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by Werner Güth
and Eric van Damme

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by

Werner Güth, **)
Eric van Damme, ***)

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**) Dept. of Economics, University of Frankfurt (Main).

***) CentER for Economic Research, Tilburg University and SFB 303, University of Bonn.

Institut für Gesellschafts- und Wirtschaftswissenschaften
Abteilung Wirtschaftstheorie II
Rheinische Friedrich-Wilhelms-Universität Bonn
Adenauerallee 24-26
D-5300 Bonn 1
West Germany
Telefon (0228) 73 92 42

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Contents

Abstract

The paper studies the most simple version of the Spence job market signaling model in which there are just 2 types of workers while education is not productivity increasing. To eliminate the multiplicity of equilibria, the general equilibrium selection theory of John Harsanyi and Reinhard Selten is applied. It is shown that this theory selects Wilson's E_2-equilibrium as the solution.

1. INTRODUCTION

Recently, signaling games have been extensively studied by economic theorists and game theorists. In such games, there is an informed party who sends a message to which one or several uninformed players react; the payoffs of the participants depend on the private information of the informed party, the signal that this player sends and the responses that the uninformed players take. Many models of economies with informational asymmetries contain a signaling game as an essential ingredient. For a (very) partial overview of this huge literature, see Van Damme [1987, Sect. 10.4].

A signaling game typically admits a great multiplicity of equilibria. This is caused by the fact that the signal space is usually larger than the type space of the informed player so that there exist unused signals. The Nash equilibrium concept does not tie down the uninformed agents' beliefs and actions at such unreached information sets, and this arbitrariness of non-equilibrium responses in turn allows many outcomes to be sustained in equilibrium. As a consequence, the class of signaling games has provided fertile playing ground for game theorists working on equilibrium refinements. Indeed many refinements of the Nash concept have been defined initially only for such games although in most cases the concepts can be extended to general extensive form games. Again, see Van Damme [1987, Sect. 10.5] for a partial survey.

To reduce the multiplicity of equilibria in signaling games arising in economic contexts, variations on the theme of iterative elimination of dominated strategies have been most popular. Some of such (sometimes ad hoc) procedures can be justified by refering to the general notion of stable equilibria introduced in Kohlberg and Mertens [1986]. Indeed, various authors claim additional virtue for the result they obtain by stating that the outcome obtained is the only one satisfying the Kohlberg/Mertens stability criterion. However, the latter stability notion has its share of counterintuitive examples (see, for instance, Van Damme [1988]), and even the seemingly unobjectionable notion of iterative elimination of ordinary weakly dominated strategies is not without pitfalls as Binmore [1987] has argued. As a consequence, the present authors are not convinced that the theory that is currently accepted by the majority of researchers in the field will survive in the long run; there is scope for alternative theories.

Our aim in this paper is to illustrate such an alternative theory, viz. the equilibrium selection theory of John Harsanyi and Reinhard Selten [1988](the HS theory), by means of applying it to a simple signaling game arising in an economic context. Specifically, we will study the most basic version of the job market signaling model introduced in the seminal work of Spence [1973,1974]. We take this well-known model rather than an abstract signaling game for didactical reasons. Since many economic signaling models have a mathematical structure similar to Spence's model, arguments similar to the ones we will use will come up in their analyses and the reader can get a good idea for how the HS theory works by reading the present paper. In the literature, the reader can already find several applications of the HS theory, however, all of these are to games that admit multiple strict equilibria, and in these stability *a la* Kohlberg/Mertens is not very powerful. (Recall that Kohlberg and Mertens do not claim that their theory is a selection theory, cf. Fn. 2 of their paper.) To our knowledge, the present paper is the first to apply the HS theory to a game in which the multiplicity of equilibria is caused solely by the existence of unreached information sets, i.e. by the perfection problem.

There is no doubt that there are fundamental differences between the HS (general) theory and alternative (partial) theories such as Kohlberg/Mertens stability. We urge the reader to read the postscript of Harsanyi and Selten [1988] in which some of these are described. Perhaps the most profound difference is that HS work with the standard form of the game (this is basically the agent normal form), whereas stability is a normal form concept. Accordingly, HS reject ideas of 'forward induction' since they conflict with their 'subgame and truncation consistency'. Since in our game no player has 2 strategic moves along the same path[1)] , this difference is not relevant for our model, however. (Also, cf. Mertens [1988]).

On the other hand, there are common elements in the various theories: HS iteratively eliminate inferior strategies of the perturbed standard form, Kohlberg/Mertens stability allows iterative elimination of dominated strategies in the normal form, and dominated strategies are inferior. Apart from the difference in game form, the main difference here is in the order of operations: HS perturb first and thereafter eliminate, whereas Kohlberg/Mertens follow the

[1] The informed player moves twice, but the second move (the choice of employer) is not really strategic, subgame perfectness determines this choice uniquely.

opposite order. A simple example in Appendix A illustrates that the order in which the operations are carried out may make a difference in general. However, in this paper we show that this is not so in the present case, i.e. if the unperturbed Spence game is dominance solvable (which holds if the proportion of low quality workers is relatively high), the Harsanyi/ Selten solution is the equilibrium that remains after all dominated strategies have been iteratively eliminated, i.e. the Pareto best separating equilibrium. Actually, the proof of this proposition constitutes the heart of the present paper, hence, the parameter constellation that is trivially to solve according to conventional methods poses the greatest difficulty for the HS theory. The case where the proportion of low ability workers is small (so that the game is not dominance solvable) is relatively easy to solve by means of the HS theory. In this case, we show that only the Pareto best pooling equilibrium spans a primitive formation, therefore, the HS theory determines this equilibrium as the solution. To summarize, in our simple model, the solution obtained by the HS theory coincides with the E_2-equilibrium notion proposed by Wilson [1977]. It should be stressed that to obtain this result we do not make use of Pareto comparisons. Although such comparisons play a role in the HS theory, in our analysis we will never be in the position that the HS theory allows us to make such a comparison. Finally it should be remarked that it remains to be investigated for which class of games the Wilson and HS solutions coincide.

The remainder of the paper is organised as follows. In Sect. 2 we introduce the model and derive the Nash equilibria of the unperturbed game. (The model is chosen so that every Nash equilibrium is sequential). The HS theory cannot be applied to this game directly, rather one has to apply the theory to a sequence of uniformly perturbed games and then let the perturbances go to zero. In the Sects. 3 and 4 it is investigated which equilibria of the original game are uniformly perfect, i.e. which equilibria can be approximated by equilibria of such uniformly perturbed games. HS consider as the initial set of solution candidates of a perturbed game the so called primitive equilibria, i.e. those equilibria that span primitive formations. A formation is a subset of strategy combinations that is closed under taking best replies. A primitive formation is a minimal one, this concept generalizes the notion of a strict equilibrium point. In Sect. 5 we investigate the formation structure of the uniformly perturbed games. We show that, if the proportion of able workers is sufficiently high, only the Pareto best pooling equilibrium spans a primitive formation, hence, this equilibrium is the HS solution if there are many able workers. If there are only few high able workers, then there are multiple primitive formations, in particular, the Pareto best pooling equilibrium as well as all separating equilibria in which the able worker does not invest too much in education span primitive formations. In this case, the HS solution is determined by applying the risk dominance criterion (in which use is made of the tracing procedure). In Sect. 6 it is shown that the Pareto best separating equilibrium risk dominates all other equilibria, hence, this equilibrium is the solution if there are few high able workers. Sect. 7 offers a brief conclusion.

2. MODEL AND EQUILIBRIA

Let Y be a finite set (of possible education choices) and let W be the finite set of possible wages that firms can offer. The entire paper is devoted to the analysis of the signaling game $\Gamma(Y, W, \lambda)$ described by the following rules:

(2.1) A chance move determines the type t of player 1 (the worker); with probability λ the type is 1, with probability $1-\lambda$ the type is 0; only player 1 gets to hear his type.

(2.2) Player 1 chooses $y \in Y$.

(2.3) Two firms (the players 2 and 3) observe the $y \in Y$ chosen and they then simultaneously offer wages $w_2, w_3 \in W$.

(2.4) The worker observes the wages offered and chooses a firm.

(2.5) The payoff (von Neumann-Morgenstern utility) is 0 for a firm that does not attract the worker, it is $t - w$ for a firm that pays the wage w to a worker of type t, and it is $w - y$ (resp. $w - y/2$) for a worker of type 0 (resp. type 1) that receives the wage w after having chosen the education level y.

Several comments are in order concerning the above specification

(i) The main difference between our game and the basic model of Spence [1973] is that in our case the least able worker is not productive. This assumption simplifies the analysis somewhat since it ensures that every

Nash equilibrium is a sequential equilibrium. With minor modifications our arguments also apply to Spence's specification, however.

(ii) To reduce the number of parameters, we have normalised the education cost such that these are twice as high for the type 0 worker as for the type 1 worker. Given our other assumptions, this normalisation is without loss of generality; the reader may verify that the HS theory yields the Wilson E_2-equilibrium as long as the least able worker has higher education cost.

(iii) Competition between the firms is modeled as a Bertrand game between two firms. It may be checked that the same results would be obtained with n firms, $n \geq 2$. Of course, if there is just one firm the solution is completely different (the firm offers a wage of zero and the workers do not invest in education).

(iv) Since the HS theory aplies only to finite games, we are forced to work with finite sets Y and W, hence, we discretize the continuous specification of Spence. Throughout, we will assume that this discretization is sufficiently fine, and at the end we will let the diameter of the grid go to zero.

