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Sensitivity to prior independence via Farlie-Gumbel-Morgenstern model

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Abstract

When the parameter space is multidimensional, the prior marginals are sometimes simple to specify, whereas the joint distribution may be extremely hard to elicit; as a consequence of this, prior independence is often assumed. When this is the case, it seems convenient to perform a sensitivity analysis, i.e., to consider a class of prior distributions (containing the independence as a particular case) and to compute the range of variation of the posterior quantity of interest, when the prior moves in that class. In this paper, the well-known Farlie-Gumbel-Morgenstern system (or a generalization of it) is proposed as the class of priors. We show that the robustness analysis is easy to carry out and, although this family may sometimes seem too narrow, it could be used as a first step in a robustness study.

1. Introduction

Let x be an observation from a sampling model $l(x|\theta)$, where $\theta = (\theta_1, \ldots, \theta_n) \in \Theta = \Theta_1 \times \ldots \times \Theta_n$. In order to perform a Bayesian analysis we need to specify a prior density $\pi(\theta_1, \ldots, \theta_n)$ on the parameters. The posterior density will then take the form

$$\pi(\theta_1, \ldots, \theta_n|x) \propto l(x|\theta_1, \ldots, \theta_n)\pi(\theta_1, \ldots, \theta_n).$$

Suppose that it is relatively simple to elicit our prior marginal densities $g_1(\theta_1), \ldots, g_n(\theta_n)$. If the parameters could be considered independent, the prior density would be $\pi(\theta_1, \ldots, \theta_n) = \Pi_{i=1}^n g_i(\theta_i)$. However, in some cases we may not have a strong belief in the latter assumption and we will want to carry out a study of robustness with respect to departures from prior independence. For this purpose it will be convenient to consider a class of prior densities, all having the specified marginals $g_1(\theta_1), \ldots, g_n(\theta_n)$, while containing the density under independence as a particular case, and to compute the ranges of the posterior quantities of interest as the prior moves in that class.

The paper by Lavine, Wasserman and Wolpert (1991) follows these lines. They consider an ϵ-contamination class where the contaminations allowed are all probability distributions with the given marginals. Although their approach is very interesting, it presents some inconveniences: in some cases the class may be too large, containing distributions that are not reasonable in view of our prior beliefs, resulting into a misleading impression of lack of robustness. Furthermore, in order to solve the problem, they need to simplify it, considering a finite partition for each component of the parameter space and assigning the probability corresponding to each set of the partition induced on Θ, to a single point in that set. Moreno and Cano (1992) propose other classes of prior distributions that could also be used as approximations to this ϵ-contamination class. Other studies of Bayesian robustness in multidimensional parameter spaces can be found in Lavine (1991a), and Berger and Moreno (1992).

Families of multivariate distributions with fixed marginals have been investigated in the literature for a long time. See, for instance, Mardia (1970) and Whitt (1976). There are well-known, simple models that, keeping the marginals fixed, allow for different degrees of dependence between the components. It seems quite reasonable to use some of these models as classes of prior distributions to analyze robustness with respect to departures from the assumption of prior independence. In particular, the Farlie-Gumbel-Morgenstern (FGM) family is very simple and could be appropriate for our purposes. It will be described in the next section. As a matter of fact, the FGM system is a particular case of a model with a very clear and intuitive meaning. This more general model will be introduced in Section 3. Ranges of posterior expectations as the prior moves over this class are computed in Section 4. Finally, some examples are analyzed in Section 5.

2. The Farlie-Gumbel-Morgenstern model

The FGM system was discussed by Morgenstern (1956), Gumbel (1958) and Farlie (1960); see also Johnson and Kotz (1975, 1977). This model is described next. For notational convenience, we shall restrict ourselves to the bidimensional case; however, the FGM system and the results in this paper can easily be extended to higher dimensions.

Let $F(\theta_1)$ and $G(\theta_2)$ be fixed marginal distribution functions. The Farlie-Gumbel-Morgenstern model corresponds to the following distribution function:

$$\Pi(\theta_1, \theta_2) = F(\theta_1)G(\theta_2)\big[1 + \lambda\{1 - F(\theta_1)\}\{1 - G(\theta_2)\}\big], \quad \text{where} \quad \lambda \in [-1, 1].$$

(Of course, $\Pi(\theta_1, \theta_2)$ depends on λ but, for the sake of simplicity, this dependence will not be explicitely expressed.)

