

# AN EMPIRICAL INVESTIGATION OF THE FACTORS THAT DETERMINE THE PRICING OF DUTCH INDEX WARRANTS

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First draft: November 30, 1994

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\*We are grateful to Theo Nijman and Marno Verbeek for providing many helpful comments. The usual disclaimer applies.

# AN EMPIRICAL INVESTIGATION OF THE FACTORS THAT DETERMINE THE PRICING OF DUTCH INDEX WARRANTS

## Abstract

This paper investigates the pricing of Dutch index warrants. It is found that when using the historical standard deviation as an estimate for the volatility, the Black and Scholes model underprices all put warrants and call warrants on the FT-SE 100 and the CAC 40, while it overprices the warrants on the DAX. When the implied volatility of the previous day is used the model prices the index warrants fairly well. When the historical standard deviation is used the mispricing of the call and the put warrants depends in a strong way on the mispricing of the previous trading day, and on the moneyness (in a nonlinear way), the volatility and the dividend yield. When the implied standard deviation of the previous trading day is used the mispricing of the call warrants is only related to the moneyness and to the estimated volatility, while the mispricing of put index warrants depends in a strong way on the moneyness, the volatility, the dividend yield and the remaining time to maturity.

## 1. Introduction

During the last 5 years index warrants have become very popular. This is especially true for the Nikkei Put Warrants (NPWs), which are mainly traded on the American and Canadian markets. Chen et al. (1992) even argue that in 1990, on average, about 15% of the American Stock Exchange's (AMEX) trading volume was represented by NPWs. Besides NPWs, the AMEX also lists call warrants on the Nikkei index, as well as warrants on other foreign indexes, such as the British FT-SE 100 index and the French CAC 40 index (Wei, 1992). In the Netherlands index warrants were first introduced in 1992, when the Dutch ING Bank introduced call and put warrants on the FT-SE 100, the CAC 40 and the German DAX indexes. In 1993 Citibank also introduced warrants on these indexes, as well as warrants on the S&P 500 and on baskets of Japanese and Hong Kong shares.

Until now, empirical research on index warrants has been concentrated mainly on the Nikkei Put warrants, traded on the American and the Canadian markets (Chen et al., 1992 and Wei, 1994). The empirical research in this paper focusses on warrants on the FT-SE 100, CAC 40 and DAX indexes, which are traded on the Amsterdam Stock Exchange. The methodology comprises the calculation of model prices for the index warrants and the analysis of the differences between market and model prices using regression analysis, taking into account the fact that the mispricings are often highly autocorrelated and possibly heteroskedastic. These problems are usually ignored in empirical studies on the pricing of options and warrants.

We find that when using the historical standard deviation, the Black and Scholes model underprices the warrants on the dividend paying FT-SE 100 and CAC 40 indexes, while it overprices the warrants on the DAX, which is a performance index. The mispricing depends in a strong way on the mispricing of the previous trading day, and on the moneyness, the volatility and the dividend yield. When using the implied standard deviation of the previous trading day the mispricing is very small, and is only related to the moneyness (in a nonlinear way) and to the estimated volatility. Put index warrants are highly underpriced by the model when the historical standard deviation is used. Here also the mispricing depends in a strong way on the mispricing of the previous trading day and on the moneyness (in a nonlinear way), the volatility and the dividend yield. As with the call warrants the mispricing is very small when the implied volatility of the previous

trading is used. The mispricing then depends on the moneyness (in a nonlinear way), the volatility, the dividend yield and the remaining time to maturity.

The remainder of this paper is organized as follows. Section 2 describes the nature and valuation of index warrants. Section 3 describes the data and test methodology. Section 4 captures the results of our tests. Finally, section 5 concludes the paper.