Let $g > 0$ denote the smallest money unit, all wages have to be quoted in integer multiples of g, hence

(2.6) $$W = \{kg;\ k = 0, 1, 2, ..., K\}.$$

To simplify the analysis somewhat, it will be assumed that

(2.7) $$Y \subset W \text{ and } 0, \lambda, 1, 2 \in Y\ ,$$

an assumption that is, however, not essential for our results to hold (Details are available from the authors upon request). To avoid some further technical uninteresting difficulties, let us assume that the grid of W is finer than that of Y, specifically

(2.8) $$\text{if} \quad y, y' \in Y \quad \text{and} \quad y \neq y', \quad \text{then} \quad |y - y'| > 4g\ .$$

Finally, we introduce the following convenient notation:

(2.9) $$\text{If} \quad x \in \mathbf{R}, \text{then} \quad x^- = max\{w \in W | w < x\}\ .$$

Next, let us start analysing the game $\Gamma(Y, W, \lambda)$. Note that this game admits several (trivial) subgames in stage (2.4): Subgame perfection requires that at this stage the worker chooses the firm offering the highest wage, and symmetry implies that the worker should randomize evenly over the firms in case they offer the same wage. Hence, it is natural to analyse the truncated game obtained by constraining the worker to behave in this way in stage (2.4). The HS theory allows this procedure to be followed. [2] Therefore, from now on, attention will be restricted to the truncated game. This game will be denoted $G(Y, W, \lambda)$

Denote by $s_t(y)$ the probability with which the type t worker takes the education choice y. Hence, (s_0, s_1) is a behavioral strategy of player 1 in $G(Y, W, \lambda)$. Let $s(y)$ be the probability that education level y is chosen

(2.10) $$s(y) = \lambda s_1(y) + (1 - \lambda)s_0(y),$$

and let $\mu(y)$ be the expected productivity if y is chosen with positive probability. By Bayes' rule

(2.11) $$\mu(y) = \lambda s_1(y)/s(y) \qquad \text{if} \quad s(y) > 0.$$

Assume $s(y) > 0$. In any Nash equilibrium of $G(Y, W, \lambda)$, firms, in response to y, play a Bertrand equilibrium of the game in which it is known that the surplus (i.e. the worker's productivity) is $\mu(y)$. The following Lemma specifies the equilibria of this game.

Lemma 2.1. *The game in which two firms compete for a surplus μ by means of wage offers $w_i \in W$ has the following equilibria:*

(a) *If $\mu \in W$, both firms either offer μ, or μ^-, or μ^{--}.*

(b) *If $\mu \notin W$, both firms either offer μ^-, or μ^{--} or they randomize between μ^- and μ^{--} choosing μ^- with probability $(g - \mu + \mu^-)/g$, where g is as in (2.6) (i.e. $g = \mu^- - \mu^{--}$).*

---

[2] Actually, the HS theory requires to first perturb the game before one starts replacing subgames (which are cells in the perturbed game) by their solutions. To simplify the exposition, we have interchanged the operations of perturbing and decomposition, what, for the case at hand, does not influence the final result.

Proof. A straightforward argument establishes that, if $\mu \notin W$, only μ^- and μ^{--} can be in the support of an equilibrium. By investigating the 2x2 game in which each firm chooses between μ^- and μ^{--}, conclusion b) follows easily from (2.6). If $\mu \in W$, the equilibrium support now possibly also includes μ and conclusion a) follows easily by investigating the 3x3 game in which each firm chooses between μ, μ^- and μ^{--}.

□

Remark 2.2. Note that in case (a) of Lemma 2.1, the equilibrium in which both firms offer μ is not perfect if $\mu > 0$ (bidding μ is dominated by bidding μ^-), while the other two equilibria are perfect (even uniformly perfect) if $\mu^{--} > 0$. At a certain stage of the solution process, the HS theory eliminates the equilibrium μ^{--} since it is not primitive (when my opponent offers μ^{--}, I am indifferent between μ^{--} and μ^-, also see Sect. 5). The equilibrium (μ^-, μ^-), being strict is definitely primitive. Consequently, in case (a), we will restrict attention to this equilibrium right from the beginning. (Also note that μ^{--} cannot occur in equilibrium if there are more than two firms). In case (b) all three equilibria are (uniformly) perfect. The mixed equilibrium is not primitive. To simplify the statements of results to follow somewhat, and without biasing the final result, we will restrict attention to the equilibrium (μ^-, μ^-) also in this case.

Let $s = (s_0, s_1)$ be an equilibrium strategy of player 1 in $G(Y, W, \lambda)$ and write $Y(s) = \{y;\ s(y) > 0\}$. The above lemma and remark, with $\mu = \mu(y)$ as in (2.11) determines the equilibrium response of the firms at y. Assume $y, y' \in Y(s)$ with $y < y'$. From optimizing behavior of the worker, one may conclude that $\mu(y) < \mu(y')$. Since it cannot be the case that both types of worker are indifferent between y and y' we must have $\mu(y) = 0$ or $\mu(y') = 1$, hence, in any equilibrium at most three education choices can occur with positive probability. The Nash concept does not specify the equilibrium responses of firms at education levels $y \in Y/Y(s)$. To generate the set of all equilibrium outcomes we may put $w_i(y) = 0$ at such y, since this is the best possible threat of the firms to avoid that y will be chosen. Using this observation it is not difficult to prove that the set of Nash equilibrium paths of $G(Y, W, \lambda)$ is as described in the following Proposition.

Proposition 2.3. *(s, w) with $s = (s_0, s_1)$ and $w = (w_2, w_3)$ with $w_i : Y(s) \to W$ is a Nash equilibrium path of $G(Y, W, \lambda)$ if and only if*

$$(2.12) \qquad \text{if} \quad s_t(y) > 0, \text{ then} \begin{cases} y \in \arg\max_{Y(s)} \; w(y) - y/(t+1) \\ \text{and} \quad w(y) - y/(t+1) \geq 0 \end{cases}$$

$$(2.13) \qquad w(y) = w_1(y) = w_2(y) = \mu(y)^- \text{ for all } y \in Y(s)$$

In the following sections, emphasis will be on those equilibria in which player 1 does not randomize. (The other equilibria are not primitive, hence, the HS theory eliminates them, see Sect.6). These equilibria fall into two classes:

(a) pooling equilibria, in which the two types of worker take the same education choice, hence $Y(s) = \{\overline{y}\}$ for some $\overline{y} \in Y$. Then $\mu(\overline{y}) = \lambda$ and $w(\overline{y}) = \lambda^-$.Therefore, (2.12) shows that $\overline{y} \in Y$ can occur in a pooling equilibrium if and only if $\overline{y} < \lambda$.

(b) separating equilibria, in which the education choice y_0 of type 0 is different from the choice y_1 of type 1. Proposition (2.3) shows that the pair (y_0, y_1) can occur in a separating equilibrium if and only if $y_0 = 0$ and

$$(2.14) \qquad 1^- - y_1 \leq 0 \leq 1^- - y_1/2$$

Hence, we have

Corollary 2.3. *For any $\overline{y} \in Y$ with $\overline{y} < \lambda$, there exists a pooling equilibrium of $G(Y, W, \lambda)$ in which both players choose $\overline{y}$. There exists a separating equilibrium in which type t chooses y_t if and only if $y_0 = 0$ and $1 \leq y_1 < 2$.*

3. THE UNIFORMLY PERTURBED GAME

$G(Y, W, \lambda)$ is a game in extensive form. The HS theory is based on a game form that is intermediate between the extensive form and the normal form, the so called standard form. In our case, the standard form coincides with the agent normal form (Selten [1975]) since no player moves twice along the same path. The agent normal form has $2|Y| + 2$ active players, viz. the 2 types of player 1, and for each $y \in Y$ and each firm i an agent iy that is responsible for the firm's wage offer at y. This agent normal form will again be denoted by $G(Y, W, \lambda)$. Note that, if (s, w) is a pure strategy combination

($s = (s_0, s_1)$ with $s_t(y_t) = 1$ for some $y_t \in Y$, $w = (w_2, w_3)$ with $w_i : Y \rightarrow W$), then the payoffs to agent t (the type t worker, $t = 0, 1$) are

$$(3.1) \qquad H_t(s, w) \; = \; max_{i=1,2}\, w_i(y_t) - y_t/(t+1),$$

while the payoffs to agent iy are

$$(3.2) \qquad H_{iy}(s, w) \; = \begin{cases} 0 & \text{if } \; s(y) = 0 \text{ or } w_i(y) < w_j(y) \\ \mu(y) - w_i(y) & \text{if } \; s(y) > 0 \text{ and } w_i(y) > w_j(y) \\ (\mu(y) - w_i(y))/2 & \text{if } \; s(y) > 0 \text{ and } w_i(y) = w_j(y) \end{cases}$$

To ensure perfectness of the final solution, the HS theory should not be directly applied to $G(Y, W, \lambda)$, but rather to a sequence of uniformly perturbed games $G_\epsilon(Y, W, \lambda)$ with $\epsilon > 0$, ϵ small, ϵ tending to zero. The latter games differ from the original one in that each agent cannot completely control his actions. Specifically in $G_\epsilon(Y, W, \lambda)$, if agent t intends to choose y_t, then he will by mistake also choose each $y \in Y$, $y \neq y_t$ with a probability ϵ and he will actually play the completely mixed strategy s_t^ϵ given by

$$(3.3) \qquad s_t^\epsilon(y) \; = \begin{cases} \epsilon & \text{if } y \neq y_t \\ 1 - (|Y| - 1)\epsilon & \text{if } y = y_t, \end{cases}$$

More generally, if the type t worker intends to choose the mixed strategy s_t then he will actually play the completely mixed strategy s_t^ϵ given by

$$(3.4) \qquad s_t^\epsilon(y) = s_t(y)(1 - |Y|\epsilon) + \epsilon$$

(Of course, ϵ should be chosen so small that $|Y|\epsilon < 1$). Similarly, if agent iy intends to choose $w_i(y)$, he will actually also choose all different wages with the same positive probability ϵ. Formally, the uniformly perturbed game $G_\epsilon(Y, W, \lambda)$ has the same agents as players, these have the same strategies available (which are now interpreted as intended choices), but the payoff function H^ϵ is slightly different from that in (3.1) – (3.2) and takes into account the above described mistake technology. If ϵ is small, the payoffs H^ϵ of those agents moving on the equilibrium path are close to the payoffs as described by H, however, for agents iy that cannot be reached intentionally there is a discontinuity. Namely, since both types of player 1 choose y with probability ϵ in the perturbed game, such agents play a Bertrand game with surplus λ. It will be convenient to assume that actually firms never make mistakes. The reader may verify that this assumption does not influence our results, it just simplifies notation. In this case, the perturbed game payoff to agent iy is given by

$$(3.5) \qquad H^{\epsilon}_{iy}(s,w) = \begin{cases} \mu^{\epsilon}_{s}(y) - w_i(y) & \text{if } w_i(y) > w_j(y) \\ (\mu^{\epsilon}_{s}(y) - w_i(y))/2 & \text{if } w_i(y) = w_j(y) \\ 0 & \text{if } w_i(y) < w_j(y) \end{cases}$$

where the expected productivity at y is given by

$$(3.6) \qquad \mu^{\epsilon}_{s}(y) = \frac{\lambda s^{\epsilon}_{1}(y)}{\lambda s^{\epsilon}_{1}(y) + (1-\lambda)s^{\epsilon}_{0}(y)}$$

with $s^{\epsilon}_{t}(y)$ as in (3.4).

