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INFORMATION MATRIX TEST, PARAMETER HETEROGENEITY AND ARCH: A SYNTHESIS

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Information Matrix Test, Parameter Heterogeneity and ARCH: A Synthesis*

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Abstract: We apply the White information matrix (IM) test to the linear regression model with autocorrelated errors. A special case of one component of the test is found to be identical to the Engle Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH). Given Chesher's interpretation of the IM test as a test for parameter heterogeneity, this establishes a connection among the IM test, ARCH and parameter variation. This also enables us to specify conditional heteroskedasticity in a more general and convenient way. Other interesting byproducts of our analysis are tests for the variation in conditional and static skewness which we call tests for "heterocliticity."

JEL Classification No. 210/Key Words: Autoregressive Conditional Heteroskedasticity; Information Matrix Test; Lagrange Multiplier Test; Parameter Heterogeneity.

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1. INTRODUCTION

In a pioneering article, White (1982) suggested the information matrix (IM) test as a general test for model specification. In recent years, this test has received a lot of attention. In particular, Chesher (1984) demonstrated that this test can be viewed as a Lagrange multiplier (LM) test for specification error against the alternative of parameter heterogeneity. As a byproduct of this analysis, Chesher (1983) and Lancaster (1984) provided a "nR^2" version of the IM test. An application of the IM test to the linear regression model by Hall (1987) led to a very interesting result that the test decomposed asymptotically into three components, one testing heteroskedasticity and the other two testing some forms of normality. Engle (1982), in an apparently unrelated influential paper, introduced the autoregressive conditional heteroskedasticity (ARCH) model which characterizes explicitly the conditional variance of the regression disturbances. He also suggested an LM test for ARCH. The purpose of this paper is to establish a connection among the IM test, parameter heterogeneity and ARCH. And as far as the IM test is concerned, we examine only the algebraic structure of the test.

An important finding by Hall (1987) was that the components of the IM test are insensitive to serial correlation. Hall also commented "had our original specification included first order autoregressive errors, then the IM test does not decompose asymptotically into the sum of our original three component test . . . plus the LM test against first-order serial correlation. In this more general framework the indicator vector no longer has a block diagonal covariance matrix due to the inclusion of the autoregressive coefficient in the parameter vector." (p. 262). In the next section, we start with a linear regression model with autoregressive (AR) errors and apply the IM test to it. The indicator vector is found to have a block diagonal covariance matrix. And as the null model now has more parameters, naturally we get a few extra components in the IM test. From the additional components of the statistic, we can also obtain the Engle's LM test for ARCH as a special case. The implication of this result is discussed in detail in section 3. Given Chesher's interpretation of the IM test as a test for parameter heterogeneity or random coefficient, it is now easy to give a random coefficient interpretation to ARCH. This fact has been noted recently by several authors [see, e.g., Tsay (1987)]. This provides us with a convenient framework to extend ARCH so that interaction factor between past residuals could also be considered and as a consequence we suggest an augmented ARCH (AARCH) model. The last section of the paper contains some concluding remarks.

2. THE IM TEST FOR THE LINEAR REGRESSION MODEL WITH AR ERRORS

We consider the linear regression model

$$y_t = x_t'\beta + \varepsilon_t \tag{1}$$

where y_t is the t-th observation on the dependent variable, x_t is a k×1 vector of <u>fixed</u> regressors and the ε_t are assumed to follow a stationary AR(p) process

$$\varepsilon_t = \sum_{j=1}^{p} \varphi_j \varepsilon_{t-j} + u_t \tag{2}$$

with $u_t \sim NIID(0,\sigma_u^2)$. We will write this AR(p) process as $\varepsilon_t = \underline{\varepsilon}_t'\phi + u_t$ where $\underline{\varepsilon}_t = (\varepsilon_{t-1},\ldots,\varepsilon_{t-p})'$ and $\varphi = (\varphi_1,\ldots,\varphi_p)'$. Assuming that $\underline{\varepsilon}_1$ is given, the log-likelihood function for this model can be written as

$$L(\theta) = \sum_{t=1}^{n} \ell_t(\theta) = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log \sigma_u^2 - \frac{1}{2\sigma_u^2}\sum_{t=1}^{n}(\varepsilon_t - \underline{\varepsilon}_t'\phi)^2$$

where $\theta = (\beta',\phi',\sigma_u^2)'$ is a $q\times 1$ vector of parameters with $q = k+p+1$. Note that $(\varepsilon_t - \underline{\varepsilon}_t'\phi)$ involves β since $\varepsilon_t - \underline{\varepsilon}_t'\phi = (y_t - \underline{y}_t'\phi) - (x_t - \underline{x}_t'\phi)'\beta$, where $\underline{y}_t = (y_{t-1},\ldots,y_{t-p})'$ and $\underline{x}_t = (x_{t-1},\ldots,x_{t-p})'$.

Let $\hat{\theta}$ denote the maximum likelihood estimate (MLE) of θ. Then White's IM test is constructed based on

$$d(\hat{\theta}) = \text{vech } C(\hat{\theta}) = \frac{1}{n}\sum_{t=1}^{n} d_t(\hat{\theta}) \qquad \text{(say)}$$

where

$$C(\hat{\theta}) = \frac{1}{n}\sum_{t=1}^{n}\left[\frac{\partial^2 \ell_t(\hat{\theta})}{\partial\theta\partial\theta'} + \left(\frac{\partial \ell_t(\hat{\theta})}{\partial\theta}\right)\left(\frac{\partial \ell_t(\hat{\theta})}{\partial\theta}\right)'\right] = A(\hat{\theta}) + B(\hat{\theta}) \qquad \text{(say)}$$

Note that $-A(\hat{\theta})^{-1}$ and $B(\hat{\theta})^{-1}$ are the two different estimators for the asymptotic variance of $\sqrt{n}\,\hat{\theta}$ using the Hessian matrix and the outer product form, respectively. Therefore, the IM test principles can also be viewed as a test based on the difference of two estimators.

