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by Marno Verbeek
and Theo Nijman

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TESTING FOR SELECTIVITY BIAS IN PANEL DATA MODELS

Marno Verbeek*)
Theo Nijman*)

Tilburg University
Department of Economics
P.O. Box 90153
5000 LE Tilburg
The Netherlands

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Abstract

Missing observations are a rule rather than an exception in panel data. In this paper we discuss several tests to check for the presence of selectivity bias in regression estimates based on panel data. One approach to test for selectivity bias in these estimates is to specify the missing data mechanism explicitly and to estimate the response mechanism and the regression equation jointly. Alternatively, one can derive the asymptotically efficient Lagrange Multiplier test once an assumption on the response mechanism has been made. Both approaches are computationally demanding as e.g. multivariate probit models have to be estimated. We propose the use of simple variable addition and (quasi) Hausman tests to test for selectivity bias and compare the power of these tests with the asymptotically efficient tests using Monte Carlo methods.

1. Introduction

Missing observations are a rule rather than an exception in panel data sets. It is common practice in applied economic analysis of panel data to analyze only the observations on units for which a complete time series is available. Since the seminal contributions of Heckman [1976, 1979] and Hausman and Wise [1979] it is well known that inferences based on either the balanced sub-panel (with the complete observations only) or the unbalanced panel without correcting for selectivity bias, may be subject to bias if the nonresponse is endogenously determined. Because the estimation of the full model including a response equation explaining the missing observations, is, in general, rather cumbersome (cf. Ridder [1990], Verbeek [1989]), it is worthwhile to have some simple tests to check for the presence of selectivity bias which can be performed first. An obvious choice for such a test is the Lagrange Multiplier test, which requires estimation of the model under the null hypothesis only. As will be shown in this paper, the computation of the LM test statistic is still rather cumbersome and, in addition, its value is highly dependent on the specification of the response mechanism and the distributional assumptions (cf. Manski [1989]). In this paper we will therefore consider several simple tests to check for the presence of selectivity bias without the necessity to estimate the full model or to specify a response equation. A consequential advantage of these tests is that they can be performed in a simple way in cases with wave nonresponse, where all information on individuals is missing in some periods, as well as unit nonresponse, where only information on the endogenous variable may be missing.

For ease of presentation we will in this paper restrict attention to the linear regression model, although several of the tests can straightforwardly be generalized to nonlinear models. Consider

$$y_{it} = X_{it}\beta + \alpha_i + \varepsilon_{it}, \qquad t = 1,\dots,T;\ i = 1,\dots,N \tag{1}$$

where X_{it} is a k dimensional row vector of exogenous variables of individual i in period t, β is a column vector of unknown parameters of interest, α_i

and ε_{it} are unobserved random variables and T and N denote the number of periods and the number of individuals (households, firms) in the panel, respectively. We assume that observations on y_{it} are available if a latent variable r^*_{it} is nonnegative only, for which we assume

$$r^*_{it} = Z_{it}\gamma + \xi^*_i + \eta_{it}, \qquad t = 1,\ldots,T;\ i = 1,\ldots,N \tag{2}$$

with Z_{it} a row vector of exogenous variables, possibly containing (partly) the same variables as X_{it}, and η_{it} an unobserved random variable. Following Chamberlain [1984] we assume that,

$$\xi^*_i = Z_{i1}\pi_1 + Z_{i2}\pi_2 + \ldots + Z_{iT}\pi_T + \xi_i, \tag{3}$$

where ξ_i is independent of all Z_{it}'s, in order to account for possible correlation between ξ^*_i and the explanatory variables in Z_{it}. Substitution in (2) yields

$$r^*_{it} = Z_{it}\gamma + Z_{i1}\pi_1 + \ldots + Z_{iT}\pi_T + \xi_i + \eta_{it}. \tag{4}$$

The (observed) indicator variable r_{it} is defined as $I(r^*_{it} \geq 0)$ and we define $c_i = \Pi^T_{t=1} r_{it}$, so that $c_i = 1$ if and only if y_{it} is observed for all t. We assume that r_{it} and Z_{it} are always observed and that y_{it} and X_{it} are observed as well if $r_{it} = 1$. Normality of the error terms in (1) and (4) is assumed for convenience. In particular, we assume that the unobserved random variables are normally distributed according to

$$\begin{bmatrix} \varepsilon_i \\ \eta_i \\ \xi_i \\ \alpha_i \end{bmatrix} \sim N\left[0, \begin{bmatrix} \sigma^2_\varepsilon I_T & & & \\ \sigma_{\varepsilon\eta} I_T & \sigma^2_\eta I_T & & \\ 0 & 0 & \sigma^2_\xi & \\ 0 & 0 & \sigma_{\alpha\xi} & \sigma^2_\alpha \end{bmatrix}\right] \tag{5}$$

with $\varepsilon_i = (\varepsilon_{i1},\ldots,\varepsilon_{iT})'$ and $\eta_i = (\eta_{i1},\ldots,\eta_{iT})'$. For identification of the probit model we will impose (as usual) $\sigma_\eta^2 + \sigma_\xi^2 = 1$. Throughout, we impose the (important) assumption that $(\varepsilon_i', \eta_i', \xi_i, \alpha_i)$ is independent of X_{jt} and Z_{jt} $(\forall j,t)$.

The fully efficient maximum likelihood estimator of the parameters in the model can be derived using e.g. the results in Ridder [1990], but this will in general be a computationally highly demanding and time consuming estimation procedure, which is unattractive in applied work (cf. Nijman and Verbeek [1989]). In this paper attention will be paid to several simple testing procedures that can be used to check whether selectivity bias is seriously present, and thus to decide whether such a complicated estimation procedure is required. First, we shall have a look at the consequences of selective nonresponse on four commonly used estimators for the parameter vector β in (1), namely the fixed effects and the random effects estimators based on the balanced sub-panel and the unbalanced panel. Second, we suggest several simple tests for selectivity bias and compare the power properties of these tests with those of the Lagrange Multiplier test.

The paper is organized as follows. In Section 2, where analytical conditions for the fixed effects and random effects estimators on either balanced or unbalanced panels to be (asymptotically) unbiased are derived, we show that the fixed effects (FE) estimator is more robust to nonresponse biases than the random effects (RE) estimator. In Section 3 numerical results from a Monte Carlo study are presented to illustrate these findings. In Section 4 we show how differences between the FE and RE estimators from a balanced and unbalanced design can be used to construct simple (quasi) Hausman tests of selectivity bias, while Section 5 discusses other tests for selectivity bias, including some simple variable addition tests and the (asymptotically efficient) Lagrange Multiplier test. In Section 6 a Monte Carlo study is used to investigate the power properties of the proposed tests in comparison with the LM test. Finally, Section 7 contains some concluding remarks.

2. Selectivity bias in the fixed and random effects estimators

In this section we will derive conditions for the (asymptotic) bias of the fixed effects (or "within") estimator of the regression coefficients β in (1) to be zero. Subsequently, we will consider the random effects estimator. If we define $\tilde{X}_{it}$ as the value of X_{it} in deviation from its (observed) individual mean, i.e.

$$\tilde{X}_{it} = X_{it} - \sum_{s=1}^{T} X_{is} r_{is} \Big/ \sum_{s=1}^{T} r_{is} \quad \text{if} \quad \sum_{s=1}^{T} r_{is} > 0 \tag{6}$$

$$= 0 \qquad\qquad \text{otherwise}$$

and analogously for $\tilde{y}_{it}$, the FE estimator based on the unbalanced panel is given by (cf. Hsiao [1986], p. 31)[1)]

$$\hat{\beta}_{FE}(U) = \left[\sum_{i=1}^{N} \sum_{t=1}^{T} \tilde{X}'_{it} \tilde{X}_{it} r_{it} \right]^{-1} \left[\sum_{i=1}^{N} \sum_{t=1}^{T} \tilde{X}'_{it} \tilde{y}_{it} r_{it} \right] \tag{7}$$

and the one based on the balanced sub-panel by

$$\hat{\beta}_{FE}(B) = \left[\sum_{i=1}^{N} \sum_{t=1}^{T} \tilde{X}'_{it} \tilde{X}_{it} c_i \right]^{-1} \left[\sum_{i=1}^{N} \sum_{t=1}^{T} \tilde{X}'_{it} \tilde{y}_{it} c_i \right] . \tag{8}$$

