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## A STOCHASTIC ANALYSIS OF AN INPUT-OUTPUT MODEL: COMMENT

by Thijs ten Raa and Mark F.J. Steel

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#### Abstract

Assuming that the input coefficients matrix has normally distributed errors, West (1986) derives formulae for the approximation of the mean and the variance of the input-output multipliers. We cast doubt on these formulae by showing that the moments do not exist under the normality assumption and by exposing an inconsistency in his derivation in the multidimensional case. We remedy these shortcomings by respecification of the stochastic structure and by direct evaluation of the moments through Monte Carlo calculations. Surprisingly, West's formulae are found to be quite accurate for an aggregated version of his data set.


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## 1. Introduction

Input-output analysis builds on a square, nonnegative matrix of input coefficients, A. Each column of A lists the input requirements per unit of output of a particular sector. The main tool of impact analysis is the so called Leontief inverse, $(I-A)^{-1}$, the matrix of input-output multipliers of changes in final demand into levels of outputs. West (1986) investigates the sensitivity of the input-output multipliers with respect to the input coefficients. Assuming that the A matrix has normally distributed errors with zero mean and known variance, he derives formulae for the approximation of the mean and the variance of $(I-A)^{-1}$, as well as for its confidence intervals.
We will cast doubt on this result by showing that the moments of the Leontief inverse do not exist under West's normality assumption. We will attempt to salvage his formulae by respecifying the error distribution in a way that is more in the spirit of input-output analysis and consistent with the theoretical notions of means and variances. Surprisingly, Monte Carlo estimates confirm West's formulae as quite accurate approximations. In deterministic input-output analysis, input coefficients are such that the Leontief inverse is nonnegative as well as A itself. If the A matrix is in value terms, column sums represent total input costs per unit of output and must, for economies with value added, be less than unity. This condition is a special case of the Hawkins-Simon (1949) conditions which are necessary and sufficient for nonnegativity of the inverse. Basically, they give an explicit account of the spectral radius of $A$ being less than unity. In the hairline case of these conditions, I - A turns singular and the Leontief inverse goes off to infinity. By assuming that $A$ has a normal distribution, I - A can take all values, including singular ones, and infinite values of the Leontief inverse are not excluded. We will show that this fact destroys the existence of the means and the variances of the multipliers, depriving approximations of a foundation.
To avoid the existence problem of the moments, it is natural to confine input coefficients to the unit interval. We do so by imposing a Beta distribution on the input coefficients with the same means and variances as before. In other words, we rectify West's (1986) sensitivity analysis to
the extent that we adjust the nature of the error distribution. We maintain its location and spread, but we prevent the tail from creeping into the singular region where Leontief inverse moments do not exist. Then we calculate the means, variances and confidence intervals of the multipliers, and contrast them with West's (1986) expressions.
In principle, we could calculate the moments by adapting West's formulae to the Beta distribution. However, since we criticize them not only on the above grounds, but also for reasons of inconsistency of derivation which will be exposed below, we rather make Monte Carlo computations. Notwithstanding the theoretical flaws in West's (1986) formulae, our results will confirm them for an aggregated version of his data set. The support we lend is pragmatic. We suspect that West's formulae are not robust with respect to disaggregation and the resulting increase of variances. We were unable to check this for the full of model of West (1986), since the data are not available. Anyway, his general presentation of the formulae, is severely qualified in the present paper.

## 2. Input-output multipliers formulae

The heart of the matter described above is independent of the dimension of the model and, therefore, best addressed in the context of a single-sector economy. Input coefficients collapse into a single scalar, $\tilde{a}$, distributed normally about $a$, with some standard error, $\sigma$. The so called observed Leontief inverse is denoted $b=(1-a)^{-1}$, following West (1986, p. 365). It is not equal to the mean of the Leontief inverse defined by $\tilde{b}=(1-\tilde{a})^{-1}$, where the difference is due to the nonlinearity of Leontief inversion. This difference is the bias induced by ignoring the stochastics. The multiplier density, moments and confidence interval can be derived from the normal density of $\tilde{a}-a, \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left[\frac{-1}{2 \sigma^{2}}(\tilde{a}-a)^{2}\right\}$, and applying the transformation of the Leontief inverse, $\tilde{b}=(1-\tilde{a})^{-1}$. Since $1-\tilde{a}=\tilde{b}^{-1}$, the Jacobian is $\tilde{b}^{-2}$ and $\tilde{a}-a=-\tilde{b}^{-1}+b^{-1}$. This yields the following density of $\tilde{b}-\mathrm{b}$,

