

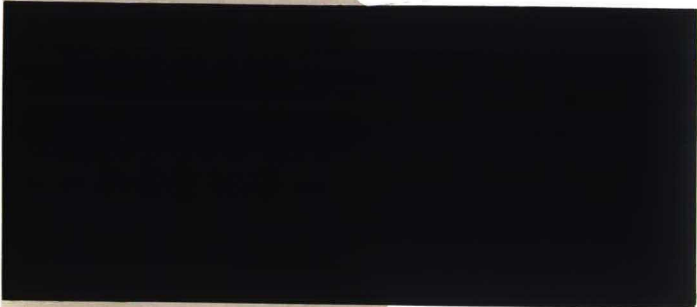
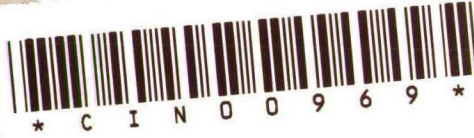
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PERFECT EQUILIBRIUM IN A MODEL OF
COMPETITIVE ARMS ACCUMULATION

by

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PERFECT EQUILIBRIUM IN A MODEL OF COMPETITIVE ARMS ACCUMULATION*

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ABSTRACT

This paper is concerned with the classic "guns versus butter" dilemma for two countries engaged in an arms race. The "West" is a decentralised market economy whose government uses optimal taxation to provide a public good, defence, and the "East" is a centrally planned economy. Utilities of the two countries depend on consumption, leisure and defence. Defence is a characteristic, which is an increasing function of the difference between home and foreign weapon stocks. In this way the problem of competitive arms accumulation is modelled as a differential game. The cooperative outcome leads to a moratorium on investment in weapons. Two non-cooperative solutions are compared. The first one is the standard open-loop Nash equilibrium solution, which presumes that the countries cannot condition their investment in arms on the rival's weapon stock. The second one is the subgame-perfect Nash equilibrium solution, which presumes that countries can monitor foreign weapon stocks. The perfect equilibrium leads to lower levels of arms, so that it is more efficient to allow countries to monitor each other's weapon stocks. Moreover, the perfect equilibrium strategies lead to a more satisfactory strategic underpinning of the Richardson equations. The full characterisation of the two non-cooperative equilibria also allows for comparative statics with respect to the underlying parameters of the model. Finally, it is shown what happens when one of the countries tries to acquire leadership by announcing its strategy beforehand.

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1. INTRODUCTION

Conflict over arms accumulation has in recent years become a more prevalent feature of relations between West and East. The political aspects of the arms race receive a great deal of attention both in the press and in academic studies (e.g. Richardson [1960], Boulding [1961], McGuire [1965] and SIPRI [1982]). Much of the theoretical analysis of arms conflict uses game theory (e.g. Schelling [1980]). The welfare of one country depends on the level of security which is perceived to be an increasing function of its own weapon stock and a decreasing function of the foreign weapon stock. This may be because any imbalance in weapon stocks increases the likelihood of loosing a possible war and increases the likelihood that a war might in fact be initiated. Alternatively, a country may simply feel that it gains international prestige from having a more superior army than its rivals. Both of these factors can in principle lead to a balance of terror. Such defence externalities can also be shown to lead to prisoner's dilemma situations. In the absence of cooperation each country builds up a larger weapon stock than with cooperation, because in the absence of commitments no country trusts the other countries to stick to a negotiated level of lower or zero weapon stocks. Other studies concentrate on the technological and strategic aspects of arms and the relationship to the probability that war breaks out (Saaty [1968], Intriligator [1975], Intriligator and Brito [1976, 1982]).

From the point of view of an economist the purely political analyses of conflict over arms do not pay adequate attention to the "guns versus butter" dilemma. A higher level of investment in weapons eventually increases the feeling of security and thus welfare, but it also means that there are less resources available for private sector consumption and therefore welfare

diminishes. A variety of studies employ optimal control and differential game theory to analyse the intertemporal trade-offs inherent in such "guns versus butter" dilemmas (e.g. Brito [1972], Deger and Sen [1984]). The problem with the differential game studies is that they consider open-loop Nash equilibrium solutions whereas feedback Nash equilibrium solutions are more appropriate for two reasons. Firstly, the feedback model employs more realistic information patterns, since each country can nowadays be assumed to be able to monitor the current levels of each other's weapon stocks rather than only the initial levels. Secondly, the linear-quadratic feedback model provides a strategic underpinning of the Richardson equations, which show up as first-order conditions for optimal investment behaviour in arms. The informational non-uniqueness resulting from closed-loop information patterns with memory (Başar and Olsder [1982]) is resolved when the principle of subgame-perfectness (Selten [1975]) is imposed, which has the added advantage that the resulting feedback equilibrium strategies are credible. The feedback approach to the problem of competitive arms accumulation was proposed before (Simaan and Cruz [1975]), but that paper does not give a full characterisation of the strategic equilibrium, so that it was not possible to compare the levels of weapon stocks in the feedback approach with the levels in the open-loop approach and to perform comparative statics with respect to the underlying parameters of the behavioural model.

