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Tilburg University

Center
for
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No. 95104

OWEN'S COALITIONAL VALUE AND AIRCRAFT LANDING FEES

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November 1995

ISSN 0924-7815

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October 6, 1995

Abstract

This paper considers the determination of aircraft landing fees. It is proposed to model the situation at an airport as a game with a system of unions and to use the Owen value for this type of games to determine the fees for movements at the airport. Such a modelling creates the possibility to take into account the fact that airplanes are organized in airlines. The ideas in this paper are illustrated by the description of the situation at the airport Labacolla, which is the airport of Santiago de Compostela, Spain. Further, a characterization of the Owen value is provided that is applicable in situations where only the systems of unions are subject to change.

[1] We thank Xunta de Galicia and DGICYT for financial support through the projects XUGA20702B93 and PB94-0648-C02-02.

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1 Introduction

Cooperative game theory has proved to be a useful tool in analyzing cost allocation situations. There is a whole literature dealing with cost allocation methods that are based on game theoretic concepts. Examples include Billera *et al.* (1978), who apply game theory to determine internal billing rates for long-distance telephone calls that are placed through WATS (Wide Area Telecommunication Service) at Cornell University, and Straffin and Heaney (1981), who apply game theory to the cost allocation problem faced by the Tennessee Valley Authority in the 1930's.[5] Another well-known application of game theory is the use of the game theoretic solution concept of the Shapley value to determine aircraft landing fees. The so-called airport games were studied in Littlechild and Thompson (1977), Littlechild and Owen (1973), and others.

In this paper we focus on the determination of aircraft landing fees. Although the model of airport games that was studied in the literature untill now turned out to be quite valuable, we believe that there is an important aspect in the determination of aircraft landing fees that is ignored in the model that is currently used, namely the fact that airplanes are organized in airlines. From our point of view, airplanes should not be considered as isolated units but as a part of an airline and one can imagine that larger airlines have more possibilities to negotiate discounts or other cost advantages than smaller ones.

In this paper, we propose to use a model and a corresponding solution concept that give us the possibility to take into account the organizations of airplanes in airlines. The model and solution concept that we consider are the model of coalitional games with a priori unions and the extension of the Shapley value to this richer model that was introduced by Owen (1977) (the Owen value). Since we want to argue that the Owen value provides an appropriate method to determine aircraft landing fees, we are interested in axiomatic characterizations of this value. However, the axiomatic characterizations of this value that already exist in the literature (see Owen (1977), Hart and Kurz (1983), and Winter (1992)) are only valid when the system of unions is fixed and the coalitional games are variable. But in the context of the determination of aircraft landing fees when the organization of airplanes into airlines is taken into account, it is more appealing to characterize the Owen value in terms of changing systems of unions. This becomes even more appealing when we realize that the importance of the Owen value is partially due to

[5] For an overview of applications of game theory in cost allocation situations we refer the reader to Young (1985) and to Tijs and Driessen (1986).

the fact that it has revealed to be a useful tool to analyze the process of union formation in coalitional games (see, for instance, the study of a political scenario in Carreras and Owen (1988)). In this paper we provide a characterization of the Owen value that is valid when the coalitional game is fixed and the system of unions is subject to changes.

The structure of the paper is as follows. In section 2 we describe the model of airport games that is currently in the literature and we illustrate this model by the situation at Labacolla, the airport of Santiago de Compostela, Spain, during the months January, February, and March of 1993. In section 3 we describe the model of games with a priori unions and we model the airport cost allocation problem as such a game. We clarify the modelling process again with the description of the situation at the airport of Labacolla. Finally, in section 4 we provide an axiomatic characterization of the Owen value that is appealing in the context of the determination of aircraft landing fees.

