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## BAYESIAN INFERENCE IN TIME SERIES

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# BAYESIAN INFERENCE IN TIME SERIES 

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## BAYESIAN INFERENCE IN TIME SERIES

Whenever decisions under uncertainty have to be made, a statistical paradigm becomes an essential tool for extracting information from observed data and using this to improve our knowledge about the world (inference), thus eventually guiding us in our decision problem. The Bayesian paradigm is certainly not as heavily used as its frequentist or "classical" counterpart in the realm of econometrics, yet it seems ideally suited to address several questions of great interest to applied researchers in this field. It provides us with a formal way of incorporating the prior information we so often possess before actually seeing the data, it fits perfectly with sequential learning and decision theory, and directly leads to exact small sample results. In addition, it naturally gives rise to predictive densities, where all nuisance parameters are integrated out: the latter is exactly what we need for forecasting.

Let us briefly consider an example. A very simple model for the transactions demand for money $m_{t}$, observed at time $t$, is the following:

$$
\begin{equation*}
m_{t}=\alpha_{0}+\alpha_{1} i_{t}+\alpha_{2} p_{t}+\alpha_{3} r_{t}+\varepsilon_{t} \tag{1}
\end{equation*}
$$

where all variables are in logarithms and $i_{t}, p_{t}$ and $r_{t}$ respectively denote real income, price and interest rate; $\varepsilon_{t}$ is just an unobserved error term. Before seeing any data, a lot of prior information on the coefficients in (1) will typically exist: $\alpha_{1}$ and $\alpha_{2}$ will be believed to be positive and the interest elasticity $\alpha_{3}$ will often be constrained to $(-1,0)$. Furthermore, values of $\alpha_{2}$ around one are deemed likely if one believes in the absence of money illusion, and both $\alpha_{1}$ and $\alpha_{2}$ close to one reflects a prior belief in a constant velocity of circulation. These beliefs will then have to be formalized in a prior distribution on the coefficients.

For the main features of both paradigms, let me refer the reader to an excellent survey by Poirier (1988). Suffice it to say here that the basic underlying probability interpretation for a Bayesian is a subjective one, referring to a personal degree of belief. The fundamental rules of probability calculus are then used to examine how prior beliefs are transformed to posterior beliefs by the data information. The researcher learns from observing the world, and the only restriction on her subjective beliefs is consistency with the axioms of probability. Having obtained some actual data on ( $m_{t}, i_{t}, p_{t}, r_{t}$ ) she will be able to monitor how each new data point revises her beliefs sequentially.
These posterior distributions can be used to integrate out the coefficients when she is interested in forecasting future values of money demand through (1). As she does not know the $\alpha$ 's, it seems natural to average out over them, using her knowledge of that moment, rather than conditioning on particular values. She can also test e.g. the hypothesis of no money illusion ( $\alpha_{2}=1$ ). One way is to use highest posterior density intervals: "Does the shortest interval which covers, say, $95 \%$ of the posterior probability mass for $\alpha_{2}$ include the value one?" Given a proper prior on $\alpha_{2}$ (i.e. one that integrates to one) she can also use a posterior odds test, which is directly interpretable in terms of the posterior probability of the hypothesis.

Clearly, in reporting such results it will often be good practice to conduct a sensitivity analysis with a range of different prior distributions, which will typically contain a "noninformative" or "reference" prior distribution as well. Finally, the mapping from prior to posterior uncertainty can only depend on the likelihood yielded by the data actually observed: observations that might have occurred, but did not, carry no weight whatsoever. This so-called likelihood principle is crucial for appreciating the differences between both paradigms.

Within the vast literature on models for time series data, we select two broad groups of models that seem of particular relevance for applications in macroeconomics and finance: univariate ARMA models and structural econometric models. The symbol $p$ is used generically to denote density functions, whether proper or not.

