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TRADE DYNAMICS AND ENDOGENOUS GROWTH - AN OVERLAPPING GENERATIONS MODEL

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Trade Dynamics and Endogenous Growth – An Overlapping Generations Model

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Abstract

This paper developes a two sector endogenous growth model which can generate multiple steady state equilibria each with their own rate of growth. It applies this model to a global analysis of international trade and shows that the world economy too may have multiple steady state equilibria each with their own rate of growth. Furthermore the transition to one of these steady states is interesting and may involve reversals in trade direction, growth rates and income levels. Specific cases are examined where international trade reduces a country's growth rate to zero, where international trade pulls up the average growth rate in the world economy and also where international trade pulls down the average growth rate in the world economy and finally where international trade allows one country's growth rate to catch up and overtake that of another.

Introduction

This paper derives a two sector model of endogenous growth with the property that there may exist multiple steady state equilibria each with their own rate of growth. This paper then applies this model to the *global* analysis of trade dynamics in a two country general equilibrium world and shows that there may exist multiple steady states for the world economy also. As a result of this multiplicity of equilibria the transition to a steady state in the world economy may involve reversals of trade direction, income levels and relative growth rates. In specific examples this paper shows how international trade may cause the average rate of growth in the world economy to rise (or alternatively, fall), how trade may cause an individual economy to stagnate and finally how trade can cause a relatively low income, slow growing economy to catch up and overtake the income and growth rate of a higher income, faster growing country ! The global nature of these autarkic and trade dynamics is interesting because much of the endogenous growth literature restricts itself to analyzing steady state growth paths.

What is the intuition or philosophy behind these results ? It is that, due to the existence of multiple equilibria, the 'fundamental' characteristics of an economy will not be able to determine the relative growth rates of two countries in autarky. However when these economies are linked by a common price under international trade, these 'fundamentals' will be able to determine which country will grow faster in the long run. This is why the transition from autarky to a long run trade equilibrium may involve reversals of relative growth rates. It is also intuitive that the prevalence of multiple steady state equilibria is not restricted to the autarkic model and that, although which country grows fastest under international trade is determined, the absolute value of a country's rate of growth is not determined.

This paper contributes to the literature in two main areas, the dynamic analysis of international trade and growth theory. The contribution to the dynamic analysis of international trade is threefold. Firstly, this paper is a dynamic Heckscher-Ohlin model and so extends the work of Onika and Uzawa [15] both to the endogenous growth literature and to the overlapping generations framework. Secondly, although the intuition may seem similar to Grossman and Helpman [10] chaper 8, Krugman [11] and Young [20], in that this paper is a 'learning by doing' generated growth and trade model, there is an important difference between those models and this model. In those papers, comparative advantage is not driven by fundamentals, it is arbitrary. Learning by doing will reinforce any initial international pattern of specialization. This is not the case in this paper, which follows Stokey [19] in assuming that sectoral learning by doing effects spill over sufficiently to the rest of the economy to ensure that the direction of trade is not a trivial problem. Thus in this paper fundamental characteristics of an economy influence the trade dynamics and long run equilibrium. Thirdly the modelling of a dynamic world economy with multiple steady state growth equilibria is itself a significant contribution.

The contribution of this paper to endogenous growth theory can also be placed under three headings. Firstly it extends and complements the literature using the Lucas [12] model of endogenous growth. Chamley [4] has shown how that model can also generate multiple steady state growth paths with different rates of growth, however preferences have to be of the C.E.S. form and tractable analysis of the transitional dynamics have to be restricted to a local analysis near a steady state. The advantage of the overlapping generations framework is that the analysis is global and preferences can be more general. Secondly, the modelling of technical progress is noteworthy. Many papers in the literature, for example see Boldrin [2] or Boldrin and Rustichini [3], model the growth externality as being a function of present day variables, usually the aggregate capital stock or capital to labor ratio in a given period. These models are problematic in the sense that the explaination for the growth externality should not be 'learning', since if the capital stock is decreasing along its path to the steady state then 'knowledge' will be decreasing also ! In this paper the growth externality is modelled so that knowledge can never be decreasing and thus the learning metaphor is consistent. Thirdly the paper can provide a model which agrees with De Long and Summers' [5] empirical finding that faster growing countries tend to have lower relative prices of investment goods.

This paper is organised as follows. In the first two sections I follow Galor [8] very closely in describing and deriving the autarkic growth model and its global dynamics. In section 3 I gradually build up the dynamics of the model under international trade with no transfer of knowledge between countries. This section first analyzes the dynamics of a small open economy, then the dynamics between a small and a large economy and finally the global convergence to an international trade steady state from any initial starting point. In section 4 I briefly look at the world economy under perfect transfer of knowledge and show that the phenomena of multiple steady states for the world economy can occur in this case too. It derives a simple model under which the world economy can be modelled as a single closed economy and so may have multiple steady states.

1 Description of the Model.

1.1 Production

In this economy there are two goods, a perishable consumption good, X_t, and a non-perishable investment good, Y_t. The state of technology or 'knowledge' at time t is denoted by λ_t and enters in a labor augmenting way into the producation functions of both goods. Both goods are produced with constant returns to scale production functions with the properties described below.

$$X_t = F_x(K_t^x, \lambda_t.L_t^x) = f_x(k_t^x)\lambda_t L_t^x$$

$$Y_t = F_y(K_t^y, \lambda_t.L_t^y) = f_y(k_t^y)\lambda_t L_t^y$$

$$\text{or } x_t \equiv \frac{X_t}{\lambda_t L_t} = f_x(k_t^x)l_t^x \text{ and } y_t \equiv \frac{Y_t}{\lambda_t L_t} = f_y(k_t^y)l_t^y$$

where $k_t^x \equiv \frac{K_t^x}{\lambda_t L_t^x}$ is the capital:efficiency labor ratio in the x sector at time t, $k_t^y \equiv \frac{K_t^y}{\lambda_t L_t^y}$ is the capital:efficiency labor ratio in the y sector at time t, and $l_t^x \equiv \frac{L_t^x}{L_t}, l_t^y \equiv \frac{L_t^y}{L_t}$ are the fraction of the labor force employed in the X and Y sectors respectively. Full employment of factors will hold so that $K_t^x + K_t^y = K_t$ and $L_t^x + L_t^y = L_t$ where K_t and L_t are the total stocks of capital and labor at time t, respectively.

For simplicity capital depreciates fully after one period and so next years capital stock is this years production of the investment good, that is

$$K_{t+1} = Y_t$$

I will assume that both $f_x(k_t^x)$ and $f_y(k_t^y)$ have the following properties.

$$f_i(k_t^i) > 0 \quad f_i'(k_t^i) > 0 \quad f_i''(k_t^i) < 0 \quad \forall k_t^i > 0$$

$$\lim_{k_t^i \to 0} f_i(k_t^i) = 0 \quad \lim_{k_t^i \to 0} f_i'(k_t^i) = \infty \quad \lim_{k_t^i \to \infty} f_i'(k_t^i) = 0 \text{ where } i = x, y$$

Finally I will assume, as in Galor [8], that the investment good is more capital intensive than the consumption good, for all factor price ratios.

