\lambda=\lambda^*$ for the model-based hedge and $\lambda=0$ for the unhedged spot position.

2.1 The minimum-variance model

Ederington (1979) defines hedging effectiveness as the reduction in the variance of the value of a position hedged with futures. The objective of a hedge is to minimize the risk of a given position. This risk is represented by the variance of the returns. Ederington derives a closed-form solution for the optimal hedge ratio. This minimum-variance hedge ratio has the following form:

$$\lambda^* = \frac{\sigma_{SF}}{\sigma_F^2} \tag{1}$$

where: λ^* = the minimum-variance model-based hedge ratio;

σ_{SF} = the covariance of the changes in the spot and futures prices;

σ_F^2 = the variance of the change in the futures price.

It is assumed that σ_{SF} and σ_F^2 are time-invariant. The model-based hedge ratio minimizes the variance of the value of a portfolio, which includes a long (short) position in the spot currency and a short (long) position in the futures. The measure for hedging effectiveness can be defined as the percent reduction of the variance. The minimum-variance measure for the model-based hedged position is:

$$HE_{mv} = 1 - \frac{\sigma^2_{r_P}}{\sigma^2_{r_S}} \qquad (2)$$

where: HE_{mv} = the minimum-variance measure for the model-based hedged position;[3]

$\sigma^2_{r_P}$ = the variance of the portfolio returns, hedged with futures ($\lambda = \lambda^*$);

$\sigma^2_{r_S}$ = the variance of the returns of the spot position.

The minimum-variance measure for the naively hedged position is:

$$HE_{mv,\ naive} = 1 - \frac{\sigma^2_{r_{Naive}}}{\sigma^2_{r_S}} \qquad (3)$$

where: $HE_{mv,\ naive}$ = the minimum-variance measure for the naively hedged position;

$\sigma^2_{r_{Naive}}$ = the variance of the portfolio returns, naively hedged with futures (λ=1).

The minimum-variance model has been tested empirically for currency futures by Naidu and Shin (1981) and Hill and Schneeweis (1982). Both papers report substantial risk reduction in comparison with the unhedged spot position. Naidu and Shin (1981) conclude that with two out of three contracts it is possible to reduce variance with 70 to 90%. Hill and Schneeweis (1982) compare the hedging effectiveness of the minimum-variance hedge with the naively hedged position. They find that both techniques yield a high effectiveness. The largest reduction of variance is achieved with the minimum-variance hedge. The theory of Ederington and its empirical tests have two drawbacks.

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[3] Notice that the hedging effectiveness is the R^2 of a regression where the returns of the spot position are regressed against the returns of a futures position.

The first is that it is assumed that the portfolio should be adjusted continuously. The second is the assumption that hedgers have a mean-variance utility function with infinite risk-aversion.[4]

2.2 The α-t model

The minimum-variance model discussed above is based on the mean-variance portfolio theory of Markowitz (1952). In this theory risk is perceived as the dispersion of returns. Deviations below and above a specific return are assumed to be undesirable. Crum, Payne and Laughhunn (1981) state that managers perceive risk as the failure to obtain a specific aspiration level, the target return. This is relevant for the treasurer of a firm who uses futures to anticipate a loss caused by volatile currency prices or who tries to buy insurance against his risk exposure. Fishburn (1977) has formalized this perception of risk in his α-t model. Fishburn describes the expected utility of an outcome under a target return (t), weighted by a measure for risk-aversion (α). This measure for risk is defined as:

$$G_{\alpha}(t) = \int_{-\infty}^{t} (t-Y)^{\alpha} dF(Y) \tag{4}$$

where: $G_{\alpha}(t)$ = the expected utility of the loss;
t = the target return;
α = a measure for risk aversion for below target returns;
Y = the return below the target return;
$F(Y)$ = the probability distribution of Y.

An individual who has an α between 0 and 1 is risk seeking, if α is equal to 1 the individual is risk neutral and if α is larger than 1 the individual is risk averse.[5]

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[4] Alternatively it can be assumed that hedgers have a mean-variance utility function and that the expected change in the futures price is zero.