If $F(\theta_1)$ and $G(\theta_2)$ have density functions $f(\theta_1)$ and $g(\theta_2)$ respectively, the density corresponding to $\Pi(\theta_1, \theta_2)$ is easily shown to be

$$\pi(\theta_1, \theta_2) = f(\theta_1)g(\theta_2)\big[1 + \lambda\{1 - 2F(\theta_1)\}\{1 - 2G(\theta_2)\}\big].$$

Notice that for $\lambda = 0$ we obtain $\pi(\theta_1, \theta_2) = f(\theta_1)g(\theta_2)$, that is, θ_1 and θ_2 are independent. The cases $\lambda = 1$ and $\lambda = -1$ represent the maximum degrees of positive and negative

dependence, respectively, allowed in this model. In our Bayesian analysis this translates into prior beliefs about the parameters. An advantage of the FGM system is that only moderate departures from independence are permitted. We next give two examples of this assertion (see Johnson and Kotz (1977)):
a) If the marginals are $N(0,1)$, the correlation is $\lambda\pi^{-1}$ (so it ranges from -0.318 to 0.318).
b) If the marginals are uniform distributions over $(0,1)$, the correlation is $\lambda/3$ (so it ranges from -0.333 to 0.333).
The FGM model can also be expressed as follows.
For $\lambda \in [0,1]$:

$$\pi(\theta_1,\theta_2) = (1-\lambda)f(\theta_1)g(\theta_2) + \lambda f(\theta_1)g(\theta_2)[1+\{1-2F(\theta_1)\}\{1-2G(\theta_2)\}],$$

and for $\lambda \in [-1,0]$:

$$\begin{aligned}\pi(\theta_1,\theta_2) &= (1+\lambda)f(\theta_1)g(\theta_2) - \lambda f(\theta_1)g(\theta_2)[1-\{1-2F(\theta_1)\}\{1-2G(\theta_2)\}] \\ &= (1-\mu)f(\theta_1)g(\theta_2) + \mu f(\theta_1)g(\theta_2)[1-\{1-2F(\theta_1)\}\{1-2G(\theta_2)\}]\end{aligned}$$

where $\mu = -\lambda \in [0,1]$.
This representation naturally suggests the generalization that we describe in the next section.

3. A more general model

Let us assume that in the process of prior elicitation we come up with the marginal densities $f(\theta_1)$ and $g(\theta_2)$ but we are not entirely comfortable with the assumption of prior independence between θ_1 and θ_2. In this case, a sensible and very simple class of prior densities is

$$\Gamma = \Gamma^+ \cup \Gamma^- \tag{3.1}$$

where

$$\begin{aligned}\Gamma^+ &= \{\pi(\theta_1,\theta_2) = (1-\lambda)\pi_I(\theta_1,\theta_2) + \lambda\pi^+(\theta_1,\theta_2),\ \lambda \in [0,1]\}, \\ \Gamma^- &= \{\pi(\theta_1,\theta_2) = (1-\lambda)\pi_I(\theta_1,\theta_2) + \lambda\pi^-(\theta_1,\theta_2),\ \lambda \in [0,1]\},\end{aligned} \tag{3.2}$$

$\pi_I(\theta_1,\theta_2) = f(\theta_1)g(\theta_2)$ is the density obtained under independence and $\pi^+(\theta_1,\theta_2)$ and $\pi^-(\theta_1,\theta_2)$ are fixed densities with marginals $f(\theta_1)$ and $g(\theta_2)$, representing some degree of positive and negative dependence, respectively.
Obviously, the FGM model is a special case of this one, with
$\pi^+(\theta_1,\theta_2) = f(\theta_1)g(\theta_2)[1+\{1-2F(\theta_1)\}\{1-2G(\theta_2)\}]$,
$\pi^-(\theta_1,\theta_2) = f(\theta_1)g(\theta_2)[1-\{1-2F(\theta_1)\}\{1-2G(\theta_2)\}]$.

The class Γ^+ is quite similar to the class of distributions considered in De la Horra and Ruiz-Rivas (1988).

It is easy to see that for $\lambda = 0$ we are in the case of prior independence between θ_1 and θ_2. As λ increases, we obtain a bigger degree of dependence, always keeping the marginal densities fixed at $f(\theta_1)$ and $g(\theta_2)$. The maximum degree of dependence allowed for is controlled by the choice of π^+ and π^-.