2 Airport Games

Several authors have studied the problem of allocating the costs of building and exploiting an airport movement area[6] from a game theoretic angle. These costs have a simple but interesting structure: the cost of building a runway depends essentially on the "largest" aircraft for which the runway is designed, while the cost of subsequently using the runway is proportional to the number of movements of each type of aircraft. Hence, the costs can be divided into two parts: the variable costs that are incurred when airplanes arrive at or leave from the airport, and the fixed costs of constructing the runway. In general, there is no problem in assigning the variable costs, because they are generated by individual airplanes. The fixed costs, however, are harder to allocate, because they are more or less independent of movements by individual airplanes. The game theoretic approach to the allocation of the fixed costs is as follows: first a coalitional game (N, c) is defined in which the individual movements by airplanes are considered to be the players in the game and the cost for a group of movements S is defined as the (fixed) costs that would be incurred when an airport had to be constructed that could accomodate all the movements in the set S. These costs will essentially be determined by the "largest" airplane that is in the set S, because this plane will need the longest runway.

Suppose there are T types of airplanes that use the runway and let N_t be the (finite)

[6] This terminology is taken from Littlechild and Thompson (1977). They explain: "A movement is a take-off or landing. The movement area includes the runways, taxiways, and apron areas, as distinguished from the terminal area."

set of movements that are made by airplanes of type t. Hence, $\{N_1, N_2, \ldots, N_T\}$ is a partition of the player set N. Let c_t be the cost of constructing a runway that is suitable for airplanes of type t. Furthermore, we assume (without loss of generality) that the types are numbered in such a way that types with a higher number generate higher costs, i.e., $c_1 < c_2 < \ldots < c_T$. Naturally, a runway that accomodates airplanes of a type t will also accomodate airplanes of a smaller type $\tau < t$. Therefore, the costs of constructing a runway that accomodates all the movements in a set $S \subseteq N$ equals the costs of constructing a runway to accomodate the largest airplane that is represented in the set of movements S. In formula,

$$c(S) = \max \{c_t \mid S \cap N_t \neq \emptyset\}.$$

Once the airport game is defined, one can apply game theoretic solution concepts to find allocations of the costs. A game theoretic allocation rule that turned out to be especially interesting for this type of problems is the Shapley value, that was introduced by Shapley (1953) and further studied in the context of airport games by Littlechild and Thompson (1977), Littlechild and Owen (1973), Owen (1982), and others.[7] The cost allocation rule that is defined by the Shapley value was also proposed by the airport economists Baker and Associates (1965) and Thompson (1971), who approached the problem from an economic point of view. The Shapley value was axiomatically characterized within the context of airport cost allocation problems by Dubey (1982).

The Shapley value of the airport game described above assigns to each movement by an airplane of type t the same cost, namely

$$\phi_t(N, c) = \sum_{\tau=1}^{t} \frac{c_\tau - c_{\tau-1}}{|N_{\geq\tau}|},$$

where $c_0 := 0$ and $N_{\geq\tau} := \cup_{k=\tau}^{T} N_k$, the set of all the movements made by planes of type τ or larger planes. This allocation has the following interpretation: the costs of constructing the first part of the runway, the cost c_1 that is incurred by all types of airplanes, is divided equally among all the movements at the airport. Then, the costs of constructing the second part of the runway, the cost $c_2 - c_1$ that is incurred by all types of airplanes except for the first type, is divided equally among all the movements by airplanes of types $2, 3, \ldots, T$. Continuing in this way, the total cost c_T is allocated to all the movements at the airport.

[7] Also, the nucleolus of airport games was studied by several authors. We mention Littlechild (1974), Littlechild and Owen (1976), and Owen (1982). However, we will focus on the Shapley value in this paper.

To illustrate the problem of airport cost allocation and the application of game theory to this type of problems, we consider the situation at Labacolla, the airport of Santiago de Compostela, Spain, in the first three months of 1993.[8] In Table 2.1 we provide the types of airplanes that use Labacolla, and the number of movements made by these types of airplanes. Further, we also give the costs for the types of airplanes and the allocation of the costs corresponding to the Shapley value. The costs in the table are given in thousands of Pesetas.

Type	t	Number of movements	Cost	Shapley value
CESSNA	1	10	8,120	6.455
LEARJET-25	2	6	15,134	12.075
B-757	3	78	32,496	26.054
DC-9	4	464	34,265	27.574
B-737	5	232	39,494	35.044
B-727	6	438	44,850	46.488
DC-10	7	30	50,000	218.150

Table 2.1

Although the approach of cost allocation in airports that is described above (and studied extensively in several papers) is quite interesting and useful, it ignores one important aspect of the situations that are described, namely that the movements by aircrafts at airports are (in general) not individual movements, since the airports in reality have agreements with *airlines*. Hence, the movements of airplanes at a certain airport are grouped according to the airlines they belong to. One can easily imagine that this consideration may have an impact on the allocation of the costs, since airlines that have a larger number of movements at a certain airport may have more opportunities to negotiate discounts on landing fees or other cost advantages than airlines with less movements.