## **2. Index warrants and their valuation**

### **2.1. Index warrants**

Veld (1992) defines call (put) warrants as options to buy (sell) underlying values that will become (are existing) liabilities to the issuer. Therefore Veld (1992) argues that index warrants are not warrants, but that they are in fact "options in a strict sense". Despite the fact that index warrants are options instead of warrants, it is still possible to point out some differences between index warrants and exchange traded index options. The first difference is that index warrants are traded on the Stock Exchange, instead of on the Options Exchange<sup>1</sup>. Thus, the credit risk of index warrants is not taken over by a Clearing Organization, as is the case for exchange traded index options. The second difference is that the issuer of index warrants issues a fixed amount of contracts, whereas the number of exchange traded options is flexible. A third difference is that it is not possible to write index warrants. It is possible to take a short position in these warrants, but that includes the obligation to deliver the index warrants in a short period of time. The fourth difference is that index warrants usually have an initial maturity of 2-4 years, while exchange traded options have maturities of up to 9 months. However, this difference is not applicable everywhere. For example, on the Chicago Board of Options Exchange (CBOE), LEAPS are traded. These are exchange traded index options on the S&P 500, the S&P 100 and the

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<sup>1</sup> This difference is not entirely applicable in the Netherlands. The index warrants issued by the ING Bank are indeed traded on the Amsterdam Stock Exchange. On the other hand, the index warrants issued by Citibank are traded on the European Options Exchange. However, they are traded in a special category. Therefore, not all rules applicable to the exchange traded options are relevant for index warrants. For example, the credit risk of the Citibank index warrants is not taken over by the clearing organization of the options exchange.

Major Market indexes. LEAPS have maturities of up to 3 years (Hull, 1993). On the EOE also index options are traded on the EOE index, with maturities of up to 3 years. The fifth difference is that index warrants are usually traded in the local currency (in the Netherlands, that is Dutch guilders), while they have an exercise price denominated in a foreign currency. This may cause valuation problems. These problems are discussed in the next section.

## **2.2. The valuation of index warrants**

Wei (1992, 1994) argues that for valuation purposes the fifth difference mentioned above is the main difference between index warrants and index options. He identifies a number of possible pay-off specifications, depending on how the foreign currency is converted into local currency. All Dutch warrants have in common that, at the exercise date, the foreign currency pay-off of the warrant is converted into guilders at the then prevailing exchange rate. In such a case the model price can be calculated in the foreign currency and converted into guilders at the current exchange rate. This can be shown by a simple no arbitrage argument<sup>2</sup>.

## **2.3. Empirical research on index warrants**

Until now, empirical research on index warrants has been concentrated on Nikkei Put Warrants (NPWs). The most important results from this research can be summarized as follows. Chen et al. (1992) have investigated the pricing characteristics of NPWs traded on the AMEX. They tested both a European pricing model (the Square Root CEV model) and an American pricing model (the Barone-Adesi and Whaley (1987), BAW model). Two estimates were used for the volatility. The first estimate is the implied standard deviation for the same warrant, calculated on the day before the measurement day. This estimate was used for both models. The second estimate is the historical standard deviations over the last 120 days. This estimate was used only for the BAW model.

Chen et al. find that for both volatility estimates the BAW model overestimates warrant

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<sup>2</sup> See Wei (1992).

prices, while the Square Root CEV model underestimates warrant prices. Another finding is that the errors for the Square Root CEV model are smaller than for the BAW model. This is the case for both volatility estimates for the BAW model. According to Chen et al. (1992) the underestimation of the warrant prices for the Square Root CEV model is caused by the omission of the early exercise possibility. However, since Chen et al. use the implied standard deviation of the previous day to calculate the model prices, it should be expected that the effect of early exercise is implicit in this standard deviation. It is hard to see therefore how this can explain the low model prices. The last notable feature from their research is that the errors are higher just after the issuance of the NPWs, indicating a "market learning effect".

Wei (1994) examines the pricing of NPWs on the Toronto Stock Exchange (TSE). Wei uses Binomial and Trinomial lattices. The use of lattices enables the inclusion of the possibility of early exercise. The volatility is estimated as a weighted average of the implied volatilities of the NPWs which were traded on the previous day. Like Chen et al. (1992), Wei (1994) finds that his American model overestimates warrant prices. He argues that the overpricing may be caused by two factors, i.e. the fact that the credit risk on the index warrants is not guaranteed by a Clearing Organization and the inclusion of a so-called "Extraordinary Event Clause" in the conditions of all NPWs. This clause prevents the exercise of NPWs upon the occurrence of certain abnormal events. In effect the clause protects the issuers of the warrants against undesirable market conditions.