1.1.1 Factor Prices

The capital and labor markets are perfectly competitive. Thus, when both goods are produced, the return to capital, r_t, is given by the following

$$r_t \;=\; f_y'(k_t^y) = p_t f_x'(k_t^x).$$

where p_t is the price of the consumption good; the investment good is numeraire.

Similarly the return to efficiency Labor w_{e_t}, is given by

$$w_{e_t} \;= f(k_t^y) - k_t^y f'(k_t^y) = p_t[f(k_t^x) - k_t^x f'(k_t^x)]$$

As is well known in the two sector model when both goods are produced, that given p_t, k_t^x, k_t^y, w_{e_t} and r_t can be uniquely determined. When both k_t and p_t are given, x_t and y_t are uniquely determined and so we can write,

$$x_t = x(p_t, k_t) \text{ and } y_t = y(p_t, k_t)$$

It can also be shown that under the above assumptions that we can sign the derivatives of these functions and that in particular $y_p < 0$, $y_k > 0$, $x_p > 0$ and $x_k < 0$. In this paper h_i will denote the partial derivative of function h with respect to argument i.

A useful property of the two sector model is the ability to depict in (p,k) space the areas where the economy chooses to specialize in the production of X or Y, and where it chooses to produce both goods. Galor [8] shows that a $p_{max}(p_{min})$ schedule exists above(below) which the economy produces only the consumption(investment) good. The p_{min} and p_{max} schedules are both increasing functions of k and are depicted in Figure 1. Our assumptions about preferences below, imply that the economy will always produce both goods in autarkic equilibrium, and so the economy must lie between the p_{min} and p_{max} schedules. However as we shall show, an open economy may choose to specialize in production.

1.1.2 Technological Progress

Technical progress is due to a 'learning by doing' externality in one of the two sectors. Following the idea of Romer [18] and others, knowledge is modelled as a stock which may be added to but not reduced in each time period. Specifically I model the increase in knowledge in each time period as being proportional to the amount of the per capita production in one of the two sectors. Thus knowledge in time t is the cumulative sum of past per capita outputs in that sector, that is,

$$\lambda_t = \sum_{i=0}^{t-1} \alpha \frac{Y_i}{L_i} \text{ or } \sum_{i=0}^{t-1} \alpha \frac{X_i}{L_i}$$

This specification can be criticised on the grounds that one might expect larger countries to produce more knowledge. Appendix B shows that most of the analysis goes through with a size dependant rate of growth of knowledge function - $\lambda_t = \sum_{i=0}^{t-1} \alpha Y_i$ or $\sum_{i=0}^{t-1} \alpha X_i$ - though in the two country model, the world economy will tend to be dominated by large countries.

1.2 Consumers and Preferences

Agents live in the typical overlapping generations world. They live for two periods and have perfect forsight. They are endowed with one unit of physical labor only in their first year of life and they supply it perfectly inelastically. Individuals born in period t are characterised by a utility function $u(c_t^t, c_{t+1}^t)$ where c_t^t, c_{t+1}^t are the consumption when young and old respectively. I assume u to be strictly monotonic, strictly quasi-concave and to have the usual boundary conditions $\lim_{c_t^t \to 0} u_{c_t^t} = \lim_{c_{t+1}^t \to 0} u_{c_{t+1}^t} = \infty$ which ensures that both goods are always demanded. I also impose homotheticity, which is necessary for a balanced steady state rate of growth and that the savings function is non decreasing in the real rate of return, which is the common assumption in the single sector overlapping generations economy necessary to ensure a unique momentary equilibrium [1].

Since Y is the only non-perishable good, the only way for agents to consume in their second period of life is by buying Y when young and living off its earnings the next period when they are old. Thus $c_{t+1}^t = \frac{r_{t+1} s_t}{p_{t+1}}$, where s_t denotes the saving per person -(S_t will denote total savings).

[1] See Galor and Ryder [9] and Mountford [13] for a discussion of this for the standard and endogenous growth, single sector overlapping generations economy respectively.

Savings are chosen to maximise utility and thus

$$s_t = s(w_t, p_t, r_{t+1}, p_{t+1}) = \operatorname{argmax}[u(\frac{(w_t - s_t)}{p_t}, \frac{r_{t+1}s_t}{p_{t+1}})]$$

From homotheticity we can write

$$s(w_t, p_t, r_{t+1}, p_{t+1}) = \lambda_t s(w_{e_t}, p_t, r_{t+1}, p_{t+1})$$

and since from above we know that w_{e_t} and r_t are both functions of p_t we can write

$$s_t = \lambda_t s(p_t, p_{t+1})$$

It can be shown that $s_{p_t} > 0$ and since savings are increasing in the real rate of return, $s_{p_{t+1}} < 0$

1.2.1 Preferences in the Trade Model

In the trade model I will assume log-linear preferences. The reason for this is that, as is well known, these preferences give rise to a savings function of the form, $s_t = \beta w_t$ where $0 < \beta < 1$. Thus since this expression doesn't depend on second period variables we can write $s_t = \lambda_t s(p_t)$. As will be shown below this assumption reduces the dimensionality of the autarkic growth model from two to one, and more importantly it reduces the dimensionality of the trade model from three to two.

1.3 Dynamic Equilibrium of the Closed Economy

A dynamic equilibrium occurs when all markets clear, that is when an economy is in momentary equilibrium, and when the economy also satisfies the state equation that $K_{t+1} = Y_t$ under conditions of perfect forsight.

To ensure momentary equilibrium, using Walras' Law, we require $S_t = \lambda_t s_t(p_t, p_{t+1})L_t = Y_t$ or from homotheticity,

$$s_t(p_t, p_{t+1}) = y(p_t, k_t)$$

From our assumptions about preferences this defines an implicit function

$$p_{t+1} = \theta(k_t, p_t) \tag{1}$$

where $\theta_k = \frac{y_k}{s_{p_{t+1}}} < 0$ and $\theta_p = \frac{y_p - s_{p_t}}{s_{p_{t+1}}} > 0$

The state equation, $K_{t+1} = Y_t$, can also be written in efficiency unit form,

$$k_{t+1} = \frac{y_t}{(1+n)\frac{\lambda_{t+1}}{\lambda_t}} = \frac{y_t}{(1+n)(1+\alpha y_t)} \text{ or } \frac{y_t}{(1+n)(1+\alpha x_t)}$$

Which ever sector the technological progress takes place in, we will write this as the following function

$$k_{t+1} = \psi(k_t, p_t) \tag{2}$$

Thus the dynamical system is characterised by equations (1) and (2) and is a two dimensional non-linear difference equation in general.

2 Global Dynamics and Steady State Equilibria

This section will describe the global dynamics of the dynamical system and it will show that it is characterised by steady state equilibria which will be alternately saddlepath stable and unstable. The dynamics are best described by defining two locii in (p_t, k_t) space, the PP and KK locii. These locii are the set of points where p_t - for the PP locus - and the k_t - for the KK locus - are at a steady state level. These locii are plotted in Figure 1.