[5] The α-t model is a generalization of the mean-variance model. The models are equal for the special case that $t=\infty$ and $\alpha=2$.

Laughhunn, Payne and Crum (1980) have tested α (assuming t=0) for downside risk and found that 71% of the managers are risk seeking for below target returns. Ahmadi, Sharp and Walther (1986) derive a statistical estimate for the risk measure in the α-t model of Fishburn (1977). The α-t model model-based hedge ratio is found by minimizing risk. This problem is reflected in equation (5):

$$\lambda^* = \mathrm{Argmin}_{\lambda}\ G_{n\alpha}(t) = \frac{1}{N}\sum_{i=1}^{M} (t - Y_i)^{\alpha} \tag{5}$$

where: $G_{n\alpha}(t)$ = an estimate of the expected utility of a loss;
N = the number of returns;
M = the number of below target returns.

The returns are calculated using logarithmic notation. In the literature, so far no closed-form solution has been found for the minimalization problem. We consider it essential to calculate the measures for hedging effectiveness, using the hedge ratio where the measure for effectiveness is optimal. In this paper the model-based hedge ratios are obtained using numerical methods. Analogous to the minimum variance measure of hedging effectiveness, the measure for the α-t model can be defined as the percent reduction in risk:

$$HE_{\alpha t} = 1 - \frac{G_{n\alpha,\ \mathrm{hedged}}(t)}{G_{n\alpha,\ \mathrm{spot}}(t)} \tag{6}$$

Johnson and Walther (1984) apply the α-t model to hedging effectiveness of forwards. The hypothesis tested is that model-based hedges are more effective than naive hedges. In their study the minimum-variance model is used to obtain the hedge ratio and the α-t model is used to calculate the effectiveness of this hedge. The results suggest that a naive hedge is more effective than a minimum-variance model-based hedge. Ahmadi, Sharp and Walther (1986) perform a similar research for currency futures and options. Hedging effectiveness is defined as the ratio of the expected utility of the below target outcome of a position hedged with currency futures and a position hedged with currency options.

Futures are found to be more effective than options. Two remarks have to be made concerning these empirical studies. The first remark is that no model-based hedge ratio is calculated that optimizes utility according to the α-t model. The hedges in the empirical tests are therefore naive hedges (Ahmadi, Sharp and Walther, 1986) or minimum-variance model-based hedges (Johnson and Walther, 1984). Thereby the problem of maximizing the hedging effectiveness is avoided. Both methods imply that the measured hedging effectiveness can be improved by using a hedge ratio in accordance with the α-t model. The second remark is that, although Fishburn (1977) constructs a risk-return model, the measure for hedging effectiveness exclusively incorporates the risk element.

2.3 The Sharpe-ratio model

Howard and D'Antonio (1984) use the Sharpe-ratio to construct a risk-return measure for hedging effectiveness.[6] They define this measure as the ratio of the Sharpe-ratios for an unhedged spot position and a spot position hedged with futures. With this contribution the utility function of a hedger is extended from minimizing risk to optimizing risk and return. The authors derive a closed-form solution for the optimal hedge ratio. This solution is given in equation (7):

$$\lambda^* = \frac{(\varphi - \rho)}{\frac{P_F}{P_S} . \frac{\sigma_S}{\sigma_F} . (1 - \varphi\rho)} \;, \text{ where } \varphi = \frac{E(r_F)/\sigma_F}{(E(r_S) - r^f)/\sigma_S} \qquad (7)$$

where: P_S = the spot price of a currency;

P_F = the futures price of a currency;

r_F = the change of the futures price of a currency;

r_S = the change of the spot price of a currency;

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[6] The reward-to-variability ratio derived by Sharpe (1966) is simply the ratio of the excess return (expected return of the portfolio minus the risk-free interest rate) and the standard deviation of the portfolio.