Using linear regression analysis Wei (1994) finds that the model overprices (underprices) in the money (out of the money) warrants and that it overprices (underprices) high (low) volatility warrants. Wei (1994) finds a positive relation between the error and the time to maturity. Therefore he concludes that just after the issuance (when the maturity is longest) the warrants are most mispriced. Therefore he concludes (just like Chen et al., 1992) that there is a "market learning effect".

### **3. Data description and methodology**

### 3.1. The indexes and the model

The purpose of this study is to evaluate differences between model and market prices for Dutch index warrants. Three indexes are considered: the German DAX index, the French CAC 40 index and the British FT-SE 100 index. The DAX index is a performance index. In such an index it is assumed that dividends are reinvested in the index, which makes it similar to a non dividend paying stock. The CAC 40 and the FT-SE 100 indexes do not make corrections for dividend payments. Thus, these two indexes can be considered as dividend paying stocks.

Model prices are calculated with the Black and Scholes (1973) model (B/S-model). In case of the CAC 40 and the FT-SE 100 index warrants a correction is made for continuous dividend payments (see Merton, 1973). We have also done calculations for the Square Root CEV model (Cox and Ross, 1976). However, this led to outcomes very similar to the outcomes from the B/S-model. Therefore these results are not reported in this study.

### 3.2. Sample selection

In May 1992 the ING Bank introduced warrants on the DAX, CAC 40 and FT-SE 100 indexes. These warrants were listed on the Amsterdam Stock Exchange (ASE). On each index one series of call and one series of put warrants were introduced. Exercise prices close to the then prevailing values of the indexes were selected. In May 1993 ING Bank introduced several new series. Later in September 1993 Citibank also introduced several new series of call and put warrants on these indexes<sup>3</sup>. Unfortunately the introduction of these new series by ING Bank and Citibank led to a sharp decline in the liquidity of all series. Because of this, we have restricted our data-set to the index warrant observations from May 12, 1992 to April 13, 1993. Each warrant entitles its holder to acquire 1/100 times the difference between the value of the index and the exercise price. Like the NPWs discussed in section 2.3, the index warrants issued by the ING Bank include an "Extraordinary Event Clause". In our data-set we have included the closing prices of the index warrants on days that actual trading took place. In table 1 the index warrants outstanding

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<sup>3</sup> These warrants were listed on the European Options Exchange, see note 1.

in our research period (with their respective exercise prices, expiration dates and number of observations) are presented.

[Insert Table 1]

Information on the index warrant prices and the prices of the underlying indexes is derived from Datastream. The exercise prices and the expiration dates are derived from the respective issuance prospectuses.

### **3.3. The estimation of the unobservable variables**

In empirical research on option prices, the volatility is generally estimated using either a historical or an implied estimate of the standard deviation. In this paper both estimates are used. The historical standard deviation (HSD) is calculated over the 50 day period preceding each measurement date. In order to translate the daily estimate of the volatility into a yearly estimate it is assumed that one year comprises 250 trading days (Hull, 1993). The implied standard deviation is estimated using data for the previous trading day ( $ISD_{t-1}$ ). On day  $t-1$  the observable variables and the market price for the warrant are gathered in order to calculate the  $ISD_{t-1}$ . In case of call (put) warrants, the market price of a call (put) warrant is used on day  $t-1$  in order to calculate the  $ISD_{t-1}$ .

Because we do not have dividend yields available of the CAC 40 and FT-SE 100 indexes, these dividend yields are estimated using the dividend yields that are calculated for the Datastream indexes with regard to the French (Société des Bourses Françaises, SBF) and British (London Stock Exchange, LSE) stock exchanges. The riskless interest rates are estimated using yields on British, French and German government bonds with the same maturity as the index warrants. Both the dividend yields and the riskless interest rates are gathered from Datastream.