As we have already pointed out, the class defined in (3.1) and (3.2) is very adequate for studies of robustness to departures from the hypothesis of prior independence. Its main advantage is the simplicity of elicitation and computation. In the next section we provide analytical results for the supremum and the infimum of the posterior expectation of a function of interest, when the prior ranges over this class. This greatly alleviates the robustness analysis, which typically is computationally very demanding. We think this is an important point in favour of this class (see Example 5.1 below).

On the other hand, in some cases we may find that this class is too narrow, leaving out densities that we may not wish to discard a priori. Even when that is the case, the simplicity of its analysis still makes this class worth to be used at a first step of a robustness study. If we find absence of robustness with this class, the same conclusion would be obtained for a wider class of priors (see Example 5.2).

4. Sensitivity of posterior quantities

Consider the class of priors Γ introduced in (3.1) and (3.2). We are interested in the sensitivity of the posterior expectation of some function, $h(\theta_1, \theta_2)$, as the prior ranges over that class. We therefore need to obtain

$$\sup_{\pi\in\Gamma} \int_\Theta h(\theta)\pi(\theta|x)d\theta \quad \text{and} \quad \inf_{\pi\in\Gamma} \int_\Theta h(\theta)\pi(\theta|x)d\theta. \tag{4.1}$$

As Γ in (3.1) is the union of Γ^+ and Γ^-, the quantities in (4.1) are equal to

$$\begin{aligned} &\max\left\{ \sup_{\pi\in\Gamma^+} \int_\Theta h(\theta)\pi(\theta|x)d\theta,\ \sup_{\pi\in\Gamma^-} \int_\Theta h(\theta)\pi(\theta|x)d\theta \right\} \\ \text{and} \quad &\min\left\{ \inf_{\pi\in\Gamma^+} \int_\Theta h(\theta)\pi(\theta|x)d\theta,\ \inf_{\pi\in\Gamma^-} \int_\Theta h(\theta)\pi(\theta|x)d\theta \right\}, \end{aligned} \tag{4.2}$$

respectively.

Some common choices for $h(\theta)$ are $h(\theta_1, \theta_2) = \theta_1$, for studying the posterior mean of the parameter of interest, and $h(\theta_1, \theta_2) = I_C(\theta_1, \theta_2)$, for the posterior probability of a set C of interest.

In the next theorem, we give the supremum and the infimum of the posterior expectation of $h(\theta)$ when the prior $\pi(\theta)$ ranges in Γ^+. Similar results can be obtained for Γ^- changing π^+ by π^-.

Theorem. *With Γ^+ defined as in (3.2), if*

$$\frac{\int_\Theta h(\theta)l(x|\theta)\pi^+(\theta)d\theta}{\int_\Theta l(x|\theta)\pi^+(\theta)d\theta} \geq \frac{\int_\Theta h(\theta)l(x|\theta)\pi_I(\theta)d\theta}{\int_\Theta l(x|\theta)\pi_I(\theta)d\theta},$$

then

$$\sup_{\pi\in\Gamma^+} \int_\Theta h(\theta)\pi(\theta|x)d\theta = \frac{\int_\Theta h(\theta)l(x|\theta)\pi^+(\theta)d\theta}{\int_\Theta l(x|\theta)\pi^+(\theta)d\theta}$$

and

$$\inf_{\pi\in\Gamma^+}\int_\Theta h(\theta)\pi(\theta|x)d\theta = \frac{\int_\Theta h(\theta)l(x|\theta)\pi_I(\theta)d\theta}{\int_\Theta l(x|\theta)\pi_I(\theta)d\theta}.$$

If the inequality sign is reversed, the roles of the infimum and the supremum are interchanged.