ρ = the correlation coefficient of the changes of the futures and the spot price;

r^f = the risk-free interest rate.

The returns of the spot and the futures positions are defined as the logarithmic price changes. In a reply to a comment of Chang and Shanker (1987) the original measure is amended by Howard and D'Antonio (1987). The measure they suggest, gives the extra return that can be realized using futures in a position with the risk of the spot position without these futures. This risk-equivalent extra return is expressed per unit risk. The measure for hedging effectiveness is given in equation (8):

$$HE_{Sr} = \frac{r^f + \dfrac{\overline{r_\lambda} - r^f}{\sigma_\lambda}\sigma_S - \overline{r_S}}{\sigma_S} \tag{8}$$

where : r_λ = the return of a spot position hedged with a fraction of λ futures contracts.

Chang and Shanker (1986) applied the Sharpe-ratio model in order to test the hedging effectiveness of currency futures and of an option synthetic futures contract.[7] Chang and Shanker assume that the portfolios can be adjusted continuously. It is concluded that the synthetic instrument is less effective than the futures contract. Therefore the futures contract cannot be considered to be redundant.

3. Methodology and data description

In the empirical studies discussed in the previous section it was assumed that the model-

[7] An option synthetic futures contract can be created by buying a call option and selling a put option with the same exercise price and by lending or borrowing certain amounts. The synthetic instrument has the same characteristics as a futures contract.

based hedge ratio and the hedging effectiveness are based on data from the same period. A related assumption is that the model-based futures positions are continuously adjusted. However, for most market participants this is not a realistic assumption. Therefore in this study an out-of-sample test is performed.[8] In this test positions are assumed to be fixed for the entire hedging period. It is assumed that a hedger first uses a period to calculate the model-based hedge ratio. After this period the amount of futures contracts, as calculated by the model-based hedge ratio, is used to hedge a spot position. The returns of this model-based hedged position will be compared with a naively hedged position in the same period. In this study model-based hedge ratios are calculated for each of the three previously discussed measures of hedging effectiveness. In the literature until now model-based hedge ratios have not been calculated for the α-t model. We calculate these ratios numerically.

In this study we use a data set from the Futures Industry Institute, which contains daily spot and futures prices of the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME). The currency futures on this exchange are quoted in US dollars. For the hedges on the British Pound and the German Mark data are available from December 1976 to October 1993. For the Japanese Yen we use data from April 1977 to October 1993. Currency futures for these three currencies are available with delivery dates in March, June, September and December. The data set for the risk free interest rate contains monthly data of US Treasury Bills with a remaining maturity of one month. For each currency we create a data set using the principle of rolling the hedge forward (see Hull, 1993, page 39-40). For each contract the prices of the three months before the delivery month and the price of the first day of the delivery month are used. On the first day of the delivery month the hedger closes the futures contracts that will expire in the same month. Also a new contract is opened on the same day. In other words, from the expiration date of the previous contract until the first day of the month, the nearest-to-maturity contract is used. The data set obtained in this way is then split

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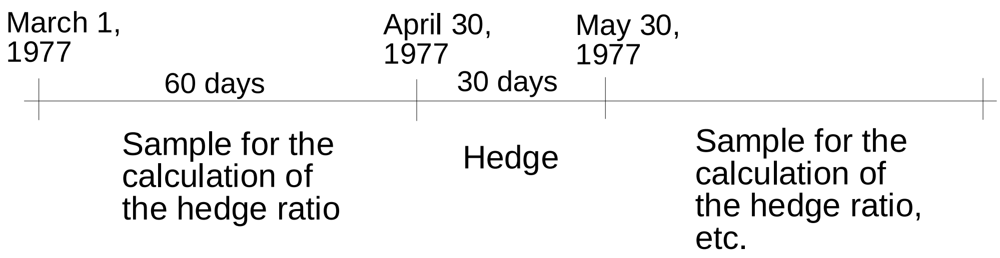
[8] In another context, out-of-sample hedging is studied by Benet (1990).

into periods of 90 days. Model-based hedge ratios are calculated using daily data for the first 60 days of a period. Then a hedge is simulated with a hedge period of 30 days. By calculating hedges in this way we avoid the overlapping samples problem. This approach is illustrated in figure 1.