### **3.4. Methodology**

In studies, in which model and market prices for options (or warrants) are compared, at least two possible measures for the deviation between market and model prices can be



used:

$$\text{Deviation 1 (D}_1\text{)} = \frac{\text{market price} - \text{model price}}{\text{model price}}$$

$$\text{Deviation 2 (D}_2\text{)} = \frac{\text{market price} - \text{model price}}{\text{market price}}$$

It appears that in most studies  $D_2$  is used. For example, from a review of warrant pricing studies, Veld (1994) concludes that in all studies in which a comparison of model and market prices is made, deviations of the  $D_2$  type are being used. However, as pointed out by Whaley (1982), the deviations of the  $D_1$  type are preferable in a regression of the deviation on variables that determine the option price, in order to reduce the possible effect of heteroskedasticity.

It is interesting to find out whether deviations are related to specific characteristics of the underlying options. This may be helpful when constructing improved option pricing models. Some studies use a "rough classification" of deviations. In such studies the sample is split up according to the moneyness (in the money, at the money and out of the money) of the option<sup>4</sup>. One obvious problem is that in case e.g. a relation is found between the deviation and the moneyness of the option, it may be the case that the deviation and the moneyness have a causal relation with a third variable (e.g. the maturity) but not with each other. In other words the relation between the deviation and the moneyness may be spurious. Therefore some studies combine the classifications according to different characteristics. For example Leonard and Solt (1990) divide deviations in 9 categories according to moneyness and maturity. Of course, this does not fully resolve the above mentioned problem. Besides that, it becomes very difficult to interpret these tables. This is one reason that we prefer the use of regression analysis in order to analyze the deviations. A second reason is that in using such classifications the independent variables become discrete, while in fact they are continuous. Regression analysis makes use of the continuity of both the dependent and independent variables.

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<sup>4</sup> Examples of these studies include Sterk (1982) for stock call options, Bodurtha and Courtadon (1987) for foreign currency call and put options, Blomeyer and Johnson (1988) for stock put options and Kremer and Roenfeldt (1993) for equity warrants.

Some studies use simple regression analysis to analyze the deviations. In such a case the deviation is regressed upon a constant and one variable (e.g. the moneyness)<sup>5</sup>. Although in such studies the problem of making the independent variable discrete disappears, the above mentioned problem of spurious relations still remains. For this reason we will use multiple regression in order to analyze the deviations<sup>6</sup>.

## 4. Empirical results

### 4.1. The call warrants

In table 2 the average deviations for the call warrants are included, both for model prices calculated with historical standard deviations (HSDs) and model prices calculated with implied standard deviations from the previous day ( $ISD_{t-1}$ ).

[Insert table 2]

For the HSD deviations are smallest for the DAX warrants. This should come as no surprise, because the DAX warrants are equal to warrants on non dividend paying stocks. Therefore they can be considered as European call options. The fact that the average deviations are positive for the CAC 40 and FT-SE 100 indexes may be attributed to the omission of the early exercise possibility. The negative deviation for the DAX warrants indicates that the model overprices these warrants. This may be caused by the existence of the "Extraordinary Event Clause" and by the fact that the credit risk of the warrants is not guaranteed by a Clearing Organization.

In case the  $ISD_{t-1}$  is used instead of the HSD, the deviations get much smaller. This is no surprise either, because the  $ISD_{t-1}$  only includes information from day  $t-1$ , while the HSD is based on information of the last 50 days.

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<sup>5</sup> This is done e.g. by Whaley (1982) for stock call options, Chance (1986) for index call options, Chesney and Scott (1989) for foreign exchange call and put options and Wei (1994) for NPWs.

<sup>6</sup> Studies that have also used multiple regression techniques for the analysis of deviations include Schulz and Trautmann (1994) for equity warrants, Chesney et. al. (1994) for index call and put options, Chesney and Scott (1989) for foreign currency call and put options and Loudon (1990) for stock put options.