[8]The situation that we describe here is taken from Bergantiños *et al.* (1995) and it is based on the data that they were able to gather. Although we do not know all the movements in the airport, we believe that the data that we do have are sufficient to make an example that illustrates the ideas that we want to express in this paper. Further, we restrict the scope of our analysis to the months January, February, and March of 1993. In particular, we consider only the depreciation of the runway during these months. The costs for certain types of airplanes are computed using specifications for the types of airplanes and data on the costs of constructing a square meter of a runway. We computed the fees over a period of three months, but this period can of course be varied. Typically, computing the fees over a different time span will result in different fees.

Therefore, we propose to use a model that takes into account the fact that movements of airplanes are organized in airlines.

3 Games with a Priori Unions and Aircraft Landing Fees

The model and solution concept that we propose to use are the model of games with a priori unions and the extension of the Shapley value to these games as defined by Owen (1977). This value is usually referred to as the *Owen value* and we will adopt this terminology. A system of (a priori) unions for a coalitional (cost) game is a partition of its player set which provides a prior description of the cooperative structure of the players. The Owen value is a cost allocation rule for games with a priori unions that is based on marginal contributions, just like the Shapley value is. It first allocates the total costs among the unions as the Shapley value of the induced game played among the unions. Further, within each of the unions it re-allocates the costs that are to be paid by the union among its members, taking into account their possibilities for joining other unions. We formally introduce the model of games with a priori unions and the Owen value in this section.

A game with player set N and a system of unions P is a triple (N, c, P), where c is the characteristic function of a (cost) game (N, c) and $P = \{P^1, P^2, \ldots, P^A\}$ is a partition of the player set N into a priori unions. We will denote the set of all such triples (N, c, P) by $U(N)$ and we will denote by U the class of all sets $U(N)$ for any finite N. The Owen value allocates the total cost among the unions as the Shapley value of the induced game played among the unions. The game played among the unions is called the ***quotient game*** and it is the game (P, c^P) where the characteristic function c^P is defined by

$$c^P(\tilde{P}) := c\left(\cup_{P^a \in \tilde{P}} P^a\right)$$

for all $\tilde{P} \subseteq P$. This means that the cost of a (sub)set of unions equals the cost of the set of all players that belong to either one of these unions. The Shapley value of the game (P, c^P) assigns a part of the total cost to each of the unions P^a. The part of the cost that is assigned to the union P^a has to be paid by the members of this union. The Owen value allocates the cost assigned to the union among its members again according to the philosophy of the Shapley value. Hence, the share of the cost that each member of the union has to pay is determined using marginal costs. For the sake of completeness, the formula to compute the Owen value for general games with a priori unions is given in the appendix.

In the context of airport games, however, the formula of the Owen value can be simplified. First, we model the airport cost allocation problem as a game with a priori unions. In addition to the description of the airport cost allocation problem described in section 2, we now also take into account the fact that the movements in the set N are grouped according to the airlines they belong to. Suppose there are A airlines that use the airport. Then we have a system of a priori unions $P = \{P^1, P^2, \ldots, P^A\}$, where P^a consists of those movements in the set N that are made by airplanes of airline a. The triple (N, c, P), where N and c are defined as in section 2 and P is defined as above, models the airport cost allocation problem as a game with a priori unions. The Owen value of (N, c, P) assigns to each movement by an airplane of type t and of airline a the cost[9]

$$\psi_{a,t}(N,c,P) = \sum_{\tau=1}^{t} \frac{c_\tau - c_{\tau-1}}{|\mathcal{A}_{\geq\tau}| \cdot |N^a_{\geq\tau}|}, \tag{1}$$