$$
\begin{equation*}
\frac{\tilde{b}^{-2}}{\sqrt{2 \pi \sigma^{2}}} \exp \left[\frac{-1}{2 \sigma^{2}}\left(\tilde{b}^{-1}-\mathrm{b}^{-1}\right)^{2}\right] \tag{2.1}
\end{equation*}
$$

This agrees with (2.1) of West (1986), by substitution of his implicitly defined $y=\tilde{b}-b$ and, in his notation (only with superscripts added to avoid confusion),

$$
\begin{align*}
& A^{*}=b^{4} \sigma^{2}  \tag{2.2}\\
& B^{*}=b^{3} \sigma^{2}  \tag{2.3}\\
& C^{*}=b^{2} \sigma^{2} . \tag{2.4}
\end{align*}
$$

West (1986) then approximates the mean and variance of $\tilde{b}-b$ by

$$
\begin{align*}
& E(\tilde{b}-b) \doteq \frac{b^{3} \sigma^{2}}{\left(1-7 b^{2} \sigma^{2}\right)^{3 / 7}}  \tag{2.5}\\
& V(\tilde{b}-b)=V(\tilde{b}) \doteq b^{4} \sigma^{2}\left(1+\frac{59}{16} b^{2} \sigma^{2}\right)^{128 / 59} \tag{2.6}
\end{align*}
$$

To calculate confidence intervals for $\tilde{b}-b$, the transformation can be seen as

$$
\tilde{b}-b=\frac{1}{1-\tilde{a}}-\frac{1}{1-a}=\frac{\tilde{a}-a}{(1-a)(1-\tilde{a})}=\frac{b^{2}(\tilde{a}-a)}{b(1-\tilde{a})}=\frac{b^{2}(\tilde{a}-a)}{1-b(\tilde{a}-a)}
$$

where the last equality rests on the fact that $b=1+b a$, which can be derived from the definition, $b=(1-a)^{-1} . \tilde{b}-b$ is a monotonic transformation of $\tilde{a}-a$ provided that the denominator remains positive. The latter condition is fulfilled if $\tilde{a}<1$, using $b>0$ (since $a<1$ ) and the next to last expression of the above string of equalities. Consequently, any (1- $\alpha$ ) confidence interval for the input coefficient, $\left[a-z_{\alpha / 2} \sigma, a+z_{\alpha / 2} \sigma\right]$, situated to the left of unity $\left(a+z_{\alpha / 2} \sigma<1\right)$ is transformed to

$$
\begin{equation*}
\left[b-\frac{b^{2} z_{\alpha / 2}{ }^{\sigma}}{1+b z_{\alpha / 2}{ }^{\sigma}}, b+\frac{b^{2} z_{\alpha / 2}{ }^{\sigma}}{1-b z_{\alpha / 2}}\right] \tag{2.7}
\end{equation*}
$$

This agrees with (2.7) of West (1986), by substitution of (2.2) and (2.3). The provision $a+z_{\alpha / 2} \sigma<1$ must hold for column totals in the multisector case, where West's formulae involve sums over the sectors.