A consistent estimator of the variance matrix of $\sqrt{n}d(\hat{\theta})$ is [see White (1982, p. 11)]

$$\hat{V}(\hat{\theta}) = \frac{1}{n} \sum_{t=1}^{n} a_t(\hat{\theta})a_t'(\hat{\theta}) \tag{3}$$

where $a_t(\hat{\theta}) = d_t(\hat{\theta}) - \nabla d(\hat{\theta})A(\hat{\theta})^{-1}\nabla \ell_t(\hat{\theta})$ with $\nabla d(\hat{\theta}) = \frac{1}{n} \sum_{t=1}^{n} \frac{\partial d_t(\hat{\theta})}{\partial \theta}$ and $\nabla \ell_t(\hat{\theta}) = \frac{\partial \ell_t(\hat{\theta})}{\partial \theta}$. Then the White IM test takes the form of

$$T_W = nd'(\hat{\theta})\hat{V}(\hat{\theta})^{-1}d(\hat{\theta}) \tag{4}$$

When the model (1) is correct, T_W follows an asymptotic χ^2 distribution with $\frac{q(q+1)}{2}$ degrees of freedom. If there is an intercept term in the regression model (1), the χ^2 degrees of freedom should be reduced by one. Similar adjustments are necessary if the regressors contain some polynomial terms and a constant, or if some of the exogenous variables are binary [see White (1980, p. 825)]. It should also be noted that White (1982) derived the IM test for IID observations. However, as shown in White (1987), the IM equality holds under fairly general conditions. For our autoregressive case, mixing conditions stated in White (1987) are satisfied, and therefore the IM test remains valid.

After some algebra and rearranging the terms in $d(\hat{\theta})$, we can write (for algebraic derivations, see Appendices A and B), suppressing θ such that writing $\hat{d}$ for $d(\hat{\theta})$,

$$\hat{d} = (\hat{d}_1', \hat{d}_2', \hat{d}_3, \hat{d}_4', \hat{d}_5', \hat{d}_6')' \tag{5}$$

where

$$\hat{d}_1:\quad [\frac{1}{n\hat{\sigma}_u^4}\sum_{t=1}^{n}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_{ti}-\underline{x}_{ti}'\hat{\phi})(x_{tj}-\underline{x}_{tj}'\hat{\phi})]_{i,j=1,2,\ldots,k;\quad i\leq j}$$

$$\hat{d}_2:\quad [\frac{1}{n\hat{\sigma}_u^4}\sum_{t=1}^{n}(\hat{u}_t^2-\hat{\sigma}_u^2)\hat{\varepsilon}_{t-i}\hat{\varepsilon}_{t-j}]_{i,j=1,2,\ldots,p;\quad i\leq j}$$

$$\hat{d}_3:\quad [\frac{1}{4n\hat{\sigma}_u^8}\sum_{t=1}^{n}(\hat{u}_t^4-3\hat{\sigma}_u^4)]$$

$$\hat{d}_4:\quad [\frac{1}{n\hat{\sigma}_u^4}\sum_{t=1}^{n}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_{ti}-\underline{x}_{ti}'\hat{\phi})\hat{\varepsilon}_{t-j}-\frac{1}{n\hat{\sigma}_u^2}\sum_{t=1}^{n}\hat{u}_t x_{t-ji}]$$

$$i=1,2,\ldots,k;\quad j=1,2,\ldots,p$$

$$\hat{d}_5:\quad [\frac{1}{2n\hat{\sigma}_u^6}\sum_{t=1}^{n}\hat{u}_t^3(x_{ti}-\underline{x}_{ti}'\hat{\phi})]_{i=1,2,\ldots,k}$$

$$\hat{d}_6:\quad [\frac{1}{2n\hat{\sigma}_u^6}\sum_{t=1}^{n}\hat{u}_t^3\hat{\varepsilon}_{t-i}]_{i=1,2,\ldots,p}$$

Our expressions for $\hat{d}_1$, $\hat{d}_3$ and $\hat{d}_5$ are identical to those of Δ_1, Δ_3 and Δ_2 of Hall (1987, pp. 259-260) if we put $\hat{\phi} = 0$. If it is desirable to test only in a certain direction, we can premultiply $\hat{d}$ by a selection matrix whose elements are either zero or unity [see White (1982, pp. 9-10) and Hall (1987, p. 258)].

Now to obtain the IM test statistic, all we need is to derive the variance matrix of $\hat{d}$. We find that the variance matrix is block diagonal (for detailed derivation, see Appendix B). Denote the estimator of the variance of $\sqrt{n}\hat{d}_i$ as $\hat{V}(\hat{d}_i) \equiv \hat{V}_i$, $i = 1,2,\ldots,6$. To express $\hat{V}_i$'s succinctly, we define the vectors whose typical elements are described as

$$\underline{\chi}_t: \; [(x_{ti} - \underline{x}_{ti}'\hat{\phi})(x_{tj} - \underline{x}_{tj}'\hat{\phi}) - \frac{1}{n}\sum_{t=1}^{n} (x_{ti} - \underline{x}_{ti}\hat{\phi})(x_{tj} - \underline{x}_{tj}\hat{\phi})]_{i,j=1,2,\ldots,k;\;\; i\leq j}$$

$$\underline{\xi}_t: \; [\hat{\varepsilon}_{t-i}\hat{\varepsilon}_{t-j} - \frac{1}{n}\sum_{t=1}^{n} \hat{\varepsilon}_{t-i}\hat{\varepsilon}_{t-j}]_{i,j=1,2,\ldots,p;\;\; i\leq j}$$

$$\underline{s}_t: \; [(x_{ti} - \underline{x}_{ti}'\hat{\phi})\hat{\varepsilon}_{t-j}]_{i=1,2,\ldots,k;\;\; j=1,2,\ldots,p}$$

$$\underline{z}_t: \; [x_{t-ji}]_{i=1,2,\ldots,k;\;\; j=1,2,\ldots,p}$$

$$\underline{r}_t: \; [x_{ti} - \underline{x}_{ti}'\hat{\phi}]_{i=1,2,\ldots,k}$$