2.1 The PP Locus

The steady state level of p_t will be where $p_{t+1} - p_t = 0$ or where $\theta(k_t, p_t) - p_t = 0$. Using the implicit function theorem, the slope of the PP locus is given by

$$\frac{dp}{dk}|_{PP} = \frac{\theta_k}{1-\theta_p} = \frac{y_k}{s_{p_{t+1}} + s_{p_t} - y_p}|_{PP}$$

The 'Inada Conditions' on the utility ensure that both goods will always be demanded. This implies that the PP locus, which is derived from the momentary equilibrium conditions, must always lie between the p_{min} and p_{max} locii, which are upward sloping. Thus although the above expression for the slope of the PP locus can be negative over some intervals of k_t, over the whole real line the PP locus must tend upwards.

Given p_t there is a unique level of k_t that lies on the PP locus and since $\theta_k < 0$, the direction of the vector field points downwards when k is to the right of the PP locus and upward when k is to the left.

2.1.1 The PP Locus under Log-Linear Preferences

Under log-linear preferences the PP curve ceases to exist in an intertemporal sense because $s(p_t) = y(p_t, k_t)$ is not an intertemporal equation. However, the locus $s(p_t) = y(p_t, k_t)$ can still be plotted and is useful, since it depicts the position of the momentary equilibrium of the economy for every level of k_t. This locus is always upward sloping.

2.2 The KK Locus

The steady state level of k_t will be where $k_{t+1} - k_t = 0$ or where $\psi(k_t, p_t) - k_t = 0$. Using the implicit function theorem, the slope of the KK locus is given by

$$\frac{dp}{dk}\,|_{KK}= \frac{(1+n)(1+\alpha y)^2 - y_k}{y_p}\,|_{KK}$$

when technological progress is in the Y sector, and

$$\frac{dp}{dk}\,|_{KK}= \frac{(1+n)(1+\alpha x)^2 - [(1-\alpha x)y_k - \alpha y x_k]}{(1-\alpha x)y_p - \alpha y x_p}\,|_{KK}$$

when the externality is in the X sector.

From the Rybcynski Theorem which tells us that that $y_k \frac{k}{y} > 1$ and from $\frac{k}{y} = \frac{1}{(1+n)(1+\alpha y)}$ or $\frac{1}{(1+n)(1+\alpha x)}$ along the KK curve, we know that when the externality is in the X sector the KK locus slopes upward, and when the externality is in the Y sector the KK locus will also slope upward if α is sufficiently small.

The concavity of the production functions, together with the "Inada Conditions", implies that there is a maximum sustainable k_t. For this level of k to be maintained all the economy's resources must be placed into producing Y. This implies the KK locus must hit the p_{min} locus at this level.

Above the KK locus, since $\psi_p < 0$ for both versions of technological progress, the vector field will be pointing to the left, and below the KK the vector field will be pointing to the right.

2.3 Existence of Steady State Equilibria

The economy will be in a steady state dynamic equilibrium where the KK and PP locii intersect, this may occur many times or not at all. We know at the maximum sustainable k_t that the PP locus will be above the KK locus because the PP locus must always lie above the p_{min} locus.

Therefore a condition for the existence of at least one intersection of the PP and KK curves is that the slope of the KK curve is greater than the slope of the PP curve at the origin. However, as in Galor [8], there may be any odd number of intersections.

The existence of multiple steady state equilibria is a general result of the overlapping generations framework. It occurs because the derivatives of the slopes of the two locii depend on the third derivatives of the two production functions, $f_x(k_t^x), f_y(k_t^y)$, which are not restricted and so the two locii may intersect many times[2].

The existence of multiple steady state equilibria is important because it means that countries identical in every respect except for their initial conditions can end up in very different steady states. Thus 'fundamentals' cannot determine a country's long run rate of growth. It is even possible for a country with a high propensity to consume the good with the growth externality to grow slower than a country with a low propensity to consume this good!

2.3.1 Stability of Steady State Equilibria

Equations (1) and (2), show that the model can be described by the following the dynamical system.

$$\begin{bmatrix} k_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} \psi(k_t, p_t) \\ \theta(k_t, p_t) \end{bmatrix}$$

To analyse the stability of this system around a steady state equilibrium $(\bar{k}, \bar{p})$ we need to examine the eigenvalues of the Jacobian matrix, J, of this system.

$$J = \begin{bmatrix} \psi_k & \psi_p \\ \theta_k & \theta_p \end{bmatrix}$$

Appendix A.1 and A.2 show that when the learning by doing is in either sector that for α sufficiently small, this system is alternately saddlepath stable and unstable and that saddlepaths occur if and only if the PP curve is steeper than the KK curve.

Furthermore, from the direction of the vector field and the continuity of the system it is true that there must exist a global stable manifold which connects all the steady state equilibria. This

[2] The ability of the third derivative of the production function in the single sector overlapping generations model to cause multiple steady state equilibria is demonsrated by Galor and Ryder [9] for the standard model and Mountford [13] for the endogenous growth model.

global stable manifold will be a monotonically increasing curve in (p_t, k_t) space with the direction of motion along it taking points towards the saddlepath stable steady states, as shown in Figure 1.

2.3.2 The Rate of Growth in the Steady State

At a steady state we know that $k_t = \bar{k}$ and $p_t = \bar{p}\ \forall t$. This implies that k_t^x, k_t^y, l_t^x and l_t^y will also be constant. National income per capita in the the steady state is given by

$$\frac{\bar{p}X_t + Y_t}{L_t} = \bar{p}f_x(k_t^x)\lambda_t l_t^x + f_y(k_t^y)\lambda_t l_t^y$$

and so the growth rate in the steady state will be given by $\frac{\lambda_{t+1}}{\lambda_t} = (1 + \alpha y)$ or $(1 + \alpha x)$.

If the externality is in the Y sector then the model produces an interesting empirically verified result. This is so because if there are multiple steady state equilibria, a steady state with a faster rate of growth will also have a larger $\bar{k}$ and $\bar{p}$. Thus the country with the lower relative price of capital goods - higher p - will have a faster growth rate. This is precisely the finding of the study by Delong and Summers [5], who found that there was a strong negative correlation between growth rates and relative prices of capital goods.

2.4 Discussion of Assumptions and Summary of Results

This section has derived a two sector endogenous growth model along the lines of Galor's [8] overlapping generations model with multiple steady state equilibria each with their own rate of growth and where the growth rates of these steady states can accord with the empirical study of Delong and Summers [5]. The assumptions needed to generate these results are not strong. Preferences must be homothetic in order to ensure a balanced growth rate steady state. On the production side, the assumption that $k_t^y > k_t^x$ is the standard assumption of the two sector overlapping generations model. Galor [8] has shown how the assumption that $k_t^x < k_t^y$ leads to indeterminacy [3].

The assumption which needs the most discussion is the α is small assumption. This is used alot because its intuition, that learning is gradual, is clear. However if you would like to restrict

[3] Drugeon [6] demonstrates the indeterminacy of the model when $k_t^y < k_t^x$ for the triple cobb-douglas economy.

$\alpha = 1$, there are alternative assumptions. Firstly one should note that the model works in this case, for the learning externality in either sector for log-linear preferences. If you want to retain more general preferences and if the learning externality is in the Y sector, then you must set $s_{p_{t+1}} + s_{p_t} + y_{p_t} > 0$. This implies that the responsiveness of savings to changes in the real rate of return are limited so that the PP curve is always upward sloping. If the learning externality is in the X sector and $\alpha = 1$ then you must also assume that k^y is sufficiently greater than k^x. The reason for this is that, as the appendix A.2 shows, the only use of the 'α is small' assumption is to limit the size of the term $\alpha y(y_k x_p - y_p x_k)$ and the absolute size of the term in brackets is negatively related to the size of $k^y - k^x$, see Ryder [18].