Table 2 also suggests that the deviations for at the money (ATM) warrants are larger than for in the money (ITM) or out of the money (OTM) warrants. However, we have already argued that this result may be misleading. Therefore we will continue our analysis with a regression analysis. In table 3 the results of the following regression equation are presented:

$$D_{1,t} = \beta_0 + \beta_1 D_{1,t-1} + \beta_2 M_t^c + \beta_3 (M_t^c)^2 + \beta_4 \sigma_{isd,t-1}^c + \beta_5 q_t + \beta_6 r_t + \beta_7 (T-t) + \varepsilon_t ,$$

where:

$D_{1,t}$	=	deviation of the $D_1$ type on day $t$ ;
$M_t^c$	=	the moneyness of a call warrant on day $t$ ; this is defined as $(I_t - X)/X$ ;
$I_t$	=	the closing value of the index on day $t$ ;
$X$	=	the exercise price of the index warrant;
$\sigma_{isd,t-1}^c$	=	the implied standard deviation for the same call warrant on day $t-1$ ;
$q_t$	=	the (continuous) dividend yield on the index on day $t$ ;
$r_t$	=	the (continuous) riskless interest rate on day $t$ ;
$T-t$	=	the remaining maturity of the warrant.

The reason that  $(M_t^c)^2$  is included in the regression besides  $M_t^c$  is that the rough classification in table 2 suggests a nonlinear relation between  $D_{1,t}$  and moneyness.

We started by estimating the regression equation without the deviation of the preceding day ( $D_{1,t-1}$ ). In this case, the residuals still show strong signs of autocorrelation. Therefore  $D_{1,t-1}$  is added to the regression equation. Even after including  $D_{1,t-1}$  the DAX index warrants still show some signs of autocorrelation. This does not disappear when adding lagged independent variables ( $M_{t-1}^c$ ,  $\sigma_{isd,t-2}^c$ ). However, the autocorrelation does disappear when the two period lagged dependent variable ( $D_{1,t-2}$ ) is added to the regression equation.

Thus the equation for the DAX warrant is estimated as:

$$D_{1,t} = \beta_0 + \beta_1 D_{1,t-1} + \beta_2 M_t^c + \beta_3 (M_t^c)^2 + \beta_4 \sigma_{isd,t-1}^c + \beta_5 q_t + \beta_6 r_t + \beta_7 (T-t) + \beta_8 D_{1,t-2} + \varepsilon_t .$$

The t-values in table 3 are based on White-standard errors to account for possible conditional heteroskedasticity. We first go into the results for the model prices based on the HSD.

[Insert Table 3]

The significant positive values for the coefficient of  $D_{1,t-1}$ , which are found for all three call warrants, indicate that a positive (negative) deviation on day t-1 is followed by a positive (negative) deviation on day t. A possible explanation is the finding that stock returns are characterized by conditional heteroskedasticity (Bollerslev, Chou & Kroner, 1992). Thus, if the volatility on day t-1 is large (small) than it will probably also be large (small) on day t. The HSD is calculated over the last 50 days. If the volatility on day t increases, the HSD will underestimate the true volatility. So, an increase in volatility will lead to a model price that is too low. In other words, the delayed reaction of the HSD on an increase in volatility combined with the fact that a high (low) volatility on day t-1 will be followed by a high (low) volatility on day t, can explain the fact that a positive (negative) deviation on day t-1 will be followed by a positive (negative) deviation on day t.

The negative coefficients for the variable  $\sigma_{isd,t-1}^c$ , which are also found for all three index warrants, seem surprising. Contrary to the HSD, the ISD may be expected to react immediately to an increase (decrease) of the true volatility. An increase (decrease) is expected to lead to a higher (lower) warrant price, which will also lead to an increase (decrease) of the ISD. If due to an increase of the volatility, the HSD underestimates the true standard deviation. This leads to a model price that is too low which implies a positive value for  $D_{1,t}$ . Therefore we would expect a positive relation between the  $ISD_{t-1}$  and the deviation. Of course, here we should realize that this is the effect that we would expect conditional upon all other variables being constant. Since in reality however,  $\sigma_{isd,t-1}^c$  will be negatively correlated with  $D_{1,t-1}$  this may account for the negative coefficient of  $\sigma_{isd,t-1}^c$ .