The main objectives of this paper are to provide a more satisfactory strategic foundation of the Richardson model and to show that the subgame-perfect Nash equilibrium leads to less weapon accumulation in both countries than the open-loop Nash equilibrium. This means that the subgame-perfect Nash equilibrium is more efficient, since both countries obtain higher

welfare as they can consume more goods and leisure without feeling less secure. The policy conclusion is that both countries should be encouraged to monitor each other's weapon stocks. The analysis is set up as follows. There are two countries involved in the arms race. The West is a decentralised market economy whose government maximises the discounted utility of a representative household and levies lump-sum taxes in order to finance investment in arms and provide a public good, defence. The East is a centrally planned economy. Utility in both countries is a function of consumption, leisure and defence. Defence is a characteristic which depends on the difference between home and foreign weapon stocks. When consumption and leisure are normal goods, there is a "guns versus butter" dilemma as more taxes lead to more weapons at the expense of less consumption and leisure. Section 2 formulates this two-country model. The model is kept as simple as possible. Extensions to more general utility functions, distortionary taxation in the West or other formulations, which pay more attention to the different economic systems of the two countries, do not change the results of this paper on the impact of information. Section 3 derives the main cooperative outcome of the resulting differential game and shows that cooperation leads to a moratorium on investment in weapons. Section 4 gives the non-cooperative Nash equilibrium for the case that countries cannot observe their rival's current weapon stock. Section 5 gives the perfect equilibrium, which corresponds to the case that countries can monitor their rival's current weapon stock. It is shown that this approach is more efficient, leads to less weapon accumulation and provides a more satisfactory strategic underpinning of the Richardson equations. The resulting parameters of the Richardson equations are compared with what would result with the open-loop approach and a sensitivity analysis for

these parameters with respect to the underlying parameters of the model is performed. Section 6 attends to the case in which one of the countries tries to become a Stackelberg leader by announcing its policy beforehand. It is shown that the open-loop Stackelberg equilibrium leads to less weapon accumulation than the corresponding Nash equilibrium and makes the leader worse off than the follower. The perfect Stackelberg equilibrium, as well as the perfect consistent conjectures equilibrium, coincides with the subgame-perfect Nash equilibrium. Section 7 concludes the paper and contains some suggestions for further research.

2. THE "GUNS VERSUS BUTTER" DILEMMA

The West is a decentralised market economy with a representative household, a representative firm and a government. There are no domestic or foreign financial assets and the economy does not engage in international trade. There is no private capital accumulation, although the government does invest in weapon stocks. There is only one domestically produced commodity, which can be used for both consumption and investment purposes. The government demands goods for investment, the household supplies labour and demands goods for consumption, and the firm demands labour and supplies goods. The real wage adjusts in order to ensure labour market equilibrium. The government finances the investment in arms, i.e., the provision of the public good defence, by means of non-distortionary taxation and maximises the utility of the representative household. The household maximises utility $u(c, l, d)$, where c , l and d denote consumption, labour supply and defence, subject to its budget constraint $0 \leq c \leq wl + \pi - \tau$, where w , π and τ denote the real wage, profits and lump-sum taxes, respectively. Utility is assumed to be separable in defence. Defence is a characteristic (cf. Lancaster

[1966]), which is an increasing function of the own weapon stock, a , and a decreasing function of the foreign weapon stock, a^* , that is $d = D(a, a^*)$. Furthermore, it is assumed that an equal increase in both home and foreign weapon stocks leaves the level of defence or security unaffected, that is $D_a(a, a^*) = -D_{a^*}(a, a^*) > 0$. For an interior solution, the marginal rate of substitution between leisure, $1-l$, and consumption equals the real opportunity cost of leisure, that is $-u_l/u_c = w$. The firm maximises profits $\pi = f(l) - wl$, where f is a concave production function, which yields $w = f'(l)$. Goods market equilibrium implies $f(l) = c + g$, where g denotes the level of government investment, and the government's budget constraint is $g = \tau$. It follows that the indirect utility function for the government can, without loss of generality, be written as

$$u(C(g), L(g), d) = U(g) + D(a, a^*) \quad (1)$$

where $U' = u_c C' + u_l L'$, $C' = (u_c f'' + u_{ll} + u_{cl} f')/\Delta$, $L' = -(u_{lc} + u_{cc} f')/\Delta$ and $\Delta = -[u_c f'' + u_{ll} + 2u_{cl} f' + u_{cc} (f')^2] > 0$. It will be assumed that consumption and leisure are normal goods, so that an increase in taxes reduces consumption, leisure and thus utility ($C' < 0$, $L' > 0$, $U' < 0$). A sufficient condition for this assumption is that utility is also separable in consumption and leisure. The assumption that utility is separable in defence is primarily made for methodological reasons. It can be argued that the problem of arms accumulation should be modelled as an insurance where the level of defence decreases the probability of being attacked and therefore increases the probability that nobody survives and that the utility of all the current and all future generations from then on is zero (Shepherd [1988]). This argument suggests that, if an attack only affects the utility of the current generation, an appropriate utility function might be $P[a-a^*] \tilde{U}(g)$, where $P[.]$ denotes the instantaneous probability of not

being attacked, $P' > 0$, and $\tilde{U}(g)$ denotes the indirect utility function. Taking the logarithm yields (1) with $U(g) = \log(\tilde{U}(g))$ and $D(a, a^*) = \log(P(a - a^*))$. A proper analysis of the probabilities of survival, when an attack destroys the current and all future generations, requires an intertemporal stochastic framework, but this leads to a differential game formulation which is extremely difficult to solve. In any case, such an intergenerational analysis is more appropriate for a nuclear arms than for a conventional arms build-up. However, if the analysis allows for nuclear attacks where all future generations are wiped out, then the only credible, non-cooperative equilibrium is for neither country to accumulate missiles. When the build-up of nuclear weapons leads to a finite probability of an attack which is too horrendous to consider and when there exists a zero probability of attack, there is no incentive for arms build-up. In other words, deterrence requires the probability of commitment to investments which may imply launching missiles and blowing up the world and which are therefore not rational to carry out if called upon to do so. This seems to exclude perfect equilibrium as an appropriate solution concept for deterrence games.