Figure 1 Sample using non-adjusted positions and model-based hedge ratios



For each hedge the returns of three positions are calculated:

- spot position without a hedge (λ=0);
- spot position with a naive hedge (λ=1);
- spot position with a model-based hedge ($\lambda=\lambda^*$).

The results are dividend into three subperiods. Each subperiod contains an equal number of simulated hedges:

- period I: December 2, 1976 to July 6, 1982;
- period II: July 7, 1982 to February 12, 1988;
- period III: February 16, 1988 to October 1, 1993.

Finally the measures of hedging effectiveness are applied to calculate the effectiveness of hedging with currency futures.

4. Results

The results of this study regard three models, three currencies and three periods. For each model a naive hedge is compared with a hedge using the in-sample model-based hedge ratio. This is done in an out-of-sample setting. It is assumed that the portfolio consisting of the spot position and the futures position is not adjusted during the hedge period. Two questions are answered. Does hedging improve the utility of a hedger in this setting? Is it more effective to hedge naively or to hedge with an in-sample model-based hedge ratio?

4.1 The minimum variance model

The results of the application of the minimum-variance model are summarized in table 1. In this table the hedging effectiveness is given for each currency in the three periods.

Table 1 Minimum variance hedging effectiveness

Currency	Period	HE_{mv}	$HE_{mv,\ naive}$
British Pound	I	0.9461	0.9662
	II	0.9622	0.9887
	III	0.9637	0.9892
German Mark	I	0.9223	0.9774
	II	0.9666	0.9948
	III	0.9627	0.9796
Japanese Yen	I	n.a.	n.a.
	II	0.8816	0.9700
	III	0.9539	0.9923

HE_{mv} is the minimum-variance hedging effectiveness with model-based hedge ratios (equation (2)); $HE_{mv,naive}$ is the minimum-variance hedging effectiveness with naive hedges (equation (3)); n.a.=not available, that is for this period not enough data were available to calculate the hedging effectiveness; period I runs from December 2, 1976 to July 6, 1982; period II runs from July 7, 1982 to February 12, 1988; period III runs from February 16, 1988 to October 1, 1993.

In the column entitled HE_{mv} the effectiveness of the minimum-variance hedge with fixed positions is presented. To calculate the numbers in this column, first the model-based hedge ratio is determined using equation (1). The hedging effectiveness is calculated using equation (2). In the column entitled $HE_{mv,\ naive}$ naive hedges are used. In such a hedge the hedge ratio is 1 and the hedging effectiveness is calculated using equation (3). For example, in period I the hedging effectiveness of the British Pound futures is 94.61%, respectively 96.62%. In this example the hedger would have preferred the naive hedge, because the risk is reduced by 96.62%. Using a fixed minimum-variance hedge ratio risk is only reduced by 94.61%. In general, both methods are effective in reducing the currency risk. The results imply that for every period and every currency studied, naive hedging is more effective than hedging using the fixed model-based hedge. This conclusion differs from the empirical results of Hill and Schneeweis (1982). This is mainly due to the replacement of the assumption of continuous adjustment by the assumption of no adjustments during the hedging period. In the situation where the latter assumption holds a hedger should not bother to use the minimum-variance model. A naive hedge is more effective.

4.2 The α-t model

The α-t model has been tested for α's from 0.25 to 5.00 with intervals of 0.25. The target return is assumed to be zero. This seems to be a plausible target, because treasurers will at least try to prevent the occurrence of a loss. The hedging effectiveness is given in tables 2 to 4.