Evidently, $\hat{\beta}_{FE}(.)$ is consistent (for $N \to \infty$) if selection given Z_{it} is completely random, i.e. if the hypothesis H_0: $\sigma_{\varepsilon\eta} = \sigma_{\alpha\xi} = 0$ holds. Moreover, as already noted by Heckman [1979] in the cross sectional sample selection model, no bias in the slope parameters will occur if $\pi_1 = \ldots = \pi_T = \gamma_1 = 0$ or if Z_{is} is independent of X_{it}. We shall now turn to the question whether weaker conditions for consistency can be derived. Letting $r_i = (r_{i1},\ldots,r_{iT})'$, it is straightforward to show that $\hat{\beta}_{FE}(U)$ and $\hat{\beta}_{FE}(B)$ are consistent estimators (for $N \to \infty$) of β if[2)]

$$E\{ \tilde{\varepsilon}_{it} \mid r_i \} r_{it} = 0, \qquad t = 1,\dots,T,\ i = 1,\dots,N \tag{9}$$

or

$$E\{ \tilde{\varepsilon}_{it} \mid c_i \} c_i = 0, \qquad t = 1,\dots,T,\ i = 1,\dots,N, \tag{10}$$

respectively. Consequently, a sufficient condition[3] for both conditions (9) and (10) to hold is that

$$E\{ \tilde{\varepsilon}_{it} \mid r_i \} = 0, \qquad t = 1,\dots,T,\ i = 1,\dots,N. \tag{11}$$

In Appendix A we show that for the case of normally distributed errors

$$E\{ \tilde{\varepsilon}_{it} \mid r_i \} = \sigma_{\varepsilon\eta}\sigma_{\eta}^{-2} \Big[E\{ \xi_i+\eta_{it} \mid r_i \} - \sum_{s=1}^{T} r_{is}\, E\{ \xi_i+\eta_{is} \mid r_i \} \Big/ \sum_{s=1}^{T} r_{is} \Big]. \tag{12}$$

Equation (12) implies that $\hat{\beta}_{FE}$ is consistent not only if $\sigma_{\varepsilon\eta} = 0$, but also if $E\{ \xi_i+\eta_{it} \mid r_i \}$ does not vary over time. The latter condition implies (see Appendix A) that there is no selectivity bias if the probability of an individual of being observed is constant over time, even if $\sigma_{\varepsilon\eta} \neq 0$. This is caused by the fact that the correction term for selectivity in (1) is absorbed in the fixed effect if it is constant over time. This was noted earlier in a different model by Meghir and Saunders [1987]. Since (12) does not contain $\sigma_{\alpha\xi}$, a correlation between the individual effects in the structural equation (1) and the probit equation (4) does not result in a bias in the fixed effects estimator. In this case selectivity has an effect on the structural equation which is fixed for a given individual over all periods in which its dependent variable is observed.

Next we consider the random effects estimator (cf. Hsiao [1986, p. 34 ff.]). Defining $\iota = (1, 1,\dots, 1)'$ of dimension T,

$$\Omega = V\{\iota\alpha_i + \varepsilon_i\} = \sigma_\alpha^2 \iota\iota' + \sigma_\varepsilon^2 I, \quad \Omega_i^r = r_i r_i' * \Omega,$$

$$X_i = \begin{bmatrix} X_{i1} \\ \vdots \\ X_{iT} \end{bmatrix}, \quad X_i^r = \begin{bmatrix} r_{i1}X_{i1} \\ \vdots \\ r_{iT}X_{iT} \end{bmatrix}, \quad y_i = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \text{and} \quad y_i^r = \begin{bmatrix} r_{i1}y_{i1} \\ \vdots \quad \vdots \\ r_{iT}y_{iT} \end{bmatrix},$$

where $*$ denotes the Hadamard (elementwise) product, the random effects estimator based on the unbalanced panel can be written as[4)]

$$\hat{\beta}_{RE}(U) = \left[\sum_{i=1}^{N} X_i^{r'} (\Omega_i^r)^+ X_i^r \right]^{-1} \left[\sum_{i=1}^{N} X_i^{r'} (\Omega_i^r)^+ y_i^r \right], \tag{13}$$

where A^+ indicates the Moore-Penrose inverse of A. If only the complete observations in the panel are used the random effects estimator is given by

$$\hat{\beta}_{RE}(B) = \left[\sum_{i=1}^{N} X_i' \Omega^{-1} X_i c_i \right]^{-1} \left[\sum_{i=1}^{N} X_i' \Omega^{-1} y_i c_i \right], \tag{14}$$

Note that these estimators can easily be computed using OLS on transformed data even if the unbalanced panel is used (see, e.g., Baltagi [1985] or Wansbeek and Kapteyn [1989]).

The estimators $\hat{\beta}_{RE}(.)$ are (asymptotically) unbiased if

$$E\{ \alpha_i + \varepsilon_{it} \mid r_i \} = 0 \qquad t = 1,\dots,T,\ i= 1,\dots,N. \tag{15}$$

In case of normally distributed errors, the expectation of ε_{it} given selection is given by (see Appendix A)

$$E\{ \varepsilon_{it} \mid r_i \} = \sigma_{\varepsilon\eta}\sigma_\eta^{-2} \left[E\{ \xi_i + \eta_{it} \mid r_i \} - \frac{\sigma_\xi^2}{\sigma_\eta^2 + T\sigma_\xi^2} \sum_{s=1}^{T} E\{ \xi_i + \eta_{is} \mid r_i \} \right], \tag{16}$$

while the conditional expectation of α_i given selection is given by (see Appendix A)

$$E\{ \alpha_i \mid r_i \} = \frac{\sigma_{\alpha\xi}}{\sigma_\eta^2 + T\sigma_\xi^2} \sum_{s=1}^{T} E\{ \xi_i + \eta_{is} \mid r_i \}. \tag{17}$$

Note that the second term on the righthand-side of (16) and the righthand-side of (17) cancel out in (15) and hence have no effect on the bias if $\sigma_{\alpha\xi}/\sigma_{\varepsilon\eta} = \sigma_\xi^2/\sigma_\eta^2$, which restriction was implicitely imposed by Hausman and Wise [1979]. Clearly, $\sigma_{\alpha\xi} = \sigma_{\varepsilon\eta} = 0$ implies that (15) will hold. However, the condition that $E\{ \xi_i+\eta_{it} \mid r_i \}$ does not vary with t which is sufficient for the consistency of β_{FE} is not sufficient for the consistency of $\hat{\beta}_{RE}$. For the latter we either need that $E\{ \xi_i+\eta_{it} \mid r_i \}$ is constant and $T \to \infty$ (since the FE-estimator and the RE-estimator are equivalent when T tends to infinity[5]) or that $E\{ \xi_i+\eta_{it} \mid r_i \}$ is constant and $\sigma_{\alpha\xi} = -\sigma_{\varepsilon\eta}$, which does not seem to be very likely in practice. Thus, the fact that for small T the estimator $\hat{\beta}_{FE}$ is more robust to selective nonresponse than $\hat{\beta}_{RE}$ might be a reason to prefer the fixed effects estimator although of course some efficiency is lost by this choice if in fact $\sigma_{\varepsilon\eta} = \sigma_{\alpha\xi} = 0$ (and the model is correct).

The size of the bias is determined by the projection of the conditional expectation that was derived above on the (transformed) X_{it}'s. Although it is possible to analyze the effects of changes in model parameters on the conditional expectation of the (transformed) error term analytically (cf. Ridder [1988]), it is, in general, virtually impossible to give analytical expressions in terms of the model parameters for projections of these expectations on the explanatory variables, i.e. of the biases in the estimators. To obtain some insight in the numerical importance of the bias in the four estimators discussed above, we will present some numerical results in the next section.

3. Some numerical results on the pseudo true values of the RE and FE estimators.

In this section we will present some numerical results on the pseudo true values of $\hat{\beta}_{FE}$ and $\hat{\beta}_{RE}$ for a simple model consisting of equations (1) and (4) with only one exogenous variable included besides the constant term. This exogenous variable ($z_{it} = x_{it}$) is assumed to be generated by a Gaussian AR(1) process with mean zero, autocorrelation coefficient ρ_x and variance σ_x^2. For simplicity we have imposed equality of all π_t's in (3). The model used for simulation is thus given by

$$y_{it} = \beta_0 + \beta_1 x_{it} + \alpha_i + \varepsilon_{it} \tag{18}$$

$$r_{it}^{*} = \gamma_0 + \gamma_1 x_{it} + \pi \bar{x}_i + \xi_i + \eta_{it} \tag{19}$$

where $\bar{x}_i$ is the average value of the x_{it}'s over time.