## 3. Comment

The density of the Leontief inverse, as deviation from the observed one, (2.1), behaves like $\tilde{\mathrm{b}}^{-2}$ for $\tilde{\mathrm{b}} \rightarrow \pm \infty$. This is perfectly integrable, as it should be for any proper density function. It admits no moments, however. The first moment has a density which behaves like $\tilde{b}^{-2}=\tilde{b}^{-1}$ and the second moment has a density which behaves like $\tilde{b}^{2} \tilde{b}^{-2}=1$, both for $\tilde{b} \rightarrow \pm \infty$. Neither expression is summable. Consequently, the moments diverge. The mean and the variance of the Leontief inverse do not exist under the assumption of normality.
A general result regarding the existence of such, so called negative moments is given in Lehmann and Shaffer (1988), who prove that if the underlying probability density function (pdf) fulfills $0<p(0+)<\infty$, the pdf of the inverse will have Cauchy tails, and will thus allow no moments for the inverse. A necessary, though not sufficient, condition for negative moments is that $p\left(0^{+}\right)=0$. They also show that the pdf of the inverse will of ten be (at least) bimodal if negative values are not excluded. The normal distribution, in particular, suffers from nonexistence of negative moments as well as bimodality of the pdf for the inverse.
If we restrict ourselves to continuous pdf's, say $p(x)$, the positive real line, Piegorsch and Casella (1985) find that if $\lim p(x) / x^{\alpha}<\infty$ for some $\alpha>0$, then the inverse $x^{-1}$ admits a finite mean, showing also that $\mathrm{p}\left(0^{+}\right)=0$ in itself is not sufficient.
Basically, the Leontief inverse goes off to infinity (plus of minus) when the input coefficient becomes unity. This value is excluded in deterministic input-output analysis, for example by Hawkins and Simon (1949), but not by West (1986). The normal distribution attaches positive mass to any neighborhood of unity. Any distribution with this property prevents existence of the moments of the Leontief inverse. West's (1986) mean and variance formulae, (2.5) and (2.6), approximate moments that do not exist. At best, they are approximations to infinity.

The density formula itself, (2.1), is correct for single-sector economies. "Under the assumptions noted previously," a multi-dimensional version holds according to West (1986, p. 373). These assumptions are

$$
\begin{equation*}
\delta\left(a_{q s}\right) \delta\left(a_{r m}\right)=\delta\left(a_{q m}\right) \delta\left(a_{r s}\right) \text { for all } q, s, r \text {, } m \text {, where } \delta\left(a_{i j}\right) \text { is } \tag{I}
\end{equation*}
$$ the $(i, j)$ th element of $\tilde{A}-A, \tilde{A}$ being the random matrix of input coefficients distributed about $A$.

(II) $\quad \delta\left(a_{i j}\right) \sim N\left(0, \sigma_{i j}^{2}\right)$

Pieter Kop Jansen of Tilburg University observed that by the first assumption, the rank of the matrix $\widetilde{A}-A$ is at most one. Typically, the error structure must be proportional for all sectors of the economy. This rules out independence of errors. Therefore, assumption $I$ is inconsistent with the independence implicit in assumption II. Thus, West's claim that the density formula holds for generic input coefficients and standard errors is based on inconsistent assumptions.
In addition, the assumption $A^{*} C^{*}=B^{* 2}$, given by (2.2-4) for the dimensional case, on which West's approximation of confidence intervals is founded, is generally no longer valid with more than one sector.
In short, West's (1986) formulae do not hold for multi-sector economies and his stochastic assumptions admit no mean or variance, not even for single-sector economies. Note that, in the absence of a mean, West's (1986, p. 365) claim that observed multiplier values, though biased, should be consistent, is not meaningful.

## 4. Alternative stochastics

In this section we examine an alternative stochastic structure for the input coefficients, that does not preclude the existence of moments for the multipliers. This is achieved by restricting the support of the pdf on $\tilde{a}$ to the unit interval, on which we specify a Beta density. Thus, in the single-sector case, $\tilde{a} \sim \beta(p, q)$ with pdf

$$
B(p, q)^{-1} \tilde{a}^{p-1}(1-\tilde{a})^{q-1}, 0 \leq \tilde{a} \leq 1 \text { and } p, q>1
$$