If firms do not make mistakes, the workers' payoffs are (up to a positive affine transformation, depending on the worker's own mistakes) exactly as in (3.1). Since affine transformations leave the solution invariant, we will consequently analyse the game in which the types of the worker have pure strategy set Y, the agents of the firms have strategy set W and in which the payoffs are as in (3.1), (3.5). This game will be denoted $\tilde{G}_{\epsilon}(Y, W, \lambda)$. Note that the perturbed game $\tilde{G}_{\epsilon}(Y, W, \lambda)$ is an ordinary agent normal form game of which we will analyse the Nash equilibria. There is no need to consider a more refined solution concept: All "trembles" have already been incorporated into the payoffs.

After having uniformly perturbed the game, the next step, in applying the HS theory consists in checking whether the game is decomposable, i.e. whether there exist (generalized) subgames, so called cells. (See the diagram on p. 127 of Harsanyi and Selten [1988]). It may be thought that, for each $y \in Y$, the agents $\{2y, 3y\}$ constitute a cell. Indeed these agents do not directly compete with an agent iy' with $y' \neq y$. However, they directly interact with both types of the worker and, hence, through the worker they indirectly compete also with agent iy'. To put it differently, the game does not contain any (proper) subcells, it is indecomposable. Therefore, we move to the next stage of the solution procedure, the elimination of inferior choices from the game.[3)]

[3] It will turn out that this step is actually redundant in the case at hand, i.e. the reader may move directly to Sect. 5 if he prefers. Since one of our aims is to illustrate the various aspects of the HS theory, we thought it best to include the discussion on inferior strategies.

4. ELIMINATION OF INFERIOR STRATEGIES

A strategy is inferior if its stability set, i.e. the region of opponents' strategy combinations where this strategy is a best reply, is a strict subset of another strategy's stability set. It can be verified that in our model, the inferior strategies are exactly those that are weakly dominated. Clearly, such strategies exist in our model: For any agent iy offering a wage greater than or equal 1 is weakly dominated. Furthermore, for the type t worker, choosing y with $y/(t+1) \geq maxW$ is dominated by choosing 0. HS require that one eliminates all such choices, and that one continues this process until there are no inferior choices left. (Along the way, one should also check whether the reduced game obtained contains cells, but this will never be the case in our model, see also Fn. 3). The following Proposition describes the irreducible game that results after all inferior strategies have been successively eliminated.

Proposition 4.1. *If ϵ is sufficiently small, then by iterative elimination of inferior strategies, the game $\tilde{G}_\epsilon(Y, W, \lambda)$ reduces to the game $\tilde{G}^r_\epsilon(Y, W, \lambda)$ in which the following strategies are left for the various agents:*
For type 0 : $Y^r_0 = \{y \in Y;\ y < 1\}$
For type 1 : $Y^r_1 = \{y \in Y;\ y < 3 - max(2\lambda, 1)\}$
For agent iy with $y < 1$: $W^r_{iy} = \{w \in W;\ w < 1\}$
For agent iy with $1 \leq y < 3 - max(2\lambda, 1)$: $W^r_{iy} = \{w \in W; w \in [\lambda^-, 1)\}$
For agent iy with $y \geq 3 - max(2\lambda, 1)$: $W^r_{iy} = \{\lambda^-\}$

Proof. Clearly $w \in W$ with $w \geq 1$ is inferior for any agent iy. Once these inferior strategies have been eliminated, choosing $y \geq 1$ becomes inferior for type 0 (it is dominated by choosing $y = 0$) and $y \geq 2$ becomes inferior for type 1. We claim that in the resulting reduced game an agent iy with $y < 1$ does no longer have any inferior actions. Namely, $w_i(y) = 0$ is the unique best response against $w_j(y) = 0$ if $s_0(y) = 1$ and $s_1(y) = 0$ (hence $\mu^\epsilon_s(y) < g$), so that offering zero is not inferior (Here we need that ϵ is small). Furthermore, if $w \leq 1^{--}$, then w is the unique best response of agent iy against $w_j(y) = w^-$ if $\mu^\epsilon_s(y) \approx 1$ (i.e. $s_0(y) = 0$ and $s_1(y) = 1$). Therefore, such wages also are not inferior. Finally, since both firms offering $w = 1^-$ at y is a strict equilibrium if $\mu^\epsilon_s(y) > 1^-$, also $w = 1^-$ is not inferior, which establishes the claim. Next, turn to an agent iy with $y \geq 1$. Since the type 0 worker cannot choose y voluntarily, we either have $s(y) = 0$, hence $\mu^\epsilon_s(y) = \lambda$, or the type 1 worker chooses y voluntarily, in which case $\mu^\epsilon_s(y) \approx 1$. Consequently, at y the firms play a Bertrand game for a surplus of λ or of 1. The standard Bertrand argument implies that, starting with $w = 0$, all wages with $w < \lambda$ will be iteratively eliminated. On the other hand, the same argument that was used for the case $y < 1$ establishes that wages strictly between λ^{--} and 1 cannot be eliminated. The reduction of the firms' strategy sets obtained in this way does not introduce new inferior strategies for the type 0 worker. Namely, if this worker expects that $y^* < 1$ will result in the wage 1^- while all other $y < 1$ yield wage 0, then the unique best response is to choose y^*, so that y^* cannot be inferior. If $\lambda > \frac{1}{2}$ the reduction leads to new inferior strategies for the type 1 worker, however. Namely, by choosing $y = 1$, he receives an income of at least $\lambda^- - \frac{1}{2}$ so that any eduction choice $y \geq 3 - 2\lambda$ becomes inferior. Finally, if y is inferior for both the type 0 and the type 1 worker, then firms at y play a Bertrand game for surplus λ and we already argued above that in this case only λ^- is not (iteratively) inferior. Hence, we have shown that $\tilde{G}_\epsilon(Y, W, \lambda)$ can be reduced to at least $\tilde{G}^r_\epsilon(Y, W, \lambda)$ as specified in the Lemma. Since the latter game is irreducible, the proof is complete.

□

The following Figure displays the result of Proposition 4.1. Along the horizontal y-axis, we indicate the noninferior actions of the worker. For each value of y, the shaded area displays the noninferior responses of the firms. The Figure is drawn for the case $\lambda > \frac{1}{2}$.

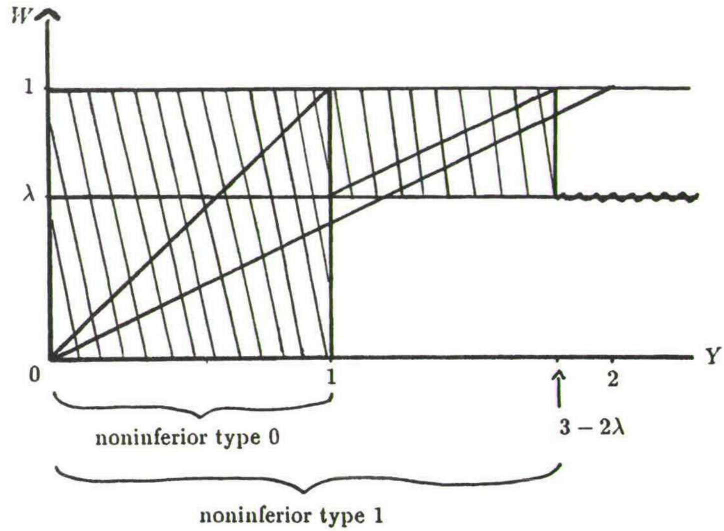

Figure 1. The reduced perturbed game $\tilde{G}^r_\epsilon(Y, W, \lambda)$.

By comparing Proposition 4.1 with Proposition 2.2 it is seen that, at least if $\lambda > \frac{1}{2}$, some equilibria of the original game are eliminated by considering the reduced perturbed game. Specifically, separating equilibria in which type 1 invests very much in education as well as some equilibria involving randomization are no longer available in $\tilde{G}^r_\epsilon(Y, W, \lambda)$. All in all, however, this step of the HS solution procedure is not very successfull in cutting down on the number of equilibrium outcomes.

It is also instructive to compare Proposition 4.1 with the result that would have been obtained by iterative elimination of dominated strategies in the unperturbed game $G(Y, W, \lambda)$. In the latter, after wages $w \geq 1$ have been eliminated, choosing $y \geq 1$ becomes dominated for type 0, but not for type 1, hence, whenever such y is chosen, firms play a Bertrand game with surplus 1 so that all wage offers except 1^- are dominated. Hence, by choosing $y = 1$, the type 1 worker guarantees a utility of (about) $\frac{1}{2}$. In particular, choosing $y > 1$ is now dominated for this type. We see that the reduced game associated with the unperturbed game is strictly smaller than $\tilde{G}^r_\epsilon(Y, W, \lambda)$. The difference is especially dramatic if $\lambda < \frac{1}{2}$. Proposition 2.2 shows that in this case there is (essentially) only one equilibrium of the unperturbed game that remains in the reduced unperturbed game, viz. the separating equilibrium with $y_1 = 1$. Hence, if $\lambda < \frac{1}{2}$, the unperturbed game is dominance solvable [4] , and indeed, the conventional analysis generates the efficient separating equilibrium ($y_0 = 0, y_1 = 1$) as the outcome when $\lambda < \frac{1}{2}$. The reduced perturbed game, however, does not force agents iy with $y \in (1, 2)$ to offer the wage of 1 since they may still think that they are reached by mistake. As a consequence, $\tilde{G}^r_\epsilon(Y, W, \lambda)$ still admits many equilibria even if $\lambda < \frac{1}{2}$, and the HS theory does not immediately yield efficient sepapration as the solution. Still we will show that, the risk dominance criterion of HS forces efficient separation as the solution in this case.