We consider two possible specifications for the selection equation, one in which π is a priori set to zero (in which case selection in period t is determined by x_{it}), and one in which γ_1 is a priori set to zero such that the average value of x_{it} over time determines selection. Given this choice of specification, the relative biases of the estimators for β_1 in this model, defined as $(\bar{\beta}_1-\beta_1)/\beta_1$, where $\bar{\beta}_1$ is the pseudo true value of the respective estimators for β_1, depend on

- T, the number of time periods;
- $\rho_\alpha = \sigma_\alpha^2(\sigma_\alpha^2+\sigma_\varepsilon^2)^{-1}$, the importance of the individual effect in equation (18);
- $\rho_\xi = \sigma_\xi^2$, importance of the individual effect in the selection equation;
- ρ_x, the autocorrelation coefficient of x_{it};
- $p_0 = \Phi(\gamma_0)$, the (unconditional) probability of observation when $x_{it} = 0$ for all t;
- $R_y^2 = \beta_1^2\sigma_x^2(\beta_1^2\sigma_x^2+\sigma_\alpha^2+\sigma_\varepsilon^2)^{-1}$, the (theoretical) R^2 of equation (1);

- R_r^2, the R^2 of the selection equation; $R_r^2 = \gamma_1^2\sigma_x^2(\gamma_1^2\sigma_x^2+1)^{-1}$ if $\pi = 0$, or $R_r^2 = \pi^2\sigma_{\bar{x}}^2(\pi^2\sigma_{\bar{x}}^2+1)^{-1}$ if $\gamma_1 = 0$ with $\sigma_{\bar{x}}^2 = \sigma_x^2(3 + 4\rho_x + 2\rho_x^2)/9$ (the variance of $\bar{x}_i$);

and

- $\rho_{\varepsilon\eta} = \sigma_{\varepsilon\eta}/\sigma_\varepsilon\sigma_\eta$, the correlation between the error shocks in (18) and (19);
- $\rho_{\alpha\xi}$, the correlation between the individual effects in (18) and (19).

If we assume that all correlations are nonnegative, all parameters, except T, are restricted to the interval [0,1]. Without loss of generality, it is assumed that $\gamma_1 \geq 0$ or $\pi \geq 0$. In Table 1 estimated relative biases (relative differences between the estimated pseudo true values and the true values) of the four estimators discussed above are given for several combinations of parameter values and T = 3. The number of replications is chosen in such a way that all standard errors are smaller than .005. Although, as always, it is difficult to draw definitive conclusions from results for specific parameter values the results in Table 1 suggest the following points.

- The biases in the estimators can be substantial. In some cases it is even possible that the sign of the pseudo true value is opposite to the sign of the true value of β_1. Moreover, like other simulation results (not reported in this paper) suggest, if the true β_1 parameter is equal to zero (which implies that $R_y^2 = 0$), a significant effect of the explanatory variable on y_{it} can be found. This phenomenon is also known from the standard (cross section) sample selection model of Heckman [1979].
- The bias is negative in all cases for which a selectivity bias is expected from the analytical results of the previous section. Thus, as one would expect, the parameter of interest is underestimated if selection is positively affected by the corresponding variable ($\gamma_1 > 0$, $\pi > 0$) and if the covariances between the error terms in the structural equation and the probit equation are positive ($\sigma_{\varepsilon\eta} > 0$, $\sigma_{\alpha\xi} > 0$). If γ_1 (or π) and the covariances have opposite signs, the parameter β_1 would be overestimated.

Table 1. Relative bias (in %) in the FE and RE estimators from a balanced and an unbalanced panel

Reference situation: $T = 3$, $R_y^2 = .1$, $R_r^2 = .9$, $\rho_\alpha = .1$, $\rho_x = .7$, $p_0 = .5$, $\rho_\xi = .1$ and $\rho_{\alpha\xi} = .5$

A] $\pi = 0$, $\rho_{\varepsilon\eta} = .9$

Estimator	REF.	$R_y^2=.9$	$R_r^2=.1$	$\rho_\alpha=.9$	$\rho_x=.3$	$p_0=\Phi(1)$	$\rho_\xi=.9$	$\rho_{\alpha\xi}=.9$
FE(B)	-78	-8	-49	-25	-90	-61	-28	-77
RE(B)	-79	-9	-49	-27	-93	-61	-39	-81
FE(U)	-98	-10	-50	-33	-101	-77	-37	-98
RE(U)	-116	-13	-53	-39	-115	-88	-56	-121

B][3)] $\pi = 0$, $\rho_{\varepsilon\eta} = 0$

Estimator	REF.	$R_y^2=.9$	$R_r^2=.1$	$\rho_\alpha=.9$	$\rho_x=.3$	$p_0=\Phi(1)$	$\rho_\xi=.9$	$\rho_{\alpha\xi}=.9$
RE(B)	-6	-1	-5	-2	-6	-6	-17	-11
RE(U)	-6	-1	-4	-6	-7	-5	-19	-12

C][3)] $\gamma_1 = 0$, $\rho_{\varepsilon\eta} = .9$

Estimator	REF.	$R_y^2=.9$	$R_r^2=.1$	$\rho_\alpha=.9$	$\rho_x=.3$	$p_0=\Phi(1)$	$\rho_\xi=.9$	$\rho_{\alpha\xi}=.9$
RE(B)	-34	-3	-38	-1	-17	-27	-17	-35
RE(U)	-74	-7	-44	-4	-41	-61	-32	-75

Notes: 1) The number of replications in each situation is chosen in such a way that all standard errors are smaller than 0.5 %.

2) All simulation results are obtained using the NAG-library subroutines G05CCF and G05DDF.

3) From the analytical results we know that the fixed effects estimators are consistent in this case, which was confirmed by the Monte Carlo results.

- Although the fact that the conditions for the fixed effects estimator to be consistent are weaker than those for the random effects estimator does not necessarily imply that the bias in the latter is always larger than that in the first, our simulations show that this is in fact the case. If there is a difference between the RE and FE pseudo true values, it is in favor of the latter estimator. This result is caused by the fact that we have assumed that $\rho_{\alpha\xi} > 0$. In the not very likely situation where $\rho_{\alpha\xi} < 0$ and $\rho_{\varepsilon\eta} > 0$, the bias in the random effects estimator may in fact be smaller. If the amount of bias is used as criterion for choosing an estimator, it is obvious from our analytical and numerical results that the fixed effects estimator is likely to be preferable to the random effects estimator.
- For almost all situations we consider, the bias in the estimator based on the unbalanced panel is larger (in absolute value) than that in the same estimator based on the balanced panel; if it is smaller the difference between the two estimates is negligible given the size of the Monte Carlo experiment. This somewhat surprising result suggests that a balanced panel may be preferred to an unbalanced panel. A possible explanation for this result might be that the individuals that are not observed in all periods have on average a lower probability of being observed, thus also a lower probability in those periods they are observed, implying a larger correction term in the regression equation. In the standard sample selection model of Heckman this would mean that for those individuals Heckman's lambda deviates more from zero.

Keeping all parameters fixed at some level except one, it may be possible to say something about the change of the bias if that one parameter is changed. It is evident from the analytical results and also from the numerical results above that a rise in R_y^2 will cause a decrease in the absolute value of the bias, simply because a rising R_y^2 diminishes the role of the error terms α_i and ε_{it}. On the other hand, a rise in R_r^2 increases the absolute value of the bias, since it increases the correlation between the probabilities of being observed and the explanatory variable(s) x_{it}. For $p_0 \geq \frac{1}{2}$ ($\gamma_0 \geq 0$), an increase in p_0 diminishes this correlation and therefore decreases the absolute value of the bias. Obviously, increasing the (absolute values of the) correlation coefficients $\rho_{\varepsilon\eta}$ or $\rho_{\alpha\xi}$ (already being nonnegative) causes a rise in the absolute value of the bias of all estimators. A more important individual effect in equation (18), ρ_{α}, seems to reduce the absolute value of the bias; the effect of ρ_x and ρ_{ξ} however is ambiguous.

4. Hausman tests for selective nonresponse

In Section 2 four estimators of β have been presented which are all consistent in the absence of selective nonresponse (i.e. if $\rho_{\varepsilon\eta} = \rho_{\alpha\xi} = 0$), namely the fixed effects estimators based on the balanced (sub)panel and the unbalanced panel and the random effects estimators based on the balanced and unbalanced panel. It will be clear from the analytical and numerical results in the previous sections that it is quite unlikely that the pseudo true values of either two estimators are identical, unless both estimators are consistent. Therefore, it is possible to construct a test for selectivity bias based on the differences between either two, three or four estimators. Letting

$$\hat{\beta} = (\ \hat{\beta}'_{FE}(B),\ \hat{\beta}'_{FE}(U),\ \hat{\beta}'_{RE}(B),\ \hat{\beta}'_{RE}(U)\ )' \xrightarrow{P} \bar{\beta},\quad N \to \infty \tag{20}$$

and V the corresponding asymptotic variance covariance matrix, the hypothesis $R\bar{\beta} = 0$ can be tested using

$$\xi_R = N\ \hat{\beta}'R'\ (R\hat{V}R')^{-}\ R\hat{\beta}, \tag{21}$$

which is asymptotically distributed as a central Chi-square with d degrees of freedom under $R\bar{\beta} = 0$ (the null hypothesis), where A^{-} denotes a generalized inverse of A and d is the rank of RVR'. Note that the asymptotic Chi-square distribution is also valid in cases with non-normal errors in (1) and (4) if some regularity conditions are met (see, e.g., Newey [1985]).