The significant values for the coefficient  $q_t$ , which are found for the CAC 40 and the FT-

SE 100 call warrants, emphasize that a European option pricing model is used for the pricing of American warrants. The non linear relation between  $D_{1,t}$  and  $M_t^c$  which was indicated by table 2 is confirmed for the CAC 40 warrants by the significance of the coefficient  $(M_t^c)^2$ . Thus, the deviation decreases if the option is more ITM or OTM.

With regard to the tests for the ISD the following remarks can be made. The coefficient of determination ( $\bar{R}^2$ ) is much lower than in case of the HSD. This should come as no surprise of course, since when using the ISD of the previous day the model does much better in explaining the observed market prices. The factor  $D_{1,t-1}$  is no longer significant. If the tests are repeated without  $D_{1,t-1}$ , there are no signs of autocorrelation. The only variables that are significant are the intercept,  $M_t^c$  (except for the FT-SE 100 warrants),  $(M_t^c)^2$  (except for the DAX warrants) and  $\sigma_{isd,t-1}^c$ .

The negative sign of the coefficient for  $\sigma_{isd,t-1}^c$  can be explained by writing the warrant price  $C_t$  as a function of the volatility  $\sigma_t$ :  $C_t = C(\sigma_t)$ . If we choose for  $C$  the Black and Scholes formula, the volatility  $\sigma_t$  is by definition equal to the implied volatility  $\sigma_{isd,t}^c$ . Using a first order Taylor expansion, we can then write:

$$C(\sigma_t) = C(\sigma_{isd,t-1}^c) = \frac{\partial C}{\partial \sigma} \Big|_{\sigma_{isd,t-1}^c} * (\sigma_t - \sigma_{isd,t-1}^c) .$$

Note that  $C(\sigma_{isd,t-1}^c)$  is just the model price that we are testing. Thus, it is possible to write the relative deviation  $D_{1,t}$  as:

$$D_{1,t} = \frac{C(\sigma_t) - C(\sigma_{isd,t-1}^c)}{C(\sigma_{isd,t-1}^c)} = \frac{\partial C / \partial \sigma \Big|_{\sigma_{isd,t-1}^c}}{C(\sigma_{isd,t-1}^c)} * \sigma_t - \frac{\partial C / \partial \sigma \Big|_{\sigma_{isd,t-1}^c}}{C(\sigma_{isd,t-1}^c)} * \sigma_{isd,t-1}^c .$$

Since the warrant price  $C_t$  is a strictly increasing function of the volatility,  $\partial C / \partial \sigma$  must be positive. Also, since the warrant price can never be negative, excluding the case where  $C(\sigma_{isd,t-1}^c) = 0$ , it must be true that:

$$\frac{\partial C / \partial \sigma \Big|_{\sigma_{isd,t-1}^c}}{C(\sigma_{isd,t-1}^c)} > 0 .$$

Combining the last two equations it follows immediately that the deviation  $D_{1,t}$  is negatively related to the ISD of the previous day,  $\sigma_{isd,t-1}^c$ . Therefore, the negative sign for  $\sigma_{isd,t-1}^c$  in table 3b can be expected and is caused by the fact that we have to use an

estimate for  $\sigma_t$ , rather than the true value of  $\sigma_t$  itself.

## 4.2. The put warrants

In table 4 the average deviations for the put warrants are included, both for model prices calculated with HSDs and model prices calculated with the  $ISD_{t-1}$ 's for the same put warrant.

[Insert Table 4]

The deviations for the HSD are extremely large. The reason is that in some cases extremely low model prices are calculated. Because the difference between the market price and the model price is divided by the model price, this leads to a very large deviation<sup>7,8</sup>. The deviations seem especially large for OTM and ATM warrants. Just as in case of the call warrants, the deviations are significantly positive for the CAC 40 and FT-SE 100 warrants. This can be attributed to the omission of the early exercise possibility. However, the deviations are no longer significant if the  $ISD_{t-1}$  is used.