The last inequality is necessary and sufficient for unimodality; see, e.g., Johnson and Kotz (1970, p. 41). Note that $q>1$ implies that $\lim p(1-\tilde{a}) /(1-\tilde{a})^{\alpha}=0$, so that the first negative moment should then $\tilde{a} \uparrow 1$ exist, according to Piegorsch and Casella (1985). $\tilde{\mathrm{b}}=(1-\tilde{\mathrm{a}})^{-1}$ has the following density,

$$
\begin{equation*}
B(p, q)^{-1}\left(\tilde{b}^{-1}\right)^{q-1}\left(1-\tilde{b}^{-1}\right)^{p-1} \tilde{b}^{-2}=B(p, q)^{-1} \tilde{b}^{-q-1}\left(1-\tilde{b}^{-1}\right)^{p-1} \tag{4.1}
\end{equation*}
$$

from which its $r$-th moment has a density which behaves like $\left(1-\tilde{b}^{-1}\right)^{p-1}$ for $\tilde{b} \rightarrow 1$ and $\tilde{b}^{-q-1+r}$ for $\tilde{b} \rightarrow \infty$. It is integrable if $-q-1+r<-1$. (The part near $\tilde{b}=1$ is automatically integrable by $p \geq 1$.) Since we want at least two moments, we need $q>2$.
Parameters $p$ and $q$ are determined by the observed mean and standard error of the input coefficient. The moments of $\tilde{a}$ are easily evaluated and equated with a and $\sigma^{2}: \frac{p}{p+q}=a$ and $\frac{p q}{(p+q)^{2}(p+q+1)}=\sigma^{2}$.
This is obtained by putting

$$
\begin{equation*}
p=a\left(\frac{a-a^{2}}{\sigma^{2}}-1\right) \text { and } q=(1-a)\left(\frac{a-a^{2}}{\sigma^{2}}-1\right) \tag{4.2}
\end{equation*}
$$

They are bigger than 1 and 2, respectively, if and only if $\sigma^{2}$ is small enough, more precisely, if and only if $\sigma^{2} \leq a \frac{a-a^{2}}{1+a}$ and $\sigma^{2}<a \frac{(1-a)^{2}}{3-a}$, respectively. It is easy to see that the first constraint dominates the second on $0 \leq a \leq 1 / 3$ and that, there, a sufficient condition is $\sigma / a \leq \sqrt{1 / 2}$, which is tight at $a=\frac{1}{3}$. On $1 / 3 \leq a \leq 1 / 2$ the second constraint dominates and a sufficient condition is $\sigma / a<\sqrt{1 / 5}$, which is tight at $a=\frac{1}{2}$. The region $\frac{1}{2}<a<1$ is ignored, since it contains no data in the present example.
Although West's formulae, (2.5) and (2.6), are theoretically flawed, they generate results which are close to our Monte Carlo estimates, as we shall see shortly. Let us also compare them to the theoretical results in our present setting. Since $\tilde{\mathrm{b}}$ has its density given by (4.1), it is straightforward to derive the mean,

$$
E(\tilde{b})=B(p, q)^{-1} B(p, q-1)=\frac{p+q-1}{q-1}
$$

Substituting $(4.2)$ and $b=(1-a)^{-1}$ and subtracting $b$, we obtain

$$
\begin{equation*}
E(\tilde{b}-b)=\frac{b^{3} \sigma^{2}}{1-\frac{b+1}{b-1} b^{2} \sigma^{2}} \tag{4.3}
\end{equation*}
$$

Similarly, we derive

$$
\begin{align*}
& E\left(\tilde{b}^{2}\right)=B(p, q)^{-1} B(p, q-2)=\frac{(p+q-1)(p+q-2)}{(q-1)} \text { and } \\
& V(\tilde{b}-2)  \tag{4.4}\\
& \left(1-\frac{b+1}{b-1} b^{2} \sigma^{2}\right]^{2}\left[1-\frac{2 b+1}{b-1} b^{2} \sigma^{2}\right]
\end{align*}
$$

West's approximation formulae, (2.5) and (2.6), and our exact formulae, (4.3) and (4.4), are close for $b^{2} \sigma^{2}$ small. Then $E(\tilde{b}-b)$ tends to $b^{3} \sigma^{2}$, both in (2.5) and in (4.3), and $V(\tilde{b}-b)$ tends to $b^{4} \sigma^{2}$, both in (2.6) and in (4.4). Note that the stochastic assumptions underlying both sets of formulae are different, however.