The separable specifications of utility investigated so far (see van der Ploeg and de Zeeuw [1989]) show the same role of information, so that the main result of this paper seems to be robust with respect to alterations in the utility function. In order to be able to obtain analytical solutions a second-order Taylor series approximation of indirect utility is adopted. If preferences are quadratic and technology is linear, the approximation is exact. This yields a strategic underpinning of the Richardson equations and enables a comparison of different game equilibria as well as a sensitivity analysis with respect to the underlying parameters of the model. The quadratic approximation is given by

$$\begin{aligned}
 U(g) + D(a, a^*) &= \tilde{\theta}_0 + \theta_1 g - 1/2 \theta_2 g^2 + \theta_3 (a - a^*) - 1/2 \theta_4 (a - a^*)^2 \\
 &= \theta_0 - 1/2 \theta_2 (g - \bar{g})^2 - 1/2 \theta_4 (a - a^* - \bar{m})^2, \quad \theta_2, \theta_4 > 0 \quad (2)
 \end{aligned}$$

where $\bar{g} = \theta_1/\theta_2$ and $\bar{m} = \theta_3/\theta_4 > 0$ can be interpreted as the target level of public spending and the desired lead in weapon stocks, respectively. The assumption of normal goods, $U' = \theta_1 - \theta_2 g < 0$, implies that $g > \bar{g}$ for all $g \geq 0$, so that $\theta_1 \leq 0$ must hold. The intertemporal utility of the West for the problem starting at time t is given by the infinite-horizon value function

$$V(t, a, a^*) = \int_t^{\infty} [U(g) + D(a, a^*)] \exp[-r(s-t)] ds \quad (3)$$

where r is the rate of time preference. The West maximises $V(0, a_0, a_0^*)$, where a_0 and a_0^* are the initial weapon stocks, subject to the arms accumulation for home weapons

$$\dot{a} = g - \delta a, \quad a(0) = a_0 \quad (4)$$

where δ is the depreciation rate, and similarly for foreign weapons. The dilemma of "guns versus butter" is that high taxes are required to finance a large build-up of weapons, but this necessarily implies less private consumption and leisure.

The East is a command or centrally planned economy. The variables in the East are denoted by an asterisk. Because the purpose of the paper is only to show the impact of monitoring, it is assumed that the East has the same technologies and preferences as the West. The government plans c^* , l^* and g^* to maximise utility, $u(c^*, l^*, d^*)$, subject to the material balance condition, $f(l^*) = c^* + g^*$. This yields the same indirect utility function as in the West, $U(g^*) + D(a^*, a^*)$.

The decentralised market economy of the West and the centrally planned economy of the East are identical, because identical technologies and preferences have been assumed and because no distortions or market

imperfections have been considered and therefore the fundamental theorem of welfare economics holds. If the West had to levy distortionary taxes on labour income, there would be asymmetries and the East and West would not have the same indirect utility function. With identical technologies and preferences, the tax distortions in the West imply lower levels of employment, output and consumption for a given level of government investment in arms. However, the conclusions with respect to the comparison of different game equilibria will be the same (see van der Ploeg and de Zeeuw [1989]). Another form of asymmetry between the two economies occurs when one allows for rigid wages and prices in the short run, because then the West is likely to be in a regime of Keynesian unemployment and the East in a regime of repressed inflation (see Malinvaud [1977]).

3. COOPERATIVE BEHAVIOUR

Pareto-efficient outcomes for the differential game formulated in section 2 are found from the maximisation with respect to g and g^* of

$$\int_0^{\infty} \{ \alpha [U(g) + D(a, a^*)] + (1-\alpha) [U(g^*) + D(a^*, a)] \} \exp(-rt) dt \quad (5)$$

subject to (4) and $\dot{a}^* = g^* - \delta a^*$, $a^*(0) = a_0^*$, where $0 \leq \alpha \leq 1$. It follows that the marginal disutilities of government investment in arms in terms of foregone consumption and leisure ($-\alpha U'(g)$ and $-(1-\alpha)U'(g^*)$ for the West and the East, respectively) should equal the marginal values of weapon stocks, which are denoted by λ and λ^* , respectively, if this is feasible. Otherwise, if the marginal disutility of government spending exceeds the marginal value of weapons, the complementary slackness conditions imply that no investment in weapons take place ($g = 0$ if $-\alpha U'(g) > \lambda$). The marginal values of the weapon stocks must satisfy

$$\dot{\lambda} = (r+\delta)\lambda - \alpha D_a(a, a^*) - (1-\alpha)D_a(a^*, a), \lim_{t \rightarrow \infty} \exp(-rt)\lambda(t)a(t) = 0$$

and

(6)

$$\dot{\lambda}^* = (r+\delta)\lambda^* - \alpha D_a^*(a, a^*) - (1-\alpha)D_a^*(a^*, a),$$

$$\lim_{t \rightarrow \infty} \exp(-rt)\lambda^*(t)a^*(t) = 0.$$

(7)

One interpretation of (6)-(7) is that the "rental" charge plus the depreciation charge minus the capital gains term defines the user cost of weapons and should match the marginal utility of weapons to the world. If equal weights are attached to the West and the East ($\alpha = 1/2$), it follows that in the steady state $\lambda = \lambda^* = 0$, as in the steady state (or when the initial weapon stocks of the two countries are the same) the game is zero-sum at the margin with respect to a and a^* (i.e., $D_a(a, a^*) + D_a(a^*, a) = -2\theta_4(a-a^*) = 0$ and similarly the sum of marginal utilities of defence with respect to the foreign weapon stock is zero). For $\theta_1 < 0$ the steady state cooperative outcome is a corner solution, but for $\theta_1 = 0$ the corner solution coincides with the unconstrained solution. To avoid corner solutions, both in this section and in later sections, and to ensure that the assumption of normal goods is satisfied for all $g \geq 0$, the value of θ_1 can be taken to be zero. It follows that in the steady state $g = g^* = a = a^* = 0$, so that the cooperative outcome is to have a moratorium on investment in weapons and to run down weapon stocks until these have fallen to zero.