where $c_0 := 0$, $N^a_{\geq\tau} := \cup_{k=\tau}^{T} N_k \cap P^a$, the set of planes of airline a that are of type τ or of a larger type, and $\mathcal{A}_{\geq\tau} := \{\alpha \in \{1, 2, \ldots, A\} \mid N^\alpha_{\geq\tau} \neq \emptyset\}$, the set of airlines that do own airplanes of type τ or larger types. This allocation has the following interpretation: the costs of constructing the first part of the runway, the cost c_1 that is incurred by all types of airplanes, is divided equally among all the airlines and within each airline the allocated costs are reallocated equally among all the airplanes. Then, the costs of constructing the second part of the runway, the cost $c_2 - c_1$ that is incurred by all the types of airplanes except for the first type, is divided equally among all the airlines that own airplanes of type 2 or larger types and within each airline the allocated costs are reallocated equally among all the airplanes of types $2, 3, \ldots, T$. Continuing in this way, the total cost c_T is allocated to all the movements at the airport.

Note that when the fees are computed according to the Owen value, the total fee paid by an *airline* only depends on the types of airplanes of this airline that make movements at the airport and not on the number of airplanes of the airline. But for a larger airline the total fees to be paid can be distributed among more movements. As a result, the fee *per movement* will be lower for larger airlines. Note that this does not tell us what will happen when airlines merge. Rather, it compares the specific fees for different airlines

[9]Since the derivation of this formula is similar to the derivation of the Shapley value for airport games as performed by Littlechild and Owen (1973), we do not include this derivation in the paper. The derivation can be obtained from the authors upon request.

in an existing situation. It is not possible to make general statements about what will happen when airlines merge. To analyze such a merger one has to take into account explicitly the specific decomposition of the airlines into movements.

It may seem strange that when the fees are computed according to the Owen value, the total fee paid by an *airline* does not change when this airline decides to make more movements at the airport with airplanes of types that are smaller than or as large as the ones it already uses at the airport. However, one should realize that the fees we compute using the Owen value are only a part of the total fees that have to be paid, namely the part that is meant to cover the fixed costs of constructing and maintaining the airport movement area. The variable costs that are incurred when airplanes arrive at or leave from the airport constitute another part of the total fees, and this part of the fees causes the total fee paid by an airline to be higher when it decides to make more movements at the airport.

We continue the example of the airport Labacolla that we started in section 2. Table 3.1 provides a description of the airlines that use Labacolla and of the grouping of movements at the airport according to airlines.

Airline	CESSNA	LEARJET 25	B-757	DC-9	B-737	B-727	DC-10
Air Europa			36		172		
Aviaco				12			
Britannia					6		
British Airways			2				
Condor Flugdienst			2				
Caledonian Airways			2				
Eurobelgian Airlines					2		
Futura					32		
Gestair Executive Set	2						
Iberia				452		438	
Air Charter					2		
Corse Air					4		
Air UK Leisure					2		
Ibertrans	2						
LTE			36				
Mac Aviation		6					
Monarch Airlines Ltd					2		
Sobelair					6		
Trabajos Aéreos	2						
Tea Basel LTD					2		
Oleohidráulica Balear SA	4						
Viasa							30
Spanair					2		

Table 3.1

Airline	CESSNA	LEARJET 25	B-757	DC-9	B-737	B-727	DC-10
Air Europa			8.110		11.183		
Aviaco				151.093			
Britannia					369.224		
British Airways			843.378				
Condor Flugdienst			843.378				
Caledonian Airways			843.378				
Eurobelgian Airlines					1107.673		
Futura					69.230		
Gestair Executive Set	176.522						
Iberia				2.037		9.070	
Air Charter					1107.673		
Corse Air					553.836		
Air UK Leisure					1107.673		
Ibertrans	176.522						
LTE			46.854				
Mac Aviation		120.367					
Monarch Airlines Ltd					1107.673		
Sobelair					369.224		
Trabajos Aéreos	176.522						
Tea Basel LTD					1107.673		
Oleohidráulica Balear SA	88.261						
Viasa							334.778
Spanair					1107.673		

Table 3.2

In Table 3.2 we give the Owen value for each movement at Labacolla, specified by the type of airplane and airline. We conclude from this table that the fees for movements are higher for airlines that use Labacolla incidentally and that they are advantageous for airlines that use the airport intensively. When reading Table 3.2 one should remember that these fees are per movement and that an airline with a lot of movements can spread the costs among all these movements. This is the reason why the fee per movement is lower. Further, we again remind the reader that these fees only represent the contribution to the fixed costs of constructing and maintaining the airport movement area and that there is also a variable cost per movement that has to be paid.