3 Trading Equilibria with No Transfer of Knowledge

In this section as discussed above, I make the additional assumption of log-linear preferences. International trade changes the constraints that an economy faces. A closed economy can only increase it's k by producing more Y, but an open economy can also import Y. I assume a world economy consisting of two countries where there is no international borrowing available to either country and therefore trade must balance in each period, that is

$$p_t M_t^x + M_t^y = 0$$

where M_t^y, M_t^x are the imports of the investment good and consumption good respectively.

This section firstly proves that there exists a unique momentary trading equilibrium by examining the reciprocal demand curve. The next subsection analyzes the effect of trade on a small open economy. The following subsection then looks at the trade dynamics between a small and a large economy. Finally the last subsection shows that from any initial conditions, the world economy must converge to one of the steady state equilibria described in the previous subsection.

3.1 The Reciprocal Demand Curves

Following Onika and Uzawa [15], a country's reciprocal demand curve for imports is derived and it is shown that, given k, it is weakly monotonic in p. Thus two countries' reciprocal demand curves will intersect only once and the intersection will be between the autarkic prices of the two economies.

I will look at the demand for imports by each country of the investment good, $M_t^y = -p_t M_t^x$. From homotheticity and the definition of the production function for Y we can say

$$M_t^y = \lambda_t s(w_{e_t}(k_t, p_t))L_t - f_y(k_t^y)l_t^y \lambda_t L_t \text{ or } m_t^y = s(w_{e_t}(k_t, p_t)) - y_t$$

Where m_t^y is the level of imports of the investment good per efficiency units of labor. The derivative of m_t^y with respect to p_t is always non-negative,

$$\begin{aligned}
&\text{For } p_t \leq p_{min}(k_t) && \Longrightarrow m_t^y = s(w_{e_t}(k_t)) - f_y(k_t) && \Longrightarrow \frac{\partial m_t^y}{\partial p_t} = \frac{\partial s_t}{\partial p_t}\frac{\partial w_{e_t}}{\partial p_t} = 0 \\
&\text{For } p_t \geq p_{max}(k_t) && \Longrightarrow m_t^y = s(w_{e_t}(p_t, k_t)) && \Longrightarrow \frac{\partial m_t^y}{\partial p_t} = \frac{\partial s_t}{\partial p_t}\frac{\partial w_{e_t}}{\partial p_t} > 0 \\
&\text{For } p_{min}(k_t) \leq p_t \leq p_{max}(k_t) && \longrightarrow m_t^y = s(w_{e_t}(p_t)) - y(k_t, p_t) && \Longrightarrow \frac{\partial m_t^y}{\partial p_t} = \frac{\partial s_t}{\partial w_{e_t}}\frac{\partial w_{e_t}}{\partial p_t} - y_p > 0
\end{aligned}$$

Figure 2 plots the reciprocal demand curves of two countries, names A and B. The intersection of the $(m_t^y)^A$ and $-(m_t^y)^B$ curves is the international price. Note that it is unique and lies between the two countries' autarkic prices.

3.2 Small Open Economy

A small open economy is a trading country that takes the equilibrium international price as given. It is useful to define a new locus to analyse this case, the SS locus. This is the open economy equivalent of the KK locus. It is the level of p such that next period's k will equal this period's k. The SS curve has an interesting relationship to the PP curve, which as explained in Section 2.1.1., due to log-linear preferences depicts the locus of the momentary equilibrium of the closed economy. When the the PP curve lies above the KK curve the SS curve will be above the PP curve and when the PP curve is below the KK curve then the SS curve will be below the PP curve. These relationships are depicted in Figure 3 and are demonstrated below.

3.2.1 Properties of the SS Locus

The SS locus gives the level of p_t, given k_t^i such that $k_{t+1}^i = k_t^i$, where i stands for country i. We can write the equation of motion of the economy as $k_{t+1} = \frac{s(w_{e_t}(p_t))}{1+n}\frac{\lambda_t}{\lambda_{t+1}}$ and thus the SS locus is given by

$$\frac{s_t}{(1+n)}\frac{\lambda_t}{\lambda_{t+1}} - k_t^i = 0$$

The slope of the SS locus in the diversified area of production, is

$$\frac{\partial p_t}{\partial k_t^i}\,|_{SS} = \frac{(1+n)(1+\alpha y_t)^2 + \alpha s_t y_k}{(1+\alpha y_t)(\frac{\partial s_t}{\partial w_{e_t}}\frac{\partial w_{e_t}}{\partial p_t}) - \alpha s_t y_p}$$

when learning is in the Y sector and

$$\frac{\partial p_t}{\partial k_t^i}\,|_{SS} = \frac{(1+n)(1+\alpha x_t)^2 + \alpha s_t x_k}{(1+\alpha x_t)(\frac{\partial s_t}{\partial w_{e_t}}\frac{\partial w_{e_t}}{\partial p_t}) - \alpha s_t x_p}$$

when learning is in the X sector. Both of these expressions will be positive if α is small [4].

The position of the SS locus relative to the PP and KK locii can also be determined for sufficiently small α. When the PP locus lies above the KK locus the SS locus will be above the PP locus and when the PP locus is below the KK locus then the SS locus must be below the PP locus. To see that this is true remember all points below the KK locus are where $\frac{y_t}{(1+n)\frac{\lambda_{t+1}}{\lambda_t}} > k_t$ and points on the PP locus are where $s(w,p) = y$. Thus to make $\frac{s_t}{(1+n)\frac{\lambda_{t+1}}{\lambda_t}} = k_t$, we need to lower s, which since $\frac{\partial s_t}{\partial p_t} > 0$, implies p must fall. The assumption of a small α is needed only in the case of learning in the X sector. It is sufficient to ensure that the savings effect dominates the externality effect.

3.2.2 The Effects of Trade on the Small Open Economy

Using the SS curve and Figure 3 we can analyse the trade dynamics, the direction of trade and the effect of trade on growth in the small open economy.

The steady state equilibrium for this economy is the intersection of the SS curve with the exogenously given world price. The trade dynamics are interesting. The appendix A.3 shows that if the learning externality is in the Y sector, in the non-specialized production area the economy will oscillate towards the steady state equilibria and if the X sector has the externality, the economy will monotonically converge on the steady state. In areas of specialised production, assuming a unique steady state, there will be monotonic convergence to the steady state.

[4] See Mountford [14] for an examination of small country dynamics when α is large.

We also know the direction of trade. Since the PP locus represents the locus where $s(p_t) = y(p_t, k_t)$ then if the steady state is where the SS curve is above the PP curve it must be that the economy is importing the investment good.