## Univariate ARMA Models

Ever since the seminal work of Box and Jenkins (1970) the class of autoregressive movingaverage ( $\Lambda R M \wedge$ ) processes has been popular to describe the variation of macroeconomic time series like GNP or stock prices. If $y_{t}$ denotes the observed value (often in logs) for a scalar variable at time $t$, the ARMA $(p, q)$ sampling model can be defined as

$$
\begin{equation*}
y_{t}=\sum_{i=1}^{p} \varphi_{i} y_{t-i}+\varepsilon_{t}+\sum_{j=1}^{q} \theta_{j} \varepsilon_{t-j} \tag{2}
\end{equation*}
$$

for $t=1, \ldots T$ and where the $\varepsilon_{t}$ 's are independent random shocks all drawn from the same Normal distribution with mean zero and variance $\sigma^{2}$. Stationarity of (2) implies e.g. that the influence of initial observations $y_{(0)}=\left(y_{1-p}, \ldots, y_{0}\right)^{\prime}$ dies out as $t$ becomes large, and restricts the parameter space of $\varphi=\left(\varphi_{1}, \ldots, \varphi_{p}\right)^{\prime}$. Invertibility lends an infinite order autoregressive (i.e. $p$ infinite and $q=0$ ) representation to (2) and puts conditions on the parameter space of $\theta=\left(\theta_{1}, \ldots, \theta_{q}\right)^{\prime}$. The latter conditions ensure a unique parameterization of (2) in terms of $\left(\varphi, \theta, \sigma^{2}\right) \equiv\left(\zeta, \sigma^{2}\right)$, given $(p, q) \equiv \mu$. Geweke (1988) investigates various other forms of nonlinear restrictions in modelling real GDP series for various countries with $p=3$ and $q=0$, and adding a constant to (2). Like in many other models (e.g. ARCH models as treated in Geweke 1989) these constraints cast serious doubt on the applicability of classical methods, yet pose no conceptual problem whatsoever in a Bayesian analysis using Monte Carlo integration (see e.g. Kloek and van Dijk 1978).
Defining $y \equiv\left(y_{1}, \ldots, y_{T}\right)^{\prime}$, the likelihood from (2), $p\left(y \mid \zeta, \sigma^{2}, \mu\right)$, is combined with the prior $p\left(\zeta, \sigma^{2}, \mu\right)$, which will typically impose invertibility and often stationarity as well. Let us now sketch an illustrative Bayesian analysis, which will be conducted conditionally upon $y_{(0)}$ (without explicitly indicating this to save on notation). A so-called natural conjugate prior for $\sigma^{2}$ is chosen, which retains its convenient form after updating by the sample, so that it can be integrated out analytically from the posterior, given the other parameters. The rest of the analysis requires numerical integration, as explained in detail in Monahan (1983). The resulting joint density $p(y, \zeta, \mu)$ can be factorized into
the predictive $p(y)$ and the posterior $p(\zeta, \mu \mid y)$ which is well-defined if $p(y)$ evaluated at the observed sample is finite. Post-sample prediction can then be conducted for, say, $\tilde{y}=\left(y_{T+1}, \ldots, y_{T+s}\right)^{\prime}$ on the basis of

$$
\begin{equation*}
p(\tilde{y} \mid y)=\int p(\tilde{y} \mid y, \zeta, \mu) p(\zeta, \mu \mid y) d \zeta d \mu \tag{3}
\end{equation*}
$$

where $p(\tilde{y} \mid y, \zeta, \mu)$ takes a convenient multivariate Student $t$ form. Note that (3) no longer involves any parameters and is perfectly suited to forecasting, often the ultimate aim of ARMA models. The order of the process $\mu$ can be formally treated as a parameter in the space of pairs of positive integers (as in Monahan 1983) with elements $\mu_{i}(i=$ $1, \ldots, K)$ and if we wish to select one particular value for $\mu$, we can consider the posterior odds ratio $(i, j=1, \ldots, K)$ :

$$
\begin{equation*}
\frac{p\left(\mu_{i} \mid y\right)}{p\left(\mu_{j} \mid y\right)}=\frac{p\left(\mu_{i}\right)}{p\left(\mu_{j}\right)} \cdot \frac{p\left(y \mid \mu_{i}\right)}{p\left(y \mid \mu_{j}\right)} \tag{4}
\end{equation*}
$$

the product of the prior odds $p\left(\mu_{i}\right) / p\left(\mu_{j}\right)$ and the Bayes factor, which is the ratio of predictive density values given the respective orders and which completely summarizes the relevant evidence in the data. However, if our interest is in forecasting or in certain (nonlinear) transformations of the parameters that have a direct economic interpretation, e.g. related to secular and cyclical behaviour, it is not required to pretest using (4), since we can just mix over the uncertain order as in (3). Our uncertainty about $\mu$ is then formally taken into account, resulting in a weighted average over different orders.