5. UNIFORMLY PERFECT EQUILIBRIA

As argued before, the multiplicity of equilibria in our model is caused solely by the imperfectness problem, i.e. by the fact that there necessarily exist unreached information sets at which the firms' beliefs are undetermined. At such education choices, firms may threaten to offer a wage of zero which in turn implies that the worker will indeed not take such a choice. The final consequence is that there exist equilibria with unattractive payoffs for the worker.

The HS theory solves the game via its uniformly perturbed game to avoid the imperfectness problem. The natural question to ask is how successfull this step of the solution procedure is to reduce the number of equilibria. At first glance it appears as if this step is very successfull, it seems that the uniformly perturbed game admits just a single equilibrium. Namely, assume that the equilibrium payoff of the type 0 worker would be strictly less than λ^-. Motivated by Proposition 2.3, it seems natural to assume that there exists y sufficiently

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[4] One could also perform this elimination in the extensive form. Since $y \geq 1$ is dominated for type 0, one eliminates the branch in which type 0 chooses such y. The resulting extensive game then has a subgame at each $y \geq 1$ and subgame perfectness forces the firms to offer the wage 1^-. This argument might suggest that in the perturbed game, for each $y \geq 1$ there is a cell consisting of the agents $1, 2y$ and $3y$. If this would hold, it would follow immediately that efficient separation is the Harsanyi/Selten solution if $\lambda < \frac{1}{2}$. However, it is not the case that the agents $1, 2y$ and $3y$ form a cell. Even though the cell condition (see Harsanyi and Selten [1988, p.95] is satisfied for the agents $2y$ and $3y$, it is violated for the type 1 worker: His payoff depends in an essential way on how firms react at other values of y. The reduced perturbed game does not contain any cells, it is itself indecomposable.

close to zero that is not intentionally chosen by either type of worker. According to (3.5) and (3.6) firms will offer the wage λ^- at such y, but then the type 0 worker profits by deviating to y. The apparent contradiction shows that the type 0 worker should have an equilibrium payoff of at least λ^-. Now inspection of Proposition 2.3 shows that the unperturbed game has only one equilibrium that satisfies this condition, viz. the pooling equilibrium in which both types of workers choose $\overline{y} = 0$. It seems that only the Pareto best pooling equilibrium can be an equilibrium of the uniformly perturbed game, and that using uniform perturbations completely solves the selection problem.

The fallacy in the above argument is that there may not exist y close to zero with $s(y) = 0$. Even though it is true that in the unperturbed game at most 3 education levels can occur with positive probability in an equilibrium, this structural property no longer holds in the perturbed game. In most equilibria of the latter game, the type 0 worker is forced to randomize intentionally over many education levels including levels close to zero. Once, the type 0 worker chooses y intentionally, firms will not have unbiased beliefs at y and they may offer wages strictly below λ. In particular, the wage may be so low that the type 0 worker becomes indifferent between choosing y and taking any equilibrium education level, and in this case there is no reason why he should not intentionally choose y. To put it differently, the Pareto best pooling equilibrium of $G(Y, W, \lambda)$ is the only equilibrium that can be approximated by pure equilibria of $\tilde{G}_\epsilon(Y, W, \lambda)$. For later reference we list this result as Proposition 5.1.

Proposition 5.1. *Only the pooling equilibrium outcome in which both types of the worker do not invest in education can be approximated by pure equilibria of uniformly perturbed games.*

An equilibrium outcome of the unperturbed game $G(Y, W, \lambda)$ is said to be uniformly perfect if, for $\epsilon > 0$, there exists an equilibrium (s^ϵ, w^ϵ) of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ that produces this outcome in the limit as ϵ tends to zero. Hence, Proposition 5.1 may be paraphrased as "the pooling equilibrium outcome at $y = 0$ is uniformly perfect". In the remainder of this section we first derive a condition that is necessary for an equilibrium outcome to be uniformly perfect (Corollary 5.3), thereafter we show (Proposition 5.6) that this condition is sufficient as well. The overall conclusion (Proposition 5.7) will be that relatively many equilibrium outcomes of the original game are uniformly perfect.

The next Lemma states a lower bound on the wage that firms may offer in an equilibrium of the uniformly perturbed game, as well as a derived lower bound on the utility of the type 1 worker.

Lemma 5.2. *If (s, w) is an equilibium of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ and if u_t is the equilibrium payoff of the type t worker, then*

$$\text{if} \quad y \geq \lambda^- - u_0 \ , \quad \text{then} \quad w(y) \geq \lambda^- \ , \tag{5.1}$$

$$u_1 \geq (\lambda^- + u_0)/2 \ . \tag{5.2}$$

Proof. Assume $y \geq \lambda^- - u_0$ but $w(y) < \lambda^-$. Then $w(y) - y < u_0$, hence, the type 0 worker cannot choose y voluntarily. Therefore, $s(y) = 0$ and $\mu_s^\epsilon(y) \geq \lambda$. The Bertrand competition (Lemma 2.1), however, then forces firms to offer a wage of at least λ^-, hence $w(y) \geq \lambda^-$. The contradiction proves (5.1). To prove (5.2), assume (without loss of generality) that $\lambda^- - u_0 \in Y$. By choosing, $y = \lambda^- - u_0$, the type 1 worker guarantees a wage of λ^-, hence, he can guarantee a payoff $\lambda^- - (\lambda^- - u_0)/2$. This establishes (5.2).

□

As a direct consequence of Lemma 5.2, we have

Corollary 5.3. *If (s, w) is a uniformly perfect equilibrium of $G(Y, W, \lambda)$ with u_t being the equilibrium payoff of the type t worker, then*

$$u_1 \geq (\lambda^- + u_0)/2 \ . \tag{5.3}$$

If (s, w) is a separating equilibrium with $s_1(y_1) = 1$, then $u_0 = 0$ and $u_1 = 1^- - y_1/2$, hence the above Corollary implies that $y_1 \leq 2^- - \lambda$. Consequently we have

Corollary 5.4. *If (s, w) is a uniformly perfect separating equilibrium and $s_1(y_1) = 1$, then $y_1 < 2 - \lambda$.*

It is easily checked that Corollary 5.3 does not allow us to eliminate any pooling equilibrium (in this case, (5.3) is always satisfied with equality), hence, let us turn to equilibria in which the worker randomizes. First, consider the case where only the type 1 worker randomizes, say between $\bar{y}$ and y_1 with $\bar{y} < y_1$. Then type 0 chooses $\bar{y}$ for sure, and $u_0 = w(\bar{y}) - \bar{y}$, $u_1 = w(\bar{y}) - \bar{y}/2$.

Furthermore, $w(\bar{y}) < \lambda^-$ since $\mu_s^\epsilon(\bar{y}) < \lambda$. One may conclude that (5.3) is always violated. Next, assume that type 0 randomizes, say between y_1 and $\bar{y}$. Then $y_1 = 0$ and type 1 also chooses $\bar{y}$ with positive probability. Furthermore, $u_0 = 0$ and $u_1 = \bar{y}/2$, hence, we must have $\bar{y} \geq \lambda^-$. Therefore, $\bar{y} \geq \lambda$ in view of (2.8). We have shown:

Corollary 5.5. *Equilibria in which only the type 1 worker randomizes are not uniformly perfect. Equilibria in which the type 0 worker randomizes between $y_1 = 0$ and $\bar{y} > 0$ uniformly perfect only if $\bar{y} \geq \lambda$, hence $u_1 \geq \lambda/2$.*

The following Figure graphically displays the results obtained thus far.

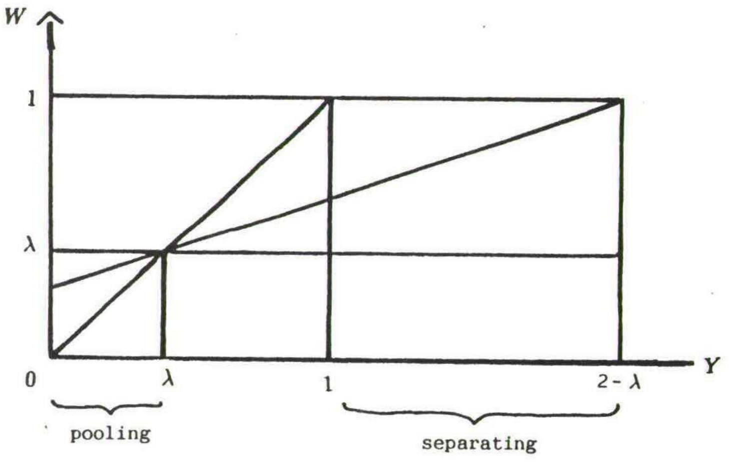

Figure 2. Uniformly Perfect Equilibrium Outcomes.

We now come to the main result of this section, which states that condition (5.3) is also sufficient for an equilibrium outcome to be uniformly perfect. We will give the formal proof only for separating equilibria. The reader may easily adjust the proof to cover the other classes of equilibria not excluded by Corollary 5.3 .