The effect of trade on growth is less transparant. If the steady state international price is above the steady state autarky price then we know that $\bar{k}$ will be higher than in autarky but we cannot say whether the rate of growth is higher or not since the price and capital effects work against each other, ($y_p < 0$, $y_k > 0$) or ($x_p > 0$, $x_k < 0$). However if the intersection takes place in the area of specialization of the good with no learning by doing effects, then we know that in the long run there will be no per capita growth. Thus in this case we can say that, since there is always positive growth in the steady state in autarky, international trade has caused the small open economy's growth to decrease. In this case therefore international trade has had a very detrimental effect on the welfare of future generations of this economy. This is the case that is depicted in Figure 3.

To summarize, although statically we know from the gains from trade theorem of the standard $2 \times 2 \times 2$ trade model, that the small open economy gains from trade, dynamically this may not be the case and the small open economy may lose out.

3.3 Trade Dynamics Between Small and Large Economies

I consider a two country world where countries, A and B, only differ in their discount rates and in their initial conditions. This means that both countries will have the same KK locus but will have different PP locii. The relative positions of the two countries' PP and SS locii are very important. The country with the higher savings rate will have a lower PP and SS curve for each level of k. This will be used frequently in the analysis below. At first I will only consider the case where learning is in the Y sector. The other case is straightforward and is briefly discussed at the end of this subsection. I will proceed by considering the following two examples.

3.3.1 Both Countries Have a Unique Autarkic Steady State

In this example the only steady state for the world economy is where the high saving propensity country is very near to its autarkic steady state level of k and is large relative to the other country which behaves therefore like a small open economy. The world price will thus be very close to the autarkic steady state price of the high saving propensity country. We know from the properties of the SS locus that the low saving country will have a lower k than the high saving country and so since $y_k > 0$, the higher savings' propensity country will be growing faster than the other country thus maintaining their relative sizes. This situation is therefore stable. This is depicted in Figure 4, where Country B is the relatively small, low saving propensity country. Country B's SS curve is above it's PP curve and so it is importing the capital good in this steady state.

To see that this is the only steady state equilibrium, suppose that country B is the large economy and the world price is its autarkic steady state price. In this case country A will converge towards the intersection of this price with it's SS curve. But this equilibrium will not hold indefinately since at this point country A has a higher k than B and is therefore growing faster than B. Eventually country A will become the large economy and the world will tend towards the previously described steady state equilibrium.

3.3.2 Both Countries Have Multiple Autarkic Steady State Equilibria.

This subsection shows how trade can pull up or pull down the average rate of growth in the world economy and how one country can catch up and overtake the growth rate and income level of another country. The latter possibility is interesting also because of the reversal of trade direction which occurs during this transition.

Suppose that the low saving country in autarky is at its highest growth rate steady state equilibrium and that the high saving country is at its lowest growth rate steady state equilibrium. This can mean that the low saving rate country is growing faster than the high saving rate country ! Now we know from the previous subsection that in the long run trading equilibria the high savings rate country must be close to one of its autarkic steady state equilibria. But which one ? If initially the low saving country is relatively economically large then it can pull up the high savings country from a low to a high growth steady state, however, if the high savings rate country is initially economically large then it will pull down the growth rate of the low savings rate economy.

The first possibility was also the catching up and overtaking example. Initially the high savings rate country was poorer and growing slower than the low savings rate country. But we know in long run equilibrium from the previous subsection that the high savings rate economy will end up larger and growing faster than the low savings rate economy. Notice along this transition the direction of trade may reverse. Initially the high savings rate economy will operate near the autarkic steady state price of the low savings rate country, this may initially be above the PP curve of the high savings country which implies that this country imports capital goods. As the high savings rate economy approaches its SS locus its k will increase and it will cross beneath the PP curve and so export capital goods.

3.3.3 If Learning is in the X Sector

If the learning externality is in the X sector precisely the same sort of dynamics will occur, only now the faster growing country in the long run, will be in the country with the *lowest* saving rate. This is because in equilibrium the country with the largest k will be growing slowest, since $x_k < 0$. Thus now the only stable equilibrium point is where the high saving rate country is operating near an autarkic steady state price of the low saving rate country, on its SS locus which is to the right of the low savings rate country's autarkic steady state level of capital. The rest of the analysis goes through in this mirror image fashion.

3.4 Global Trade Dynamics

The previous section showed that in a steady state of the world economy it must be the case that one country becomes economically large relative to the other and that this large economy must be growing faster than the other economy. The previous section also showed that convergence to a trading steady state equilibrium may be a gradual process involving changing relative income levels and growth rates between countries and also changing trade patterns. This section gives the sufficient conditions under which, regardless of the initial sizes of the two countries and the initial levels of k_t^A and k_t^B, the world economy will converge to one of these trading steady state equilibria. The appendix A.4 shows that these conditions will be met with the learning externality in either sector if α is sufficiently small.

The two sufficient conditions for global convergence to a trading steady state equilibria are
(i) If $s^A < s^B$ and $k_t^A < k_t^B$ then $k_j^A < k_j^B \quad \forall j \geq t$

(ii) If one country is economically large relative to the other and if $s^A < s^B$ then $k_t^A < k_t^B$

The intuition behind these conditions is straightforward. Again I will only go through the case for the learning externality in the Y sector, the other case again just being the mirror image. Condition (ii) is simply the condition that the SS locii are well behaved and lie beneath the PP locus when the PP locus is below the KK locus, as described in section (3.2.1). Condition (i) says that once the country with the higher savings rate has a higher level of k, that it will always have a higher level of k.

Why does this ensure global convergence to a trading steady state equilibrium ? Condition (i), since $y_k > 0$, implies that once the country with the higher savings rate has a higher level of k, that it will always grow faster than the other country and thus will eventually become large relative to the other country. Condition (ii) ensures that at some point the high savings country must have a higher level of k, since if the other country had a larger k for a long time it would grow relatively economically large, but condition (ii) ensures that if this happened then the higher saving country must tend to a higher level of k.

3.5 Discussion of Assumptions

In the dynamic trade model the assumptions that require the most discussion are those about capital mobility, the form of technological progress and technological spillovers and again the size of α. It is well known that capital mobility in the two sector model can cause indeterminacy. Under factor price equalization for example, capital can flow from one country to the other without altering factor prices. This means that the actual levels of sectoral production in a country are indeterminate. Thus one must either follow Onika and Uzawa [15] in assuming no capital mobility or Fischer and Frenkel [7] in assuming an "adjustment cost" investment function. To concentrate on trade dynamics I chose the former. The assumption that technological progress spills over across both sectors is necessary so that the direction of trade is not arbitrarily determined by the initial direction of trade, see Stokey [19] for a similar argument. The assumption that learning occurs in only one sector is also common in the literature, see Krugman [11] or Grossman and Helpman [10] for example. If the externality were a "learning by using" instead of "learning by producing" capital goods, then this would be a different model, but intuitively one would expect very similar results. In the present model the relative savings rate of an economy will determine that economy's relative economic position in long run equilibrium. A higher saver will end up producing more capital goods and so, if the externality is in the Y sector, will grow faster. Since one would expect a higher saving economy, in a world with no lending or borrowing, also to use more capital goods, one would expect similar results to the present model.