We also denote

$$\hat{W} = \nabla\hat{d}_{41}\hat{A}_{11}^{-1}\nabla'\hat{d}_{41} + \frac{1}{n\hat{\sigma}_u^2}\sum_{t=1}^{n} \underline{z}_t\underline{z}_t'$$

where ∇d_{41} is the (4,1) block of $\nabla d(\theta)$ and A_{11} is the upper left-hand corner block of $A(\theta)$ [see Appendix B]. Then we have very concise forms of $\hat{V}_i$'s as follows:

$$\hat{V}_1 = \frac{2}{n\hat{\sigma}_u^4}\sum_{t=1}^{n} \underline{\chi}_t\underline{\chi}_t', \qquad \hat{V}_2 = \frac{2}{n\hat{\sigma}_u^4}\sum_{t=1}^{n} \underline{\xi}_t\underline{\xi}_t', \qquad \hat{V}_3 = \frac{3}{2\hat{\sigma}_u^8}$$

$$\hat{V}_4 = \frac{2}{n\hat{\sigma}_u^4}\sum_{t=1}^{n} \underline{s}_t\underline{s}_t' + \hat{W}, \quad \hat{V}_5 = \frac{3}{2n\hat{\sigma}_u^6}\sum_{t=1}^{n} \underline{r}_t\underline{r}_t', \quad \hat{V}_6 = \frac{3}{2n\hat{\sigma}_u^6}\sum_{t=1}^{n} \hat{\underline{\varepsilon}}_t\hat{\underline{\varepsilon}}_t'$$

Given the block diagonality of the variance matrix of $\hat{d}$, we can write the IM test as

$$T_W = \sum_{i=1}^{6} T_i = n\sum_{i=1}^{6} \hat{d}_i'\hat{V}_i^{-1}\hat{d}_i \qquad (6)$$

that is, the derived IM test statistic is found to be decomposed as the sum of six quadratic forms. In the next section, we analyze these components of T_W in detail.

3. INTERPRETATION OF THE COMPONENTS OF THE IM TEST

Using Chesher's analysis, we can say the statistic T_1 is a test for randomness of the regression parameters in the presence of autocorrelation. If we put $\hat{\phi} = 0$, then this reduces to the White (1980) test for heteroskedasticity [and T_{1n} in Hall (1987, p. 261)]. Recently, there have been some robustness studies of various tests for heteroskedasticity in the presence of autocorrelation [see, e.g., Epps and Epps (1977), Bera and Jarque (1982), Godfrey and Wickens (1982), Bumb and Kelejian (1983), Bera and McKenzie (1986)] and their general conclusion is that various tests for heteroskedasticity are sensitive to the presence of autocorrelation. A byproduct of our analysis is that we have a simple test for heteroskedasticity in the presence of autocorrelation. All we need to do is to modify the White test slightly. Instead of regressing the squares of the least squares residuals on the squares and cross products of x_t's, we should regress $\hat{u}_t^2$ on the squares and cross products of $(x_t - \underline{x}_t'\hat{\phi})$ after estimating the model with an appropriate AR process. For example, if there is AR(1) error, then the regressors should be the squares and cross products of $(x_t - \hat{\phi}_1 x_{t-1})$. Similarly, the modification of T_{2n} in Hall (1987), which is our T_5, requires that we should replace x_t by $(x_t - \underline{x}_t'\hat{\phi})$. Our T_3 is a (kurtosis) test for normality, and it utilizes the conditional mean corrected residuals rather than the OLS residuals.

Let us now concentrate on the new test statistics we obtain by including ϕ in our model. The statistic T_2 tests the randomness of $\phi = (\phi_1, \phi_2, \ldots, \phi_p)'$. Suppose that the parameters of autoregressive errors are varying around a mean value with finite variances. This can be formulated as $\phi_t \sim (\phi,\Omega)$, where $\phi_t = (\phi_{1t},\phi_{2t},\ldots,\phi_{pt})'$. Then T_2 is the LM statistic for testing H_0: $\Omega = 0$. Let us first consider a very special case in which $\phi = 0$ and Ω is diagonal. Therefore, we have $\hat{\phi}_1 = \hat{\phi}_2 = \ldots = \hat{\phi}_p = 0$, and $\hat{u}_{t-i} = \hat{\varepsilon}_{t-i}$ $(i=1,2,\ldots,p)$, where the $\hat{\varepsilon}_t$ are the OLS residuals. Consequently, T_2 reduces to

$$T_2 = \frac{1}{2}\,[\sum_{t=1}^{n} \underline{\hat{u}}_t^2(\frac{\hat{u}_t^2}{\hat{\sigma}_u^2} - 1)]'[\sum_{t=1}^{n} \underline{\xi}_t\underline{\xi}_t']^{-1}[\sum_{t=1}^{n} \underline{\hat{u}}_t^2(\frac{\hat{u}_t^2}{\hat{\sigma}_u^2} - 1)] \tag{7}$$