In order to be able to compute the test statistics in (21) for the restrictions we would like to test, the full matrix V is needed. Using the following property (see, e.g., Hausman [1978]) of an efficient estimator $\hat{\vartheta}_E$ for ϑ and any consistent estimator $\hat{\vartheta}_C$ for ϑ[6),]

$$\operatorname{Cov}\{ \hat{\vartheta}_E - \hat{\vartheta}_C , \hat{\vartheta}_C \} = 0, \tag{22}$$

and the definitions of the four estimators it can be shown that all blocks in the matrix V are a function of the variance covariance matrices of the four estimators in $\hat{\beta}$ only. In particular, it holds that

$$V = \begin{bmatrix} V_{11} & V_{22} & V_{33} & V_{44} \\ & V_{22} & V_{22}V_{11}^{-1}V_{33} & V_{44} \\ & & V_{33} & V_{44} \\ & & & V_{44} \end{bmatrix} \tag{23}$$

where $V_{11} = V\{\hat{\beta}_{FE}(B)\}$, $V_{22} = V\{\hat{\beta}_{FE}(U)\}$, $V_{33} = V\{\hat{\beta}_{RE}(B)\}$ and $V_{44} = V\{\hat{\beta}_{RE}(U)\}$. Using (23) any test statistic given in (21) can easily be computed. Two obvious candidates from the tests that compare two out of four possible estimators, are those comparing the fixed or random effects estimators from the balanced sub-panel and the unbalanced panel, where $R = R_1 = [\ I\ {-I}\ 0\ 0\ ]$ or $R = R_2 = [\ 0\ 0\ I\ {-I}]$, respectively. Two other choices, $R_3 = [\ I\ 0\ {-I}\ 0\ ]$ and $R_4 = [\ 0\ I\ 0\ {-I}\ ]$, result in the standard Hausman specification test for uncorrelated individual effects (see, e.g., Hsiao [1986, p. 48]) and its generalization to an unbalanced panel, respectively. A fifth test compares the FE estimator in the balanced sub-panel and the RE estimator in the unbalanced panel ($R_5 = [\ I\ 0\ 0\ {-I}\ ]$), while for the last possible test $R_6 = [\ 0\ I\ {-I}\ 0\ ]$. The first five tests are easy to compute since the variance covariance matrix RVR' in the test statistics is simply the difference between two diagonal blocks of V, in particular, the difference between the variance of the consistent estimator and the efficient estimator. For the sixth test neither of the two estimators is efficient and the variance covariance matrix is somewhat more complicated. Any test statistic based on comparison of three or four estimators tests two or three of the hypotheses $R_i'\bar{\beta} = 0$. Note that any combination of three restrictions from $R_i'\bar{\beta} = 0$ yields an equivalent null hypothesis with equivalent test statistics.

Unlike in the standard case the Hausman tests proposed above are based on estimators which are all inconsistent under the alternative. In the very unlikely case where all estimators would have identical asymptotic biases these tests will have no power at all. Keeping this in mind the null hypotheses (H_0^i: $R_i\bar{\beta} = 0$) of the tests above can be translated into hypotheses in terms of the model parameters. If we define

H_0^{FE}: $\rho_{\varepsilon\eta} = 0$ or $Z_{it}\gamma$ is constant over t (the fixed effects estimators are consistent), and

H_0^{RE}: [$\rho_{\varepsilon\eta} = \rho_{\alpha\xi} = 0$] or [$Z_{it}\gamma$ is constant over t and $\sigma_{\varepsilon\eta} + \sigma_{\alpha\xi} = 0$] (the random effects and the fixed effects estimators are consistent),

then the following relationships hold:

$$
\begin{array}{lllll}
 & \Rightarrow & H_0^{RE} & \Rightarrow & H_0^i,\ i = 1,\dots,6 \\
H_0 & & \Downarrow & & \\
 & \Rightarrow & H_0^{FE} & \Rightarrow & H_0^1
\end{array}
$$

For conducting inferences it is not relevant whether H_0: $\sigma_{\alpha\xi} = \sigma_{\varepsilon\eta} = 0$ is true or not, but whether H_0^{RE} or H_0^{FE} is correct, since inferences will be based on either the random effects or the fixed effects estimator. Therefore, the Hausman tests may be appropriate instruments for checking the consistency of these estimators, although they are only able to test for the stronger hypotheses H_0^i. If both H_0^{RE} and H_0^{FE} are false all estimators are inconsistent. In this case one can choose for the fixed effects estimator from the balanced sub-panel (since it probably has - given our Monte Carlo results - the smallest bias), or compute an efficient random effects or fixed effects estimator correcting for selectivity using the results from Ridder [1990] or Verbeek [1989], respectively.

Note that only the first test statistic (based on R_1) is appropriate for checking H_0^{FE}, while any other test statistic can be used for H_0^{RE}. The optimal testing procedure seems to be to test for the stronger hypothesis first (H_0^{RE}), and, if this test rejects, test subsequently for the weaker one (H_0^{FE}). Of course, it is preferable to use the most powerful test out of all possible tests for the hypothesis H_0^{RE}. However, the analysis of statistical power is extremely difficult if not impossible, not only because the test statistics are not mutually independent, but also because we are working with Hausman specification tests for which the null hypotheses H_0^i cannot be written down in a simple parametric form. Therefore, standard results on the power of Hausman tests (cf. Holly [1982]) and on sequential testing (see, e.g., Mizon [1977], Holly [1987]) are not applicable in this situation.

With respect to the question whether a single $R_i\bar{\beta} = 0$ $(i = 1,\ldots,6)$ or a combination of two or three restrictions should be tested, one can note that which of these testing procedures is more powerful depends on the differences between the possible pairings of estimators. If, e.g., $R_k\bar{\beta} \approx 0$ and $R_\ell\bar{\beta} > 0$ then the test based on R_ℓ will be more powerful than the test based on both R_k and R_ℓ and also more powerful than the one based on R_k only. Thus, to obtain some ideas about the power properties of the tests, we are forced to numerical analyses, which is the subject of the Section 6. If only a single restriction is tested (one R_i is chosen), nothing can be said analytically about the power properties either. The test with the highest power is the one which is based on the estimators with the largest possible difference between the pseudo true values and the smallest difference between the corresponding variances. Again, numerical analysis should answer the question.

5. Some other tests for selective nonresponse

Given the model in (1) and (4) and the assumed normality of the error terms in (5) is it possible to write down the likelihood function (cf. Ridder [1990]) and to derive the Lagrange Multiplier test statistic for the null hypothesis that $\sigma_{\varepsilon\eta} = \sigma_{\alpha\xi} = 0$ (H_0). Computation of this test statistic requires estimation of the complete model under the null, which necessitates numerical integration (over one dimension) for the response equation (4), which is a random effects probit model. In addition, the scores with respect to all parameters in the model are required to compute the test statistic from the first derivatives of the loglikehood, since there does not appear to be any form of block diagonality of the Fisher information matrix under the null.

If we define the (endogenously determined) transformation matrix R_i such that it transforms $\iota_T\alpha_i + \varepsilon_i$ into a T_i vector of observed elements, the loglikelihood contribution of individual i is given by

$$L_i = \log f(R_iy_i,\ r_i) = \log f(r_i|R_iy_i)\ f(R_iy_i). \tag{24}$$

Because under H_0 the two components in the right hand side of (24) depend on non-overlapping subsets of the vector of parameters, the score contributions with respect to the parameters in (1) can be found in Hsiao [1985, p. 39][7)], while those for the parameters in (4) can be derived from a standard random effects probit likelihood (see Appendix B). The most difficult score contributions are those with respect to the two covariances $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$; the latter even requires double numerical integration (see Appendix B). Because estimation under H_0 requires numerical integration (for each individual) for the probit part of the model and computation of each score contribution also requires numerical integration over one or two dimensions, the LM test is rather unattractive in applied work.