This analysis leans heavily on the property that the game is zero-sum at the margin, which is satisfied because the defence characteristic depends upon the difference in arms levels. For example, when it depends also upon the sum of arms levels and is given by

$$D(a, a^*) = \theta_3(a-a^*) - 1/2 \theta_4(a-a^*)^2 + \theta_5(a+a^*) - 1/2 \theta_6(a+a^*)^2 \quad (8)$$

with $\theta_5, \theta_6 \geq 0$, the game is no longer zero-sum at the margin in the long run. This set-up can easily be shown to result in

$$a(\infty) = a^* = 2\theta_5 / [(r+\delta)\delta\theta_2 + 4\theta_6] > 0, \quad (9)$$

so that when both countries want a positive stock of weapons between the two of them ($\theta_5 > 0$) their steady-state levels of weapons will be positive. This defence characteristic may be realistic when the two countries want a positive stock of weapons to act as a deterrence for third countries. The transient cooperative solution is best obtained by solving for the global averages and global differences separately. This is possible, because $(a-a^*)$ and $(\lambda-\lambda^*)$ on the one hand and $1/2 (a+a^*)$ and $1/2 (\lambda+\lambda^*)$ on the other hand form two decoupled sub-systems of differential equations. Application of this procedure and some algebraic manipulations yield the cooperative trajectory

$$a(t) = a(\infty)[1 - \exp(\omega_a t)] + 1/2 (a_0 + a_0^*) \exp(\omega_a t) + 1/2 (a_0 - a_0^*) \exp(\omega_d t) \quad (10)$$

and similarly for $a^*(t)$, where $\omega_a = 1/2 [r - \sqrt{(r+\delta)^2 + 16\theta_6/\theta_2}] < 0$ denotes the stable eigenvalue associated with the system of global averages and $\omega_d =$

$1/2 [r - \sqrt{(r+\delta)^2 + 16\theta_4/\theta_2}] < 0$ denotes the stable eigenvalue associated with the system of global differences. Again it can be seen that, if the initial arms levels are the same ($a_0 = a_0^*$) and countries only care about differences

in arms levels ($\theta_5 = \theta_6 = 0$), the cooperative outcome is to have a moratorium on investment in arms and to run down stocks via wear and tear until these have fallen to zero ($a(t) = a^*(t) = a_0 \exp(-\delta t)$). In general, the adjustment speeds up when the relative priorities of "guns" rather than

"butter" (θ_4/θ_2 and θ_6/θ_2) increase. The level of investment in arms in the cooperative outcome can be written as $g = g^C - 1/2 (\omega_d + \delta)(a^* - a) + 1/2 (\omega_a + \delta)(a + a^*)$, where $g^C = \delta a(\infty)$, so that investment in home arms is a

negative function of the global stock of arms and of the excess of the stock of home arms over foreign arms.

In the absence of a mechanism which enforces the cooperative outcome, each country has an incentive to deviate by increasing its security at the expense of its rival, if the desired lead in weapons is positive ($\theta_3 > 0$). Therefore the cooperative outcome will only be considered as a benchmark for the relative efficiency of the different non-cooperative outcomes, which will be considered in the next sections.

4. OPEN-LOOP NASH EQUILIBRIUM

Consider the situation where the West and the East do not cooperate and where neither country dominates the arms race, so that a Nash equilibrium is appropriate. The Nash equilibrium concept can lead to different types of solutions when applied to differential games (e.g. Starr and Ho [1969a,b]). In order to analyse the problem of competitive arms accumulation Brito [1972] employed the open-loop Nash equilibrium concept. This concept presumes that the investments in arms at each point in time are only conditioned on the initial weapon stocks, a_0 and a_0^* , and that each country pre-commits itself to a path of investment in arms. It follows that the expected investments of the rival do not depend on past or current weapon stocks or on past or current investments of the country under consideration. The expectations of each other's path of investment are correct in equilibrium. In order to be able to compare the open-loop Nash equilibrium with other equilibria in the next sections it will be fully characterised in this section. The first-order conditions, which result from Pontryagin's maximum principle, give rise to

$$\dot{a} = (\lambda + \theta_1)/\theta_2 - \delta a, \quad a(0) = a_0 \quad (11)$$

$$\dot{a}^* = (\lambda^* + \theta_1)/\theta_2 - \delta a^*, \quad a^*(0) = a_0^* \quad (12)$$

$$\dot{\lambda} = (r + \delta)\lambda - \theta_3 + \theta_4(a - a^*), \quad \lim_{t \rightarrow \infty} \exp(-rt)\lambda(t)a(t) = 0 \quad (13)$$

$$\dot{\lambda}^* = (r + \delta)\lambda^* - \theta_3 + \theta_4(a^* - a), \quad \lim_{t \rightarrow \infty} \exp(-rt)\lambda^*(t)a^*(t) = 0 \quad (14)$$

where λ and λ^* denote the marginal values of their own weapon stocks for the West and the East, respectively. The marginal disutility of public spending, $-U'(g) = \theta_2 g - \theta_1$, has to match the marginal value of weapons, λ , which gives investment in arms as an increasing function of its marginal value, $g = (\lambda + \theta_1)/\theta_2$. The steady state of (11)-(14) yields

$$g(\infty) = g^*(\infty) = \delta a(\infty) = \delta a^*(\infty) = (\theta_1/\theta_2) + \theta_3/[(r + \delta)\theta_2] \equiv g^0 > 0. \quad (15)$$

The steady-state levels of weapon stocks are positive, which can be interpreted as the familiar deterrence or "balance of terror" argument. They increase when the discount rate or the depreciation rate decreases, when the relative priority of "butter" rather than "guns" (θ_2/θ_4) decreases, and when the desired lead in weapon stocks over the rival country (θ_3/θ_4) increases. The steady state is a saddlepoint, since there are two stable eigenvalues