where $\underline{\hat{u}}_t^2 = (\hat{u}_{t-1}^2,\hat{u}_{t-2}^2,\ldots,\hat{u}_{t-p}^2)'$ and a typical elements of $\underline{\xi}_t$ is now $(\hat{u}_{t-i}^2 - \frac{1}{n}\sum_{t=1}^{n}\hat{u}_{t-i}^2)$, for $i = 1,2,\ldots,p$. This is identical to the Engle (1982) LM statistic for testing the pth-order linear ARCH disturbances, i.e., testing H_0: $\alpha_1 = \alpha_2 = \ldots = \alpha_p = 0$ in the ARCH process specified as

$$\mathrm{Var}(u_t|\underline{u}_t) = \sigma_u^2 + \alpha_1 u_{t-1}^2 + \ldots + \alpha_p u_{t-p}^2$$

where $\underline{u}_t = (u_{t-1},u_{t-2},\ldots,u_{t-p})'$. An asymptotically equivalent form of this statistic is nR^2 where R^2 is the coefficient of multiple determination from the regression of $\hat{u}_t^2$ on a unit term and $(\hat{u}_{t-1}^2,\hat{u}_{t-2}^2,\ldots,\hat{u}_{t-p}^2)$.

From our representation of test for ARCH as a test for randomness of φ parameters and its equivalence to one component of the IM test, the consequence of the presence of ARCH is that the "usual" estimators for variance of $\hat{\phi}$ will be inconsistent if ARCH is ignored. This is similar to the case that the standard variance estimator for $\hat{\beta}$ is inconsistent in the presence of static heteroskedasticity. Therefore, the standard tests for autocorrelation are not valid in the presence of ARCH [see, e.g., Diebold (1986) and Bera et al. (1990)]. This result is not entirely obvious since under ARCH, the disturbances are still unconditionally homoskedastic. Although the above point could be made without an IM test interpretation, IM test framework provides an easy guidance for checking whether the standard inference procedures fail.

We now relax the assumption of the diagnolaity of Ω. The structure of the test statistic will remain the same except R^2 will be obtained by regressing $\hat{u}_t^2$ on a constant and the squares and cross products of the lagged residuals. T_2 will then be a LM statistic for testing H_0: $\alpha_{ij} = 0$ $(i \geq j = 1,2,\ldots,p)$ in

$$\mathrm{Var}(u_t|\underline{u}_t) = \sigma_u^2 + \sum_{i=1}^{p}\sum_{j=1}^{p} \alpha_{ij}u_{t-i}u_{t-j} \quad i \geq j \tag{8}$$

The above specification of conditional variance generalizes the Engle ARCH model. This will be called the augmented ARCH (AARCH) process. Properties and testing of this model are discussed in Bera et al. (1990). Lastly, if we additionally relax the assumption of $\phi = 0$, $\hat{u}_t$ will no longer be equal to $\hat{\varepsilon}_t$ and T_2 will have to be calculated from the regression of $\hat{u}_t^2$ on a constant and the squares and cross products of $\hat{\varepsilon}_{t-i}$ $(i = 1,2,\ldots,p)$. This will give us the LM statistics for testing ARCH or AARCH in the presence of autocorrelation.

From the above discussion, it is clear that the Engle ARCH model can be viewed as a special case of random coefficient autoregressive (RCAR) model. To see this more clearly, let us write equation (2) as

$$\varepsilon_t = \sum_{j=1}^{p} \phi_{jt}\varepsilon_{t-j} + u_t.$$

If it is assumed that $\phi_{jt} \sim (0,\alpha_j)$ and $\text{cov}(\phi_{jt},\phi_{j't}) = 0$, for $j \neq j'$, then the conditional variance is given by

$$\text{Var}(\varepsilon_t|\underline{\varepsilon}_t) = \sigma_u^2 + \sum_{j=1}^{p} \alpha_j\varepsilon_{t-j}^2.$$

Here we observe that ARCH and the above RCAR models have the same first two conditional moments as mentioned in Tsay (1987) where it is called as second-order equivalence. If we further assume that the ϕ_{jt} are normally distributed, then all the moments of ARCH and RCAR processes will be the same, e.g., for $p = 1$, the first four moments are

$$\mu_1 = 0,\ \mu_2 = \frac{\sigma_u^2}{1-\alpha_1},\ \mu_3 = 0 \text{ and } \mu_4 = \frac{3\sigma_u^4(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}$$

[see Engle (1982, p. 992)]. Here we should note that calculation of moments are much easier under the RCAR scheme.

By comparing T_1 and T_2, we note that they test for static and conditional heteroskedasticity, respectively. Given the block diagonality of covariance matrix of the IM test in our case, we can test for static and conditional heteroskedasticity simultaneously simply by adding up these two statistics. The statistic T_4 is also related to T_1 and T_2. From the expression of $\hat{d}_4$, we note that T_4 has two components. The second component is based on $(n\hat{\sigma}_u^2)^{-1}\sum_t \hat{u}_t\underline{x}_t'$ and this can be viewed as some form of a test for exogeneity. In certain practical applications when the x_t are known to be exogenous, this part can be ignored. Then T_4 will be based only on the first component of $\hat{d}_4$ and the resulting test will be a test for conditional heteroskedasticity caused by the interaction between the disturbance term and the regressors. We should, however, note that by excluding the second component of $\hat{d}_4$ we do not get exactly an IM test but something that is very close to an IM test. For the special case,

$$\hat{V}_4 = \frac{2}{n\hat{\sigma}_u^2} \sum_{t=1}^{n} \underline{s}_t \underline{s}_t'$$

and therefore, for obtaining T_4 we run the regression of $\hat{u}_t^2$ on a constant and cross products of lagged innovations $\hat{\underline{\varepsilon}}_t$ and transformed exogenous variables $x_t - \underline{x}_t'\hat{\phi}$. As a natural consequence, a general test statistic for heteroskedasticity would be $T_1 + T_2 + T_4$ which under the null hypothesis will have an asymptotic χ^2 distribution with $(k+p)(k+p+1)/2$ degrees of freedom. To get reasonable power, we will have to make a judicious selection of the regressors from the set of squares and cross products of $x_t - \underline{x}_t'\hat{\phi}$ and $\hat{\underline{\varepsilon}}_t$, or make some adjustment to the test statistic [see Bera (1986)].