We want to conclude the example by noting that it can be advantageous for airlines to cooperate when the fees are computed using the Owen value. The 'Iberia-group' consists of the airlines Aerolíneas Argentinas, Aviaco, Binter Canarias, Binter Mediterráneo, Ladeco, Viasa, Viva, and, of course, Iberia. When the three airlines of the Iberia-group that use Labacolla, namely Aviaco, Iberia, and Viasa, act as one airline when negotiating movement fees, then the fee for a DC-9 will be 2.180, for a B-727 it will be 14.556, and for a DC-10 the fee will be 186.222 (all in thousands of Pesetas and for the Iberia-group). Hence, the fee for a DC-9 of Aviaco decreases drastically, and for a DC-9 of Iberia it increases slightly. The fee for a B-727 of Iberia increases with about 50 percent, but the fee for a DC-10 of Viasa is only about half of what it was before. In total, the fees that are to be paid by airplanes of the Iberia-group decrease from 16749.84 to 12973.708 (in thousands of Pesetas).

4 A Characterization of the Owen Value

Since we would like to propose to use the model of games with a priori unions to describe the problem of cost allocation in relation with airports and to use the Owen value as a rule, we have to justify the use of the Owen value in this context. However, all the characterizations (i.e., justifications) of the Owen value existing in game theoretical literature (see Owen (1977), Hart and Kurz (1983), and Winter (1992)) use axioms that are only related to the characteristic function of the corresponding coalitional games. This, in fact, is equivalent to justifying the Owen value for the family of all coalitional games with a fixed system of a priori unions. However, when applying the Owen value in the context of airport cost allocation, it is more appealing to have a characterization that can be applied to a situation where the coalitional game is fixed and where the unions are possibly subject to changes. In this section we provide an axiomatic characterization of the Owen value in this spirit.

We observe that the Owen value is, in fact, a generalization of the Shapley value. Namely, we can identify the set of coalitional games (without a system of unions) with the subset of U that consists of games with trivial systems of unions only. Here, by a trivial system of unions we mean that all the unions contain exactly one player or, equivalently, that each player forms a union on his or her own. Since the Owen value of such a game with a trivial system of unions coincides with the Shapley value of the corresponding coalitional game, the Owen value is a generalization of the Shapley value to games with a priori unions. Of course, the Owen value is one out of many possible answers to the question of how to generalize the Shapley value for those situations in which the system of unions is non-trivial. To capture this idea we introduce the notion of *coalitional Shapley value.*

Definition 1 *A coalitional Shapley value is an allocation rule ψ for games with a priori unions which assigns to every game with a system of unions $(N, c, P) \in U(N) \subset U$ an element of $\mathbb{R}^N$ in such a way that for all games with a trivial system of unions ψ coincides with the Shapley value of the corresponding coalitional game.*

In the remainder of this section we will restrict attention to coalitional Shapley values. One should realize that this is equivalent to restricting attention to allocation rules that satisfy the properties that characterize the Shapley value (cf. Shapley (1953) and Dubey (1982)). We will introduce two more properties of allocation rules for games with a priori unions and we will show that the Owen value is the unique coalitional Shapley value satisfying these two properties.

The first property, balanced contributions, is a property that states that if two players i and j are in the same a priori union, then the loss (or gain) that player i inflicts on player j when he decides to leave the union is the same as the loss (or gain) inflicted on player i when j leaves the union. This property reflects the idea that all players in a union should profit equally from joining the union and that it cannot be the case that one specific player extracts all the benefits that are generated by the formation of the union. In the context of airport games this means that within an airline the fees should be assigned to movements in a way that is fair in the sense that no airplane can demand a lower fee under the threat of withdrawing from the airline and acting as an isolated airplane at the airport.