## 5. A Monte Carlo experiment

Although the previous section contains explicit expressions for the moments of the multiplier in a single-sector economy using a Beta distribution, the extension to $n>1$ sectors defies exact analytical treatment. To avoid the kind of inconsistencies that West (1986) had to use (see our Section 3), we perform a Monte Carlo experiment, where the input coefficients are all drawn from independent Beta distributions with $p$ and $q$ chosen as in (4.2) for each element of $A$.
West (1986) uses his formulae to examine multipliers, more precisely, their means, standard errors and confidence intervals. To define the multipliers, let the last sector be households and consider any of the other sectors, $k$. The disaggregated multipliers of this sector are listed in the $k$-th column of the Leontief inverse. The simple sum of all these disaggregated multipliers but the last one defines the output multiplier of sector k. If the terms are weighted by the households income row coefficients (the bottom row of the full A matrix), one obtains the so called income
multiplier of sector $k$. If the weights are employment coefficients, one obtains the employment multiplier of sector $k$.

Since West (1986) contains no data, we turn to West (1982), where we find aggregated input-output data, reproduced in tables 1 and 2. Since employment figures are missing, we confine ourselves to output and income multipliers. The output multipliers are the column totals of the Leontief inverse of the full system, excepting the bottom entries. It can be shown that these bottom entries match the income multipliers, if the household sector has zero income.
We check West's formulae (with summation over all $n+1=6$ sectors) by Monte Carlo estimates of the means, standard errors and confidence intervals of the output and income multipliers under our specification of the error distribution, chosen to obtain theoretical consistency. In particular, we assume independent Beta distributions on the unit interval for the input coefficients using West's (1982) means and standard deviations. Using the original data from Tables 1 and 2, we observed that West's formulae and our Monte Carlo estimates (using 10,000 drawings) yielded very similar means, standard errors and confidence intervals. We suspected that this coincidence reflects the lack of stochastics in West (1982). Tables 1 and 2 show that the standard deviations were taken very small relative to the input coefficients. West (1982, pp. 14 and 11), however, states that "Obtaining reliable estimates of (their) standard errors, for example, is a major problem on its own, and one which obviously cannot be addressed in this paper" and "the input coefficient standard error matrix should be considered only as illustrative in this context." For reasons like these, we have quintupled the standard errors in table 2 . The consequent results are given in table 3. Relative to our Monte Carlo results, West's (1986) formulae are accurate for the means of the multipliers to the third decimal and underestimate the standard errors and confidence interval borders to the second decimal. In other words, even though West's formulae are proxies to moments which do not exist and the derivation is inconsistent in the multi-dimensional case, they perform surprisingly well in our context which admits theoretical values of the moments and uses direct Monte Carlo calculations.
6. Conclusion

From a purely conceptual point of view, we argue against the use of approximations to moments that do not exist under the assumptions made, at least when a viable alternative is available. The fact that normality of input coefficients does not admit finite moments for the elements of the Leontief inverse is felt to be very problematic if we are interested in evaluating such moments. This situation is remedied by making an alternative stochastic assumption, using the Beta distribution defined on the unit interval, which does not prevent existence of these negative moments, given certain restrictions on its parameters. In addition, the theoretical inconsistencies that West (1986) has to introduce in order to extend the analysis to a multi-sector case, lead us to adopt a Monte Carlo approach, which allows us to remain in a theoretically fully consistent framework, even to the extent of imposing restrictions like e.g. the Hawkins-Simon (1949) conditions.

Despite all these theoretical problems, the actual numerical values of the formulae suggested by West seem very good approximations in the particular, highly aggregated, case we have examined. On the basis of these results, we do not argue against the use of his formulae in aggregated input-output models, but we would suggest changing the underlying assumptions.

Table 1. Direct Input Coefficients Matrix, $a_{i j}$

| Sector | 1 | 2 | 3 | 4 | 5 | $H-H$ |
| :---: | ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | 0.0885 | 0.0172 | 0.1545 | 0.0000 | 0.0007 | 0.0366 |
| 2 | 0.0001 | 0.0225 | 0.0197 | 0.0014 | 0.0059 | 0.0000 |
| 3 | 0.1283 | 0.1421 | 0.1927 | 0.0468 | 0.1056 | 0.2723 |
| 4 | 0.0440 | 0.0731 | 0.0584 | 0.0372 | 0.0393 | 0.1600 |
| 5 | 0.0425 | 0.1148 | 0.0661 | 0.1395 | 0.0775 | 0.3525 |
| H-H | 0.0913 | 0.1300 | 0.2344 | 0.3833 | 0.4239 | 0.0000 |

Source: West (1982).