Proof: For $\pi \in \Gamma^+$,

$$\int_\Theta h(\theta)\pi(\theta|x)d\theta = \frac{(1-\lambda)\int_\Theta h(\theta)l(x|\theta)\pi_I(\theta)d\theta + \lambda\int_\Theta h(\theta)l(x|\theta)\pi^+(\theta)d\theta}{(1-\lambda)\int_\Theta l(x|\theta)\pi_I(\theta)d\theta + \lambda\int_\Theta l(x|\theta)\pi^+(\theta)d\theta},$$

where $\lambda \in [0,1]$.
Its derivative with respect to λ is

$$\frac{\int_\Theta h(\theta)l(x|\theta)\pi^+(\theta)d\theta\int_\Theta l(x|\theta)\pi_I(\theta)d\theta - \int_\Theta h(\theta)l(x|\theta)\pi_I(\theta)d\theta\int_\Theta l(x|\theta)\pi^+(\theta)d\theta}{\left\{(1-\lambda)\int_\Theta l(x|\theta)\pi_I(\theta)d\theta + \lambda\int_\Theta l(x|\theta)\pi^+(\theta)d\theta\right\}^2}.$$

If

$$\frac{\int_\Theta h(\theta)l(x|\theta)\pi^+(\theta)d\theta}{\int_\Theta l(x|\theta)\pi^+(\theta)d\theta} \geq \frac{\int_\Theta h(\theta)l(x|\theta)\pi_I(\theta)d\theta}{\int_\Theta l(x|\theta)\pi_I(\theta)d\theta},$$

the derivative is nonnegative and the result is obtained. ♣

Needless to say, the above result could also be derived by applying the elegant linearization technique (see Lavine (1991b)), which turns the nonlinear problem of optimizing a posterior expectation into a linear one; but this technique is obviously not necessary here.

The theorem together with the statement in (4.2) imply that our calculations will be reduced to evaluating the integrals

$$\int_\Theta h(\theta)l(x|\theta)\pi_I(\theta)d\theta,\ \int_\Theta h(\theta)l(x|\theta)\pi^+(\theta)d\theta,\ \int_\Theta h(\theta)l(x|\theta)\pi^-(\theta)d\theta,$$
$$\int_\Theta l(x|\theta)\pi_I(\theta)d\theta,\ \int_\Theta l(x|\theta)\pi^+(\theta)d\theta \text{ and } \int_\Theta l(x|\theta)\pi^-(\theta)d\theta.$$

These evaluations will very often have to be done by numerical methods.

We would like to remark that the FGM model is even simpler, since it can be expressed as a function of a unique parameter

$$\pi(\theta_1,\theta_2) = f(\theta_1)g(\theta_2)\left[1+\lambda\{1-2F(\theta_1)\}\{1-2G(\theta_2)\}\right],\ \lambda\in[-1,1].$$

In this case, the extrema are attained at $\lambda = -1$ and $\lambda = 1$, which correspond to π^- and π^+, respectively, and only four integrals need to be computed.
In the following section we use this class in two robustness analyses.

5. Examples

Example 5.1: ECMO data

A clinical trial was designed to compare the performance of ECMO (extracorporeal membrane oxygenation) with that of the standard therapy in the treatment of a respiratory disease in infants (see Ware (1989)). Of nine patients that were given ECMO all nine survived, whereas of ten patients in the control group, only six survived.

Let p_1 and p_2 be the probabilities of survival under the standard treatment and ECMO, respectively. We reparameterize in terms of (δ, γ), where $\delta = \eta_2 - \eta_1$, $\gamma = (\eta_1 + \eta_2)/2$ and $\eta_i = \log\{p_i/(1-p_i)\}$, i=1, 2. The goal is then to determine whether $\delta > 0$ (or equivalently $p_2 > p_1$), γ being a nuisance parameter. Therefore we will focus on the posterior probability $P(\delta > 0|data)$.

Kass and Greenhouse (1989) carry out a robust analysis of this quantity by specifying 42 different prior distributions on (δ, η_1) and on (δ, γ), assuming in all cases prior independence. The prior marginal distributions favoured in this study are a Cauchy$(0, 1.099^2)$ for δ and a Cauchy$(0, 0.419^2)$ for γ. This choice is made on the basis of the assumption that randomization is ethically justifiable and some historical control information. Under this independent prior, it is obtained that $P(\delta > 0|data) \approx 0.94$. Other studies of sensitivity of this posterior quantity to the specification of the prior distribution can be found in Lavine (1991a), Lavine, Wasserman and Wolpert (1991), Berger and Moreno (1992) and Moreno and Cano (1992). They all find small ranges of variation of the posterior probability of ECMO being superior to the standard therapy, as long as the class of priors is not unreasonably large.