Definition 2 *An allocation rule ψ on U has balanced contributions if for all $(N, c, P) \in$*

$U(N)$, all $P^a \in P$, and all $i,j \in P^a$ it holds that

$$\psi_i(N,c,P) - \psi_i(N,c,P_{-j}) = \psi_j(N,c,P) - \psi_j(N,c,P_{-i}),$$

where P_{-i} is the system of unions that results when player i separates from the union he belongs to, i.e., $P_{-i} := \{P^1, \ldots, P^{a-1}, P^a \backslash \{i\}, P^{a+1}, \ldots, P^A, \{i\}\}$, and P_{-j} is defined analogously.

The second property, the quotient game property, is a property that states that the behavior of an allocation rule is consistent in the sense that the sum of the benefits assigned to the individual players of a union is equal to the total benefit assigned to the union in the game played among the unions (cf. section 3). In the context of airport games this means that for an airline it does not matter whether the airport authorities compute the fees per movement or per airline; as long as the authorities use a rule that has the quotient game property both procedures will result in the same total fee for the airline.

Definition 3 *An allocation rule ψ on U has the quotient game property if for all $(N,c,P) \in U(N)$ and all $P^a \in P$*

$$\sum_{i \in P^a} \psi_i(N,c,P) = \psi_{P^a}(P,c^P,\mathcal{P}),$$

where $\mathcal{P}$ is the trivial system of unions for the set of players P, i.e., $\mathcal{P} := \{\{P^1\},\{P^2\},\ldots,\{P^A\}\}$.

The following theorem states that the balanced contributions property and the quotient game property characterize a unique coalitional Shapley value. The resulting rule is the Owen value.

Theorem 1 *The Owen value is the unique coalitional Shapley value satisfying balanced contributions and the quotient game property.*

The proof of Theorem 1 is included in the appendix. The proof provided in the appendix is given in the most general way, namely for the class of all games with a priori unions. However, the careful reader may note that the proof only requires changing the unions and that it leaves the characteristic function unchanged. Therefore, it is straightforward that the Owen value is the unique allocation rule for airport games that is an extension of the Shapley value satisfying balanced contributions and the quotient game property.

We end this section on axiomatic characterizations of the Owen value with the following remark. In the original characterization of the Owen value by Owen (1977) the Owen value was shown to be the unique allocation rule on U satisfying the carrier property, symmetry in the unions, symmetry in the quotient, and additivity.[10] In this characterization the property 'symmetry in the unions' can be replaced by the property 'balanced contributions'. We do not include a proof of this statement, but it can be obtained from the authors upon request.

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Appendix

This appendix contains the formula to compute the Owen value for general (cost) games and the proof of Theorem 1.

Let (N, c, P) be a game with a system of unions. Then the Owen value of this game is given by the following formula. Take $i \in N$ and let P^a be the (unique) union to which i belongs, i.e., $i \in P^a \in P$. Then

$$\psi_i(N,c,P) := \sum_{Q \subseteq P \setminus \{P^a\}} \sum_{S \subseteq P^a \setminus \{i\}} \frac{|S|!(|P^a| - |S| - 1)!|Q|!(|P| - |Q| - 1)!}{|P^a|!|P|!} \cdot M_i(Q,S),$$

where $M_i(Q,S)$ denotes the marginal contribution of player i to Q and S given by

$$(c(\cup_{P^a \in Q} P^\alpha \cup S \cup \{i\}) - c(\cup_{P^a \in Q} P^\alpha \cup S)).$$

This formula is quite complicated (which is the reason why we did not want to put it in the main text), but it has an interpretation that is quite similar to the interpretation of the Shapley value. The Owen value of $i \in P^a \in P$ is the average of all marginal contributions of i in all orderings of the players that preserve the grouping of the players into unions. Here, an ordering is said to preserve the unions if two players of the same union have no player in between them that is not a member of the same union. For a more extensive explanation of the Owen value we refer the reader to Owen (1977).

We continue with the proof of theorem 1.