For the cross sectional sample selection model Heckman [1976, 1979] proposed a simple way to test for selectivity bias and to obtain consistent estimators. As discussed in Ridder [1990] this method can be generalized to the case of panel data, where two correction terms to equation (1) are added instead of just the one variable known as Heckman's lambda (or the inverted Mill's ratio). These two correction terms are the conditional expectations of the two error terms (α_i and ε_{it}) given the sampling scheme, as given in (16) and (17) evaluated at the (consistent) parameter estimates of the probit model under the null hypothesis. The two unknown covariances $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$ are not included in these correction terms but are the corresponding true coefficients in equation (1). Obviously, consistent estimation of these coefficients $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$ allows one to check whether nonresponse is selective or not. Since estimation of the parameters in the response equation as well as computation of the conditional expectation of $\xi_i + \eta_{it}$ in (16) and (17) requires numerical integration, these generalized Heckman [1979] method is still computationally unattractive. Because the parameters of the two correction terms are not estimated efficiently, the test based on Heckman's procedure is not efficient.

Because of the computational burden of the generalized Heckman [1979] procedure, it may be worthwhile to have some simple variables that can be used instead to approximate the true correction terms to check for selective nonresponse. If the nonresponse is endogenously determined one could have the intuitive notion that the pattern of missing observations has in one way or another an influence on the relationship between the endogenous and the exogenous variables. A simple way to check whether such influence is present is to include a variable in the regression equation comprising the effect of the missing data pattern, for example the number of waves the individual is participating or a dummy indicating whether the individual is observed in all waves or not, and to check whether this variable enters the equation significantly. In fact this is just a simple way of trying to approximate the Heckman [1979] like correction terms which are known to have nonzero coefficient when the null is not true. In many cases the additional variables are constant over time for each individual implying that the corresponding parameters are not identified in the case where the individual effects α_i are treated as fixed. In this section we shall therefore restrict attention to random effects estimators. We consider three possible variables that can be included in the regression equation. First, $T_i = \Sigma_{s=1}^{T} r_{is}$, the number of waves individual i participates, second $c_i = \Pi_{s=1}^{T} r_{is}$, a 0-1 variable equal to 1 iff individual i is observed in all periods and third, $r_{i,t-1}$, indicating whether individual i is observed in the previous period or not. Note that $r_{i,0} = 0$ by assumption. We are forced to using the unbalanced panel since in the balanced panel the added variables are identical for all individuals and thus incorporated in the intercept term.

Although one could have the intuitive feeling that at least one of the added variables has an influence on the relationship between y_{it} and x_{it} if there is selective nonresponse, there is no theoretical argument for this effect being linear and thus the power of the tests may be doubtful. If we denote the coefficient for the added variable w, say, by γ_w then the null hypothesis for the variable addition test is H_0^w: $\gamma_w = 0$. Note that H_0 implies H_0^w but that the converse is not true.

In the next section where we present some numerical results on the power of the simple tests proposed in this and the previous section, we compare these tests with the Lagrange Multiplier test, which is known to be asymptotically efficient for testing the null hypothesis H_0.

6. Some numerical results on the power of the tests

In the preceding two sections a number of tests are proposed which can be used to check whether selectivity bias is present or not. In this section we present numerical results on the power properties of the Hausman tests, the variable addition tests and the LM test for the parameter values that were already used in Section 3. Note that we do not consider the generalized Heckman test which is as hard to compute but asymptotically less powerful than the asymptotically optimal Lagrange Multiplier test. Because we transform the t-statistics in the variable addition tests into the corresponding Wald tests, the large sample properties of all tests are determined by the decentrality parameter and the number of degrees of freedom of a limiting χ^2-distribution. The decentrality parameters have been estimated by Monte Carlo. For the (quasi) Hausman tests, for example, and a sequence of local alternatives, $\bar{\beta} = \beta + \bar{\zeta}/\sqrt{N}$ (where $\bar{\zeta}$ is a 4 dimensional column vector), it holds that

$$\xi_R = N\,\hat{\beta}'R'\,(R\hat{V}R')^{-}\,R\hat{\beta} \xrightarrow{L} \chi_d^2\,(\bar{\zeta}'R'(RVR')^{-}R\bar{\zeta}\,) = \chi_d^2(\delta_R),\ N \to \infty. \quad (25)$$

Using the estimated pseudo true values $\hat{\bar{\beta}}$ from Section 3 and the corresponding estimated variances $\hat{\bar{V}}$, the decentrality parameter δ_R can be estimated by

$$\hat{\delta}_R = n\hat{\bar{\beta}}'R'\,(R\hat{\bar{V}}R')^{-}\,R\hat{\bar{\beta}}. \quad (26)$$

For the variable addition tests proposed in Section 5 we extend the random effects estimator based on the unbalanced panel given in (13) to include the additional regressor w. If the random effects estimator for the coefficient γ_w is denoted by $\hat{\gamma}_w$ with asymptotic variance V_w, the Wald variable addition test satisfies

$$\xi_w = N\,\hat{\gamma}_w^2\hat{V}_w^{-1} \xrightarrow{L} \chi_1^2(\bar{\zeta}^2V_w^{-1}) = \chi_1^2(\delta_w),\quad N \to \infty \quad (27)$$

under a sequence of local alternatives $\gamma_w = \bar{\zeta}/\sqrt{N}$. The decentrality parameter δ_w can thus straightforwardly be estimated by

$$\hat{\delta}_w = n\hat{\gamma}_w^2\hat{V}_w^{-1} \quad (28)$$

from which (approximate) probabilities of rejection for sample size n can be computed.

The Lagrange Multiplier test statistic is given by

$$\xi_{LM} = \sum_{i=1}^{N}(\partial L_i/\partial\vartheta)'\left[\sum_{i=1}^{N}(\partial L_i/\partial\vartheta)(\partial L_i/\partial\vartheta)'\right]^{-1}\sum_{i=1}^{N}(\partial L_i/\partial\vartheta)'\,\Big|_{\vartheta=\hat{\vartheta}_0}, \quad (29)$$

where L_i is the loglikelihood contribution of individual i, ϑ is the full parameter vector (including $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$) and $\hat{\vartheta}_0$ the estimate for ϑ under H_0.

The decentrality parameter of the asymptotic Chi-square distribution (under a sequence of local alternatives) δ_{LM} can be estimated by

$$\hat{\delta}_{LM} = (n/N)\ \tilde{\xi}_{LM} \tag{30}$$

where $\tilde{\xi}_{LM}$ equals the expression in (29) evaluated in the ML estimator (under H_0) for β, σ_α^2 and σ_ε^2, zeroes for $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$ and the true values for σ_ξ^2, σ_η^2 and γ (π). The latter is allowed since the ML estimator for these parameters under H_0 is consistent under the alternative. Details on the computation of $\tilde{\xi}_{LM}$ are presented in Appendix B.

The estimated decentrality parameters for six Hausman tests, three variable addition tests and the LM test are presented in Table 2 for the parameter values used in Section 3 to determine the pseudo true values of the four estimators. The numbers presented in the table are the estimated decentrality parameters for a sample size of 500, i.e. n = 500, based on a sample size in the Monte Carlo experiment of N = 25,000. The implied probabilities of rejection (at a nominal size of 5%) for any number of observations can be computed using Table 3. Note that the estimators are not normally but (non-centrally) Chi-square distributed, which makes computation of confidence intervals difficult. Based on the asymptotic normality of the parameter estimators the variance of $\hat{\delta}$ approximately satisfies

$$V\{\hat{\delta}\} = n^2/N^2\ (\ d + N/n\ \delta) \tag{31}$$

where d is the number of degrees of freedom, and where we use the fact that $N/n\ \hat{\delta}$ is Chi-square distributed. It is important to note that this variance increases with the true value δ. For large enough δ the corresponding standard error for N = 25,000 and n = 500 is (approximately) given by $0.283\sqrt{\delta}$.

Looking at panel A of Table 2 first, where both H_0^{FE} and H_0^{RE} are false, we see that in this case none of the variable addition tests has any power. Obviously, these variables are under these data generating processes not capable of approximating the Heckman [1979] like correction terms or the

Table 2. Decentrality parameters of the Chi-square distributions of several tests for selective nonresponse at n = 500 and T = 3

Reference situation: $T = 3$, $R_y^2 = .1$, $R_r^2 = .9$, $\rho_\alpha = .1$, $\rho_x = .7$, $p_0 = .5$, $\rho_\xi = .1$ and $\rho_{\alpha\xi} = .5$