Proposition 5.6. *A separating equilibrium outcome (y_0, y_1) with $y_0 = 0$ and $y_1 < 2 - \lambda$ is uniformly perfect.*

Proof. The proof is by construction. Take ϵ small and define the strategies

(s, w) by means of

$$s_0(y) = \begin{cases} \frac{\epsilon(\lambda - y - g)}{(1-\lambda)(1-|Y|\epsilon)(y+g)} & \text{if } 0 < y < \lambda \\ 0 & \text{if } y \geq \lambda \\ 1 - \sum\limits_{y \neq 0} s_0(y) & \text{if } y = 0 \end{cases} \tag{5.4}$$

$$s_1(y) = \begin{cases} 0 & \text{if } y \neq y_1 \\ 1 & \text{if } y = y_1 \end{cases} \tag{5.5}$$

$$w_i(y) = \begin{cases} 0 & \text{if } y = 0 \\ y & \text{if } 0 < y < \lambda \\ \lambda^- & \text{if } y \geq \lambda, y \neq y_1 \\ 1^- & \text{if } y = y_1 \end{cases} \tag{5.6}$$

Note that $s_0(y) \to 0$ for all $y \neq 0$ as $\epsilon \to 0$, so that s_0 is a well-defined strategy if ϵ is small. This strategy has been chosen so that

$$\mu_s^\epsilon(y) = y + g \qquad \text{for } 0 < y < \lambda \tag{5.7}$$

(Direct verification using (3.4) and (3.6) is easy). Furthermore, if ϵ is small, then $\mu_s^\epsilon(0) < g$, so that (Lemma 2.1), firms' agents indeed bid equilibrium wages for $y < \lambda$. If $y \geq \lambda$ and $y \neq y_1$, then $\mu_s^\epsilon(y) = \lambda$, while $\mu_s^\epsilon(y_1) > 1^-$ if ϵ is small so that firms bid optimally also for these education choices. If wages are as in (5.6), the type 0 worker has the interval $[0, \lambda)$ as optimal education choices, while the optimal choice of the type 1 worker is y_1 since $y_1 < 2 - \lambda$. Hence, for ϵ small, (s, w) is an equilibrium of $\tilde{G}_\epsilon^r(Y, W, \lambda)$. Since (s, w) in the limit produces the separating outcome (y_0, y_1), this outcome is uniformly perfect.

□

The proof that other outcomes that are not eliminated by Corollary 5.3 are uniformly perfect proceeds similarly. The basic insight is that, if $\mu < \lambda^- - u_0$, then $y = w(y) - u_0$ for some $w(y)$ and the type 0 worker may choose y voluntarily in order to justify the wage offer $w(y)$ of firms at y. Furthermore, as long as the type 0 worker is indifferent, the type 1 worker will not have y as an optimal choice. Hence, condition (5.3) is sufficient for uniform perfectness, and we may state

Proposition 5.7. *The uniformly perfect equilibrium outcomes of $G(Y, W, \lambda)$ are*

(i) *all pooling equilibrium outcomes*

(ii) *the separating outcomes with $y_1 < 2 - \lambda$, hence, $u_1 \geq \lambda/2$*

(iii) *the equilibrium outcomes in which type 0 randomizes between $y_1 = 0$ and $\bar{y}$ with $\bar{y} \geq \lambda$, hence $u_1 \geq \lambda/2$.*

Note that all three classes of equilibria described in the above Proposition satisfy $u_1 \geq \lambda/2$. This condition is necessary for uniform perfectness (cf. Corollary (5.3)), but it is not sufficient: There exist equilibria in which only the type 1 worker randomizes that satisfy this condition and these are not uniformly perfect (Corollary 5.5).

6. FORMATIONS AND PRIMITIVE EQUILIBRIUM OUTCOMES

Given that strict equilibria (i.e. pure equilibria in which each player looses by deviating) are (at least at the intuitive level) more stable than non-strict ones, it frequently is more natural to select a strict equilibrium. (Harsanyi and Selten [1988, Sect.5.2]). Of course, strict equilibria do not always exist so that HS were led to search for a principle that generalizes (weakens) the idea of strictness as a selection criterion and that still helps to avoid those equilibria that are especially unstable. HS have come up with the concept of primitive formations. A formation of a game specifies for each agent a subset of his strategy set such that any best reply (in the original game) against any correlated strategy combination with support contained in the restricted game is again in the restricted strategy set. Primitive formations are sets that are minimal with respect to this property. HS (Lemma 5.2.1) have shown that primitive formations exist, and it is also true that any formation contains an equilibrium of the original game. If (s, w) is an equilibrium of $\tilde{G}^r_\epsilon(Y, W, \lambda)$, then we will write $F^\epsilon(s, w)$ for the primitive formation in $\tilde{G}^r_\epsilon(Y, W, \lambda)$ that contains (s, w). We will say that (s, w) spans $F^\epsilon(s, w)$. Note that $F^\epsilon(s, w) = \{(s, w)\}$ whenever (s, w) is a strict equilibrium. Hence, primitive formations are the smallest substructures with similar properties as strict equilibria. The HS theory favors the selection of equilibria which span primitive formations to retain as much as possible of the stability properties of strict equilibria. Specifically, HS consider as the natural solution candidates (the first candidate set) the set of all solutions to primitive formations. (See the flowchart on p.222 of Harsanyi and Selten [1988]). In this section, we determine the minimal formations of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ and their solutions. We will proceed by constructing for each equilibrium (s, w) the minimal formation spanned by it, and then check whether there exists an alternative equilibrium (s', w') with

$$(6.1) \qquad F^\epsilon(s', w') \underset{\neq}{\subset} F^\epsilon(s, w)$$

An equilibrium (s, w) belongs to a primitive formation if and only if no (s', w') satisfying (5.1) can be found. Such an equilibrium (s, w) will be called a primitive equilibrium. (This definition differs slightly from the HS definition, the results in this section however show that the equilibria that are primitive according to our definition are exactly the initial candidates in the HS sense.). Finally, an equilibrium outcome of the unperturbed game will be called primitive if it can be obtained as a limit of primitive equilibrium outcomes of perturbed games as the perturbations vanish.

We have already seen in Sect.4 that the efficient pooling equilibrium outcome of the unperturbed game can be approximated by pure equilibrium outcomes of the ϵ-uniformly perturbed game. Namely, if ϵ is small, the strategy combination

$$(6.2) \qquad s_0(0) = s_1(0) = 1 \ , \ w_i(y) = \lambda^- \quad \text{for all} \quad y \in Y \ ,$$

is an equilibrium of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ which produces this outcome in the limit.[5] Note that the equilibrium (6.2) is strict, hence, primitive. Consequently, the pooling equilibrium outcome at $\bar{y} = 0$ is primitive as well.

Proposition 6.1. *The equilibrium outcome in which the workers are pooled at $\bar{y} = 0$ is primitive.*

To verify which other outcomes are primitive, the following Lemma is helpful.

Lemma 6.2. *Let (s, w) be an equilibrium of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ and write $F^\epsilon_{iy}(s, w)$ for the strategy space of agent iy in the formation $F^\epsilon(s, w)$. For ϵ small, we have*

$$(6.3) \qquad \text{if} \ \ y = 0 \ \text{or} \ \ s(y) = 0, \ \text{then} \ \ \lambda^- \in F_{iy}(s, w) \ .$$

[5] The reader may recall from Lemma 2.1 that the Bertrand game for a surplus of λ, has equilibria that differ from (λ^-, λ^-). However, these are not primitive, hence, at last we can justify Remark 2.2.

Proof. If $s(y) = 0$ and workers play according to s, then firms play a Bertrand game for surplus λ at y. Remark 2.2 and the fact that each formation contains an equilibrium implies that $\lambda^- \in F^{\epsilon}_{iy}(s,w)$. Next, consider $y = 0$. The statement of (6.3) is clearly fulfilled for the equilibrium from (6.2). If (s,w) is an alternative equilibrium, then the argument from the proof of Proposition 5.6 implies that there exists an alternative best reply s' of the worker with $s'(0) = 0$. Since s' is a best reply $s' \in F^{\epsilon}(s,w)$ and $F^{\epsilon}(s,w) \subseteq F^{\epsilon}(s',w)$. The conclusion now follows from the first part of the proof.

□

The Lemma immediately implies that equilibrium outcomes of the unperturbed game with $u_1 < \lambda^-$ cannot be primitive. Namely, if (s,w) is an equilibrium of $\tilde{G}^r_\epsilon(Y,W,\lambda)$ with $u_1 < \lambda^-$, then both types of the worker have the education choice 0 as the best response whenever firms offer $w(y)$ for $y \neq 0$ and offer λ^- for $y = 0$.(The Lemma implies that this strategy of the firms belongs to $F^{\epsilon}(s,w)$). But if workers play this strategy, firms should offer $w = \lambda^-$ for all y and $F^{\epsilon}(s,w)$ contains the equilibrium (6.2), hence (s,w) is not primitive.

Corollary 6.3. *In any primitive equilibrium outcome, the payoff to the type 1 worker is at least λ^-.*

Corollary 6.4. *Equilibrium outcomes in which the workers are pooled at $\bar{y} > 0$ are not primitive.*

Next we show that the mixed equilibrium outcomes that were not yet eliminated in Sect.5 are not primitive.

Proposition 6.5. *Equilibrium outcomes in which the type 0 worker randomizes between 0 and $\bar{y}$ are not primitive.*

Proof. Let (s,w) be an equilibrium of $\tilde{G}^r_\epsilon(Y,W,\lambda)$ that produces in the limit an outcome as described in the Proposition. Then $s_0(\bar{y}) > 0$ and $s_1(\bar{y}) > 0$. Hence, the formation $F^{\epsilon}(s,w)$ also contains the strategy in which both workers choose $\bar{y}$ for sure. The firms' best response against this strategy is to offer λ^- for each education choice, and, if firms behave in this way, the workers should choose $y = 0$. Hence, $F^{\epsilon}(s,w)$ contains the equilibrium from (6.2) so that (s,w) is not primitive.

□

Figure 3 graphically illustrates the results obtained thus far.

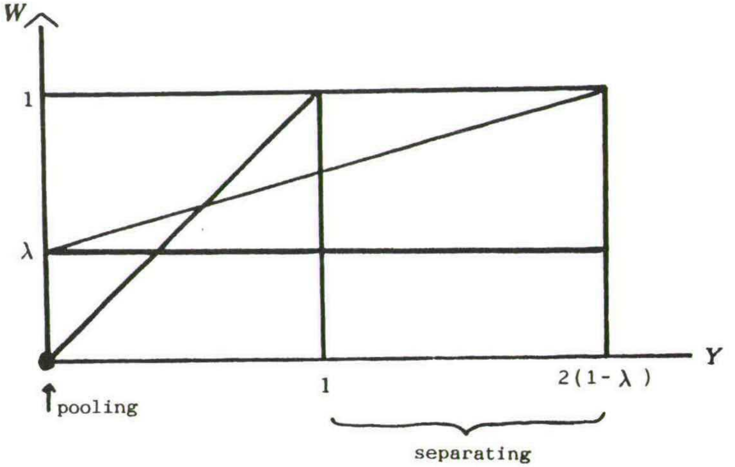

Figure 3. Primitive Equilibrium Outcomes ($\lambda < \frac{1}{2}$).