The assumption which needs the greatest discussion is the α is small assumption. If one would like to set $\alpha = 1$ then, when the learning externality is in the X sector, you must also assume a unique and stable steady state equilibrium for the small open economy, which requires that $\frac{-sx_k}{(1+n)(1+x)^2} < 1$ at the steady state. If this is true then all the above analysis will go through. However when the learning externality is in the Y sector, a small α is necessary for global convergence. For the analysis of the small open economy and the dynamics between small and large economies to remain valid when $\alpha = 1$, one must also assume stability of the steady state for the small open economy, which requires that $\frac{-sy_k}{(1+n)(1+y)^2} > -1$ at the steady state.

4 Trading Equilibrium with Perfect Transfer of Knowledge

This is the other common assumption about at the transferability of knowledge and has been analysed in the infinite horizon framework by Grossman and Helpman [10] and Rivera-Batiz and Romer [16]. Although one can show under the current specification of technical progress that there may exist multiple steady state equilibria each with their own rate of growth it is very difficult to get any general result on the dynamics between these steady states. Instead I show that by altering the specification of technical progress, under factor price equalisation the world economy can be modelled as a single closed economy and thus may exhibit multiple steady state equilibria each with their own rate of growth.

4.1 Technical Progress

I will again only describe the case where technical progress is in the Y sector. I now assume that the knowledge at time t in each country is

$$\lambda_t = \sum_{i=0}^{t-1} \alpha^i \frac{Y_i}{L_i} = \alpha^i \sum_{i=0}^{t-1} \frac{Y_i}{L_i}$$

where α^i is the proportion of the world's population that live in country i, that is $\alpha^i = \frac{L_t^i}{L_t^A + L_t^B}$. Again Appendix 2 shows that the $\lambda_t = \sum_{i=0}^{t-1} Y_i$ will also enable us to model the world economy under factor price equalisation as a single closed economy.

Given this new definition and the assumption of perfect transferability of knowledge with no duplication, the addition to knowledge in each period is the sum of the knowledge produced in both countries. That is $\alpha^A \frac{Y_t^A}{L_t^A} + \alpha^B \frac{Y_t^B}{L_t^B} = \frac{Y_t}{L_t}$ where Y_t is the total world production of Y at time t and L_t is the world working population at time t. Thus we can write in the world economy

$$\frac{\lambda_{t+1}}{\lambda_t} = 1 + y_t$$

So the growth of knowledge in the world economy looks like the rate of growth of knowledge in a closed economy with $\alpha^i = 1$.

4.2 Factor Price Equalization

It is well known that in this two sector, two country world there will be factor price equalization if both economies produce both goods. This will enable us to derive a PP and KK locus for the world economy as a whole and thus show that under factor price equalization, the world economy behaves like a single closed economy and that therefore the world economy itself has multiple steady state equilibria.

4.2.1 PP,KK Locii under of Factor Price Equalisation

The PP locus is the locus of the momentary equilibrium for the world economy, it requires

$$\lambda_t s^A(w_{e_t}(p_t))L_t^A + \lambda_t s^B(w_{e_t}(p_t))L_t^B = Y_t^A + Y_t^B$$

dividing through by $\lambda_t L_t$ gives

$$\alpha^A s^A(w_{e_t}(p_t)) + \alpha^B s^B(w_{e_t}(p_t)) = y_t^A \alpha^A + y_t^B \alpha^B = y_t$$

One can think of this as a PP curve for a closed economy with a savings function which is a weighted average of the savings functions of the two individual economies.

The KK curve for the world economy can be derived in similar fashion. We require

$$K_{t+1} = Y_t^A + Y_t^B \longrightarrow k_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\frac{L_{t+1}}{L_t} = y_t$$

From the above definition of technical progress this implies

$$k_{t+1} = \frac{y_t}{(1+n)(1+y_t)}$$

From the above analysis the right hand side of this equation is only a function of p_t and k_t and is identical to the KK locus of a closed economy with $\alpha^i = 1$.

Thus we can conclude that since the world economy, under factor price equalization, behaves like a single closed economy and so the world economy too may have multiple steady state equilibria. Thus it may be possible to increase the growth rate of the world economy by increasing world k. Such a policy may require co-ordination.

5 Conclusion

This paper has developed a two sector endogenous growth model which can generate multiple steady state equilibria each with their own rate of growth. It has applied this model to a global analysis of international trade and has shown that the world economy too may have multiple steady state equilibria each with their own rate of growth. Furthermore the transition to one of these steady states is interesting and may involve reversals in trade direction, growth rates and income levels. Specific cases were examined where international trade reduced a country's growth rate to zero, where international trade pulled up the average growth rate in the world economy and also where international trade pulled down the average growth rate in the world economy and finally where international trade allowed one country's growth rate to catch up and overtake that of another.

Appendix A.

A.1. Stability Analysis of the Steady State Under Learning by Doing in the Y Sector

The analysis of Section 1.3 gives the following two dimensional system of difference equations

$$\begin{bmatrix} k_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} \psi(k_t, p_t) \\ \theta(k_t, p_t) \end{bmatrix} = \begin{bmatrix} \frac{y_t}{(1+n)(1+\alpha y_t)} \\ \theta(k_t, p_t) \end{bmatrix}$$

To analyse the stability of this system around a steady state equilibrium $(\bar{k}, \bar{p})$ we need to examine the eigenvalues of the Jacobian matrix, J, of this system.

$$J = \begin{bmatrix} \frac{y_k}{(1+n)(1+\alpha y_t)^2} & \frac{y_p}{(1+n)(1+\alpha y_t)^2} \\ \frac{y_k}{s_{p_{t+1}}} & \frac{y_p - s_{p_t}}{s_{p_{t+1}}} \end{bmatrix}$$

Defining the trace of J as trJ and the determinant of J as detJ, it is well known that the eigenvalues of J will be real, distinct and positive if $\mathrm{trJ} > 0$, $\mathrm{detJ} > 0$ and $\triangle \equiv (\mathrm{tr(J)})^2 - 4\times \det(\mathrm{J}) > 0$. Tediuos algebra can show that these conditions hold.

Relationship Between the PP and KK Locii and Stability

Given the above, the model will be a saddle point stable if (i) $1-\mathrm{tr(J)} + \det(\mathrm{J}) < 0$ and will be unstable if (ii) $1-\mathrm{tr(J)} + \det(\mathrm{J}) > 0$ and $\mathrm{trJ} > 2$. We will look at the case where the PP curve is upward sloping, the downward sloping case is straightforward. We will show that (i) is satisfied when the slope of the PP locus is steeper than that of the KK locus and (ii) is satisfied when the KK locus is steeper.