Table 2 Target return hedging effectiveness of the British Pound

	Period I		Period II		Period III	
α	$HE_{\alpha t}$	$HE_{\alpha t,\ naive}$	$HE_{\alpha t}$	$HE_{\alpha t,\ naive}$	$HE_{\alpha t}$	$HE_{\alpha t,\ naive}$
0.25	0.4205	0.2754	0.1967	0.2789	0.0625	-0.3558
0.50	0.5758	0.5696	0.4243	0.5158	0.3343	0.1284
0.75	0.6997	0.7455	0.6202	0.6734	0.6208	0.4399
1.00	0.7701	0.8496	0.7414	0.7791	0.7360	0.6405
1.25	0.8459	0.9110	0.8397	0.8502	0.8275	0.7695
1.50	0.8956	0.9474	0.8923	0.8981	0.8928	0.8523
1.75	0.9283	0.9689	0.9280	0.9304	0.9300	0.9055
2.00	0.9471	0.9816	0.9515	0.9522	0.9525	0.9395
2.50	0.9701	0.9935	0.9772	0.9770	0.9770	0.9752
3.00	0.9834	0.9977	0.9886	0.9887	0.9883	0.9899
3.50	0.9908	0.9992	0.9938	0.9943	0.9937	0.9959
4.00	0.9948	0.9997	0.9962	0.9971	0.9961	0.9983
4.50	0.9970	0.9999	0.9974	0.9985	0.9971	0.9993
5.00	0.9982	0.9999	0.9980	0.9992	0.9976	0.9997

$HE_{\alpha t}$ is the α-t model hedging effectiveness with fixed model-based hedge ratios (equation (5) and (6)); $HE_{\alpha t,\ naive}$ is the α-t model hedging effectiveness with naive hedges (equation (6)); α is a measure for risk-aversion; period I runs from December 2, 1976 to July 6, 1982; period II runs from July 7, 1982 to February 12, 1988; period III runs from February 16, 1988 to October 1, 1993.

Table 3 Target return hedging effectiveness of the German Mark

	Period I		Period II		Period III	
α	$HE_{\alpha t}$	$HE_{\alpha t,\ naive}$	$HE_{\alpha t}$	$HE_{\alpha t,\ naive}$	$HE_{\alpha t}$	$HE_{\alpha t,\ naive}$
0.25	0.3884	0.8156	0.4265	0.7865	0.3063	-0.2915
0.50	0.6329	0.9089	0.6231	0.8785	0.5278	0.1767
0.75	0.7444	0.9558	0.6868	0.9303	0.6929	0.4786
1.00	0.8092	0.9789	0.7490	0.9600	0.8223	0.6701
1.25	0.8609	0.9900	0.8137	0.9772	0.8779	0.7908
1.50	0.8738	0.9953	0.8524	0.9871	0.9103	0.8668
1.75	0.8593	0.9978	0.8745	0.9927	0.9316	0.9148
2.00	0.8330	0.9990	0.8905	0.9996	0.9453	0.9452
2.50	0.6394	0.9999	0.9134	0.9987	0.9653	0.9770
3.00	0.2006	0.9999	0.9287	0.9959	0.9790	0.9902
3.50	-0.6051	0.9999	0.9392	0.9999	0.9877	0.9957
4.00	-1.9674	0.9999	0.9460	0.9999	0.9928	0.9981
4.50	-4.1662	0.9999	0.9494	0.9999	0.9957	0.9992
5.00	-7.6141	0.9999	0.9497	0.9999	0.9972	0.9996

$HE_{\alpha t}$ is the α-t model hedging effectiveness with model-based hedge ratios (equation (5) and (6)); $HE_{\alpha t,\ naive}$ is the α-t model hedging effectiveness with naive hedges (equation (6)); α is a measure for risk-aversion; period I runs from December 2, 1976 to July 6, 1982; period II runs from July 7, 1982 to February 12, 1988; period III runs from February 16, 1988 to October 1, 1993.