Finally, let us turn to separating equilibrium outcomes. For the outcome to be primitive it is necessary that choosing $y = 0$ is not an alternative best response for the type 1 worker whenever firms offer the wage λ^- at $y = 0$, hence, $y_1 < 2(1 - \lambda)$. (cf. Corollary 6.3). In the next proposition we show that this condition is not only necessary but that it is also sufficient for a separating outcome to be primitive.

Proposition 6.6. *A separating equilibrium outcome is primitive if and only if* $y_1 < 2(1 - \lambda)$.

Proof. It suffices to show that the condition is sufficient. The proof is constructive. Let (y_0, y_1) be a separating equilibrium outcome with $y_0 = 0$ and $y_1 < 2(1 - \lambda)$ and let the equilibrium (s, w) of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ that produces this outcome in the limit be constructed as in the proof of Proposition 5.6. It is easily seen that , for ϵ small, $F^\epsilon(s, w)$ contains the following restricted strategy sets

$$(6.4) \qquad Y_0 = \{y \in Y;\ y < \lambda\} \qquad Y_1 = \{y_1\}$$

$$
(6.5) \qquad W_{iy} = \begin{cases} \{w \in W; w \leq \lambda^-\} & \text{if } y < \lambda, \\ \{1^-\} & \text{if } y = y_1, \\ \{\lambda^-\} & \text{otherwise.} \end{cases}
$$

Furthermore, one may easily verify that the collection of strategies defined by (6.4) (6.5) is closed under taking best replies, hence, it is a formation and, therefore, exactly equal to $F^\epsilon(s, w)$. Since (s, w) is the unique equilibrium that is contained in this formation, (s, w) is primitive.

□

Since $y_1 \geq 1$ in any separating equilibrium outcome, we see from Proposition 6.6 that no such outcome is primitive if $\lambda \geq \frac{1}{2}$. By combining this observation with Proposition 5.6 and the previous results from this section, we, therefore obtain

Corollary 6.7. *(i) If $\lambda \geq \frac{1}{2}$, only the outcome in which the workers are pooled at $\bar{y} = 0$ is primitive.*
(ii) If $\lambda < \frac{1}{2}$, in addition to the efficient pooling outcome, also the separating equilibrium outcomes with $y_1 < 2(1 - \lambda)$ (hence $u_1 \geq \lambda$) are primitive.

7. RISK DOMINANCE

In the previous section, we determined all primitive equilibria of $\tilde{G}^r_\epsilon(Y, W, \lambda)$. The set of all these equilibria is what HS call the first candidate set. HS propose to refine this set by a process of elimination and substitution until finally only one candidate, the solution, is left. Loosely speaking, this process consists in eliminating all candidates that are 'dominated' by other candidates and by replacing candidates that are equally strong by a substitute equilibrium. In our application, we will not need the substitution procedure we will show that there exists exactly one equilibrium in the first candidate set that dominates all other equilibria in this set.

Attention will be confined to the case where $\lambda < 1/2$. If $\lambda \geq 1/2$, and ϵ is small, then the perturbed game $\tilde{G}^r_\epsilon(Y, W, \lambda)$ has just a single primitive equilibrium, and this induces the efficient pooling outcome, so that we have

Proposition 7.1. *If $\lambda \geq 1/2$, then the HS solution of the Spence signaling game $\Gamma(Y, W, \lambda)$ is the outcome in which the workers are pooled and do not invest in education.*

Let e, e' be equilibria of the perturbed game $\tilde{G}^r_\epsilon(Y, W, \lambda)$, write A for the set of agents participating in this game (with generic element a) and let $u_a(e)$ (resp. $u_a(e')$) be the payoff to agent a when e (resp. e') is played. Finally, denote by $A(e, e')$, the set of all agents for which $e_a \neq e'_a$, that is, who play differently in e than in e'. HS say that e payoff dominates e' if

$$(7.1) \qquad u_a(e) > u_a(e') \qquad \text{for all} \quad a \in A(e, e'),$$

and their theory requires that one first eliminates all payoff dominated equilibria from the initial candidate set. In our case, the only initial candidates that could possibly be payoff dominated are those that approximate an equilibrium in which the type 1 worker is separated at an education level y' strictly above 1. Indeed, the type 1 worker prefers to be separated at a lower level y. However, condition (7.1) requires that one also considers the payoffs of the agents of the firms at y and y', and it cannot be the case that all these agents unanimously strictly prefer the type 1 worker to choose y: If agent iy strictly prefers the worker to choose y rather than y' , then agent iy' strictly prefers this worker to choose y'. Consequently, no initial candidate is payoff dominated in the HS sense, and we have

Proposition 7.2. *The criterion of payoff dominance does not reduce the initial candidate set.*

Proposition 7.2 implies that the HS theory requires to compare equilibria using the risk dominance criterion. The notion of risk dominance is central to the HS theory; It tries to capture the idea that, in a situation where the players are uncertain about which of two equilibria should be played, the players enter a process of expectation formation that may finally lead to the conclusion that one of these equilibria is less risky than the other one, and that, therefore, they should play this less risky equilibrium. In the remainder of this section we will show that the best separating equilibrium, (i.e. type 1 chooses 1), risk dominates all other solution candidates, hence, that this equilibrium is the HS solution if $\lambda < 1/2$. Before proving this main result, we introduce some notation and formally define the concept of risk dominance.

Assume it is common knowledge that the solution of the game will be either e or e' and let $A(e, e')$ be the set of those agent whose strategy in e differs from that in e'. The restricted game generated by e and e' is the game in which the player set is $A(e,e')$ and in which the set of strategy combinations is the smallest formation generated by $\{e,e'\}$. Let $a \in A(e,e')$ and assume this agent believes that his opponents will play e with probability z and e' with probability $1-z$. Then a will play his best response $b_a(z,e,e')$ against the (correlated) strategy combination $ze_{-a}+(1-z)e'_{-a}$. Harsanyi and Selten define the *bicentric prior* of agent a as $p_a = b_a(e,e') = \int_0^1 b_a(z,e,e')\,dz$. This bicentric prior may be interpreted as the mixed strategy which an outside observer (or an opponent of a) expects a to use. (Adopting the principle of insufficient reason, the outsider considers the beliefs of a to be uniformly distributed on [0,1]). Denote by p the mixed strategy combination $p = (p_a)_{a\in A(e,e')}$. This p represents the initial expectations of the players in this situation of common uncertainty. The tracing procedure transforms these preliminary expectations into final expectations. Formally, the *tracing procedure* is a map T that converts each mixed strategy combination into an equilibrium. Risk dominance is defined by means of the tracing procedure. The equilibrium e is said to ***risk dominate*** e' if for the bicentric prior $p = b(e,e')$ generated by e and e' we have $T(p) = e$. We conclude this overview of definitions by briefly describing the operator T. In our case it turns out that the linear tracing procedure is well-behaved, so we only specify this one. Let G be a game with payoff function H, and for $t \in [0,1]$, let G_p^t be a game with the same strategy sets, but in which the payoffs are

$$H_a^t(\sigma) = tH_a(\sigma) + (1-t)H_a(\sigma_a, p_{-a}) \tag{7.2}$$

hence, for $t=0$, one plays against the bicentric prior, for $t=1$, one plays the original game. Let E_p^t be the set of equilibria of G_p^t. (In our case) it can be shown that $E_p = \{E_p^t; t\in[0,1]\}$ contains exactly one continous path connecting the unique equilibrium of G_p^0 with an equilibrium of $G_p^1 = G$. The tracing result $T(p)$ of p is the final endpoint of this continous path.

After this review of definitions, we turn to the results. We will first show that, for ϵ small, the efficient separating equilibrium risk dominates any other separating equilibrium. The intuition for this result is simple. Let e be the separating equilibrium in which type 1 chooses $y=1$ and let e' be a separating equilibrium in which this type chooses $y' > 1$. The initial beliefs of the firms' agents at y and y' will be that the type 1 worker chooses both y and y' with a probability that is bounded away from zero. Since the type 0 worker only chooses y and y' by mistake, and mistakes are rare, firms will be willing to offer a wage $w = 1^-$ at y and at y'. Since y and y' garner the same wages, the type 1 worker strictly prefers to choose y as the cost incured there are lower. This reinforces firms at $y = 1$ to offer $w = 1^-$. On the other hand firms' agents at y' will gradually update their beliefs, they become more pessimistic and finally they will conclude that y' can only occur by mistake. Hence, ultimately they will offer $w = \lambda^-$: We end up at the reparating equilibrium at $y = 1$.

The following Proposition makes this argument precise.

Proposition 7.3. *Let e' be an equilibrium of $\tilde{G}^r_\epsilon(Y, W, \lambda)$ in which the type 1 worker is separated at $y' > 1$. Then, if ϵ is small, there exists an equilibrium e in which the type 1 worker is separated at $y = 1$ that risk dominates e'. Hence, efficient separating equilibria risk dominate all other separating equilibria.*

Proof. Let e' be given. Modify e' such that the type 1 worker chooses $y = 1$ rather than $y' > 1$, and such that firms offer $w = 1^-$ at y rather than at y' (where they now offer λ^-). The resulting strategy combination e is an equilibrium. We will show that e risk dominates e'. Note that $A(e, e')$ consists of 5 agents, viz. the type 1 worker and the firms' agents at y and y'. The game relevant for the risk dominance comparison has strategy space $\{y, y'\}$ for the type 1 worker, whereas the firms may choose from $\{w \in W;\ \lambda^- \leq w \leq 1^-\}$ at y and y'. The payoff to the worker is as in (3.1), if agent iy overbids the opponent with wage w, then its payoff is

$$s^\epsilon(y)(\mu^\epsilon_s(y) - w) \tag{7.3}$$

where $\mu^\epsilon_s(y)$ is as in (3.6) (with $s_0(y) = 0$) and $s^\epsilon(y)$ is the probability that y is chosen by the worker

$$s^\epsilon(y) = (1 - \lambda)\epsilon + \lambda s^\epsilon_1(y)\ . \tag{7.4}$$

If agent iy bids lower than the opponent, its payoff is zero, if both agents at y bid the same wage w, they share the quantity from (7.3). Payoffs at y' are defined similarly.