We can rewrite $1-\mathrm{tr(J)} + \det(\mathrm{J})$ as $(1-\psi_k)(1-\theta_p) - \psi_p\theta_k$ which in this model is

$$\frac{1}{s_{p_{t+1}}(1+n)(1+\alpha y)^2}\{[(1+n)(1+\alpha y_t)^2 - y_k][s_{p_{t+1}} + s_{p_t}] - (1+n)(1+\alpha y_t)^2 y_p\}$$

$$\longrightarrow 1-\mathrm{tr(J)} + \det(\mathrm{J}) \gtreqless 0 \text{ as } s_{p_{t+1}} + s_{p_t} \gtreqless \frac{(1+n)(1+\alpha y_t)^2 y_p}{(1+n)(1+\alpha y_t)^2 - y_k}$$

but this condition can be rewritten as the condition that the KK curve is steeper than the PP curve, which is

$$\frac{(1+n)(1+\alpha y_t)^2 - y_k}{y_p} > \frac{y_k}{s_{p_{t+1}} + s_{p_t} - y_p}$$

It only remains to show that when $1-\mathrm{tr(J)} + \det(\mathrm{J}) > 0$ that $\mathrm{trJ} > 2$. This is true because

$$1-\mathrm{tr(J)} + \det(\mathrm{J}) > 0 \longrightarrow y_p - s_{p_t} > s_{p_{t+1}}[1 - \frac{y_k}{(1+n)(1+\alpha y_t)^2}] - \frac{s_{p_t} y_k}{(1+n)(1+\alpha y_t)^2}]$$

The trace can be rewritten as

$$\frac{1}{s_{p_{t+1}}}[\frac{s_{p_{t+1}} y_k}{(1+n)(1+\alpha y_t)^2} + y_p - s_{p_t}]$$

Thus substituting in for $y_p - s_{p_t}$ implies

$$\mathrm{tr}J > 1 - \frac{s_{p_t} y_k}{s_{p_{t+1}}(1+n)(1+\alpha y_t)^2}$$

This expression is > 2 since from Rybcynski and the α is small assumption, $y_k > (1+n)(1+\alpha y_t)^2$ and from the upward sloping PP locus $s_{p_t} > s_{p_{t+1}}$

A.2. Stability Analysis of the Steady State Under Learning by Doing in the X Sector

The analysis is similar to the previous section only now the two dimensional system of difference equations is the following,

$$\begin{bmatrix} k_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} \psi(k_t, p_t) \\ \theta(k_t, p_t) \end{bmatrix} = \begin{bmatrix} \frac{y_t}{(1+n)(1+\alpha x_t)} \\ \theta(k_t, p_t) \end{bmatrix}$$

To analyse the stability of this system around a steady state equilibrium $(\bar{k}, \bar{p})$ we need to examine the eigenvalues of the Jacobian matrix, J, of this system.

$$J = \begin{bmatrix} \frac{(1+\alpha x)y_k - \alpha y x_k}{(1+n)(1+\alpha x_t)^2} & \frac{(1+\alpha x)y_p - \alpha y x_p}{(1+n)(1+\alpha x_t)^2} \\ \frac{y_k}{s_{p_{t+1}}} & \frac{y_p - s_{p_t}}{s_{p_{t+1}}} \end{bmatrix}$$

Again it can be shown tr$J$ and $\triangle > 0$ but α must be small for $detJ > 0$ and thus for the eigenvalues of J to be real, distinct and positive. This is because detJ is the following expression.

$$\frac{1}{s_{p_{t+1}}(1+n)(1+\alpha x)^2}\{-s_{p_t}[(1+\alpha x_t)y_k - \alpha y x_k] + \alpha y[y_k x_p - y_p x_k]\}$$

The second expression in the curly bracket is positive and α must be small enough for the first bracket to outweigh it. Alternatively one could assume that $k^y - k^x$ is sufficiently large since this expression is negatively related to this, see Ryder [18].In the following subsection we will call the second expression in the curly bracket δy.

Relationship Between the PP and KK Locii and Stability

Again following section A.1 We will rewrite $1-\text{tr}(J) + \det(J)$ as $(1-\psi_k)(1-\theta_p) - \psi_p\theta_k$ which in this model is

$$\frac{1}{s_{p_{t+1}}(1+n)(1+\alpha x)^2}\{[(1+n)(1+\alpha x_t)^2 - [(1+\alpha x)y_k - \alpha y x_k]][s_{p_{t+1}} + s_{p_t}] - (1+n)(1+\alpha x_t)^2 y_p + \delta y\}$$

$$\longrightarrow 1-\text{tr}(J) + \det(J) \gtreqless 0 \text{ as } s_{p_{t+1}} + s_{p_t} \gtreqless \frac{(1+n)(1+\alpha y_t)^2 y_p - \delta y}{(1+n)(1+\alpha x_t)^2 - [(1+\alpha x y_k - y x_k]}$$

but this condition can be rewritten as the condition that the KK curve is steeper than the PP curve, which is

$$\frac{(1+n)(1+\alpha x_t)^2 - (1+\alpha x)y_k + y x_k}{(1+\alpha x y_p - y x_p} > \frac{y_k}{s_{p_{t+1}} + s_{p_t} - y_p}$$

The proof that when $1-\text{tr}(J) + \det(J) > 0$ that tr$J > 2$ is similar to that in section A.1.

A.3. Small Economy Dynamics Under Diversification

If there is technological progress in the Y sector we have that

$$k_{t+1} = \frac{s(p_t)}{(1+n)(1+\alpha y)}$$

Thus

$$\frac{dk_{t+1}}{dk_t} = \frac{-\alpha s(p_t) y_k}{(1+n)(1+\alpha y)^2}$$

This will be between -1 and 0 for α small enough and so the dynamics will be stable and oscillating.

If there is technological progress in the X sector we have that

$$k_{t+1} = \frac{s(p_t)}{(1+n)(1+\alpha x)}$$

Thus

$$\frac{dk_{t+1}}{dk_t} = \frac{-\alpha s(p_t) x_k}{(1+n)(1+\alpha x)^2}$$

This will be between 0 and 1 for α small enough and so the dynamics will be stable.

A.4. Global Convergence Conditions

The two conditions for global convergence given in section ( ) are

(i) If $s^A < s^B$ and $k_t^A < k_t^B$ then $k_s^A < k_s^B \quad \forall s \geq t$

(ii) If one country is economically large relative to the other and if $s^A < s^B$ then $k_t^A < k_t^B$

Section (3.2.1) explained how condition (ii) is true if α is sufficiently small. Condition (i) will also be true for sufficiently low α since

$$k_{t+1}^A = \frac{s_t^A}{(1+n)\frac{\lambda_{t+1}}{\lambda_t}} \text{ and } k_{t+1}^B = \frac{s_t^B}{(1+n)\frac{\lambda_{t+1}}{\lambda_t}}$$

This implies that condition (i) will hold if the saving effect dominates the externality effect. When the externality is in the X sector there is no conflict between the two effects but when the externality is in the Y sector this requires a small α.

Appendix B

The growth model under the alternative specification of technical progress

We now define $L_t = \bar{L}\ \forall\ t$ and knowledge at time t as $\lambda_t = \sum_{i=0}^{t-1} Y_i$. This implies, since in all periods $\lambda_t s(w_{e_t}(p_t))\bar{L} = Y_t$, that

$$\frac{\lambda_{t+1}}{\lambda_t} = 1 + s(w_{e_t}(p_t))\bar{L}$$

Thus the KK locus depicts the level of p_t given k_t such that

$$\frac{y_t}{1 + s(w_{e_t}(p_t))\bar{L}} - k_t$$

This locus has the same properties as the other specification that is the partial derivative with respect to p of the above relationship gives

$$\frac{\frac{\partial y_t}{\partial p_t}(1+s) - y_t \frac{\partial s_t}{\partial p_t}}{[1 + s(w_{e_t}(p_t))\bar{L}]^2}$$

which is negative as long as both goods are produced, and the slope of the KK curve is given by

$$\frac{dp}{dk}\Big|_{k_{t+1}-k_t=0} = \frac{[1 + s(w_{e_t}(p_t)).\bar{L}]^2 - (1+s)y_k}{(1+s)y_p - y_t \frac{\partial s_t}{\partial p_t}}$$

which is also positive near the origin again using the Rybcynski Theorem.