Table 2. Input Coefficient Standard Error Matrix, $\sigma_{i f}$

| Sector | 1 | 2 | 3 | 4 | 5 | $\mathrm{H}-\mathrm{H}$ |
| :---: | ---: | ---: | ---: | :---: | :---: | :---: |
| 1 | 0.0048 | 0.0030 | 0.0055 | 0.0000 | 0.0001 | 0.0009 |
| 2 | 0.0000 | 0.0036 | 0.0004 | 0.0001 | 0.0002 | 0.0000 |
| 3 | 0.0066 | 0.0073 | 0.0079 | 0.0045 | 0.0061 | 0.0057 |
| 4 | 0.0042 | 0.0064 | 0.0030 | 0.0048 | 0.0041 | 0.0059 |
| 5 | 0.0058 | 0.0067 | 0.0046 | 0.0049 | 0.0021 | 0.0026 |
| H-H | 0.0048 | 0.0031 | 0.0056 | 0.0070 | 0.0055 | 0.0000 |

Source: West (1982).

Table 3. Multiplier Values, Moments and Confidence Intervals

| Multiplier and Sector | Observed Value | West's Estimate (Standard Error) | Our Estimate (Standard Error) | West's 95\% <br> Confidence Interval | Our 95\% <br> Confidence Interval |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Output | 1.8870 | $\begin{gathered} 1.8955 \\ (0.1748) \end{gathered}$ | $\begin{gathered} 1.8953 \\ (0.1786) \end{gathered}$ | (1.5746, 2.2652) | (1.5934, 2.2964) |
|  | 2.1555 | $\begin{gathered} 2.1641 \\ (0.1871) \end{gathered}$ | $\begin{gathered} 2.1639 \\ (0.1902) \end{gathered}$ | (1.8196, 2.5578) | (1.8323, 2.5755) |
|  | 2.5969 | $\begin{gathered} 2.6165 \\ (0.2514) \end{gathered}$ | $\begin{gathered} 2.6145 \\ (0.2556) \end{gathered}$ | $(2.1726,3.1744)$ | (2.1930, 3.1763) |
|  | 2.3124 | $\begin{gathered} 2.3249 \\ (0.1934) \end{gathered}$ | $\begin{gathered} 2.3234 \\ (0.1970) \end{gathered}$ | (1.9766, 2.7455) | (1.9924, 2.7664) |
|  | 2.4125 | $\begin{gathered} 2.4259 \\ (0.2068) \end{gathered}$ | $\begin{gathered} 2.4247 \\ (0.2109) \end{gathered}$ | (2.0536, 2.8759) | (2.0663, 2.8943) |
| Income | 0.3386 | $\begin{gathered} 0.3415 \\ (0.0699) \end{gathered}$ | $\begin{gathered} 0.3415 \\ (0.0717) \end{gathered}$ | (0.2117, 0.4873) | $(0.2185,0.5004)$ |
|  | 0.4840 | $\begin{gathered} 0.4870 \\ (0.0676) \end{gathered}$ | $\begin{gathered} 0.4866 \\ (0.0692) \end{gathered}$ | (0.3622, 0.6290) | (0.3665, 0.6390) |
|  | 0.6799 | $\begin{gathered} 0.6867 \\ (0.1017) \end{gathered}$ | $\begin{gathered} 0.6856 \\ (0.1036) \end{gathered}$ | (0.5043, 0.9081) | $(0.5069,0.9094)$ |
|  | 0.8089 | $\begin{gathered} 0.8138 \\ (0.0933) \end{gathered}$ | $\begin{gathered} 0.8130 \\ (0.0944) \end{gathered}$ | $(0.6432,1.0125)$ | $(0.6473,1.0163)$ |
|  | 0.8656 | $\begin{gathered} 0.8708 \\ (0.0915) \end{gathered}$ | $\begin{gathered} 0.8700 \\ (0.0928) \end{gathered}$ | (0.7046, 1.0672) | (0.7053, 1.0693) |

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