The assumption of stationarity that was tacitly imposed through prior restrictions on (2) has, however, been the focus of much recent debate in macroeconomics. Adding an intercept and a deterministic trend to (2), the empirical classical results of Nelson and Plosser (1982) on fourteen US time series suggest the presence of a unit root, in which case only the first differences are stationary (difference-stationarity). In this view the stochastic shocks $\varepsilon_{t}$ have a persistent effect, in contrast to the transitory effect implicit in the trend-stationary hypothesis. A popular parameterization for such models is (usually with $q=0$ )

$$
\begin{equation*}
y_{t}=\alpha+\delta t+\rho y_{t-1}+\sum_{i=1}^{p-1} \Psi_{i} \Delta y_{t-i}+\varepsilon_{t}+\sum_{j=1}^{q} \theta_{j} \varepsilon_{t-j} \tag{5}
\end{equation*}
$$

where $\rho=\sum_{i=1}^{p} \varphi_{i}, \Psi_{i}=-\sum_{j=i+1}^{p} \varphi_{j}$ and $\Delta y_{t-i}=y_{t-i}-y_{t-i-1}$, the first difference. Although other parameterizations have been suggested (see e.g. DeJong and Whiteman 1991) we shall use (5) for expository purposes. The unit root hypothesis then has the simple form $H_{o}: \rho=1$ and the model is trend-stationary for $|\rho|<1$. Whereas sampling properties of (5) display a discontinuity at a unit root, Bayesian procedures, which follow the likelihood principle and condition on the observed sample only, are not fundamentally affected by the presence of a unit root (see Sims 1988). Accordingly, Bayesian studies, such as DeJong and Whiteman (1991) and Koop (1991) find evidence of trend-stationarity in the Nelson-Plosser series. An animated debate over the choice of a standard "reference" prior distribution was sparked by Phillips (1991) who advocates
the use of Jeffreys' [1939] (1961) principles to represent prior ignorance about the parameters. Unfortunately, such an "objective" prior suffers from major defects in these dynamic models: it violates the likelihood principle and is data-based in that it depends on the sample size $T$, leading to different results if we observe the sample in, say, two parts. Also, the prior odds implicit in this improper prior distribution very strongly favour non-stationary behaviour $(|\rho| \geq 1)$ as opposed to stationarity. Even so, results for the Nelson-Plosser series are not dramatically different from those obtained under a uniform prior on $\rho$, thus still casting doubt over the "stylized fact" of a ubiquitous unit root. Another qualification arises from including structural breaks in (5). Using classical methods, Perron (1989) favours trend-stationarity with the only persistent shocks appearing exogenously as the 1929 crash and the 1973 oil price shock.

## Structural Econometric Models

If economic theory regarding the behaviour of a certain variable over time exists, we may use this in relating this variable not just to its own past, but to past and present values of other variables as well, treating the latter as "given". These explanatory variables should then be exogenous in the sense of Engle et al. (1983). Often the parameters in such models will be of some interest in themselves, if they directly relate to competing economic theories or if they quantify some stable relationship between economic variables. In such models, therefore, posterior analysis is relatively more important than in pure ARMA models. Forecasting is also of interest, though, especially in the form of policy predictions, where different values of the exogenous variables, corresponding e.g. to different government strategies, are fed into these models.