In order to analyze the deviations, in table 5 the results of the following regression equation are presented for put warrants:

$$D_{1,t} = \beta_0 + \beta_1 D_{1,t-1} + \beta_2 M_t^P + \beta_3 (M_t^P)^2 + \beta_4 \sigma_{isd,t-1}^P + \beta_5 q_t + \beta_6 r_t + \beta_7 (T-t) + \varepsilon_t ,$$

where:

$M_t^P$  = the moneyness of a put warrant on day t; this is defined as  $(X-I)/X$ ;

$\sigma_{isd,t-1}^P$  = the implied standard deviation for the same put warrant on day t-1.

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<sup>7</sup> Similar results were found by e.g. Evnine and Rudd (1985) for short term put options on the S&P 100.

<sup>8</sup> This may also explain the fact that most studies use deviations of the  $D_2$  type instead of deviations of the  $D_1$  type. In case the difference between the market and the model price is divided by the market price, the possibility of finding extremely large deviations becomes much smaller.

We first go into the results for the HSD.

[Insert Table 5]

Just as in case of the call warrants, the deviation of the previous day ( $D_{1,t-1}$ ) is needed in order to correct for autocorrelation. This variable is highly significant for all three put warrants. With regard to the other variables, it can be noted that the squared moneyness  $(M_t^p)^2$  is significant for all three put warrants, while the moneyness  $(M_t^p)$  is only significant for the DAX warrants. Other significant variables are the variable  $\sigma_{isd,t-1}^p$  and the dividend yield for the CAC 40 and FT-SE 100 indexes. The riskless interest rate is also significant for the FT-SE 100 warrant. The Lagrange-Multiplier test shows that there is still significant autocorrelation for the DAX warrants.

With regard to the results for the  $ISD_{t-1}$  the following remarks can be made. Just as in case of the call warrants, the coefficient of determination ( $\bar{R}^2$ ) decreases compared to the results for the HSD. The factor  $D_{1,t-1}$  is now only significant for the FT-SE 100 warrants. It is remarkable however, that the sign of the coefficient is negative, whereas we would expect it to be positive. Other significant coefficients are the riskless interest rate, the moneyness (both  $M_t^p$  and  $(M_t^p)^2$ ) and  $\sigma_{isd,t-1}^p$ . The dividend yield is only significant for the FT-SE 100 warrants. The remaining time to maturity is significant for both the CAC 40 and DAX warrants. Because in most of our tests we do not find that there is a significant coefficient for the time to maturity, we do not find that there is a "market learning effect" as found earlier by Wei (1994) and Chen et al. (1992).

## 5. Summary and conclusions

In this paper we have tested the Black and Scholes model for the pricing of Dutch index warrants. We used daily data from the introduction of the index warrants in May 1993 until September 1994 when the liquidity of the market for Dutch index warrants decreased significantly, due to the introduction of many new series.

The unobserved volatility was estimated using the historical standard deviation of the last 50 trading days and using the implied volatility of the previous trading day. It is found that when using the historical standard deviation, the Black and Scholes model underprices all put index warrants and the call warrants on the FT-SE 100 and the CAC 40, while it

overprices the call warrants on the DAX. When using the implied standard deviation of the previous day, the model does fairly well.

We investigated the mispricing using regression analysis, taking into account the fact that the mispricings are often highly autocorrelated and possibly heteroskedastic. It was found that when using the historical standard deviation, the mispricing of both the call warrants and the put warrants depends strongly on the mispricing of the previous day and on the moneyness, the volatility and the dividend yield. In general the mispricing depends in a nonlinear way on the moneyness. When using the implied volatility of the previous day the mispricing of the call warrants depends only on the moneyness (in a nonlinear way) and on the estimated volatility. We show that the negative dependence on the estimated volatility can be expected given the fact this estimate is used to calculate the model price. The mispricing of the put warrants depends on the moneyness (in a nonlinear way), the volatility, the dividend yield and the remaining maturity.