The last two statistics T_5 and T_6 can be viewed as the statistics for testing variation in the third moment of u_t. In T_5, the variation is assumed to depend on the exogenous variables x_t and in T_6, on the lagged innovation process. In some sense, we could say that T_5 and T_6 test for static and conditional heteroclīticity, respectively. The term heteroclīticity is used since when the skewness coefficient is plotted against x_t or ε_{t-i}, we obtain the clitic curve [see Kendall and Stuart (1973, p. 362)]. As noted in Hall (1987), the test for normality (skewness part) proposed by Bowman and Shenton (1975) and Jarque and Bera (1987) is a special case of T_5 while T_3 which tests for the variation of σ_u^2 is a pure test for kurtosis. In this connection, let us mention that if the IM test is applied to an ARCH model, that leads to a test for heterokurticity [for details see Bera and Zuo (1991)]. This provides a specification test for an estimated ARCH model.

4. CONCLUSION

Our application of the White IM test to the linear regression model with autoregressive errors provides many interesting results. The most important result is that a special case of one components of this test is identical to the Engle LM test for ARCH. Chesher's interpretation of the IM test as the test for parameter heterogeneity leads us naturally to specify the ARCH processes as a random coefficient autoregressive (RCAR) model. From both theoretical and practical points of view, this representation of ARCH is convenient and useful. As discussed in Bera et al. (1990), we can now easily verify the stationarity condition for ARCH as a special case of RCAR model, study the robustness of test for AR process in the presence of ARCH and vice versa, and generalize the ARCH process to take account of interaction between the disturbance terms.

The difference between the static and conditional heteroskedasticity is now clear. The former could be related to the variation of the regression coefficients while the latter to the variation of the autoregressive parameters. A mixture of them is possible when the heteroskedasticity is caused by the interaction between exogenous variables and disturbances. We also discuss the possibilities of static and conditional variations in skewness, what we call heterocliticity.

APPENDIX A

The Derivatives of the Log-likelihood Function

For our model, the vector of parameters is $\theta = (\beta', \phi', \sigma_u^2)'$ and the log-likelihood function for the t-th observation conditional on the information set Ψ_{t-1}, in which $\underline{\varepsilon}_t = (\varepsilon_{t-1}, \ldots, \varepsilon_{t-p})'$ is contained, is given by

$$\ell_t(\theta) = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log \sigma_u^2 - \frac{1}{2\sigma_u^2}(\varepsilon_t - \underline{\varepsilon}_t'\phi)^2.$$

Note that $u_t = \varepsilon_t - \underline{\varepsilon}_t'\phi = (y_t - \underline{y}_t'\phi) - (x_t - \underline{x}_t'\phi)'\beta$ where $\underline{y}_t = (y_{t-1}, \ldots, y_{t-p})'$ and $\underline{x}_t = (x_{t-1}, \ldots, x_{t-p})'$. Then the first and second partial derivatives of $\ell_t(\theta)$ with respect to θ are easily obtained. The first derivatives are

$$\frac{\partial \ell_t(\theta)}{\partial \beta} = \frac{1}{\sigma_u^2} u_t (x_t - \underline{x}_t'\phi), \quad \frac{\partial \ell_t(\theta)}{\partial \phi} = \frac{1}{\sigma_u^2} u_t \underline{\varepsilon}_t \text{ and}$$

$$\frac{\partial \ell_t(\theta)}{\partial \sigma_u^2} = -\frac{1}{2\sigma_u^2} + \frac{1}{2\sigma_u^4} u_t^2.$$

And the second derivatives are

$$\frac{\partial^2 \ell_t(\theta)}{\partial \beta \partial \beta'} = -\frac{1}{\sigma_u^2}(x_t - \underline{x}_t'\phi)(x_t - \underline{x}_t'\phi)', \quad \frac{\partial^2 \ell_t(\theta)}{\partial \phi \partial \phi'} = \frac{1}{\sigma_u^2} \underline{\varepsilon}_t \underline{\varepsilon}_t',$$

$$\frac{\partial^2 \ell_t(\theta)}{\partial (\sigma_u^2)^2} = \frac{1}{2\sigma_u^4} - \frac{1}{\sigma_u^6} u_t^2, \quad \frac{\partial^2 \ell_t(\theta)}{\partial \beta \partial \phi'} = -\frac{1}{\sigma_u^2}(x_t - \underline{x}_t'\phi)\underline{\varepsilon}_t' - \frac{1}{\sigma_u^2} u_t \underline{x}_t',$$

$$\frac{\partial^2 \ell_t(\theta)}{\partial \beta \partial \sigma_u^2} = -\frac{1}{\sigma_u^4} u_t (x_t - \underline{x}_t'\phi)' \text{ and } \frac{\partial^2 \ell_t(\theta)}{\partial \phi \partial \sigma_u^2} = -\frac{1}{\sigma_u^4} u_t \underline{\varepsilon}_t'.$$