Although models that are nonlinear in the parameters can, in principle, be accommodated, let us focus on the simple dynamic linear regression model:

$$
\begin{equation*}
y=X \beta+\varepsilon \tag{6}
\end{equation*}
$$

where $\varepsilon=\left(\varepsilon_{1}, \ldots, \varepsilon_{T}\right)^{\prime}$ and usually the same Normality assumption is made as in (2). The $T \times k$ matrix $X$ groups past observations of the variable $y$ as well as present and past observations of other variables, and $\beta$ is an unknown vector of coefficients. As the functional form of the likelihood does not depend on whether $X$ includes lagges $y$ 's or not, the posterior analysis of $\left(\beta, \sigma^{2}\right)$ is not at all affected by the dynamic character of (6). Only predicting further than one period ahead becomes more complicated, as shown in Chow (1973). In particular, given a natural-conjugate Normal-inverted gamma prior $p\left(\beta, \sigma^{2}\right)$ or a diffuse reference prior, the posterior densities have analytically known properties. Independent Student-inverted gamma priors, which are often a more natural choice, will result in posteriors that have the poly-t density form (Drèze 1977) and can readily be analysed with the algorithms in Richard and Tompa (1980). Finally, other prior densities, reflecting e.g. sign restrictions on the coefficients, will require numerical integration, usually with Monte Carlo techniques (see e.g. Geweke 1986).
In the structural context of (6), the error term $\varepsilon$ is now often interpreted as a "derived"
quantity, namely everything in $y$ not explained by linear regression on (the stochastic) $X$, rather than as shocks driving the system in (2).
If the $\varepsilon_{t}$ are independent over time, the Normality assumption is crucial for the above results. However, if we allow for dependence within the vector $\varepsilon$, we obtain exactly the same results for $p(\beta \mid y)$ and $p(y)$ as under Normality with any member of the multivariate elliptical class, provided we adopt the "usual" noninformative prior on $\sigma^{2}$ (Osiewalski and Steel 1990). This gives us robustness e.g. with respect to heavy tailed multivariate Student distributions, but also with respect to the truncated Pearson II family.

If we feel that the vector $\varepsilon$ for one variable modelled through (6) is correlated with the error vector of another such variable, because e.g. both sets of errors are influenced by common (world-wide) events not captured by the information set (again denoted by $X$ ), we can use the so-called seemingly unrelated regression equations (SURE) model introduced by Zellner (1962):

$$
\begin{equation*}
Y=X B+E \tag{7}
\end{equation*}
$$

where now $Y$ and $E$ are $T \times n$ matrices with columns corresponding to $y$ and $\varepsilon$ in (6) and $B$ is a $k \times n$ matrix of unknown coefficients. $E$ is assumed to follow a matricvariate generalization of a Normal distribution (see e.g. Drèze and Richard 1983:586).
Often, (linear) prior restrictions are imposed on $B$ since not all variables in $Y$ require the same regressors; e.g. if $Y$ groups wages in different countries, we may want to include only country-specific $X$ variables. Such restrictions will, in general, preclude a naturalconjugate prior and Richard and Steel (1988) advocate Monte Carlo analysis on the covariance matrix of the columns in $E$. Using recursive properties, Steel (1991b) finds analytical expressions for posterior moments of the unrestricted elements in $B$ in some cases outside the natural-conjugate framework. However, in general cases of practical interest or if we want full posterior densities, Monte Carlo procedures seem inevitable.

The model in (6) only allows for one endogenous variable, e.g. money demanded ( $m_{t}$ ) on which, say, interest rate $\left(r_{t}\right)$ and price $\left(p_{t}\right)$ have a contemporaneous effect. However, if the implicitly underlying process of interest rate and price carries any information relating to the money demand equation, these variables can no longer be treated as exogenous, and have to be modelled jointly with money. A simultaneous equation model $(\mathrm{SEM})$ for $y_{t}=\left(m_{t}, p_{t}, r_{t}\right)^{\prime}$ then results, written as

$$
\begin{equation*}
Y \Gamma=X B+E, \tag{8}
\end{equation*}
$$

with $\Gamma$ a square nonsingular $n \times n$ matrix of coefficients, again subject to restrictions. Clearly, the SURE system in (7) is a special case of (8) with $\Gamma=I_{n}$, the identity matrix. A limited information analysis of the SEM is conducted in Drèze (1976) and in Zellner et al. (1988), whereas Bauwens (1984) and Steel (1991a) use Monte Carlo procedures to perform a Bayesian analysis under full information. An insightful survey is provided in Drèze and Richard (1983).