($-\delta$ and $1/2 [r - \sqrt{(r + \delta)^2 + 8\theta_4/\theta_2}]$) associated with the backward-looking variables, a and a^* , and two unstable eigenvalues ($r + \delta$ and $1/2$

$[r + \sqrt{(r + \delta)^2 + 8\theta_4/\theta_2}]$) associated with the forward-looking variables, λ and λ^* . Since (11)-(14) is effectively a perfect-foresight system, Buiter's [1984] method of spectral decomposition or the method of undetermined coefficients can be used to solve it. It can be shown that the stable manifold is given by $\lambda = \psi^0 \theta_2 (a^* - a) + \theta_3/(r + \delta)$, where

$$\psi^0 = -1/4 [r + \delta - \sqrt{(r + \delta)^2 + 8\theta_4/\theta_2}] > 0$$

so that $g = g^0 + \psi^0(a^* - a)$. It follows that investment in weapons is higher than its steady-state level when foreign weapon stocks exceed home weapon stocks and that the marginal increase in investment, ψ^0 , increases when the discount rate or the depreciation rate decreases and when the relative priority of "butter" rather than "guns" (θ_2/θ_4) decreases. Upon substitution one obtains

$$\dot{a} = g^0 + \psi^0(a^* - a) - \delta a, \quad a(0) = a_0 \quad (16)$$

$$\dot{a}^* = g^0 + \psi^0(a - a^*) - \delta a^*, \quad a^*(0) = a_0^* \quad (17)$$

which is a stable system as the eigenvalues associated with (16)-(17) ($-\delta$ and $-2\psi^0 - \delta$) are both negative. Note that an increase in the depreciation rate increases the magnitude of both eigenvalues and therefore speeds up the route to the steady state. Equations (16)-(17) can be looked upon as Richardson's [1960] equations, where ψ^0 is the "defence" coefficient, $\psi^0 + \delta$ the "fatigue" coefficient and g^0 the "grievance" or "hatred" coefficient. However, this interpretation seems inappropriate in view of the open-loop nature of the solution concept. In the open-loop Nash equilibrium the countries cannot condition their investments on current weapon stocks, so that $g = g^0 + \psi^0(a^* - a)$ should be interpreted as a relation between the optimal sequence of levels of investment and the resulting sequence of weapon stocks, and not as a feedback strategy for investment in arms. Olsder [1977] calls this the "open-loop, open-eye" representation of the open-loop solution, but when monitoring of weapon stocks is feasible the "closed-loop, open-eye" representation of the closed-loop solution seems more appropriate (see section 5).

Equations (16)-(17) can be integrated to give the open-loop Nash equilibrium strategy

$$g(t) = g^0 + \psi^0(a_0^* - a_0) \exp[-(2\psi^0 + \delta)t] \quad (18)$$

with trajectory

$$a(t) = a(\infty)[1 - \exp(-\delta t)] + 1/2 (a_0 + a_0^*) \exp(-\delta t) + 1/2 (a_0 - a_0^*) \exp[-(2\psi^0 + \delta)t] \quad (19)$$

and similarly for g^* and a^* , where $a(\infty) = g^0/\delta$. When both countries start with identical weapon stocks ($a_0 = a_0^*$), investment in weapons is always at its steady-state level ($g(t) = g^0$, for all $t \geq 0$) and any excess of the initial level of weapon stocks over the steady-state level is gradually eliminated at the rate of depreciation. When the rival country's initial weapon stock exceeds the home initial weapon stock, the home country's investment in weapons exceeds the steady-state level. The speed at which the difference in initial weapon stocks is eliminated, $2\psi^0 + \delta = -1/\tau$ [$\tau = \sqrt{(r+2\delta)^2 + 8\theta_4/\theta_2}$], increases when the discount rate decreases and when the depreciation rate or the relative priority of "guns" rather than "butter" increases. This speed of adjustment can easily be shown to be less than the speed of adjustment of the cooperative outcome ($-\omega_d$ in section 3), so that lack of cooperation slows down adjustment.

Since the marginal values of Eastern weapon stocks to the West and vice versa do not affect the open-loop Nash equilibrium, it does not matter whether the countries observe their own weapon stock or not. This means that the open-loop Nash equilibrium also describes the situation where each country monitors its own weapon stock, but not the weapon stock of the rival country. The next section considers the situation where each country can also monitor the foreign weapon stock.

5. PERFECT NASH EQUILIBRIUM

The closed-loop Nash equilibrium allows each country to condition its investment in weapons on the current and past stocks of weapons. This type

of information structure admits, among others, memory and threat strategies, so that the solution set is non-unique (Başar and Olsder [1982]). However, if the principle of subgame perfectness (Selten [1975]) is imposed, then uniqueness typically results. The outcome will be called the subgame-perfect Nash equilibrium. This equilibrium concept in closed-loop strategies, which will depend only upon the current weapon stocks of the two countries, requires that for each subgame the relevant part of the set of strategies is in Nash equilibrium. A subgame in this context is a game over the remainder of the time horizon, that is over $[\bar{t}, \infty)$ rather than $[0, \infty)$. The restriction of the solution to a subgame must be a Nash equilibrium for all $\bar{t} \in [0, \infty)$ and for all possible levels of weapon stocks at \bar{t} . Each country expects the other country to react rationally at time \bar{t} to the information about the current weapon stocks at time \bar{t} and in equilibrium these expectations are correct. Subgame perfectness rules out threat equilibria, which rely on information patterns with memory, and equilibria which imply future investments that are not rational to carry out if called upon to do so in the future. This set-up is analogous to the requirement that the solution to the differential game has to satisfy Bellman's principle of optimality. In that context Starr and Ho [1969b] and Simaan and Cruz [1975] refer to the outcome as the feedback Nash equilibrium. The subgame-perfect or feedback Nash equilibrium can be found by dynamic programming.