A] $\pi = 0$, $\rho_{\varepsilon\eta} = .9$

	DF	REF.	$R_y^2=.9$	$R_r^2=.1$	$\rho_\alpha=.9$	$\rho_x=.3$	$p_0=\Phi(1)$	$\rho_\xi=.9$	$\rho_{\alpha\xi}=.9$
Hausman tests:									
FE(B~U)	1	1.41	1.27	0.07	2.00	0.31	1.52	0.26	1.05
RE(B~U)	1	7.23	6.00	0.06	3.55	1.53	7.48	1.84	7.33
U(FE~RE)	1	0.85	0.72	0.03	1.76	0.60	0.43	1.13	0.72
FE(B)~RE(U)	1	2.07	1.81	0.01	3.55	0.85	1.43	1.37	1.66
B(FE~RE) U(FE~RE)	2	2.04	1.64	0.04	2.02	0.89	1.83	1.39	2.49
RE(B~U) FE(B)~RE(U)	2	7.27	6.04	0.10	4.25	1.69	7.48	2.44	7.34
Variable addition tests:									
$\Sigma_t r_{it}$	1	0.01	0.01	0.04	0.14	0.03	0.11	0.10	0.04
$\Pi_t r_{it}$	1	0.03	0.03	0.00	0.24	0.04	0.04	0.17	0.14
$r_{i,t-1}$	1	0.02	0.01	0.01	0.02	0.00	0.14	0.03	0.02
Lagrange Multiplier test:									
LM	2	55.1	49.2	5.46	31.3	58.5	57.3	14.1	66.3
Bias in the RE estimator (unbalanced panel):									
% bias		-116%	-13%	-53%	-39%	-115%	-88%	-56%	-121%

Table 2 (continued)

B] ———— $\pi = 0$, $\rho_{\varepsilon\eta} = 0$ ————

	DF	REF.	$R_y^2=.9$	$R_r^2=.1$	$\rho_\alpha=.9$	$\rho_x=.3$	$p_0=\Phi(1)$	$\rho_\xi=.9$	$\rho_{\alpha\xi}=.9$
Hausman tests:									
RE(B~U)	1	0.07	0.06	0.00	0.02	0.00	0.02	0.00	0.00
U(FE~RE)	1	0.12	0.35	0.06	0.72	0.09	0.01	0.81	0.41
FE(B)~RE(U)	1	0.06	0.45	0.04	0.18	0.04	0.00	0.81	0.38
B(FE~RE) U(FE~RE)	2	0.17	0.44	0.00	0.79	0.12	0.02	0.98	0.57
RE(B~U) FE(B)~RE(U)	2	0.15	0.36	0.07	0.73	0.11	0.05	0.84	0.46
Variable addition tests:									
$\sum_t r_{it}$	1	0.09	0.07	1.88	0.61	0.32	0.22	1.23	0.59
$\prod_t r_{it}$	1	0.06	0.09	1.31	0.39	0.17	0.21	0.98	0.64
$r_{i,t-1}$	1	0.00	0.12	0.16	0.00	0.14	0.04	0.27	0.15
Lagrange Multiplier test:									
LM *)	2	1.33	0.13	4.12	4.95	1.06	1.15	5.92	3.74
Bias in the RE estimator (unbalanced panel):									
% bias		-6%	-1%	-4%	-6%	-7%	-5%	-19%	-12%

*) If the restriction $\sigma_{\varepsilon\eta} = 0$ is imposed a priori this test has one degree of freedom.

Table 2 (continued)

C] $\gamma_1 = 0,\ \rho_{\varepsilon\eta} = .9$

	DF	REF.	$R_y^2=.9$	$R_r^2=.1$	$\rho_\alpha=.9$	$\rho_x=.3$	$p_0=\Phi(1)$	$\rho_\xi=.9$	$\rho_{\alpha\xi}=.9$
Hausman tests:									
RE(B~U)	1	19.6	19.4	0.10	1.47	11.4	20.7	8.96	17.7
U(FE~RE)	1	19.9	18.3	3.73	6.68	22.4	15.2	4.35	19.3
FE(B)~RE(U)	1	16.0	14.7	1.56	2.50	15.1	12.7	3.93	15.3
B(FE~RE) U(FE~RE)	2	30.6	29.3	3.73	7.60	27.1	28.3	11.9	29.2
RE(B~U) FE(B)~RE(U)	2	29.4	28.4	3.74	6.86	24.5	27.5	11.4	27.9
Variable addition tests:									
$\Sigma_t r_{it}$	1	29.9	27.6	0.09	3.92	36.7	27.0	16.0	24.9
$\Pi_t r_{it}$	1	22.7	21.6	0.08	3.16	30.6	21.6	13.9	18.5
$r_{i,t-1}$	1	2.80	2.29	0.05	0.04	5.85	2.10	0.59	2.19
Lagrange Multiplier test:									
LM	2	75.9	73.6	13.7	20.1	83.8	66.8	12.1	83.0
Bias in the RE estimator (unbalanced panel):									
% bias		-74%	-7%	-44%	-4%	-41%	-61%	-32%	-75%

Notes: 1. Estimated decentrality parameters are based on 25,000 individual observations.
2. Estimates for decentrality parameters for sample size n can be obtained by multiplying the numbers by n/500.

Table 3. Probabilities of rejection (at 5%) for several decentrality parameters

	decentrality parameter							
DF	0	1	2	3	4	5	10	20
1	.05	.17	.29	.41	.52	.61	.89	.99
2	.05	.13	.23	.32	.42	.50	.82	.99

power properties of the Heckman test are poor as well. With regard to the Hausman tests, the results in Table 2 suggest that the test based on comparison of the random effects estimators in the balanced and the unbalanced panel (the second test) is more powerful than all other tests based on comparison of two estimators. The same holds with respect to the two tests not presented in the Table. Looking at the tests that compare two pairs of estimators (the fifth and the sixth test in Table 2), the latter seems to perform relatively well, although it is not performing uniformly better than the best one degree of freedom test. The test statistic based on comparing all four estimators (which is not reported in the Table) does not result in a very powerful test compared to those tests based on two pairs of estimators, since the additional degree of freedom has a much more dominant effect on the power than a (fairly small) rise in the decentrality parameter. For panel A of Table 2, the LM test is obviously far more powerful then any Hausman test. Note that the power of all tests reduces substantially if the R^2 of the selection equation is reduced from .9 to .1; the bias in the estimators is however still substantial (53% for the random effects estimator from the unbalanced panel).

If $\sigma_{\varepsilon\eta} = 0$, i.e. if the error shocks in the structural equation and the selection equation are uncorrelated, but $\sigma_{\alpha\xi} \neq 0$ (so H_0^{FE} is true and H_0^{RE} is not; panel B) all tests seems to have limited power only. Even the power of the LM test is very limited in this case, in which, of course, the null hypothesis H_0 is only violated in one direction ($\sigma_{\alpha\xi} \neq 0$). Since the bias in the fixed effects estimators is zero in this case, while that in the random effects estimators is small (as is apparent from Table 1), this does not seem to be a situation to worry about.

As shown in panel C of Table 2, the power of all tests appears to be larger in the case where the response is determined by an individual effect which is correlated with the regressor ($\pi \neq 0$ and $\gamma_1 = 0$) than in the case where the regressor itself determines the response ($\pi = 0$ and $\gamma_1 \neq 0$ with the same R^2 of the probit equation). Note that for the Hausman tests comparing FE and RE estimators we have a standard situation in which one of the estimators in the test statistic is consistent even if the null hypothesis does not hold. Remarkably, the variable addition tests have fairly good power properties as well, especially the one based on adding the number of waves an individual is participating ($\Sigma_t r_{it}$). The one based on including $r_{i,t-1}$ has only very limited power. Concerning the Hausman tests, the one comparing the RE and FE estimator in the unbalanced panel, which is the standard Hausman test for uncorrelated individual effects, has the largest power of the 1 degree of freedom tests. In some cases it is worthwhile to combine two restrictions and perform a two degrees of freedom test. It should be clear from the simulation results in the table that it is well possible that the standard Hausman [1978] specification test for testing the hypothesis that the individual effects are uncorrelated with the explanatory variables rejects due to the presence of selectivity bias.

Unfortunately, none of the simple tests seems to have uniformly better power properties than the others, so we cannot recommend one particular test. The power of all tests seems to depend crucially on the fact whether H_0^{FE} is false or, if it is true, why H_0^{FE} is true ($\sigma_{\varepsilon\eta} = 0$ *or* $\gamma_1 = 0$). In the latter case ($\gamma_1 = 0$) the power of most simple tests is quite reasonable, while it is not if $\sigma_{\varepsilon\eta} = 0$. In line with the Monte Carlo results above, we are tempted to say that both the second and the third Hausman test (RE, balanced vs. unbalanced, and unbalanced, FE vs. RE, respectively) perform relatively well and may be a good choice in applied work. The best choice for a variable addition test seems to be to include $\sum_t r_{it}$ in the structural equation.

It should be noted from Table 2 that it is not the case that the power of the (asymptotically optimal) Lagrange Multiplier test (for H_0) increases with the size of the bias in the random effects estimator in the unbalanced panel. In some cases the bias in the estimators is substantial while the LM test only has limited power.