In the world economy of two countries that differ in their rate of time preference, where knowledge is perfectly transferable and where the addition to knowledge in each period is the sum of the knowledge produced in both countries, then

$$\lambda_{t+1} - \lambda_t = Y_t^A + Y_t^B$$

In equilibrium under factor price equalisation we know that $\lambda_t s^A(w_{e_t}(p_t))L_t^A + \lambda_t s^B(w_{e_t}(p_t))L_t^B = Y_t^A + Y_t^B$ and so we can say

$$\frac{\lambda_{t+1}}{\lambda_t} = 1 + s^A(w_{e_t}(p_t))L^A + s^B(w_{e_t}(p_t))L^B = \phi(p_t)$$

The analysis of the PP,KK locii of the world economy can now be carried out as in the other specification of technical progress in section 4.1

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Figure 1

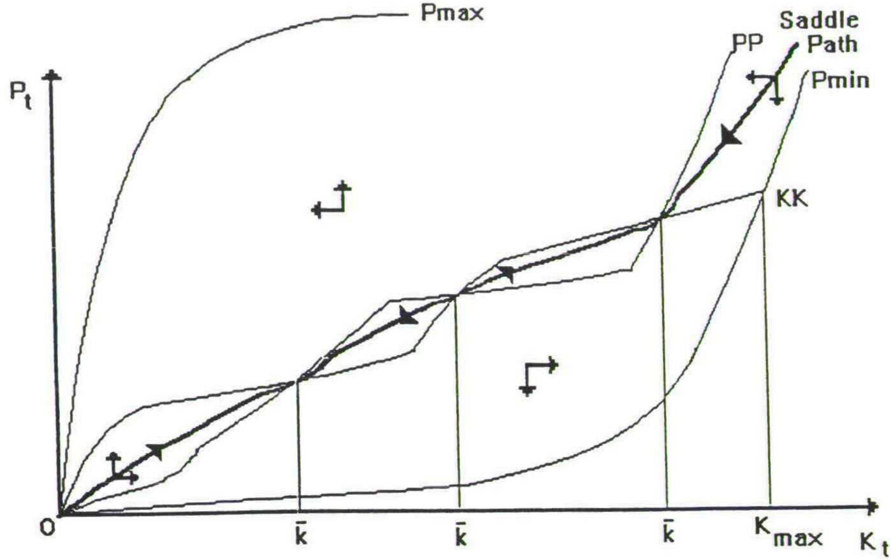

Figure 1. The autarkic growth model drawn for the case where there are three non-trivial steady states. These steady states are connected by the global stable manifold.

Figure 2

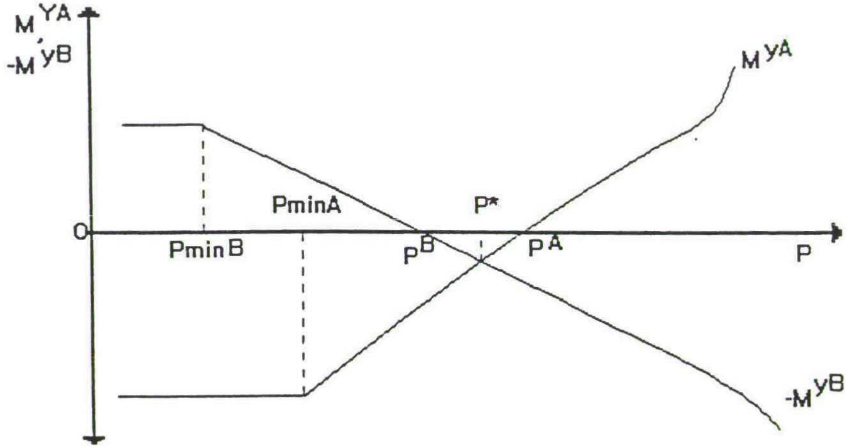

Figure 2. The reciprocal demand curves of countries A and B. The international price must be in between the two autarkic prices.

Figure 3

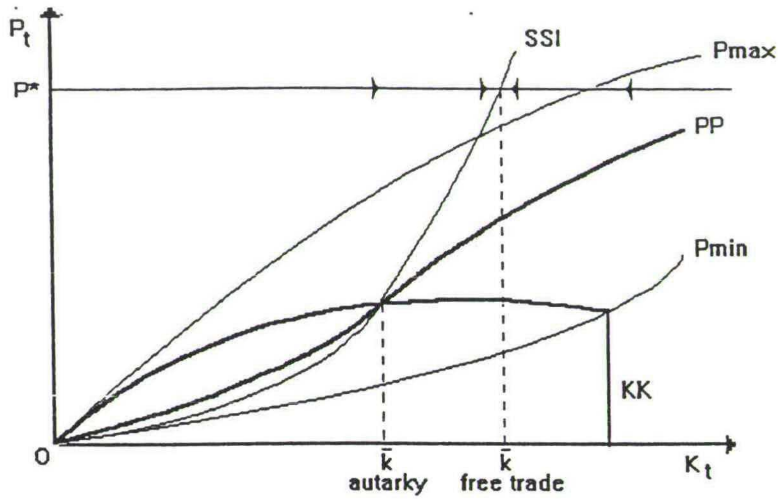

Figure 3. The small open economy drawn for the case where international trade causes specialization. Since the intersection of the SS curve with the international price lies above the p_{max} curve, the small open economy will produce only good X. If the learning externality is in sector Y, the small open economy will have zero per capita growth in the steady state.

Figure 4

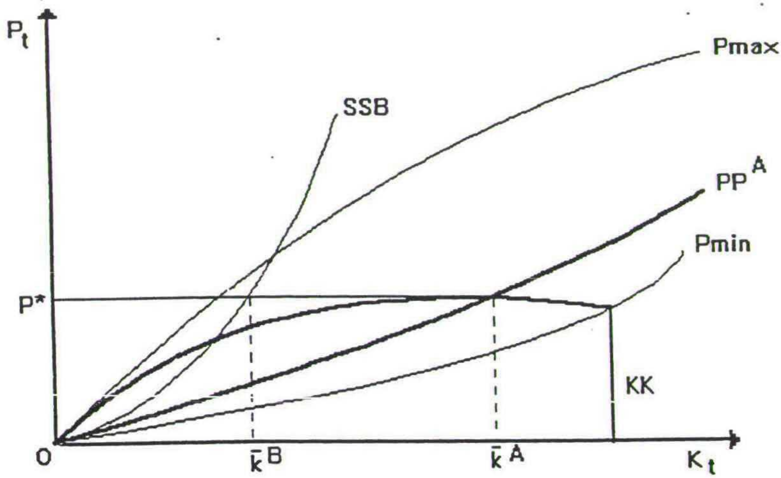

Figure 4. The steady state for the world economy for countries with different preferences and unique autarkic steady states, when the learning externality is in the Y sector. Country A is larger and is growing faster then country B. The international price is very close to Country A's autarkic price.

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