In this section, we shall use the FGM system to check for robustness to departures from prior independence, keeping the marginal distributions fixed at Cauchy$(0, 1.099^2)$ for δ and Cauchy$(0, 0.419^2)$ for γ. According to the previous section, the extrema of $P(\delta > 0|data)$ will take the form

$$\int_{\Re^2} h(\delta,\gamma)\pi^+(\delta,\gamma|data)d\delta d\gamma \quad \text{and} \quad \int_{\Re^2} h(\delta,\gamma)\pi^-(\delta,\gamma|data)d\delta d\gamma,$$

where

$$\begin{aligned} h(\delta,\gamma) &= I_{(0,+\infty)}(\delta), \\ \pi^+(\delta,\gamma) &= f(\delta)g(\gamma)[1 + \{1-2F(\delta)\}\{1-2G(\gamma)\}], \\ \pi^-(\delta,\gamma) &= f(\delta)g(\gamma)[1 - \{1-2F(\delta)\}\{1-2G(\gamma)\}], \end{aligned}$$

f, g are the density functions and F, G are the distribution functions of the fixed marginal Cauchy distributions.

Evaluating these integrals through Monte Carlo importance sampling gives

$$\int_{\Re^2} h(\delta,\gamma)\pi^+(\delta,\gamma|data)d\delta d\gamma \approx 0.96, \quad \int_{\Re^2} h(\delta,\gamma)\pi^-(\delta,\gamma|data)d\delta d\gamma \approx 0.89,$$

from which we conclude that the posterior probability of ECMO being superior to the standard treatment is reasonably robust when the prior ranges over the FGM class. Of course this is not a surprising finding in view of the previous literature, but it is included here with the purpose of illustrating the performance of our method with a well-known data set.

It is worth mentioning that although the FGM class is parametric, the bounds that we find here are not necessarily narrower than those obtained when using a more complicated, non-parametric class. For instance, the extreme values found in this particular example, are rather close to the ones in Lavine, Wasserman and Wolpert (1991) when ε lies in a neighborhood of 0.25.

Example 5.2:

Let (0.52,0), (4.95,0.03), (0.1,0.05), (-3.43,-0.01) and (3.15,-0.11) be an independent sample from a model with density

$$l(x_1, x_2|\theta_1, \theta_2) = \frac{1}{\pi\{1 + (x_1 - \theta_1)^2\}} \frac{1}{\sqrt{0.02\pi}} e^{-(x_2-\theta_2)^2/0.02},$$

that is, conditionally upon $\theta = (\theta_1, \theta_2)$, x_1 and x_2 are drawn independently from a Cauchy$(\theta_1, 1)$ and a Normal$(\theta_2, 0.1^2)$, respectively.

We wish to test the hypothesis $H_0 : \theta_1 < 1.25$ against $H_1 : \theta_1 > 1.25$. Thus we are interested in finding the posterior odds $P(H_0|data)/P(H_1|data)$.

The marginal prior for θ_1 is a mixture of two Gamma distributions, leading to the density function

$$f(\theta_1) = \left(\frac{1}{2}\left\{(0.25)^{-2} e^{-\theta_1/0.25}\theta_1\right\} + \frac{1}{2}\left\{\frac{(0.25)^{-10}}{9!} e^{-\theta_1/0.25}\theta_1^9\right\}\right) I_{(0,\infty)}(\theta_1),$$

whereas the marginal distribution for θ_2 is Exponential with density

$$g(\theta_2) = 2\, e^{-2\theta_2} I_{(0,\infty)}(\theta_2).$$

Observe that the prior chosen for θ_1 leads to $P(H_0)/P(H_1) = 1$, therefore representing indifference among the hypotheses.

Under the independent prior, $\pi_I(\theta_1, \theta_2) = f(\theta_1)g(\theta_2)$, the posterior odds are 3.17. If we now analyze sensitivity to departures from independence by embedding $\pi_I(\theta_1, \theta_2)$ in a FGM class, results in Section 4 tell us that the extrema of this posterior quantity will be attained for the priors π^+ and π^-. For π^+ it is found that $P(H_0|data)/P(H_1|data) \approx 6.14$, which strongly favours the null hypothesis, whereas for π^-, $P(H_0|data)/P(H_1|data) \approx 1.22$, which essentially does not favour any of the hypotheses. This range of variation seems too big to proceed confidently. Therefore, in this case, absence of robustness has been detected by means of a simple analysis.

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