We now compute the bicentric prior combination associated with e, e'. If the type 1 worker expects his opponents to play according to $ze + (1 - z)e'$, then his payoff if he chooses y is equal to

$$z1^- + (1 - z)\lambda^- - 1/2,$$

while if he chooses y' his payoff is

$$z\lambda^- + (1-z)1^- - y'/2.$$

His best response is to choose y whenever

$$z > 1/2 - (y'-1)/4(1-\lambda).$$

Accordingly, his bicentric prior assigns the probability

$$p_1(y) = 1/2 + (y'-1)/4(1-\lambda). \tag{7.5}$$

to choosing y and the complementary probability to choosing y'.
Next, turn to agents of the firms. Let agent iy have beliefs $ze + (1-z)e'$. If this agent chooses $w \notin \{\lambda^-, 1^-\}$, then he will only get a worker if e' is played and in this case his expected payoff is negative. Hence, such a wage cannot be optimal since by offering λ^- the agent guarantees a nonnegative payoff. In fact, λ^- yields an expected payoff of

$$(1-z)\ \epsilon\ g/2\ . \tag{7.6}$$

On the other hand, if the agent chooses 1^- his expected payoff is

$$z\lambda^\epsilon\ g^\epsilon/2 + (1-z)\ \epsilon\ (\lambda - 1^-)\ , \tag{7.7}$$

where λ^ϵ and g^ϵ are defined by

$$\left.\begin{aligned} \lambda^\epsilon &= s^\epsilon(y) \\ g^\epsilon &= \mu_s^\epsilon(y) - 1^- \end{aligned}\right\} \text{ with } s_1(y) = 1\ .$$

(Note that $(\lambda^\epsilon, g^\epsilon) \to (\lambda, g)$ as $\epsilon \to 0$). Comparing (7.6) and (7.7) we see that agent iy should choose λ^- whenever

$$z\lambda^\epsilon\ g^\epsilon/2\ <\ (1-z)\ \epsilon\ (1-\lambda-g/2)\ . \tag{7.8}$$

Write δ^ϵ for the probability that (7.8) holds when z is uniformly distributed on $[0,1]$. Note that $\delta^\epsilon \to 0$ as $\epsilon \to 0$, hence, the prior strategy of agent iy chooses

1^- with probability close to 1 if ϵ is small. The computations at y' are identical to those at y, hence, we find for the prior strategies of the firms

$$(7.9) \qquad w_{iy} = w_{iy'} = \begin{cases} 1^- \text{ with probability } 1-\delta^\epsilon \\ \\ \lambda^- \text{ with probability } \delta^\epsilon \end{cases} .$$

Given the prior strategy combination p as in (7.5), (7.9) it is easy to compute the equilibrium of G_p^0, that is, the starting point of the tracing path. Since the expected wage at y is the same as that at y', the worker chooses y since costs are lower there. Denote by $\mu^\epsilon(y)$ the expected prior productivity at y(i.e. $\mu^\epsilon(y)$ is determined by (7.4), (7.5) and (3.6)). Then $\mu^\epsilon(y) \to 1$ as $\epsilon \to 0$. The expected payoff from offering 1^- is at least equal to

$$(1-\delta^\epsilon)\big(\mu^\epsilon(y) - 1^-\big)$$

whereas the expected payoff associated to any other wage is bounded above by

$$\delta^\epsilon\big(\mu^\epsilon(y) - \lambda^-\big)$$

Consequently, only 1^- is a best reply against the prior if ϵ is small. Hence, in any equilibrium of G_p^0, the firms offer $w_{iy} = 1^-$. The same argument also generates this conclusion at y'. Hence, G_p^0 has a unique equilibrium, and this is given by

$$(7.10) \qquad s_1(y) = 1\ , \quad w_{iy} = w_{iy'} = 1^-$$

Now consider $t > 0$. If firms do not change their wage offers, there is no reason for the worker to deviate from y, and if this worker stays at y, the agents $2y$ and $3y$ should not change their wage offers either. What about the firms' agents at y'? For t small, their (subjective) payoffs from (7.2) are still largely determined by their priors and they will find it optimal to offer $w = 1^-$. Consequently, for t small, the tracing path continues with the equilibrium from (7.10). However, if the worker remains at $y = 1$, then the expected productivity at y' decreases with increasing t and the payoffs to the firms' agents at y' would become negative for t close to 1 if these agents would remain at their wage offer of 1^-. Consequently, these agents will be the first to switch, and they will switch to lower wages. This, of course , reinforces the decisions of the worker and of the firms' agents at $y = 1$: Along the tracing path, these agents will never switch, and the agents at y' have to adjust until finally an equilibrium of G is reached. In the end, these agents will, therefore, offer the wage λ^- and we see that the tracing path must end up in the equilibrium e. Hence, e risk dominates e'.

□

It is instructive to study in somewhat greater detail the adjustment process (i.e. the tracing path) that brings players from the prior p to the equilibrium e. As already remarked above only the agents at y' change behavior during the process. Take ϵ fixed and write $\tilde{G}_p^t$ for the game as in (7.2) played by these agents given that the type 1 worker chooses $y = 1$ for all t. $\tilde{G}_p^t$ resembles a standard Bertrand game for a surplus $\mu(t)$ where $\mu(t)$ is decreasing with $1^- < \mu(0) < 1$ and $\mu(1) = \lambda$. The only difference with an ordinary Bertrand game is that in $\tilde{G}_p^t$ each agent is committed to choose 1^- with probability $1-t$. Nevertheless, the equilibria of $\tilde{G}_p^t$ can be read off from Lemma 2.1. It is easily seen that there exists $t_1 > 0$ such that only $(1^-, 1^-)$ is an equilibrium of $\tilde{G}_p^t$ for $t < t_1$. Furthermore, there exists $t_2 > t_1$ such that for $t \in (t_1, t_2)$ the game $\tilde{G}_p^t$ has three equilibria, viz. $(1^-, 1^-), (1^{--}, 1^{--})$ and a mixed one. If $t > t_2$, $(1^{--}, 1^{--})$ is still an equilibrium, but the other two are not, and they are replaced by two different equilibria, viz. (1-3g,1-3g) and a mixture of 1-2g and 1-3g. Hence, a switch of behavior has to occur at or before t_2. Since the tracing path must be continuous, it cannot jump from $(1^-, 1^-)$ to $(1^{--}, 1^{--})$ at t_2, consequently it must bend backwards at t_2. Therefore, the initial segment of the tracing path looks as follows. From 0 to t_2 it consists of the equilibrium $(1^-, 1^-)$. At t_2 it bends backwards, continuing with the equilibrium in which firms randomize between $1 - g$ and $1 - 2g$ (gradually increasing the probability of $1 - 2g$ from 0 to 1), at t_1 it bends forward again and continues with the equilibrium $(1 - 2g, 1 - 2g)$. This alternation between forward and backward moving segments continues while simultaneously lowering the wages until finally the path becomes stationary at (λ^-, λ^-).

We finally come to the risk dominance comparison of the pooling equilibrium with the best separating equilibrium. The main result of this section states that the separating equilibrium dominates the pooling one if $\lambda < \frac{1}{2}$ and ϵ is small. Again the intuition is simple. Consider a situation of mutual uncertainty concerning whether, in the unperturbed game, the separating equilibrium e or the pooling equilibrium e' should be played. The type 0 worker chooses $y = 0$ in both, hence, for him the situation is unproblematic. In fact, since the wage at $y = 1$ is at most 1^-, this type will always strictly prefer to choose $y = 0$. The prior uncertainty will however lead the type 1 worker to choose both $y = 0$ and $y = 1$ with a probability that is bounded away from zero. (Below we show that the probability is approximately 1/2). Hence, firms at $y = 1$ will infer that they face the type 1 worker and they will offer $w = 1^-$. On the other hand, firms at $y = 0$ infer that the expected productivity is below (and bounded) away from λ, hence, their wage offers will be below λ as well. Given that $0 < w(y) < \lambda$ for $y = 0$ and that $w(y) = 1$ for $y = 1$, the type t worker prefers to choose $y = t$. These choices reinforce firms to offer $w = 1^-$ at $y = 1$. At $y = 0$, however, firms gradually become convinced that they face the type 0 worker and this leads them to gradually lower their wage offers, until they finally offer 0. Hence, we end up at the separating equilibrium, the separating equilibrium risk dominates the pooling one.

Formalizing the above argument is, unfortunately, rather cumbersome since the HS theory requires working with the perturbed game. The difficulty is caused by the fact that the separating equilibrium can only be approximated by mixed equilibria of the perturbed game and the latter are rather complicated (cf. Proposition 5.6). Hence, there are an enormous number of switches along the tracing path. The difficulties are not of a conceptual nature, however, formally one just has to go through a large number of steps similar to the ones described in detail in the proof of Proposition 7.3. Since already in that case the notation became cumbersome and the formal steps were not particularly illuminating, we prefer to stick to the main ideas and make the risk dominance comparison in the unperturbed game. It can be shown that this shortcut does not bias the results. (The detailed argument for the perturbed game is available from the authors upon request).