So far, we have only considered numerical analyses for a three wave panel (T = 3). If T increases, the number of individuals in the balanced subpanel (keeping all parameters fixed) will decrease, which may increase the differences found between the estimators from the balanced and the unbalanced panel. Moreover, the difference between the fixed effects estimator and the random effects estimator for a given sample will get smaller, since the weight of the between estimator in the random effects estimator is inversely related with T (cf. Hsiao [1986, p.36]). This suggests that the power of the Hausman tests comparing estimators from the balanced and unbalanced panel will increase with T and that of the standard Hausman specification tests will decrease with T. For larger T the second Hausman test (comparing the random effects estimators from the balanced and unbalanced panel) is probably the most attractive way to test hypothesis H_0^{RE}.

7. Concluding remarks

In this paper we suggested several simple tests to check the presence of selective nonresponse in a panel data model. We considered the selectivity bias of the fixed and random effects estimators and showed that the FE estimator is more robust to nonresponse biases than the RE estimator. In particular, the FE estimator will be consistent as long as the probability of being observed of a given household is constant over time. Several simple Hausman tests have been suggested which are based on the differences in the pseudo true values of these estimators. Furthermore, some variable addition tests are proposed which can be used to test for selectivity bias. Neither of these tests requires estimation of the model under selectivity nor a specification of the nonresponse mechanism.

A Monte Carlo study shows that not only the conditions for consistency of fixed effects estimator are weaker than that for a consistent random effects estimator, but also that the bias of the FE estimator is likely to be smaller than that of the RE estimator in cases where both estimators are inconsistent. The numerical results also indicate that the bias resulting from a balanced sub-panel is likely to be smaller than that from the unbalanced panel.

Although the proposed Hausman and variable addition tests have poor power properties in some cases, they may be a good instrument for checking the importance of the selectivity problem. In particular when response is partly determined by an individual effect which is correlated with the regressor the power of several Hausman tests and variable addition tests is quite reasonable in comparison with the Lagrange Multiplier test. For practical purposes at least two Hausman tests can be recommended: the one comparing the random effects estimators from the balanced and unbalanced panel, and the one comparing the RE and FE estimators in the unbalanced panel (the standard Hausman test for correlated individual effects). A test that is even simpler is the variable addition test including $T_i = \sum_t r_{it}$ in the specification of equation (1). This test also seems to perform quite reasonable in practice.

References

Baltagi, B.H. (1985), 'Pooling Cross Sections with Unequal Time Series Lengths', Economics Letters, 18, 133-136.

Chamberlain, G. (1984), 'Panel Data', in: Z. Griliches and M. Intriligator, eds., Handbook of Econometrics, Volume II, Elsevier, 1247-1318.

Hausman, J.A. (1978), 'Specification Tests in Econometrics', Econometrica, 46, 1251-1271.

Hausman, J.A. & D. Wise (1979), 'Attrition Bias in Experimental and Panel Data: the Gary Income Maintenance Experiment', Econometrica, 47, 455-473.

Heckman, J. (1976), 'The Common Structure of Statistical Models of Truncation, Sample Selection and Limited Dependent Variable Models and a Simple Estimator for Such Models', The Annals of Economic and Social Measurement, 5, 475-492.

Heckman, J. (1979), 'Sample Selection Bias as a Specification Error', Econometrica, 47, 153-161.

Heckman, J. (1981), 'Statistical Models for Discrete Panel Data', in: C.F. Manski and D. McFadden, eds., Structural Analysis of Discrete Data with Econometric Applications, MIT Press, Cambridge, 114-178.

Holly, A. (1982), 'A Remark on Hausman's Specification Test', Econometrica, 50, 749-759.

Holly, A. (1987), 'Specification Tests: An Overview', in: T.F. Bewley, ed., Advances in Econometrics, Fifth World Congress, Volume 1, Cambridge University Press, 59-97.

Hsiao, C. (1986), Analysis of Panel Data, Cambridge University Press.

Manski, C.F. (1989), 'Anatomy of the Selection Problem', The Journal of Human Resources, 24, 343-360.

Meghir, C. & M. Saunders (1987), 'Attrition in Company Panels and the Estimation of Investment Equations', working paper, University College London.

Mizon, G.E. (1977), 'Inferential Procedures in Nonlinear Models: An Application in a UK Industrial Cross Section Study of Factor Substitution and Returns to Scale', Econometrica, 45, 1221-1242.

Newey, W.K. (1985), 'Generalized Method of Moments Specification Testing', Journal of Econometrics, 29, 229-256.

Nijman, Th.E. & M. Verbeek (1989), 'The Nonresponse Bias in the Analysis of the Determinants of Total Annual Expenditures of Households Based on Panel Data', CentER Discussion Paper #8936, Tilburg University.

Ridder, G. (1990), 'Attrition in Multi-Wave Panel Data', in: J. Hartog, G. Ridder and J. Theeuwes, eds., Panel Data and Labor Market Studies, Elsevier, North-Holland (in preparation).

Verbeek, M. (1989), 'On the Estimation of a Fixed Effects Model with Selective Nonresponse', Research Memorandum FEW 376, Tilburg University.

Wansbeek, T.J. & A. Kapteyn (1989), 'Estimation of the Error Components Model with Incomplete Panels', Journal of Econometrics, 41, 341-361.

Appendix A. Some technical details on Section 3

If $\iota = (1, 1, \ldots, 1)'$ of dimension T, it is readily verified form (5) that

$$\begin{bmatrix} \alpha_i \\ \varepsilon_i \\ \xi_i \iota + \eta_i \end{bmatrix} \sim N \left[0, \begin{bmatrix} \sigma_\alpha^2 & 0 & \sigma_{\alpha\xi}\iota' \\ & \sigma_\varepsilon^2 I & \sigma_{\varepsilon\eta} I \\ & & \sigma_\eta^2 I + \sigma_\xi^2 \iota\iota' \end{bmatrix} \right] \tag{A.1}$$

which yields

$$E \{ \varepsilon_i \mid \xi_i \iota + \eta_i \} = \sigma_{\varepsilon\eta}\, \sigma_\eta^{-2} \left[I - \frac{\sigma_\xi^2}{T\sigma_\xi^2 + \sigma_\eta^2} \iota\iota' \right] (\xi_i \iota + \eta_i), \tag{A.2}$$

and proves (16) and (12) if we use the definition of $\tilde{\varepsilon}_{it}$ and take expectations conditional upon $r_{i1}, \ldots, r_{iT}$.
It also follows that

$$E\{ \alpha_i \mid \xi_i \iota + \eta_i \} = \sigma_{\alpha\xi}\, \sigma_\eta^{-2}\, \iota' \left[I - \frac{\sigma_\xi^2}{T\sigma_\xi^2 + \sigma_\eta^2} \iota\iota' \right] (\xi_i \iota + \eta_i), \tag{A.3}$$

which proves (17) after taking conditional expectations upon $r_{i1}, \ldots, r_{iT}$.

Moreover, since $E \{ \xi_i \mid r_i \}$ is fixed over time and since

$$E \{ \eta_{it} \mid r_i \} = \int \frac{\varphi((Z_{it}\gamma + \xi_i)/\sigma_\eta)}{\Phi((Z_{it}\gamma + \xi_i)/\sigma_\eta)} f(\xi_i \mid r_i)\, d\xi_i \qquad \text{if } r_{it} = 1,$$

where φ and Φ are the standard normal density and distribution function respectively, and $f(\xi_i \mid r_i)$ is the conditional density of ξ_i given selection (see Ridder [1990]), it is evident that there is no selection bias if

$Z_{it}\gamma$ is constant over time, i.e. if the probability of an individual of being observed is constant for all t.