APPENDIX B

A Consistent Covariance Matrix Estimator for the Information Matrix Test

A consistent covariance estimator for the IM test proposed by White (1982) is stated as

$$\hat{V}(\hat{\theta}) = \frac{1}{n}\sum_{t=1}^{n} a_t(\hat{\theta})a_t'(\hat{\theta}) \tag{B.1}$$

where $a_t(\hat{\theta}) = d_t(\hat{\theta}) - \nabla d(\hat{\theta})A(\hat{\theta})^{-1}\nabla \ell_t(\hat{\theta})$. Let us begin with the indicator vector $d(\hat{\theta})$ which is defined as

$$d(\hat{\theta}) = \text{vech}[C(\hat{\theta})] = \text{vech}[A(\hat{\theta}) + B(\hat{\theta})]$$

where

$$A(\hat{\theta}) = \frac{1}{n}\sum_{t=1}^{n}\left[\frac{\partial^2 \ell_t(\theta)}{\partial\theta\partial\theta'}\right]_{\theta=\hat{\theta}}$$

$$= \frac{1}{n}\sum_{t=1}^{n}$$

$$\begin{bmatrix} -\frac{1}{\sigma_u^2}(x_t-\underline{x}_t'\phi)(x_t-\underline{x}_t'\phi)' & -\frac{1}{\sigma_u^2}(x_t-\underline{x}_t'\phi)\underline{\varepsilon}_t - \frac{1}{\sigma_u^2}u_t\underline{x}_t' & -\frac{1}{\sigma_u^4}(x_t-\underline{x}_t'\phi)u_t \\ -\frac{1}{\sigma_u^2}\underline{\varepsilon}_t(x_t-\underline{x}_t'\phi)' - \frac{1}{\sigma_u^2}u_t\underline{x}_t & -\frac{1}{\sigma_u^2}\underline{\varepsilon}_t\underline{\varepsilon}_t' & -\frac{1}{\sigma_u^4}\underline{\varepsilon}_t u_t \\ -\frac{1}{\sigma_u^4}u_t(x_t-\underline{x}_t'\phi)' & -\frac{1}{\sigma_u^4}u_t\underline{\varepsilon}_t' & \frac{1}{2\sigma_u^4} - \frac{1}{\sigma_u^6}u_t^2 \end{bmatrix}_{\theta=\hat{\theta}}$$

and

$$B(\hat{\theta}) = \frac{1}{n}\sum_{t=1}^{n} [(\frac{\partial \ell_t(\theta)}{\partial\theta})(\frac{\partial \ell_t(\theta)}{\partial\theta})']_{\theta=\hat{\theta}}$$

$$= \frac{1}{n}\sum_{t=1}^{n}\begin{bmatrix} \frac{1}{\sigma_u^4} u_t^2(x_t-\underline{x}_t'\phi)(x_t-\underline{x}_t'\phi)' & \frac{1}{\sigma_u^4} u_t^2(x_t-\underline{x}_t'\phi)\underline{\varepsilon}_t' & \frac{1}{2\sigma_u^6}(x_t-\underline{x}_t'\phi)(u_t^3-\sigma_u^2u_t) \\ \frac{1}{\sigma_u^4} u_t^2\underline{\varepsilon}_t(x_t-\underline{x}_t'\phi)' & \frac{1}{\sigma_u^4} u_t^2\underline{\varepsilon}_t\underline{\varepsilon}_t' & \frac{1}{2\sigma_u^6}\underline{\varepsilon}_t(u_t^3-\sigma_u^2u_t) \\ \frac{1}{2\sigma_u^6}(u_t^3-\sigma_u^2u_t)(x_t-\underline{x}_t'\phi)' & \frac{1}{2\sigma_u^6}(u_t^3-\sigma_u^2u_t)\underline{\varepsilon}_t' & \frac{1}{4\sigma_u^4} - \frac{1}{2\sigma_u^6}u_t^2 + \frac{1}{4\sigma_u^8}u_t^4 \end{bmatrix}_{\theta=\hat{\theta}}$$

From $A(\hat{\theta})$ and $B(\hat{\theta})$, $C(\hat{\theta})$ is easily derived as

$$C(\hat{\theta}) = A(\hat{\theta}) + B(\hat{\theta})$$

$$= \frac{1}{n}\sum_{t=1}^{n}$$

$$\begin{bmatrix} \frac{1}{\sigma_u^4}(u_t^2-\sigma_u^2)(x_t-\underline{x}_t'\phi)(x_t-\underline{x}_t'\phi)' & \frac{1}{\sigma_u^4}(u_t^2-\sigma_u^2)(x_t-\underline{x}_t'\phi)\underline{\varepsilon}_t' - \frac{1}{\sigma_u^2}u_t\underline{x}_t' & \frac{1}{2\sigma_u^6}(x_t-\underline{x}_t'\phi)(u_t^3-3\sigma_u^2u_t) \\ \frac{1}{\sigma_u^4}(u_t^2-\sigma_u^2)\underline{\varepsilon}_t(x_t-\underline{x}_t'\phi)' - \frac{1}{\sigma_u^2}u_t\underline{x}_t & \frac{1}{\sigma_u^4}(u_t^2-\sigma_u^2)\underline{\varepsilon}_t\underline{\varepsilon}_t' & \frac{1}{2\sigma_u^6}\underline{\varepsilon}_t(u_t^3-3\sigma_u^2u_t) \\ \frac{1}{2\sigma_u^6}(u_t^3-3\sigma_u^2u_t)(x_t-\underline{x}_t'\phi)' & \frac{1}{2\sigma_u^6}(u_t^3-3\sigma_u^2u_t)\underline{\varepsilon}_t' & \frac{1}{4\sigma_u^8}(u_t^4-6\sigma_u^2u_t^2 + 3\sigma_u^4) \end{bmatrix}_{\theta=\hat{\theta}}$$