Table 4 Target return hedging effectiveness of the Japanese Yen

	Period II		Period III	
α	$HE_{\alpha t}$	$HE_{\alpha t,\text{ naive}}$	$HE_{\alpha t}$	$HE_{\alpha t,\text{ naive}}$
0.25	0.3627	0.7435	0.3297	0.4268
0.50	0.6100	0.7756	0.6360	0.6815
0.75	0.6697	0.7913	0.7580	0.8278
1.00	0.5071	0.7963	0.8427	0.9088
1.25	0.4350	0.7956	0.8752	0.9523
1.50	0.2524	0.7931	0.8840	0.9753
1.75	0.0613	0.7906	0.8896	0.9873
2.00	-0.1605	0.7894	0.8962	0.9934
2.50	-0.7896	0.7915	0.9082	0.9983
3.00	-2.0220	0.7988	0.9140	0.9995
3.50	-4.9407	0.8094	0.9039	0.9999
4.00	-10.9739	0.8219	0.8775	0.9999
4.50	-22.4345	0.8351	0.8440	0.9999
5.00	-43.8336	0.8484	0.8038	0.9999

$HE_{\alpha t}$ is the α-t model hedging effectiveness with model-based hedge ratios (equation (5) and (6)); $HE_{\alpha t,\text{ naive}}$ is the α-t model hedging effectiveness with naive hedges (equation (6)); α is a measure for risk-aversion; period II runs from July 7, 1982 to February 12, 1988; period III runs from February 16, 1988 to October 1, 1993.

Equation (5) is used to obtain the hedge ratio for the model-based hedged position. Equation (5) is also used to estimate the expected utility, $G_{n\alpha}(t)$, of three positions: model-based hedged, naively hedged and unhedged positions. The tables 2 to 4 relate these three risk measures, using equation (6). $HE_{\alpha t}$ is the hedging effectiveness of the model-based hedge and $HE_{\alpha t,\text{ naive}}$ is the hedging effectiveness of the naive hedge. For example, in table 4 the hedging effectiveness for the Japanese Yen in period II for an α of

0.25 is 0.3627 for the α-t model hedge and 0.7435 for the naive hedge. This indicates that the negative utility of a loss decreased with 36.27%, respectively 74.35%. As the parameter α raises from 1 to 5, hedgers become increasingly risk averse. This aversion strengthens the influence of low effectiveness of the model-based hedges and results in negative effectiveness for high α's for the German Mark in the first period and the Japanese Yen in the second period.

The results suggest that, except for a few cases, a substantial risk reduction is possible. Especially risk-averse hedgers, with high α's, can improve the hedging effectiveness by adding futures to their spot positions. In table 5 the naive and model-based hedging effectiveness per currency per period are compared for different α's.

Table 5 Comparison of naive and target return model-based hedging effectiveness

α	0.25	0.5	0.75	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Naive	5	5	6	6	6	6	7	8	8	8	8	8
Model-based	3	3	2	2	2	2	1	0	0	0	0	0
Prob.	.363	.363	.145	.145	.145	.145	.035	.004	.004	.004	.004	.004

The row 'Naive' expresses the number of observations for a specific α, where the naive hedge results in a higher hedging effectiveness than the model-based hedge; the row 'Model-based' gives the number of hedges where the model-based hedge is preferred over the naive hedge; α is a measure for risk-aversion. The row 'Prob.' gives the probability that the minimum number of simulations where the model-based hedge is preferred, is drawn from a binomial distribution under the null hypothesis that P(naive hedge is preferred)=P(model-based hedge is preferred)=0.5.

The hedging effectiveness of naively and model-based hedged positions are compared to decide which method yields the highest effectiveness. It can be concluded that in most cases the naive hedging strategy is more effective. This seems to be especially true for risk-averse hedgers. Marginal significance levels are reported to test this statement.

Under the null hypothesis that both hedges have equal probability to have the highest effectiveness, a binomial distribution can be used to calculate the probabilities of the outcomes in table 5. For α=2.5 the probability that a maximum of one model-based hedge is preferred is in that case 3.5%. Therefore, on a 3.5% significance level or higher and with an $\alpha \geq 2.5$, naive hedges have a higher effectiveness.