Appendix B. The Lagrange Multiplier test statistic for selectivity bias

The loglikelihood contribution of individual i in the full model is given by

$$L_i = \log f(r_i|R_iy_i)\ f(R_iy_i), \tag{B.1}$$

where $f(r_i|R_iy_i)$ is the likelihood function of a (conditional) T-variate probit model and $f(R_iy_i)$ is the likelihood function of a T_i-dimensional linear error components model (cf. Hsiao [1986, p. 38]). The second term is simple and can be written as

$$\begin{aligned}\log f(R_iy_i) = &- \frac{T_i}{2}\log 2\pi - \frac{T_i-1}{2}\log \sigma_\varepsilon^2 - \frac{1}{2}\log(\sigma_\varepsilon^2 + T_i\sigma_\alpha^2)\\ &- \frac{1}{2\sigma_\varepsilon^2}\sum_{t=1}^{T} r_{it}(\tilde{y}_{it} - \tilde{x}_{it}\beta)^2 - \frac{T_i}{2(\sigma_\varepsilon^2+T_i\sigma_\alpha^2)}(\bar{y}_i - \beta_0 - \bar{x}_i\beta)^2.\end{aligned} \tag{B.2}$$

The first term in (B.1) is somewhat more complicated because we have to derive the conditional distribution of the error term in the probit model. From (5) and defining $v_{it} = r_{it}(\alpha_i+\varepsilon_{it})$ (where r_{it} is treated as non-stochastic), the conditional expectation of the error term $\xi_i + \eta_{it}$ is given by

$$\begin{aligned}&E\{\xi_i+\eta_{it}|v_{i1}..v_{iT}\} =\\ &= r_{it}\frac{\sigma_{\varepsilon\eta}}{\sigma_\varepsilon^2}\left[v_{it} - \frac{\sigma_\alpha^2}{\sigma_\varepsilon^2+T_i\sigma_\alpha^2}\sum_{s=1}^{T} v_{is}\right] + \frac{\sigma_{\alpha\xi}}{\sigma_\varepsilon^2+T_i\sigma_\alpha^2}\sum_{s=1}^{T} v_{is} = c_{it},\ \text{say}.\end{aligned} \tag{B.3}$$

Using (5) the conditional variance of $\xi_i+\eta_{it}$ can also be derived. It is straightforward to show that the conditional distribution of $\xi_i+\eta_{it}$ given

$v_{i1},\ldots,v_{iT}$ corresponds with the (unconditional) distribution of the sum of three normal variables $u_{it} + \nu_{1i} + r_{it}\nu_{2i}$ whose distribution is characterized by

$$E\{\ \nu_{1i}\ \} = E\{\ \nu_{2i}\ \} = 0,\quad E\{\ u_{it}\ \} = c_{it},$$

$$V\{\ u_{it}\ \} = \sigma_\eta^2 - r_{it}\sigma_{\varepsilon\eta}^2/\sigma_\varepsilon^2 = s_t^2,\ \text{say}$$

$$V\ \{\ \nu_{1i}\ \} = \sigma_\xi^2 - T_i\sigma_{\alpha\xi}^2(\sigma_\varepsilon^2+T_i\sigma_\alpha^2)^{-1} = \omega_1,\ \text{say}$$

$$V\ \{\ \nu_{2i}\ \} = \sigma_{\varepsilon\eta}^2\sigma_\alpha^2\sigma_\varepsilon^{-2}(\sigma_\varepsilon^2+T_i\sigma_\alpha^2)^{-1} = \omega_2,\ \text{say}$$

$$\text{Cov}\ \{\ \nu_{1i},\ \nu_{21}\ \} = -\sigma_{\alpha\xi}\sigma_{\varepsilon\eta}(\sigma_\varepsilon^2+T_i\sigma_\alpha^2)^{-1} = \omega_{12},\ \text{say},$$

and all other covariances equal to zero. For notational convenience we do not explicitly add an index i to the (co)variances σ_t and $\omega_\cdot$. Note that $c_{it} = 0$, $s_t^2 = \sigma_\eta^2$, $\omega_1 = \sigma_\xi^2$ and $\omega_2 = 0$ under H_0. Like in the unconditional error components probit model (cf. Heckman [1981]), the likelihood function can be written as (dropping the $Z_{is}\pi_s$ terms for notational convenience)

$$f(r_i|R_iy_i) = E_\vartheta\{\ \prod_{t=1}^{T}\Phi(d_{it}s_t^{-1}[Z_{it}\gamma+c_{it}+\nu_{1i}+r_{it}\nu_{2i}])\ \} \tag{B.4}$$

where the expectation is taken over ν_{1i} and ν_{2i} and $d_{it} = 2r_{it}-1$. It is this likelihood function that has to be differentiated w.r.t. the unknown parameters γ, σ_ξ^2 and $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$. However, the expectation operator depends on the unknown parameter vector ϑ (because the density of ν_{1i} and ν_{2i} is not defined with respect to the same measure under H_0 and the alternative), implying that the order of taking expectations and differentiating is not interchangeable. This problem can easily be solved by defining two new integration variables that are both standard normally distributed (under the null and the alternative), τ_1 and τ_2, say. Then we obtain

$$f(r_i|R_i y_i) = \int\int \prod_{t=1}^{T} \Phi(d_{it} s_t^{-1}[Z_{it}\gamma + c_{it} + a_{it}\tau_1 + b_{it}\tau_2])\ \varphi(\tau_1)\varphi(\tau_2)d\tau_1 d\tau_2 \tag{B.5}$$

where

$$a_{it} = \omega_1^{1/2} + r_{it}\omega_{12}\omega_1^{-1/2} \quad \text{and} \quad b_{it} = r_{it}(\omega_2 - \omega_{12}^2\omega_1^{-1})^{1/2}.$$

Since $f(R_i y_i)$ does not depend on $\sigma_{\alpha\xi}$ and $\sigma_{\varepsilon\eta}$, differentiating the log of the expression above and evaluating the result under H_0 yields the scores w.r.t. the two covariances. Using the fact that for any element ψ of the parameter vector $(\gamma, \sigma_\eta^2, \sigma_{\alpha\xi}, \sigma_{\varepsilon\eta})$,

$$\partial L_i/\partial\psi = [\ \partial f(r_i|R_i y_i)/\partial\psi\ ]\ /\ f(r_i|R_i y_i) \tag{B.6}$$

with

$$\frac{\partial f(r_i|R_i y_i)}{\partial\psi} = \int\int \sum_{s=1}^{T} \prod_{\substack{t=1 \\ t\neq s}}^{T} \Phi_t(.)\ \partial\Phi_s(.)/\partial\psi\ \varphi(\tau_1)\varphi(\tau_2)\ d\tau_1 d\tau_2, \tag{B.7}$$

the score w.r.t. $\sigma_{\alpha\xi}$ can easily be derived using the following equality (under H_0)

$$\partial\Phi_t(.)/\partial\sigma_{\alpha\xi} = \varphi(d_{it}\sigma_\eta^{-1}[Z_{it}\gamma + \sigma_\xi\tau_1])\ (d_{it}/\sigma_\eta)(\ \partial c_{it}/\partial\sigma_{\alpha\xi} + \partial\omega^{1/2}/\partial\sigma_{\alpha\xi}\tau_1). \tag{B.8}$$

Similarly, for $\sigma_{\varepsilon\eta}$, we use

$$\partial\Phi_t(.)/\partial\sigma_{\varepsilon\eta} = \tag{B.9}$$

$$\varphi(d_{it}\sigma_\eta^{-1}[Z_{it}\gamma + \sigma_\xi\tau_1]).(d_{it}/\sigma_\eta).(\ \partial c_{it}/\partial\sigma_{\varepsilon\eta} + r_{it}\tau_2\sigma_\alpha^2\sigma_\varepsilon^{-2}(\sigma_\varepsilon^2 + T_i\sigma_\alpha^2)^{-1}\ ).$$

from which the score w.r.t. $\sigma_{\varepsilon\eta}$ under H_0 can easily be derived. Note that both τ_1 and τ_2 occur in the integrand such that numerical integration over two dimensions will be required.

For the scores w.r.t. γ and $\sigma_\xi^2 = 1 - \sigma_\eta^2$ it suffices under H_0 to look at $\partial f(r_i)/\partial\gamma$ and $\partial f(r_i)/\partial\sigma_\xi$, where (cf. Heckman [1981])

$$f(r_i) = \int \prod_{t=1}^{T} \Phi(d_{it}\sigma_\eta^{-1}[Z_{it}\gamma + \sigma_\xi z_1])\ \varphi(z_1)\ dz_1 \ . \tag{B.10}$$

Both scores will require numerical integration over one dimension.

Notes

1 This estimator is only defined if at least one individual is observed more than once; for finite samples there will generally be a small but nonzero probability that this is not the case, but for practical purposes this can be ignored. Similar remarks hold for all other estimators presented below.

2 The conditional expectations given in the sequel are also conditional on the exogenous variables, but for the sake of notation these are omitted.

3 A case in which this sufficient condition is not necessarily met but condition (9) holds, is the situation where observations are missing deterministically (given Z_{it}) ($E\{r_{it}|Z_{it}\} = r_{it} = 0$), for example if being on vacation implies nonresponse. Obviously, the normality assumption of the error terms in (4) is not appropriate in that case.

4 For expository purposes we ignore the fact that in practice unknown variances have to be replaced by consistent estimates.

5 This equivalency also holds when the model is not correctly specified, as in our case.

6 This property can be used for an efficient and a consistent estimator within any class of estimators satisfying: if $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$ are members of this class, then $A\hat{\vartheta}_1 + B\hat{\vartheta}_2$ is also a member of this class (for any square matrices A and B). In deriving (23) we apply (22) for the classes of (linear) estimators using the balanced sub-panel or the unbalanced panel and the class of estimators treating the individual effects as fixed constants (using the unbalanced panel).

7 Note that (3.3.20) in Hsiao (1986) contains a typing error; the first - sign on the second line should read a + sign.

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