Proposition 7.4. *If $\lambda < 1/2$, then in the unperturbed game, the best separating equilibrium risk dominates the best pooling equilibrium. The same dominance relationship exists in the perturbed game $\tilde{G}_\epsilon^r(Y, W, \lambda)$ provided that ϵ is small.*

Proof. The main advantage in working with the unperturbed game lies in the reduction in the number of agents involved in the risk dominance comparison:

We do not have to consider the type 0 worker (he chooses 0 in both equilibria), nor the firms' agents at $y \neq 0, 1$. Let e denote the separating equilibrium in which the type 1 worker chooses 1 and let e' be the pooling equilibrium in which both workers choose 0. We first compute the bicentric prior. Let players have beliefs $ze + (1-z)e'$. If the type 1 worker chooses $y = 0$ his expected payoff is $(1-z)\lambda^-$, if he chooses $y = 1$, the expected payoff is [6] $z1^- + (1-z)\lambda^- - 1/2$. Consequently, this worker will choose both $y = 0$ and $y = 1$ with a probability of approximately 1/2. Next, consider the agent of a firm at $y = 1$. This agent has to move only if the separating equilibrium is played, and in this case the agent of the competing firm offers 1^-. Hence, the only way in which this agent can make a profit is by also offering 1^-. Therefore, the bicentric prior is to offer 1^- for sure. Finally, consider a firm's agent at $y = 0$. It is easy to see that any wage $w \notin \{0, \lambda\}$ yields negative expected profits. If the agent offers the wage 0 the expected profit is 0, while the profit resulting from λ^- is equal to

$$-z(1-\lambda)(\lambda - g) + (1-z)g/2.$$

Consequently, the agent at 0 should offer the wage 0 if

$$z((1-\lambda)(\lambda - g) + g/2) > g/2.$$

Let δ be defined by

$$\delta = \frac{g/2}{(1-\lambda)(\lambda - g) + g/2} \quad , \tag{7.11}$$

then the bicentric prior of an agent at 0 chooses λ^- with probability δ and 0 with probability $1 - \delta$. Since g can be chosen arbitrarily small, the initial expectation of the worker is, therefore, that the expected wage at 0 is close to 0. Consequently, for g small, the best response of the type 1 worker is to choose $y = 1$ for sure and this reinforces the firms to offer $w = 1^-$ at $y = 1$. We claim that in any equilibrium along the path followed by the tracing procedure, the worker chooses $y = 1$. Namely, this property can only fail to hold if the firms at $y = 0$ offer a sufficiently high wage (at least equal to 1/2) and this will never be the case since the expected productivity at 0 will always be below λ and $\lambda < \frac{1}{2}$.

---

[6] The pooling equlibrium of the unperturbed game does not specify a unique wage at $y = 1$, we fix this wage at λ^-, that is, at the limit of the wages of approximating equilibria of the perturbed games.

In fact along the tracing path the wage offer at $y = 0$ will never be above $\lambda/2$ since this is the highest expected productivity at $y = 0$. The exact tracing path can be determined by using an argument as the one that follows the proof of Proposition 7.3: For tracing parameter t, the agents at $y = 0$ play a Bertrand game for a surplus of (approximately) $(1-t)\lambda/2$ modified to the extend that each agent is committed to play his prior as in (7.11) with probability $1 - t$. The unique equilibrium of this game for $t = 0$ is (g, g). As t increases, firms initially switch to higher wages (since for t small the expected surplus is large), however, for larger t, wages fall again since the surplus decreases. (Again the tracing path contains many backwards running segments). As t tends to 1 the surplus, hence, the wages tend to zero; along the way the wages never exceed the maximal surplus of $\lambda/2$. Consequently, the agents at 0 finally switch to the wage corresponding to the separating equilibrium. Since the other agents are already at this equilibrium from the beginning, the tracing path leads to the separating equilibrium. Hence, the separating equilibrium risk dominates the pooling one.

□

8. CONCLUSION

By combining the Propositions 7.1, 7.3 and 7.4, we obtain the main result of this paper.

Corollary 8.1. *The Harsanyi/Selten solution of the Spence signaling game $\Gamma(Y, W\lambda)$ is*

(i) the equilibrium outcome in which the workers are pooled at $y = 0$ if $\lambda \leq \frac{1}{2}$,

(ii) the separating equilibrium outcome in which the type t worker chooses $y = t$ if $\lambda > \frac{1}{2}$.

Even though we used the assumptions (2.6) - (2.10) to derive this result, it can be checked that at least for $\lambda \neq 1/2$, the statement of Corollary 8.1 remains correct if these assumptions are not satisfied. Hence, the result is independent of the discretization chosen. It is, therefore, justified to make a limiting argument and to talk about the HS solution of the continuum game. Hence, $g = 0$ and Y and W are continua, as in the usual specification of the Spence model found in the literature. Denote this game by $\Gamma(\lambda)$. The (limit as $g \to 0$ of the) solution found in Corollary 8.1 is known in the literature as the Wilson E_2-equilibrium (Wilson [1977]). It is that sequential equilibrium of $\Gamma(\lambda)$ that is best from the viewpoint of the type 1 worker. Hence, we have

Corollary 8.2. *For $\lambda \neq \frac{1}{2}$, the HS solution of the 2-type Spence signaling game $\Gamma(\lambda)$ is the Wilson E_2-equilibrium of $\Gamma(\lambda)$, i.e. it is that sequential equilibrium that is best for the type 1 worker.*

Given the ad hoc nature of Wilson's solution concept, it is to be expected that a coincidence as in Corollary 8.2 will not hold in general. However, the authors conjecture that for a broad class of signaling games that have similar structural properties as the game studied in this paper, the HS solution coincides with the solution proposed in Miyazaki [1977]. (The important properties are monotonicity and the single crossing condition, see Cho and Sobel [1977], it is conjectured that the number of types does not play a role). A detailed investigation into this issue will be carried out in a future paper.

The simple structure of our game enables a sensitivity analysis with respect to several assumptions made by Harsanyi and Selten that is difficult to carry out in general. We will not go into detail, but restrict ourselves to one issue. Some people have argued that the uniformity assumption made in the construction of the prior to start the tracing procedure is ad hoc. Consequently, one may ask how robust the results from Sect. 7 are with respect to this prior. The reader can easily convince himself that the outcome is very robust. Robustness especially holds for Proposition 7.4 : The separating outcome will result for $\lambda < \frac{1}{2}$ as long as the prior expectations of the players assign positive probability to this outcome (i.e. as long as the density of z is strictly positive on $[0,1]$).

The theory of Harsanyi and Selten has both evolutionary and eductive aspects. (See Binmore [1987] for a general discussion of these notions). The preference for primitive equilibria is most easily justified by taking an evolutionary perspective, the tracing procedure most certainly is eductive in nature. It is interesting to note that the HS theory ranks evolutionary considerations prior to eductive ones. This ordering of steps indeed has consequences for the final outcome since the separating equilbrium outcome with $y_1 = 1$ risk dominates any other equilibrium outcome for all values of λ. (cf the proofs of the Propositions 7.3 , 7.4) Hence, if the solution would be based on risk dominance considerations alone, the solution would always involve separation. Consequently, the solution that was initially proposed by Spence and that is defended and used in most of the subsequent literature can also be justified on the basis of risk dominance.

Finally, let us mention that recently related work has been done by Michael Mitzkewitz (Mitzkewitz [1989]). He computes the HS solution for signaling games in the following class: There are 2 players, player 1 has 2 possible types, and he can send 2 possible messages, to which player 2 can react in 2 different wages. Mitzkewitz does not assume the single crossing property, hence, he is forced to use an approach that differs from ours.

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APPENDIX A: DOMINANCE SOLVABILITY AND THE HS SOLUTION

Consider the 2-person normal form game

10 10	0 10	0 0
10 0	6 6	0 6
0 0	6 0	1 1

Table 1: G

which admits three Nash equilibria, viz. $A = (A_1, A_2)$, $B = (B_1, B_2)$ and $C = (C_1, C_2)$.

We have that A_1 (resp. A_2) is dominated by B_1 (resp. B_2), while in the reduced game B_1 (resp. B_2) is dominated by C_1 (resp. C_2). Hence, the (unperturbed) game is dominance solvable, with solution C. (This is the unique stable equilibrium of the game). HS, however, do not analyse the unperturbed game, but rather a sequence of uniformly perturbed games. In the ϵ-uniformly perturbed game $G(\epsilon)$ of G each player, when he intends to choose the pure strategy X_i, will actually choose the completely mixed strategy $(1-2\epsilon)X_i + \epsilon Y_i + \epsilon Z_i (X_i \neq Y_i \neq Z_i)$. Neglecting terms of order ϵ^2, the payoff matrix of $G(\epsilon)$ is given by

$10-30\epsilon$ $10-30\epsilon$	22ϵ $10-24\epsilon$	10ϵ 27ϵ
$10-24\epsilon$ 22ϵ	$6-8\epsilon$ $6-8\epsilon$	16ϵ $6-17\epsilon$
27ϵ 11ϵ	$6-17\epsilon$ 17ϵ	$1+2\epsilon$ $1+2\epsilon$

Table 2: $G(\epsilon)$

In $G(\epsilon)$, we have that A_1 (resp. A_2) is dominated by B_1 (resp. B_2). The reduced game in which these inferior strategies have been eliminated has 2 strict equilibria, viz. B and C. (There is a third equilibrium in mixed strategies but this is not primitive). Hence, the initial candidate set is $\{B, C\}$. Since B payoff dominates C, the HS solution of $G(\epsilon)$ is B. Consequently, B is the HS solution of G. Hence, perturbing first may make a difference since it can transfer non-strict equilibria into strict ones.

This example may lead the reader to think that the discrepancy between dominance solvability and the HS theory is caused by the fact that the latter makes use of Pareto comparisons. (Indeed, in$G(\epsilon)$, the equilibrium C risk dominates B). A second example may however show that this is not the case. The game from Table 3 is dominance solvable, with solution $D = (D_1, D_2)$. The ϵ-perturbed game has A_i and B_i as inferior strategies, while $C = (C_1, C_2)$ and $D = (D_1, D_2)$ are strict equilibria. Now C not only payoff dominates D, but it also risk dominates D, so that, even if one does not accept payoff dominance as a selection criterion, one still is lead to C as the solution as long as one accepts the other elements of the HS theory.

10 10	0 10	0 6	0 0
10 0	6 6	0 6	0 0
6 0	6 0	2 2	0 2
0 0	0 0	2 0	1 1

Table 3

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