The maximisation in the Hamilton-Jacobi-Bellman equation for the West yields $g = (V_a + \theta_1) / \theta_2 = G(t, a, a^*)$, where $V(t, a, a^*)$ is the value function for the West, and similarly for the East. Upon substitution, the Hamilton-Jacobi-Bellman equations become the set of coupled partial differential equations

$$rV - V_t = U \left[G(t, a, a^*) \right] + D(a, a^*) + V_a (\tilde{g} - \delta a) + V_{a^*} (\tilde{g}^* - \delta a^*) \quad (20)$$

$$rV^* - V_t^* = U[G(t, a, a)] + D(a^*, a) + V_a^*(g - \delta a) + V_a^*(g^* - \delta a^*) \quad (21)$$

where U and D are given by the quadratic approximation (2). In general it is very difficult to find value functions V and V^* that solve (20)-(21). For the quadratic approximation, however, quadratic value functions lead to an analytical solution. Hence, presume that V is given by

$$V(t, a, a^*) = p_0 + p_1 a + p_2 a^* - 1/2 \underline{a} P \underline{a}' \quad (22)$$

where \underline{a} is the row-vector (a, a^*) and $P = [P_{ij}]$ is a positive semi-definite symmetric matrix, and similarly for $V^*(t, a^*, a)$ with row vector $\underline{a}^* = (a^*, a)$ and parameters p_0^*, p_1^*, p_2^* and P^* . Substitution of (22) in (20)-(21) and equating coefficients on a, a^*, a^2, a^{*2} and aa^* yields $p_1^* = p_1, p_2^* = p_2, P^* = P$ and the set of coupled differential equations

$$\dot{p}_1 = (r+\delta)p_1 - \theta_3 + [(\theta_1+p_1)P_{11} + (\theta_1+p_1+p_2)P_{12}]/\theta_2 \quad (23)$$

$$\dot{p}_2 = (r+\delta)p_2 + \theta_3 + [(\theta_1+p_1)(P_{12}+P_{22}) + p_2 P_{11}]/\theta_2 \quad (24)$$

$$\dot{P}_{11} = (r+2\delta)P_{11} + (P_{11}^2 + 2P_{12}^2)/\theta_2 - \theta_4 \quad (25)$$

$$\dot{P}_{22} = (r+2\delta)P_{22} + (P_{12}^2 + 2P_{11}P_{22})/\theta_2 - \theta_4 \quad (26)$$

$$\dot{P}_{12} = (r+2\delta)P_{12} + (P_{22} + 2P_{11})P_{12}/\theta_2 + \theta_4 \quad (27)$$

There is only one steady state of (23)-(27) which ensures that the matrix P is positive semi-definite. This steady state is given by

$$P_{11} = P_{22} = -P_{12} = -1/6 \theta_2 [r+2\delta - \sqrt{(r+2\delta)^2 + 12\theta_4/\theta_2}] > 0 \quad (28)$$

$$p_1 = -p_2 = \theta_2 \theta_3 / [(r+\delta)\theta_2 + P_{11}] \quad (29)$$

It follows that the investment strategies in the perfect Nash equilibrium are given by $g = g^P + \psi^P(a^* - a)$ and $g^* = g^P + \psi^P(a - a^*)$ where

$$\psi^P = P_{11}/\theta_2 > 0 \quad (30)$$

and where the steady-state level of investment $g^P > 0$ is given by

$$g^P = g(\infty) = g^*(\infty) = \delta a(\infty) = \delta a^*(\infty) = \theta_1/\theta_2 + \theta_3/[\theta_2(r+\delta) + P_{11}] < g^0 \quad (31)$$

As in the open-loop Nash equilibrium, the steady-state levels of investment in weapon stocks, g^D (the grievance coefficient), and the marginal increase in investment, ψ^D (the defence coefficient), increase when the discount rate or the depreciation rate decreases and when the relative priority of "butter" rather than "guns" (θ_2/θ_4) decreases, and the steady-state levels of weapon stocks increase when the desired lead in weapon stocks over the rival country (θ_3/θ_4) increases. Upon substitution of the investment strategy of the West in (4) one obtains

$$\dot{a} = g^D + \psi^D(a^* - a) - \delta a, \quad a(0) = a_0 \quad (32)$$

and similarly for the East. In contrast with the results of the open-loop Nash analysis, it seems appropriate to view these equations as Richardson's [1960] equations, as investments in arms in the perfect Nash equilibrium are conditioned on the observable weapon stocks. Olsder [1977] calls the investment strategies the "closed-loop, open-eye" representation of the closed-loop solution in contrast with the "closed-loop, closed-eye" representation, which refers to the expected sequence of levels of investment in arms for the closed-loop solution. It follows that it is meaningful to consider the perfect Nash equilibrium for the differential game formulated in section 2 as the strategic underpinning of the Richardson equations with ψ^D as the defence coefficient, $\psi^D + \delta$ as the fatigue coefficient and g^D as the grievance or hatred coefficient. Obviously, it is possible to integrate (32) over time to give the analogues of (18) (the "closed-loop, closed-eye" representation) and (19) with g^O and ψ^O replaced by g^D and ψ^D .