Proof of theorem 1. (a) Uniqueness: Suppose that there exist two different coalitional Shapley values ψ^1 and ψ^2 satisfying balanced contributions and the quotient game property. Then, we can find a coalitional game (N,c) and, for this game (N,c), a system of unions $P = \{P^1, P^2, \ldots, P^A\}$ with a maximal number of unions such that $\psi^1(N,c,P) \neq \psi^2(N,c,P)$. Now, taking into account that both ψ^1 and ψ^2 satisfy the quotient game property, for all $P^a \in P$ and all $l \in \{1,2\}$ it holds that,

$$\sum_{i \in P^a} \psi_i^l(N,c,P) = \psi_{P^a}^l(P, c^P, \mathcal{P}),$$

where $\mathcal{P}$ denotes the trivial system of unions (see definition 3). But then, as ψ^1 and ψ^2 are coalitional Shapley values,

$$\sum_{i \in P^a} \psi_i^1(N,c,P) = \sum_{i \in P^a} \psi_i^2(N,c,P) = \Phi_{P^a}(P, c^P). \tag{2}$$

Hence, if $P^a \in P$ is such that P^a consists of one player (i.e. $P^a = \{i\}$), then it must hold that

$$\psi_i^1(N,c,P) = \psi_i^2(N,c,P).$$

Now, take $P^a \in P$ with at least two elements and choose $i,j \in P^a$. Then, as ψ^1 and ψ^2 satisfy balanced contributions,

$$\psi_i^l(N,c,P) - \psi_j^l(N,c,P) = \psi_i^l(N,c,P_{-j}) - \psi_j^l(N,c,P_{-i})$$

for all $l \in \{1,2\}$. But then, the maximality of P implies that

$$\psi_i^1(N,c,P) - \psi_j^1(N,c,P) = \psi_i^2(N,c,P) - \psi_j^2(N,c,P).$$

Thus, we can state that there exists a constant K^a such that

$$\psi_i^1(N,c,P) - \psi_i^2(N,c,P) = K^a$$

for all $i \in P^a$. But then, using (2), it is clear that $K^a = 0$, i.e. that $\psi_i^1(N,c,P) = \psi_i^2(N,c,P)$ for all $i \in P^a$. Consequently, it holds that $\psi^1(N,c,P) = \psi^2(N,c,P)$. This contradiction proves uniqueness.

(b) Existence: It is widely known (see, for instance Winter (1992)) that the Owen value satisfies the quotient game property. Further, it is shown in Vázquez-Brage et al. (1994) that the Owen value satisfies balanced contributions. To create a better understanding of the balanced contributions requirement in the context of airport games, we include a proof of the fact that the Owen value for airport games satisfies this requirement.

Let (N, c, P) be an airport game and let $P^a \in P$ be the movements of a fixed airline a and t_1 and t_2 two types of airplanes of airline a. We have to show that (with a slight but non-confusing abuse of notation)

$$\psi_{a,t_1}(N,c,P) - \psi_{a,t_1}(N,c,P_{-t_2}) = \psi_{a,t_2}(N,c,P) - \psi_{a,t_2}(N,c,P_{-t_1}). \tag{3}$$

Without loss of generality we assume that $t_1 < t_2$. We use formula (1) to find

$$\begin{aligned}
\psi_{a,t_j}(N,c,P) &= \sum_{\tau=1}^{t_j} \frac{c_\tau - c_{\tau-1}}{|\mathcal{A}_{\geq\tau}| \cdot |N^a_{\geq\tau}|} \\
\psi_{a,t_1}(N,c,P_{-t_2}) &= \sum_{\tau=1}^{t_1} \frac{c_\tau - c_{\tau-1}}{(|\mathcal{A}_{\geq\tau}|+1) \cdot (|N^a_{\geq\tau}|-1)} \\
\psi_{a,t_2}(N,c,P_{-t_1}) &= \sum_{\tau=1}^{t_1} \frac{c_\tau - c_{\tau-1}}{(|\mathcal{A}_{\geq\tau}|+1) \cdot (|N^a_{\geq\tau}|-1)} + \sum_{\tau=t_1+1}^{t_2} \frac{c_\tau - c_{\tau-1}}{|\mathcal{A}_{\geq\tau}| \cdot |N^a_{\geq\tau}|},
\end{aligned}$$

Where the last two equalities follow when taking into account that t_2 (respectively t_1) isolates from airline P^a. Now, simply substracting gives us equality (3).

This completes the proof of the theorem. □

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