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**Table 1: Index warrants outstanding on the Amsterdam Stock Exchange from May 12, 1992 to April 13, 1993.**

	CAC 40	FT-SE 100	DAX
<i>Call warrants</i>			
Observations	177	107	148
Exercise price	FF 1925	£ 2450	DM 1725
Expiration date	April 15, 1994	April 15, 1994	April 15, 1994
<i>Put warrants</i>			
Observations	134	180	155
Exercise price	FF 1925	£ 2450	DM 1725
Expiration date	April 15, 1994	April 15, 1994	April 15, 1994

Sources: Datastream and the respective issuance prospectuses.

**Table 2a: Deviations for the Dutch index call warrants**Based on historical standard deviations<sup>a</sup>

	<b>CAC 40</b>	<b>FT-SE 100</b>	<b>DAX</b>
<b>average <math>D_{1,t}</math><sup>c</sup></b>	20.1% (2.0)	15.0% (1.9)	-7.1% (-0.6)
<b>5%-percentile</b>	-16.7%	-5.9%	-41.2%
<b>50%-percentile</b>	22.2%	11.6%	-1.1%
<b>95%-percentile</b>	53.6%	40.7%	23.9%
<b>OTM<sup>d</sup> avg. <math>D_{1,t}</math></b>	16.2% (1.3)	22.3% (4.9)	-8.5% (-0.6)
<b>ATM<sup>d</sup> avg. <math>D_{1,t}</math></b>	30.8% (11.2)	25.4% (5.2)	3.0% (1.4)
<b>ITM<sup>d</sup> avg. <math>D_{1,t}</math></b>	18.0% (8.9)	7.1% (1.7)	0.0% (---) <sup>b</sup>

a) t-values in parentheses; t-values are based on standard deviations corrected for autocorrelation, using a Cochrane-Orcutt procedure;

b) only three observations;

c)  $D_{1,t}$  = deviation between market and model price as a fraction of the model price;

d) OTM = out of the money, ATM = at the money, ITM = in the money.

**Table 2b: Deviations for the Dutch index call warrants**Based on implied standard deviations<sup>a</sup>

	<b>CAC 40</b>	<b>FT-SE 100</b>	<b>DAX</b>
<b>average <math>D_{1,t}</math><sup>c</sup></b>	0.1% (0.4)	0.0% (0.2)	0.3% (1.0)
<b>5%-percentile</b>	-6.7%	-4.6%	-8.9%
<b>50%-percentile</b>	0.1%	-0.1%	-0.1%
<b>95%-percentile</b>	7.1%	5.6%	11.7%
<b>OTM<sup>d</sup> avg. <math>D_{1,t}</math></b>	0.2% (0.7)	-0.4% (-0.5)	0.2% (0.6)
<b>ITM<sup>d</sup> avg. <math>D_{1,t}</math></b>	-0.4% (-1.2)	0.3% (0.4)	0.3% (0.5)
<b>ITM<sup>d</sup> avg. <math>D_{1,t}</math></b>	-0.1% (-0.2)	-0.1% (-0.1)	0.1% (---) <sup>b</sup>

a) t-values in parentheses; t-values are based on standard deviations corrected for autocorrelation, using a Cochrane-Orcutt procedure;

b) only three observations;

c)  $D_{1,t}$  = deviation between market and model price as a fraction of the model price;

d) OTM = out of the money, ATM = at the money, ITM = in the money.

**Table 3a: Regression analysis of the deviations for the Dutch index call warrants model prices based on historical standard deviations<sup>a</sup>**

	<b>CAC 40</b>	<b>FT-SE 100</b>	<b>DAX</b>
$\beta_0$	2.20 (6.0)	1.60 (3.4)	0.45 (4.0)
$\beta_1 : D_{1,t-1}^b$	0.94 (31.3)	0.92 (20.8)	0.87 (10.1)
$\beta_2 : M_t^c$			