The most interesting aspect of the comparison between the open-loop Nash equilibrium and the perfect Nash equilibrium is that monitoring of foreign weapon stocks decreases the grievance coefficient $g^O > g^D$, so that

monitoring leads to less accumulation weapon stocks than in the absence of monitoring. The intuition behind this result is that, when one country considers the purchase of one additional unit of weapons, it considers the direct marginal contribution to security and welfare, D_a , but it also considers the strategic reaction of the rival. The rival will observe the additional purchase and will feel less secure, so that it will also purchase more weapons. Therefore the marginal contribution to security and welfare is reduced to $D_a + \psi^P V_a^* < D_a$, so that there is less incentive to invest in weapons than when countries cannot observe their rival's weapon stock. Since the perfect Nash equilibrium leads to more "butter" and less "guns", but with the same feeling of security, it is more efficient than the open-loop Nash equilibrium. The obvious policy implication is that countries should be encouraged to monitor each other's weapon stocks as this will lead to some unilateral disarmament and higher welfare. Another feature of monitoring is that the defence coefficient can easily be shown to be larger than without monitoring ($\psi^D > \psi^O$). It follows that the adjustment to the (lower) steady-state levels of arms is faster than in the absence of monitoring. However, this speed of adjustment can be shown to be still less than the speed of adjustment of the cooperative outcome ($2\psi^D + \delta < -\omega_d$).

Note that, when defence is a linear function of the difference in weapon stocks ($\theta_4 = 0$), the defence coefficients are zero ($\psi^D = \psi^O = 0$) and the grievance coefficient is independent of whether countries can monitor their rival's weapon stock or not ($g^D = g^O$). In fact for this special case the open-loop and subgame-perfect Nash equilibria coincide and therefore monitoring does not influence the levels of weapon stocks. This result generalises to the case where defence is separable in home and foreign weapon stocks (van der Ploeg and de Zeeuw [1989]). Finally, note that, when

neither country attempts to establish a lead in weapon stocks ($\theta_3 = 0$), the non-cooperative equilibria (with or without monitoring the rival's weapon stock) coincide with the cooperative outcome with a moratorium on investment in weapons.

6. STACKELBERG LEADERSHIP

This section considers the situation where one of the countries attempts to improve its welfare by announcing its investments in arms or its investment strategy beforehand, so that a Stackelberg equilibrium is appropriate. As for the Nash equilibrium concept it is possible to distinguish the open-loop Stackelberg equilibrium without monitoring and the subgame-perfect or feedback Stackelberg equilibrium with monitoring.

Consider first the open-loop Stackelberg equilibrium (see e.g. Basar and Olsder [1982]) with the West as the leader and the East as the follower. The leader is assumed to be able to pre-commit itself to an announced sequence of investment levels in arms. The rational reaction of the follower is $g^* = (\lambda^* + \theta_1)/\theta_2$ where λ^* is given by (14). This implies that the follower's level of investment in arms is characterised by the differential equation

$$\dot{g}^* = (r+\delta)g^* + [\theta_4(a^* - a) - \theta_3 - (r+\delta)\theta_1]/\theta_2, \quad g(0) \text{ is free} \quad (33)$$

The leader then maximises its intertemporal utility $V(0, a_0, a_0^*)$, given by (3), subject to the arms accumulation for home and foreign weapons, (4), and subject to the rational reaction of the follower, (33). The first-order conditions give rise to (4) for a and a^* , (33) and

$$\dot{g} = (r+\delta)g + [\theta_4(a - a^* + \mu/\theta_2) - \theta_3 - (r+\delta)\theta_1]/\theta_2, \quad g(0) \text{ is free} \quad (34)$$

$$\dot{\lambda}_* = (r+\delta)\lambda_* - \theta_4(a - a^* + \mu/\theta_2) + \theta_3, \quad \lambda_*(0) \text{ is free} \quad (35)$$

$$\dot{\mu} = -\delta\mu - \lambda_*, \quad \mu(0) = 0 \quad (36)$$

where λ_* and μ denote the leader's "shadowprices" of the foreign weapon stock and the foreign investment level, respectively. Note that as far as the leader is concerned, the follower's investment level, g^* , is free to jump at time zero and therefore its marginal contribution to the leader's welfare at that time must be zero, so that $\mu(0) \neq 0$. The steady state of (4) and (33)-(36) yields the steady-state levels of investment in arms

$$g^S = g^O - \theta_3 \theta_4 [\delta + \theta_4 / (r + \delta) \theta_2] [\theta_4^2 + (r + \delta)^2 \delta^2 \theta_2^2 + 3(r + \delta) \delta \theta_2 \theta_4]^{-1} \quad (37)$$

$$g^{*S} = g^O - [\theta_3 \theta_4^2 / (r + \delta) \theta_2] [\theta_4^2 + (r + \delta)^2 \delta^2 \theta_2^2 + 3(r + \delta) \delta \theta_2 \theta_4]^{-1} \quad (38)$$

and the steady-state value of the leader's shadowprice of the foreign investment level

$$\mu(\infty) = \theta_2 \theta_3 [2\theta_4 + (r + \delta) \delta \theta_2] [\theta_4^2 + (r + \delta)^2 \delta^2 \theta_2^2 + 3(r + \delta) \delta \theta_2 \theta_4]^{-1}. \quad (39)$$

Both countries accumulate less weapons than in the open-loop Nash equilibrium, but the leader accumulates less weapons than the follower ($g^S < g^{*S} < g^O$). Hence, the world is a less safe place because "balance of terror" is disturbed. The leader is always at least as well off in the open-loop Stackelberg equilibrium as in the open-loop Nash equilibrium, because it has the option not to exploit the follower's reaction curve. It follows that the loss in security, due to less arms accumulation than the rival, is outweighed by the gain in "butter". The follower also has a higher welfare than in the open-loop Nash equilibrium, because it can consume more "butter" and has a higher level of security. Finally, it can be shown, after considerable algebraic manipulation, that the leader is worse off than the

follower in the open-loop Stackelberg equilibrium, because the disadvantage of less security exceeds the advantage of more "butter" ($D(a^{*s}, a^s) - D(a^s, a^{*s}) > U(g^s) - U(g^{*s})$ where a^s and a^{*s} are the steady-state levels of weapon stocks in the open-loop Stackelberg equilibrium). Obviously, one gets the same results with the East as the leader and the West as the follower, so that there is a stalemate in that neither the West nor the East wishes to be leader.