4.3 The Sharpe-ratio model

In table 6 the hedging effectiveness is summarized, for the cases where the Sharpe-ratio model is applied.

Table 6 Sharpe-ratio hedging effectiveness

Currency	Period	HE_{Sr}	$HE_{Sr,\ naive}$
British Pound	I	-0.1113	-1.0012
	II	-0.1676	-1.9796
	III	-0.4157	-2.7537
German Mark	I	-0.0456	-0.8841
	II	0.0767	-1.1209
	III	-0.3843	-1.6720
Japanese Yen	I	n.a.	n.a.
	II	0.2143	-0.6548
	III	-0.0837	-1.4523

HE_{Sr} is the Sharpe-ratio hedging effectiveness with model-based hedge ratios (equation (7) and (8) with $\lambda=\lambda^*$); $HE_{Sr,\ naive}$ is the Sharpe-ratio hedging effectiveness with naive hedges (equation (8) with $\lambda=1$); n.a.=not available, that is for this period not enough data were available to calculate the hedging effectiveness; period I runs from December 2, 1976 to July 6, 1982; period II runs from July 7, 1982 to February 12, 1988; period III runs from February 16, 1988 to October 1, 1993.

The hedging effectiveness of the model-based hedged position, HE_{Sr}, is calculated using equation (8) with $\lambda=\lambda^*$. In order to find the model-based hedge ratio in this equation, equation (7) is used. Equation (8) with $\lambda=1$ expresses the hedging effectiveness of the naively hedged position. For example, the hedging effectiveness of the British Pound in the first period is negative for both the model-based hedge and the naive hedge. This can be explained by considering the underlying Sharpe-ratios. The Sharpe-ratio for the currency spot position is -0.4222. If this position is hedged, the ratio changes to -0.5336 for a model-based hedge and to -1.4233 for a naive hedge. In both cases the hedging effectiveness is negative because the Sharpe-ratio has declined. In table 6, only two cases are found where the effectiveness increases with the use of futures. Hedging effectiveness is negative in the other 14 cases. The use of futures has a negative impact on the risk-return ratio.[9]

5. Conclusions

In this study three models for the hedging effectiveness of futures are applied to measure the effectiveness of currency futures. Each measure compares a hedged position with an unhedged (spot) position. Two types of hedges can be distinguished. The first hedge is the naive hedge. The second is a hedge with the model-based hedge ratio, where the hedging effectiveness is maximized. In the literature it is assumed that portfolios can be adjusted continuously. This assumption is dropped, and instead it is assumed that hedgers maintain a fixed position during the hedging period. An out-of-sample test has been performed. The first model is the minimum-variance model of Ederington (1979), which minimizes the variance of a portfolio. The second model is the α-t model of Fishburn (1977), which minimizes the disutility of a loss. In the existing literature no equation for the model-based hedge ratio has been derived for this model. We used

[9] The hedging effectiveness for each observation is higher using a model-based hedge. As in the previous model, the binomial test claims that this outcome has 0.4% probability in case both hedging techniques have equal probability. Therefore naively hedged positions have a significantly lower effectiveness.

numerical analysis to obtain these ratios. The third model is the Sharpe-ratio model of Howard and D'Antonio (1984, 1987), which optimizes the risk-return trade-off. It is found, that when positions are not adjusted, hedging is only effective using the minimum-variance model and the α-t model. For the use of these two models the naively hedged position yields a higher effectiveness than the model-based hedged position. If the Sharpe-ratio model is applied hedging with currency futures does not improve the utility of a hedger. Both the methodology applied in this study and in the previous literature are extremes on a continuum. In this study no adjustments of futures positions are assumed, while in the previous literature continuous adjustment is assumed. In further research the actual hedging strategies of hedgers can be investigated. The actual frequency of adjustment can be obtained for example using questionnaires. It is then possible to investigate measures of hedging effectiveness based on the hedging strategies that are used in practice.

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