## Epilogue

More often than not, a Bayesian analysis of an empirically interesting time series model will require some form of numerical integration.
However, the spectacular increase in computing power combined with substantial progress in the implementation of numerical procedures, such as Monte Carlo importance sampling (as in e.g. Geweke 1989) and the recently introduced Gibbs sampling methods (see e.g. Gelfand and Smith 1990), will make the Bayesian framework more and more accessible to applied rescarchers in economics. I hope many of them will be prepared to explore the advantages these new and exciting possibilities hold. The Bayesian paradigm was never meant for the "chosen few", it is of far too much practical value for that!

Mark F.J. Steel

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## Bayesian Inference in Time Series

Technical terms for inclusion in the Glossary:
ARCH models
difference-stationary models exogeneity*
elliptical densities (multivariate)*
full information analysis
Gibbs sampling
invertibility (in ARMA models)
likelihood principle
limited information analysis
money illusion
Monte Carlo integration*
natural-conjugate prior*
poly- $t$ densities*
posterior odds
reference or noninformative prior
stationarity (in ARMA models)
trend-stationary models

* own definition included.


## Bayesian Inference in Time Series

Definitions for the Glossary.

Exogeneity: While modelling some variable(s) $y$ we often wish to condition on other variables, say $z$, and treat these as if they are given, without explicitly modelling them. However, we should be careful not to lose any relevant information embedded in the marginal process of $z$. Different types of exogeneity conditions essentially validate different uses of the conditional model for $y$ given $z$ : weak exogeneity allows inference on the parameters of interest, say $\varphi$, strong exogeneity sustains conditional forecasting given future values for $z$ as well, and super exogeneity validates conditional inference on $\varphi$ under interventions that affect the process of $z$.

Elliptical densities (multivariate): The family of multivariate elliptical or ellipsoidal densities has constant density values over ellipsoids in $n$-dimensional space. If $y$ has an $n$-variate continuous elliptical distribution, its density is given by

$$
p(y)=|V|^{-\frac{1}{2}} g\left[(y-m)^{\prime} V^{-1}(y-m)\right]
$$

with location vector $m \in \mathbb{R}^{n}, V$ a positive definite scale matrix of dimension $n \times n$ and where the nonnegative function $g(\cdot)$ satisfies

$$
\int_{0}^{\infty} u^{\frac{n}{2}-1} g(u) d u=\Gamma\left(\frac{n}{2}\right) \pi^{-\frac{n}{2}} .
$$

Choosing tailbehaviour by $g(\cdot)$ we can cover e.g. the Normal, the heavy-tailed Student $t$ and Cauchy, but also the truncated Pearson type II distributions.

Monte Carlo integration: An analytical integral is approximated by a sample average of drawings from numerical distributions. Clearly, by exploiting some knowledge about the problem at hand we can achieve greater accuracy with a given amount of drawings. One way to do this is through importance sampling, where we concentrate on the most informative regions. We then draw from some importance function, which should closely approximate the function we are analysing, yet should permit relatively efficient pseudorandom drawings, which is typically not the case for the actual function itself.

Natural-conjugate prior: Within Bayesian analyses, these families of prior densities often lead to a particularly tractable analysis. Essentially, they share the functional form of the likelihood (the sampling density viewed as a function of the parameters), and combine very naturally with the sample information in exponential (e.g. Normal) families of data distributions. Generally, they can be interpreted as information emanating from a hypothetical prior sample and in exponential sampling models the posterior will be of the same (tractable) functional form as the natural-conjugate prior.

Poly-t densities: Frequently, the posterior density resulting from a Bayesian regression analysis will be in this class of multivariate densities. Their kernel (i.e. the density function without integrating constant) is a product or ratio of products of multivariate Student $t$ kernels. Accordingly, such densities are denoted by product-form poly-t (also called multiple t) or ratio-form poly-t. Multimodality and skewness can easily be accommodated by this flexible family of densities. Although their integrating constants and moments are generally not known analytically, they can be calculated using efficient and reliable numerical algorithms.
(For previous papers please consult previous discussion papers.)

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