The leader's optimal sequence of levels of investment in arms is time-inconsistent (Kydland and Prescott [1977]). The leader has an incentive to announce a relatively low level of investment in arms in order to induce the follower to do the same and, once the follower has locked itself into a low level of investment in arms, it pays the leader to renege and increase its security and welfare by investing more in arms. This can be seen from the fact that the shadowprice of the foreign investment level, μ , is strictly positive for $t > 0$, whereas reoptimising at a later stage would imply that μ is reset to zero. It follows that the leader increases its investment in arms when it reneges. One solution to this problem of time-inconsistency is the "loss-of-leadership" solution (Buitier [1983]), which replaces (36) by $\mu(t) = 0$ for all $t \geq 0$. This solution coincides with the open-loop Nash equilibrium and therefore leads to a higher level of investment in arms and a loss of welfare for both countries. The "loss-of-leadership" solution is time-consistent as the leader has effectively given up its role as leader, but it is obviously not subgame perfect. Another solution to the problem of time-inconsistency is to consider the subgame-perfect or feedback Stackelberg equilibrium (Basar and Olsder [1982]), which is time-consistent by definition, but which requires closed-loop information patterns. Because the indirect utility functions, (1), do not depend upon foreign levels of

investment in weapons, the subgame-perfect Stackelberg equilibrium coincides with the subgame-perfect Nash equilibrium. It is to be expected that the enforcement of credibility of the leader's announcement again leads to higher stocks of arms as with the loss-of-leadership solution. However, there is an opposing force arising from the benefits of monitoring which leads to lower weapon stocks. It can be shown, after considerable algebraic manipulation, that when the relative priority of "butter" rather than "guns" (θ_2/θ_4) is very high the monitoring force dominates and that therefore the imposition of subgame perfectness for the Stackelberg equilibrium also leads to less weapon stocks ($g^P < g^S < g^{*S}$).

In the Stackelberg equilibrium it is assumed that one of the countries reacts rationally to the investments in arms or the investment strategy of the rival country and that this rival country chooses an optimal investment policy, which takes account of that rational reaction. The consistent conjectural variations equilibrium (Bresnahan [1981]) attempts to capture this idea for the two countries at once by introducing conjectured reaction coefficients for both countries, which have to be consistent with the actual reaction coefficients. Although this equilibrium concept is logically not very well founded (de Zeeuw and van der Ploeg [1987]), it would again lead, for the problem of competitive arms accumulation, to the same subgame-perfect equilibrium.

7. CONCLUDING REMARKS

The conflict over arms accumulation between two countries, whose governments consider a "guns versus butter" dilemma, can be modelled as a differential game. Cooperation would lead to a moratorium on investment in weapons, which corresponds to a multilateral arms treaty. The open-loop Nash

equilibrium presumes that countries cannot condition their investments in arms on the rival's current weapon stock, whereas the perfect Nash equilibrium presumes that they can. The perfect Nash equilibrium leads to lower levels of arms accumulation and more "butter", so that it is more efficient. It follows that an unilateral arms treaty should enable countries to observe their rival's weapon stock. Moreover, the perfect Nash equilibrium gives a more satisfactory strategic foundation of the Richardson equations which shows that investment in arms increases proportionately with the level of weapon stocks of the rival nation ("defence") and the desired weapon lead ("grievance" or "hatred") and decreases proportionately with the economic burden of its own weapon stock ("fatigue"). The desired lead in weapon stocks over the rival country and the relative priority of "guns" rather than "butter" positively influence the grievance coefficients and therefore the steady-state levels of weapon stocks. The discount rate, the depreciation rate and the relative priority of "butter" rather than "guns" negatively influence the defence coefficients and therefore the speed of adjustment to the steady state. The fatigue coefficients consist of the sum of the defence coefficients and the depreciation rate.

There are several interesting directions for further research. The first direction is to improve the micro-economic foundations of the economic models of the West and the East and to allow for asymmetries in these models. For example, in the present paper the government of the West uses lump-sum taxation to finance the investment in weapons. Because such taxes are non-distortionary, the two economies are identical when technologies and preferences are the same. However, when the government of the West has to resort to distortionary taxes on labour income, then output, employment and consumption are lower in the West than in the East for a given level of

investment in weapons, and also the steady-state level of weapon stocks is lower. Distortionary taxes considerably complicate the indirect utility and value functions, so that one has to resort to numerical methods for the calculation of subgame-perfect Nash equilibria (van der Ploeg and de Zeeuw [1989]). Obviously, a more interesting model would not only allow for distortionary taxes but also for money- and debt-finance of government investment in arms and for different technologies and preferences. To take another example, when wages and prices do not clear the labour and goods markets instantaneously, it may be reasonable to assume that the West is in a regime of Keynesian unemployment and the East in a regime of repressed inflation (Malinvaud [1977]). Since the West has an excess supply of labour and goods, investment in weapons not only increases the feeling of security but has also Keynesian employment generating effects. However, the East has an excess demand for labour and goods, so that investment in weapons increases the feeling of security at the expense of more rationing. The second direction for further research is to allow also for economic linkages between the two countries, due to bilateral trade flows and international capital movements. If there is nominal (real) wage rigidity in both countries and if there are floating exchange rates, government investment in weapons is a locomotive (beggar-thy-neighbour) policy. It follows that, in the absence of international policy coordination, government investment in weapons is too low (high) as the beneficial (adverse) effects on the rival country are ignored. Finally, the third direction of further research is to investigate when cooperation in arms accumulation is counter-productive. For example, when government policy is time-inconsistent due to, say, nominal wage rigidity, cooperation can exacerbate the credibility constraints with

respect to the private sector and therefore be counter-productive (Rogoff [1985]).

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