Now it is straightforward to obtain $d(\hat{\theta})$. For analytical convenience, we rearrange $d(\hat{\theta})$ as described in the paper. Then the first part of $a_t(\hat{\theta})$ defined from $d(\hat{\theta}) = \frac{1}{n}\sum_{t=1}^{n} d_t(\hat{\theta})$ can be written as

$$d_t(\hat{\theta}) = (\hat{d}_{t1}', \hat{d}_{t2}', \hat{d}_{t3}, \hat{d}_{t4}', \hat{d}_{t5}', \hat{d}_{t6}')' \tag{B.2}$$

where

$$\hat{d}_{t1} = [\hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_{t1}-\underline{x}_{t1}'\hat{\phi})^2, \ldots, \hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_{tk}-\underline{x}_{tk}'\hat{\phi})^2,$$

$$\hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_{t1}-\underline{x}_{t1}'\hat{\phi})(x_{t2}-\underline{x}_{t2}'\hat{\phi}), \ldots,$$

$$\hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_{t(k-1)}-\underline{x}_{t(k-1)}'\hat{\phi})(x_{tk}-\underline{x}_{tk}'\hat{\phi})]'$$

is a $\frac{k(k+1)}{2} \times 1$ vector,

$$\hat{d}_{t2} = [\hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)\hat{\varepsilon}_{t-1}^2, \ldots, \hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)\hat{\varepsilon}_{t-p}^2, \hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)\hat{\varepsilon}_{t-1}\hat{\varepsilon}_{t-2},$$

$$\ldots, \hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)\hat{\varepsilon}_{t-p+1}\hat{\varepsilon}_{t-p}]'$$

is a $\frac{p(p+1)}{2} \times 1$ vector,

$$\hat{d}_{t3} = (4\hat{\sigma}_u^8)^{-1}(\hat{u}_t^4 - 6\hat{\sigma}_u^2\hat{u}_t^2 + 3\hat{\sigma}_u^4)$$

is a scalar,

$$\hat{d}_{t4} = [\hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_t-\underline{x}_t'\hat{\phi})'\hat{\varepsilon}_{t-1}, \ldots, \hat{\sigma}_u^{-4}(\hat{u}_t^2-\hat{\sigma}_u^2)(x_t-\underline{x}_t'\hat{\phi})'\hat{\varepsilon}_{t-p}]'$$

$$-[\hat{\sigma}_u^{-2}x_{t-1}', \ldots, \hat{\sigma}_u^{-2}x_{t-p}']'$$

is a $kp \times 1$ vector,

$$\hat{d}_{t5} = (2\hat{\sigma}_u^6)^{-1}(\hat{u}_t^3-3\hat{\sigma}_u^2\hat{u}_t)(x_t-\underline{x}_t'\hat{\phi})$$

is a k×1 vector, and finally,

$$\hat{d}_{t6} = [(2\hat{\sigma}_u^6)^{-1}(\hat{u}_t^3-3\hat{\sigma}_u^2\hat{u}_t)\hat{\varepsilon}_{t-1},\ldots,(2\hat{\sigma}_u^6)^{-1}(\hat{u}_t^3-3\hat{\sigma}_u^2\hat{u}_t)\hat{\varepsilon}_{t-p}]'$$

is a p×1 vector.

Next we consider

$$\nabla d(\theta_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} E[\frac{\partial d_t(\theta_0)}{\partial\theta}].$$

Using the normality assumption of the u_t and taking expectation conditional on the information set Ψ_{t-1} iteratively, after some algebra we can get the following simple form of $\nabla d(\theta_0)$

$$\nabla d(\theta_0) = \begin{bmatrix} 0 & 0 & \nabla d_{13} \\ 0 & 0 & \nabla d_{23} \\ 0 & 0 & 0 \\ \nabla d_{41} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\nabla d_{13} = (mx_{11},\ldots,mx_{kk},mx_{12},\ldots,mx_{(k-1)k})'$ is a $\frac{k(k+1)}{2} \times 1$ vector with

$$mx_{ij} = -\frac{1}{\sigma_u^4} \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (x_{ti}-\underline{x}_{ti}'\phi_0)(x_{tj}-\underline{x}_{tj}'\phi_0),$$

$$i,j = 1,2,\ldots,k: \quad i \le j,$$

$\nabla d_{23} = (m\varepsilon_{11},\ldots,m\varepsilon_{pp},m\varepsilon_{12},\ldots,m\varepsilon_{(p-1)p})'$ is a $\frac{p(p+1)}{2} \times 1$ vector with

$$m\varepsilon_{ij} = -\frac{1}{\sigma_u^4} \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \varepsilon_{t-i}\varepsilon_{t-j},$$

$$i,j = 1,2,\ldots,k: \quad i \le j,$$

and $\nabla d_{41} = (w_{11}, w_{12}, \ldots, w_{kp})'$ is a kp×k matrix with

$$w_{ij} = \frac{1}{\sigma_u^2} \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (x_t - \underline{x}_t' \phi_0)' x_{t-j,i},$$

$$i = 1,2,\ldots, k, j = 1,2,\ldots,p.$$

This implies that $\nabla d(\theta_0)$ can be estimated consistently by the $\nabla d(\hat{\theta})$ which is

$$\nabla d(\hat{\theta}) = \begin{bmatrix} 0 & 0 & \nabla\hat{d}_{13} \\ 0 & 0 & \nabla\hat{d}_{23} \\ 0 & 0 & 0 \\ \nabla\hat{d}_{41} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{B.3}$$

where for example, $\nabla\hat{d}_{13} = (\overline{mx}_{11}, \ldots, \overline{mx}_{kk}, \overline{mx}_{12}, \ldots, \overline{mx}_{(k-1)k})'$ is a $\frac{k(k+1)}{2} \times 1$ vector with

$$\overline{mx}_{ij} = -\frac{1}{n\hat{\sigma}_u^4} \sum_{t=1}^{n} (x_{ti} - \underline{x}_{ti}'\hat{\phi})(x_{tj} - \underline{x}_{tj}'\hat{\phi}),$$

$$i,j = 1,2,\ldots,k: \quad i \leq j.$$

Similarly, we can simplify $A(\hat{\theta})$ as follows:

$$A(\hat{\theta}) = \begin{bmatrix} -\frac{1}{n\hat{\sigma}_u^2} \sum_{t=1}^{n} (x_t - \underline{x}_t'\hat{\phi})(x_t - \underline{x}_t'\hat{\phi})' & 0 & 0 \\ 0 & -\frac{1}{n\hat{\sigma}_u^2} \sum_{t=1}^{n} \hat{\underline{\varepsilon}}_t \hat{\underline{\varepsilon}}_t' & 0 \\ 0 & 0 & -\frac{1}{2\hat{\sigma}_u^4} \end{bmatrix} \tag{B.4}$$

For future use, let us denote the upper left-hand corner block of $A(\hat{\theta})$ as $A_{11}(\hat{\theta}) \equiv \hat{A}_{11}$. We can simplify the expression for $A(\hat{\theta})$ further by using analytic expectation of $\underline{\varepsilon}_t\underline{\varepsilon}_t'$. For examplme, when p = 1, $n^{-1} \sum_t \varepsilon_{t-1}^2$ can be replaced by $\hat{\sigma}_u^2/(1-\hat{\phi}_1)$. This might provide better finite sample performance. However, then we will loose the nR^2 interpretation of our test statistics. Also, no general expression for the analytic expectation can be given for all values of p.

Finally, $\nabla \ell_t(\hat{\theta}) = \dfrac{\partial \ell_t(\hat{\theta})}{\partial \theta}$ is easily given from Appendix A by

$$\nabla \ell_t(\hat{\theta}) = \begin{bmatrix} \dfrac{1}{\hat{\sigma}_u^2} \hat{u}_t(x_t - \underline{x}_t'\hat{\phi}) \\[2ex] \dfrac{1}{\hat{\sigma}_u^2} \hat{u}_t \hat{\underline{\varepsilon}}_t \\[2ex] -\dfrac{1}{2\hat{\sigma}_u^2} + \dfrac{1}{2\hat{\sigma}_u^4} \hat{u}_t^2 \end{bmatrix} \tag{B.5}$$

and we denote the first (k×1) vector of $\nabla \ell_t(\hat{\theta})$ as $\nabla \ell_{t1}(\hat{\theta}) \equiv \nabla \hat{\ell}_{t1}$.

For the following discussion, recall the definitions of $\underline{\chi}_t$, $\underline{\xi}_t$, $\underline{s}_t$ and $\underline{r}_t$, provided in the main text. From (B.2)-(B.5), $a_t(\hat{\theta})$ can be easily derived as

$$a_t(\hat{\theta}) = d_t(\hat{\theta}) - \nabla d(\hat{\theta}) A(\hat{\theta})^{-1} \nabla \ell_t(\hat{\theta}) = (\hat{a}_{t1}', \hat{a}_{t2}', \hat{a}_{t3}', \hat{a}_{t4}', \hat{a}_{t5}', \hat{a}_{t6}')' \tag{B.6}$$

where $\hat{a}_{t1} = \dfrac{1}{\hat{\sigma}_u^4}(\hat{u}_t^2 - \hat{\sigma}_u^2)\underline{\chi}_t$, $\hat{a}_{t2} = \dfrac{1}{\hat{\sigma}_u^4}(\hat{u}_t^2 - \hat{\sigma}_u^2)\underline{\xi}_t$, $\hat{a}_{t3} = \hat{d}_{t3}$,

$\hat{a}_{t4} = \hat{d}_{t4} - \nabla \hat{d}_{41} \hat{A}_{11}^{-1} \nabla \hat{\ell}_{t1}$, $\hat{a}_{t5} = \hat{d}_{t5}$, and $\hat{a}_{t6} = \hat{d}_{t6}$.

Now we establish the block diagonality of the covariance matrix of the IM test, say $V(\theta_0)$. It is assumed that all conditions stated in White (1982) are satisfied. Given (B.2)-(B.6) with the normality assumption of the u_t, $V(\theta_0)$ takes the form of

$$V(\theta_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n}$$

$$E\begin{bmatrix} \frac{2}{\sigma_u^4} \underline{x}_t\underline{x}_t' & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{\sigma_u^4} \underline{\xi}_t\underline{\xi}_t' & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2\sigma_u^8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\sigma_u^4} \underline{s}_t\underline{s}_t'+W & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{2\sigma_u^6} \underline{r}_t\underline{r}_t' & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2\sigma_u^6} \underline{\varepsilon}_t\underline{\varepsilon}_t' \end{bmatrix}_{\theta=\theta_0}$$

where $W = Vd_{41}A_{11}^{-1}Vd_{41}' + n^{-1}\sigma_u^{-2} \sum_t \underline{z}_t\underline{z}_t'$, and the diagonal elements are consistently estimated by $\hat{V}_i$, $i = 1,2,\ldots,6$, stated in the main text. To prove this result, let us consider

$$V(\theta_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} E[a_t(\theta_0)a_t'(\theta_0)] \tag{B.7}$$

In the first stage, we evaluate $E[a_t(\theta_0)a_t'(\theta_0)]$ conditional on the information set Ψ_{t-1} using the normality assumption of the u_t and taking expectation iteratively. In the next stage, we use the facts that at $\theta = \theta_0$, $E(\underline{\varepsilon}_t) = 0$, $E(\underline{s}_t) = 0$ and $E(\underline{\xi}_t) = 0$ for all t. Then we have the result.

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